Classification: Basic Concepts and Decision Trees

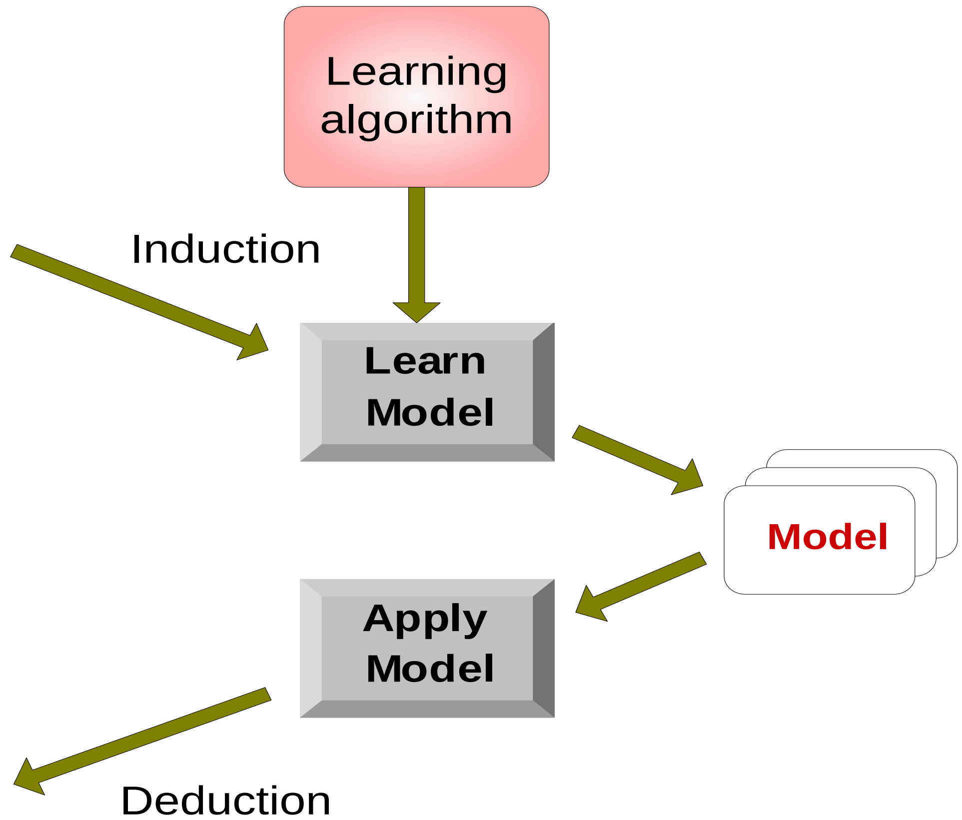
Classification: Definition

- Given a collection of records (*training set*)
  - Each record contains a set of *attributes*, one of the attributes is the *class*.
- Find a *model* for class attribute as a function of the values of other attributes.
- Goal: previously unseen records should be assigned a class as accurately as possible.
  - A *test set* is used to determine the accuracy of the model. Usually, the given data set is divided into training and test sets, with training set used to build the model and test set used to validate it.

Illustrating Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set

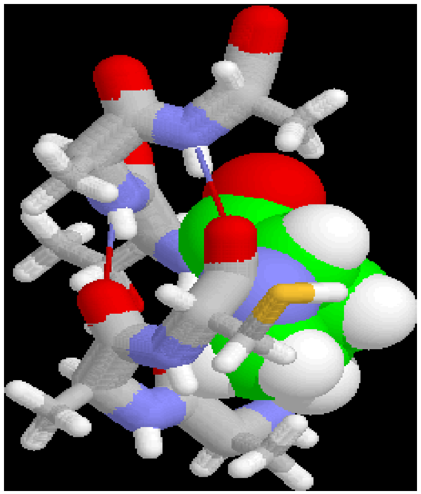


Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set

Examples of Classification Task

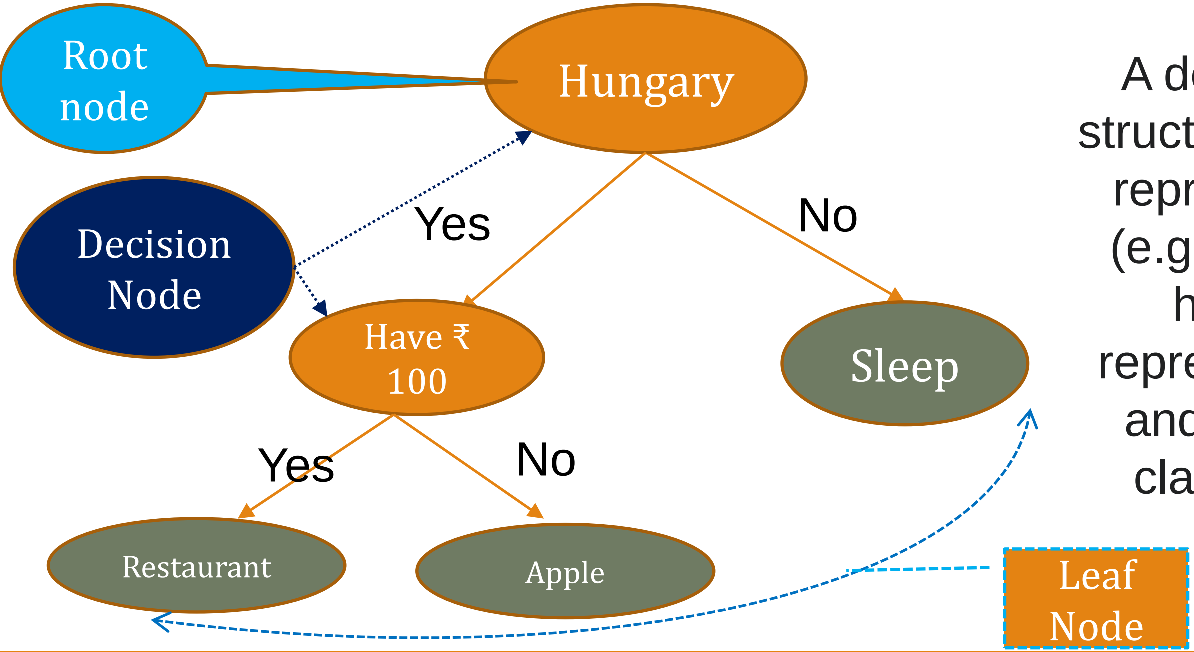
- Predicting tumor cells as **benign** or **malignant**.
- Classifying credit card transactions as legitimate or fraudulent.
- Classifying secondary structures of protein as alpha-helix, beta-sheet, or random coil.
- Categorizing news stories as finance, weather, entertainment, sports, etc.



Decision Tree: DT

- Graphical representation of all possible solutions.
- Decisions are based on some conditions.
- Decision made can be easily explained.

E.g.
Call Centre

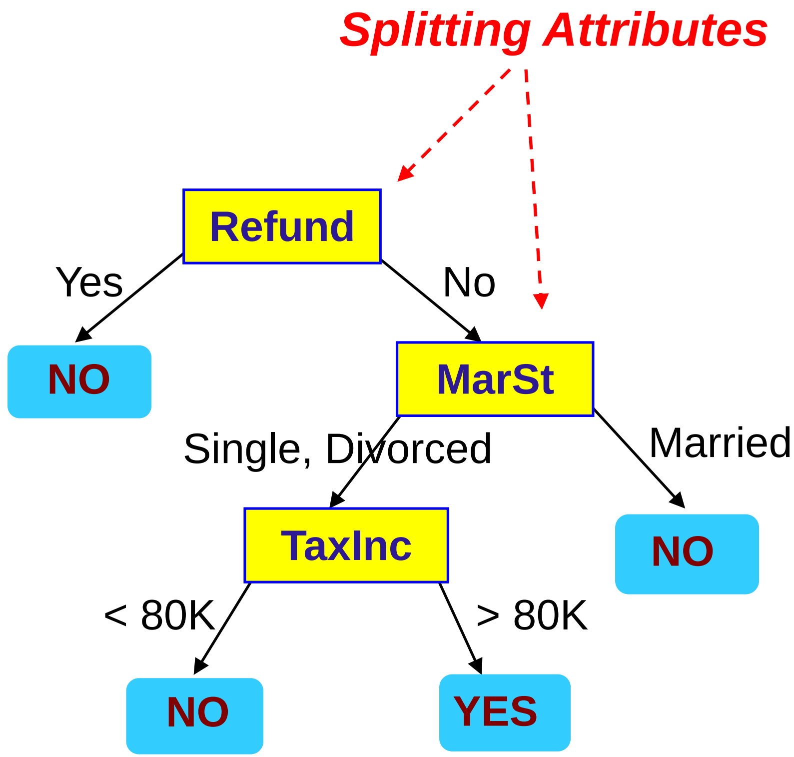


A decision tree is a flowchart-like structure in which each internal node represents a "test" on an attribute (e.g. whether a coin flip comes up heads or tails), each branch represents the outcome of the test, and each leaf node represents a class label (decision taken after computing all attributes)

Decision Tree: DT

	categorical	categorical	continuous	class
Tid	**Refund**	**Marital Status**	**Taxable Income**	**Cheat**
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Training Data

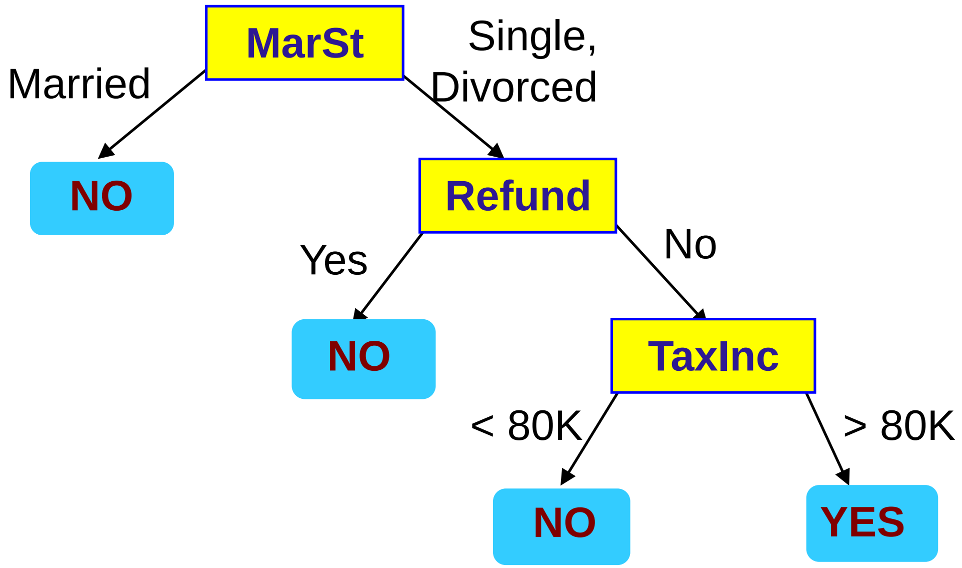


Model: Decision Tree

Another Example of Decision Tree

categorical categorical continuous class

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



There could be more than one tree that fits the same data!

Decision Tree Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
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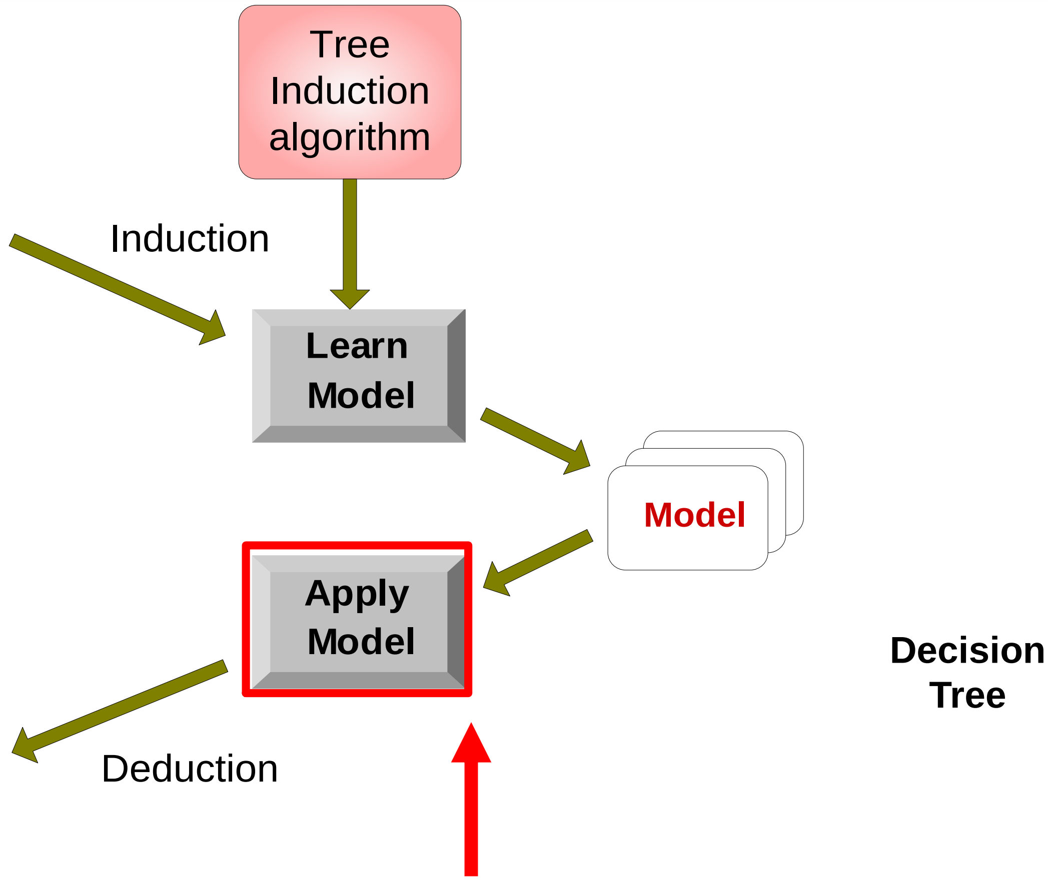
Training Set



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11	No	Small	55K	?
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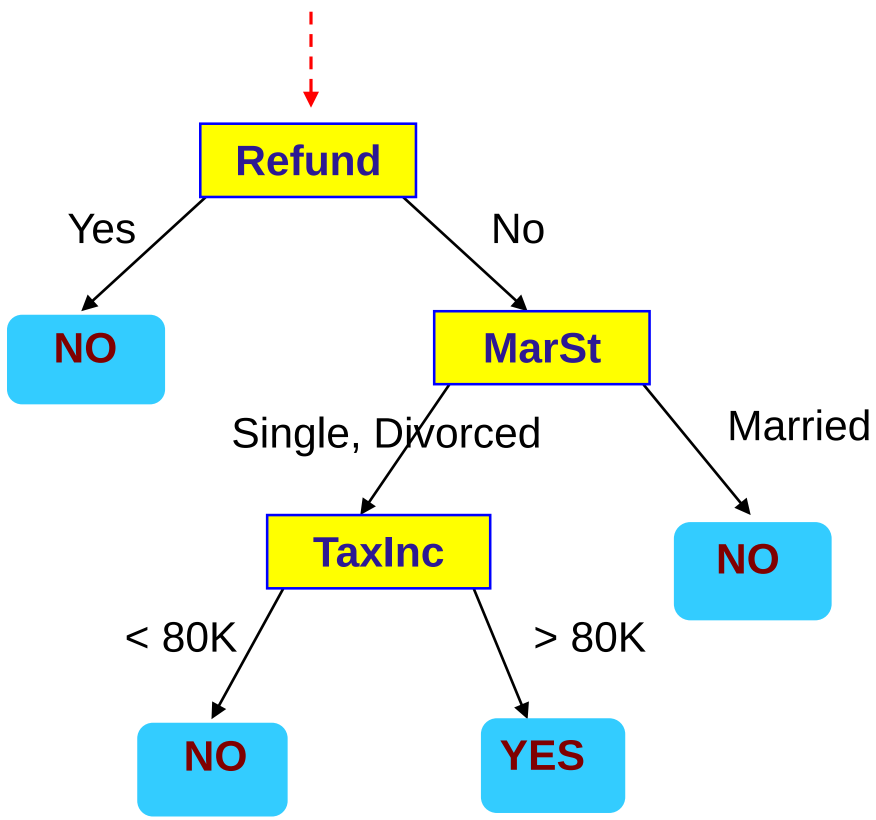
15	No	Large	67K	?

Test Set

Apply Model to Test Data

Start from the root of tree.



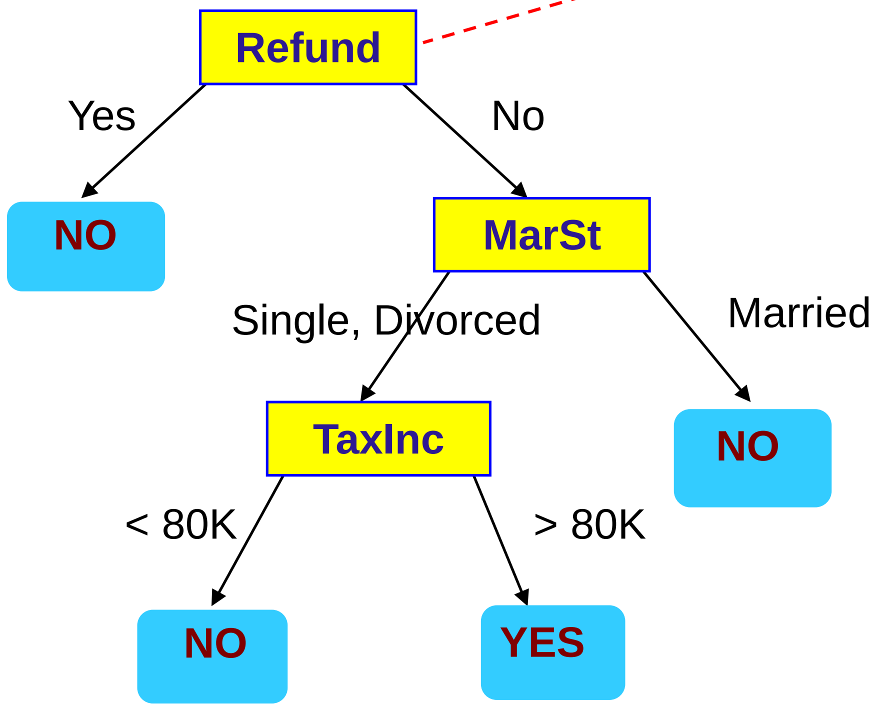
Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

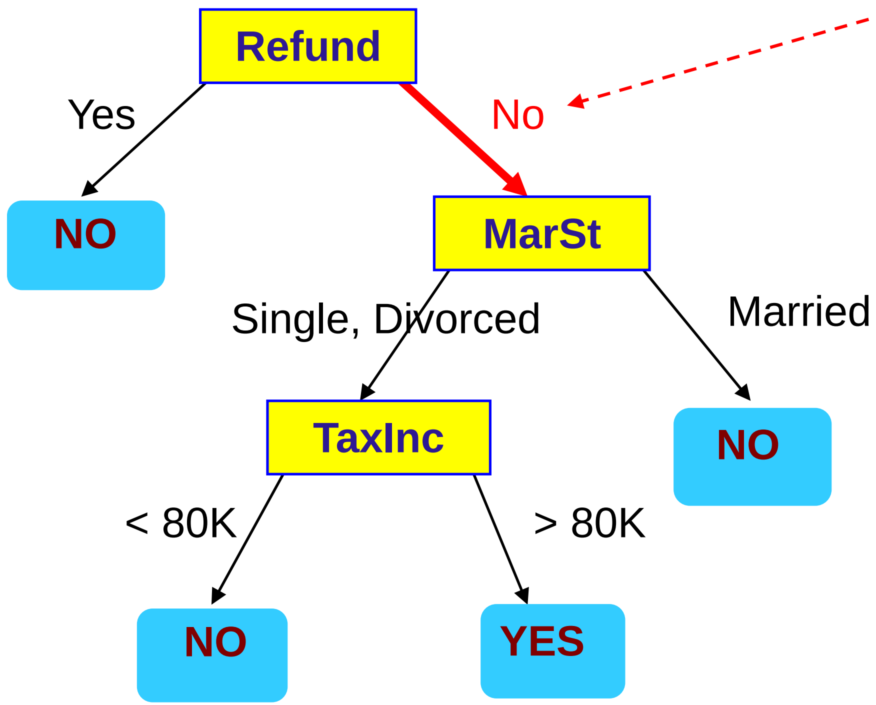
Apply Model to Test Data

Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Apply Model to Test Data



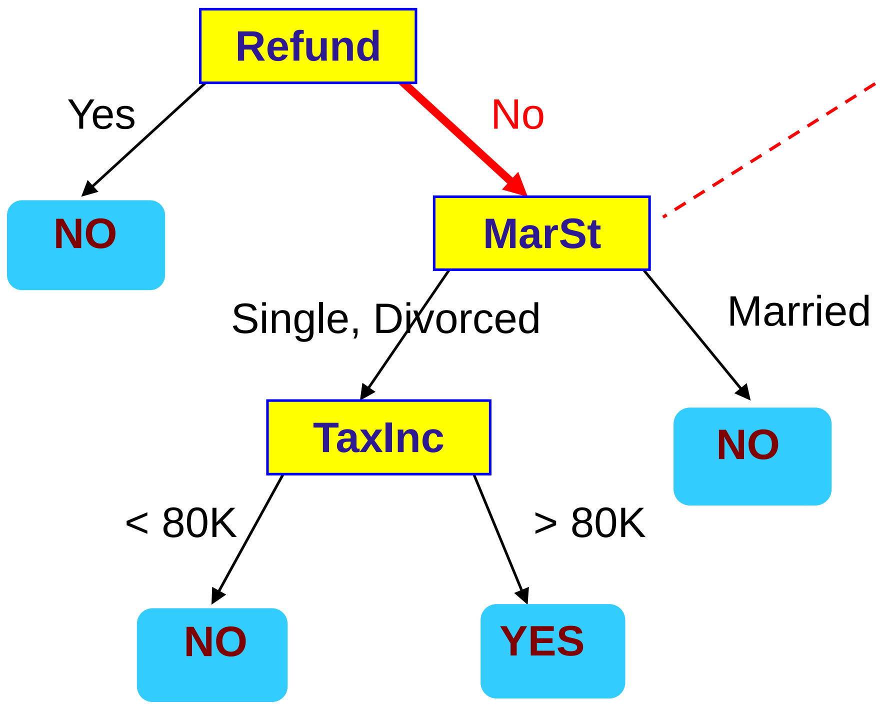
Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

Apply Model to Test Data

Test Data

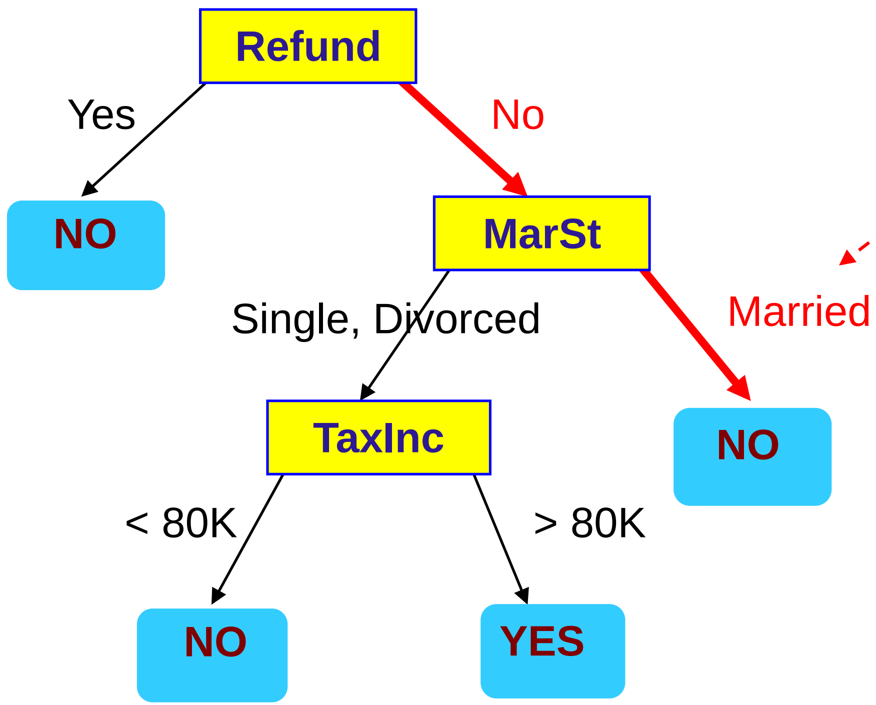
Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Apply Model to Test Data

Test Data

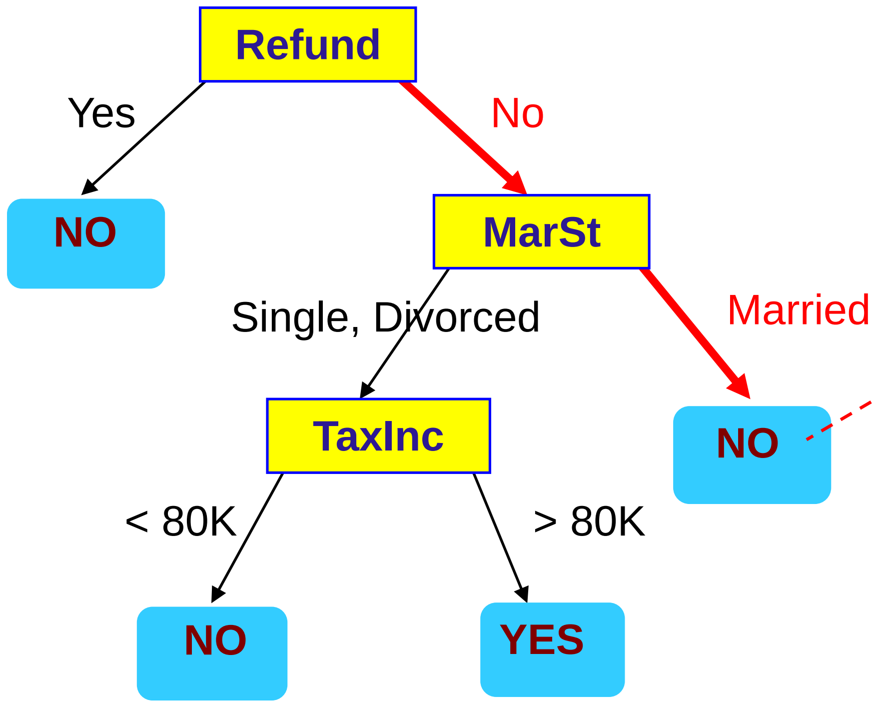
Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Apply Model to Test Data

Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Assign Cheat to “No”

Decision Tree Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
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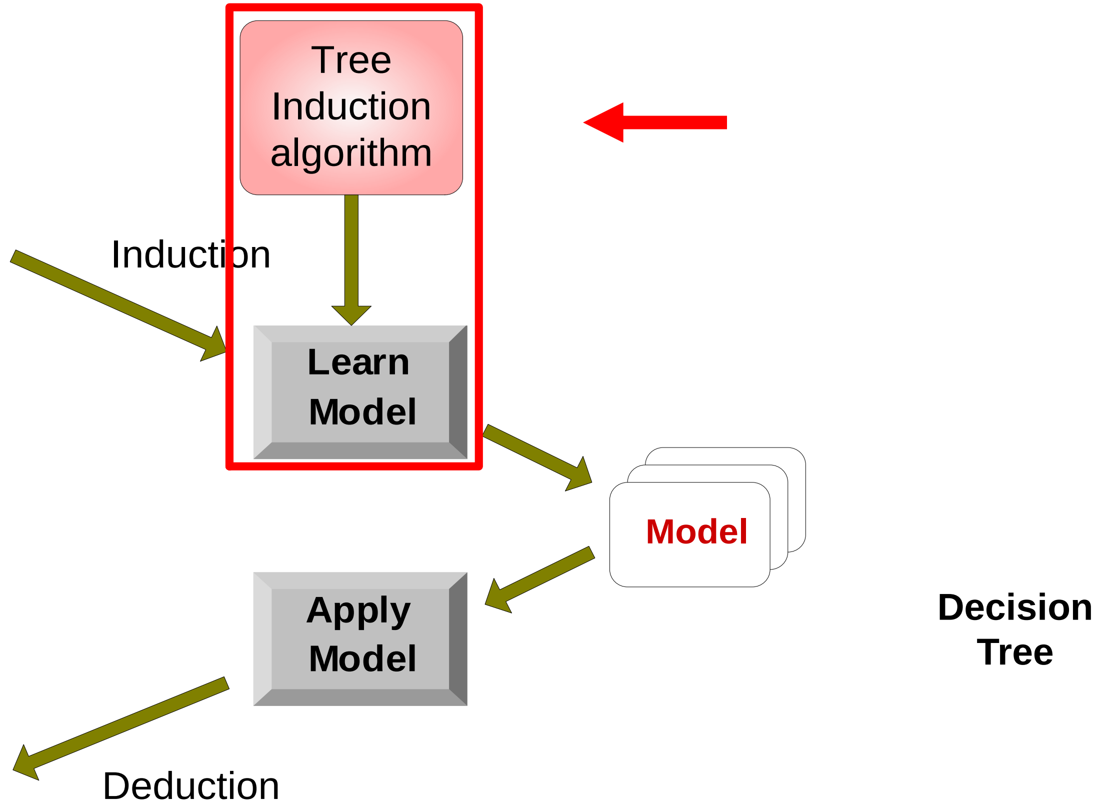
Training Set



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Test Set

Decision Tree Induction

- Many Algorithms:
  - Hunt’s Algorithm (one of the earliest)
  - CART
  - ID3, C4.5

Hunt's Algorithm

- Let D_t be the set of training records that reach a node t.
- General Procedure:
  - If D_t contains records that belong the same class y_t, then t is a leaf node labeled as y_t
  - If D_t is an empty set, then t is a leaf node labeled by the *default class*, y_d.
  - If D_t contains records that belong to more than one class, use an attribute test to split the data into smaller subsets. Recursively apply the procedure to each subset.

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
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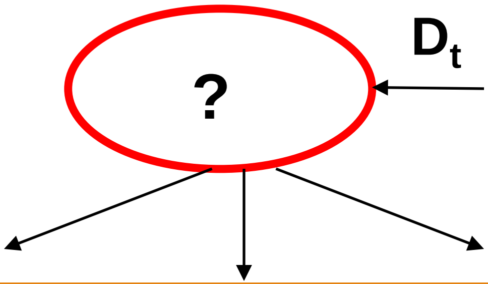
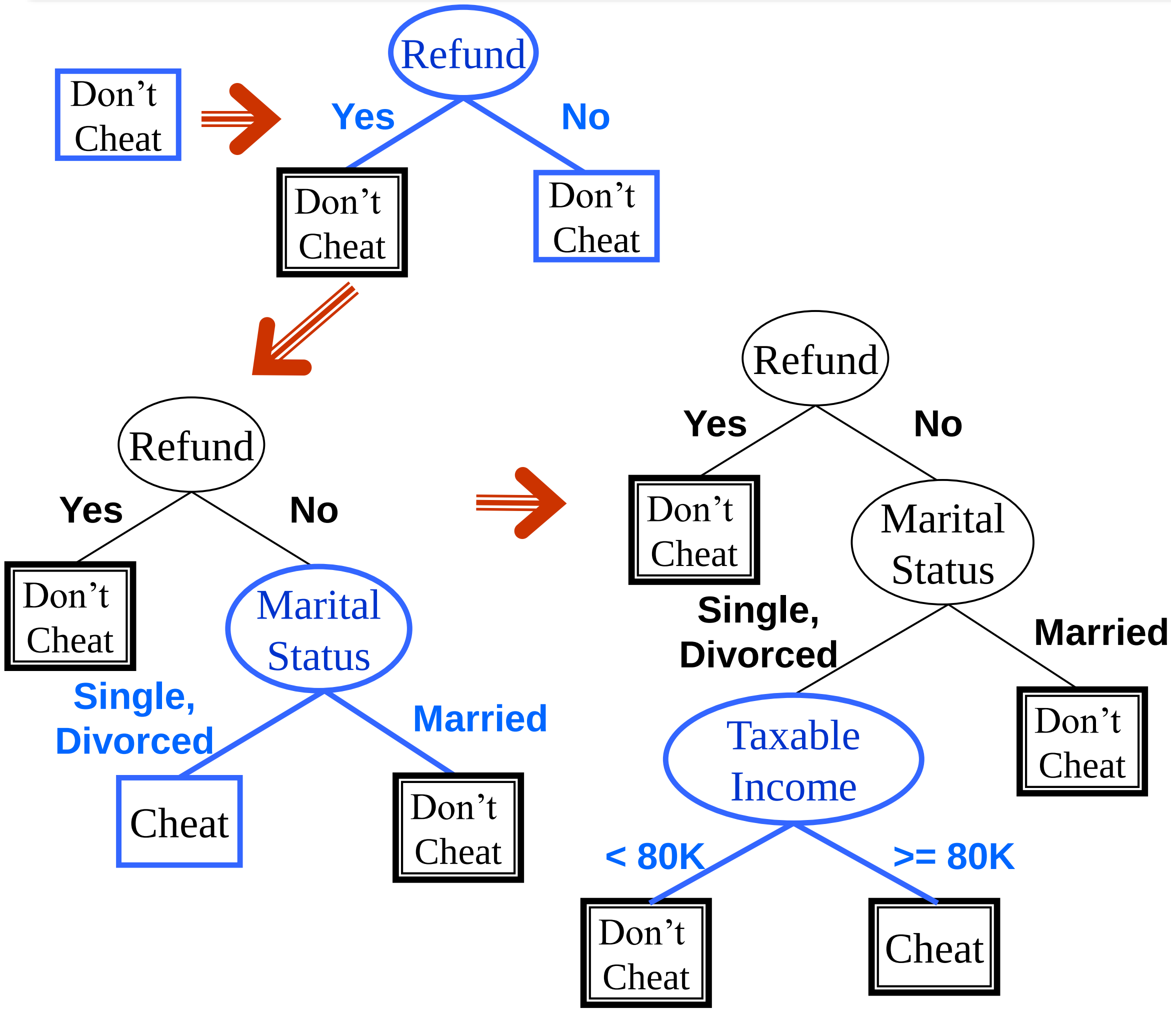
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



Hunt's Algorithm



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7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Tree Induction

- Greedy strategy.
  - Split the records based on an attribute test that optimizes certain criterion.
- Issues
  - Determine how to split the records
    - How to specify the attribute test condition?
    - How to determine the best split?
  - Determine when to stop splitting

Tree Induction

- Greedy strategy.
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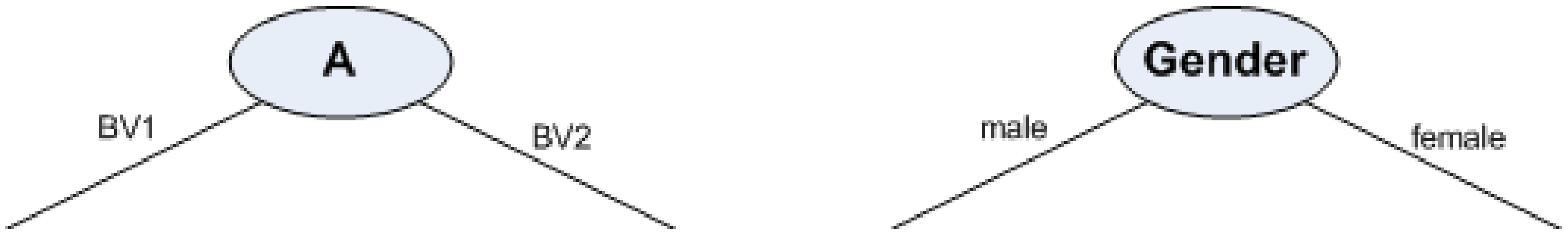
    - How to specify the attribute test condition?
    - How to determine the best split?
  - Determine when to stop splitting

How to Specify Test Condition?

- Depends on attribute types
  - Binary
  - Nominal
  - Ordinal
  - Continuous
- Depends on number of ways to split
  - 2-way split
  - Multi-way split

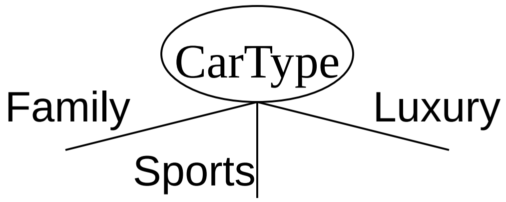
Splitting Based on Binary Attributes

- BuildDT algorithm must provides a method for expressing an attribute test condition and corresponding outcome for different attribute type
- **Case: Binary attribute**
  - This is the simplest case of node splitting
  - The test condition for a binary attribute generates only two outcomes

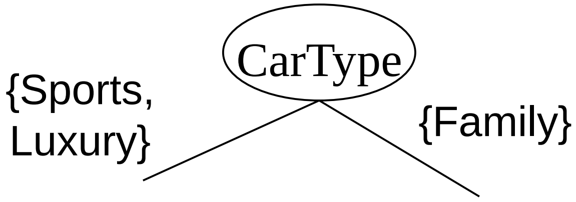


Splitting Based on Nominal Attributes

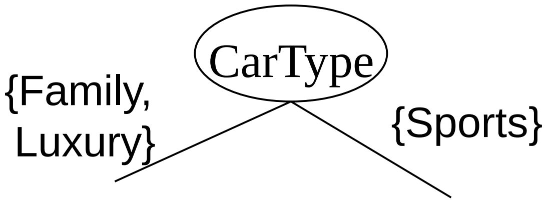
- Multi-way split: Use as many partitions as distinct values.



- Binary split: Divides values into two subsets.
  Need to find optimal partitioning.

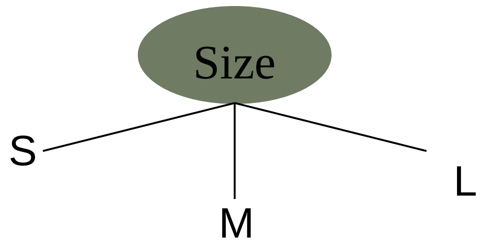


OR

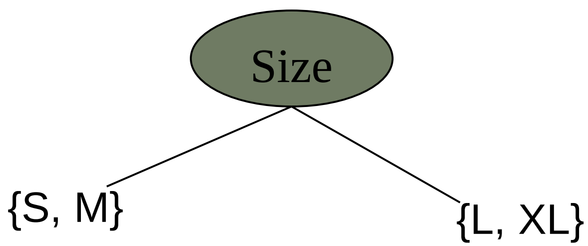


Splitting Based on Ordinal Attributes

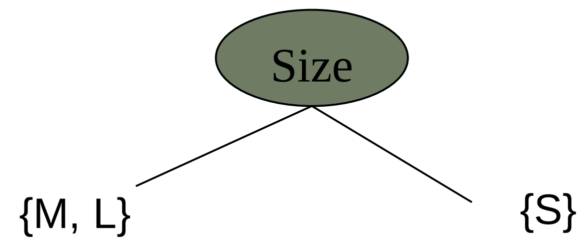
- Multi-way split: Use as many partitions as distinct values.
  - Small (S), Medium (M),
  - Large (L), Extra Large (XL)



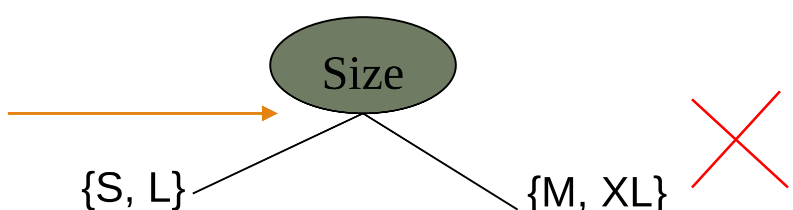
- Binary split: Divides values into two subsets. Need to find optimal partitioning.



OR



- What about this split?

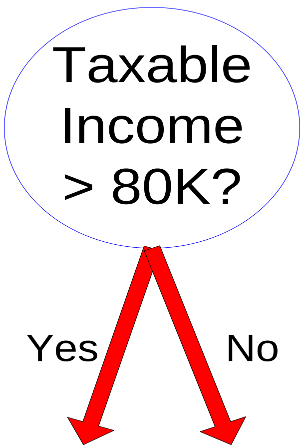


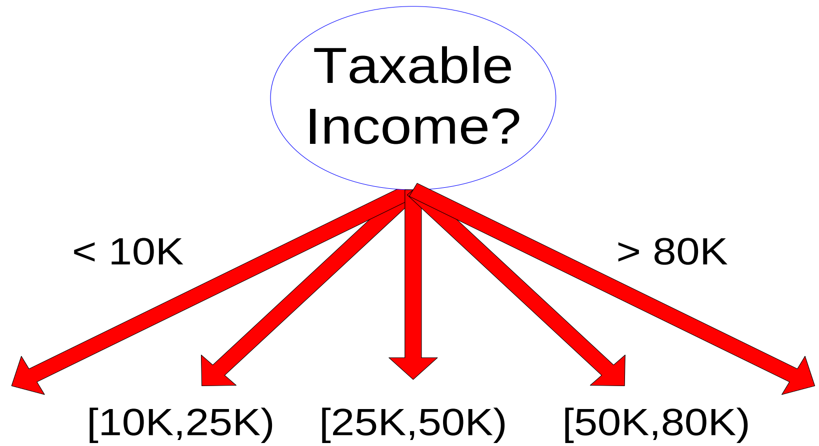
Not allowed

Splitting Based on Continuous Attributes

- Different ways of handling
  - Discretization to form an ordinal categorical attribute
    - Static – discretize once at the beginning
    - Dynamic – ranges can be found by equal interval bucketing, equal frequency bucketing (percentiles), or clustering.
  - Binary Decision: $(A < v)$ or $(A \geq v)$
    - consider all possible splits and finds the best cut
    - can be more compute intensive

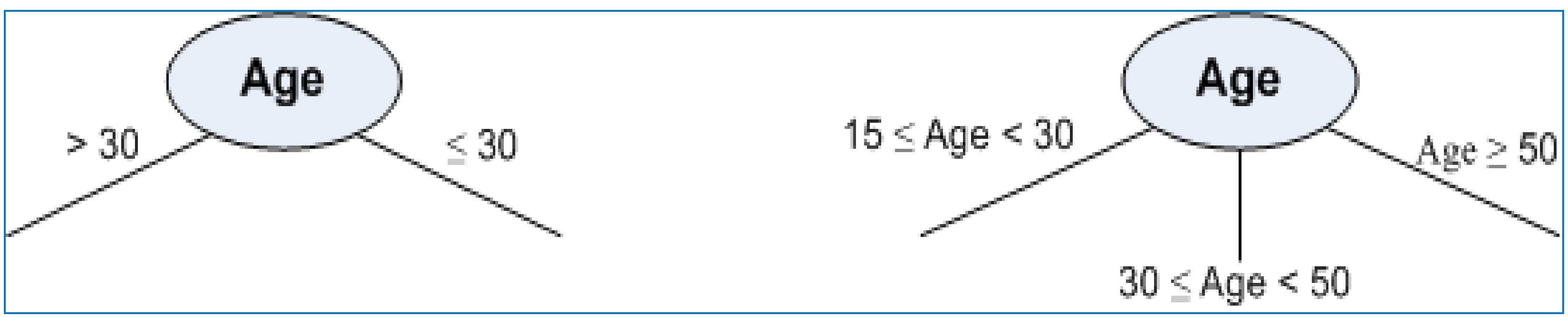
Splitting Based on Continuous Attributes…





(i) Binary split

(ii) Multi-way split



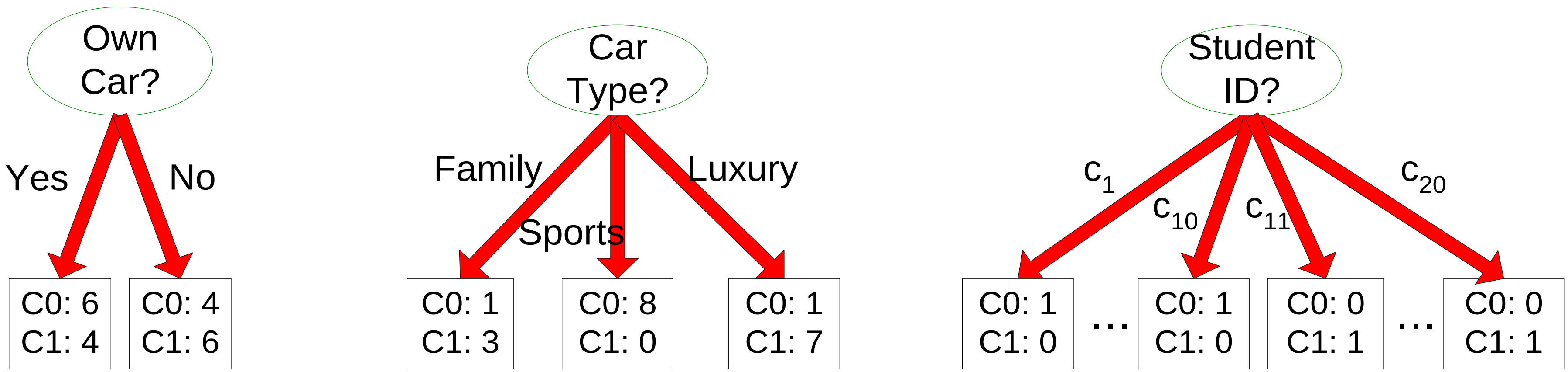
Tree Induction

- Greedy strategy.
  - Split the records based on an attribute test that optimizes certain criterion.

- Issues
  - Determine how to split the records
    - How to specify the attribute test condition?
    - How to determine the best split?
  - Determine when to stop splitting

How to determine the Best Split

Before Splitting: 10 records of class 0,
10 records of class 1



Which test condition is the best?

How to determine the Best Split

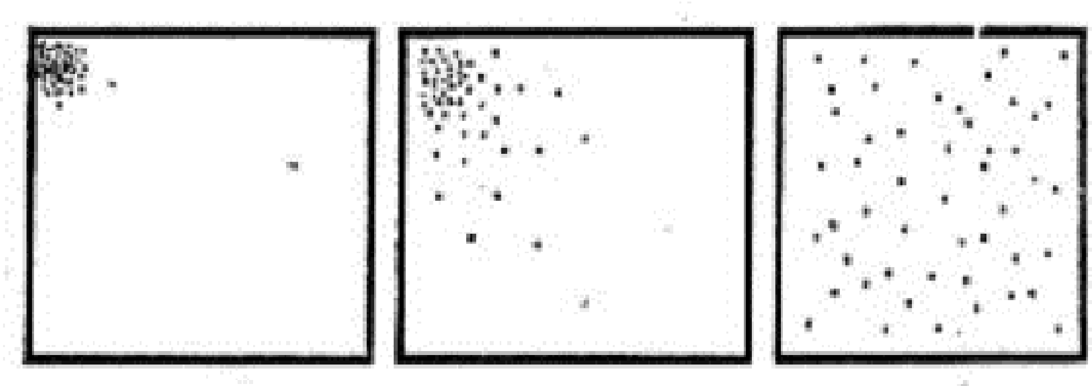
- Greedy approach:
  - Nodes with homogeneous class distribution are preferred.
- Need a measure of node impurity:

C0: 5 C1: 5	C0: 9 C1: 1
Non-homogeneous, **High degree of impurity**	**Homogeneous,** **Low degree of impurity**

Measures of Node Impurity

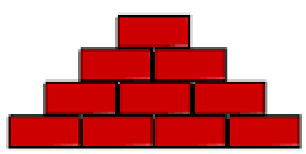
- There are three popular way to measure impurity is:
  - Information Gain
  - Gain Ratio
  - Gini Index

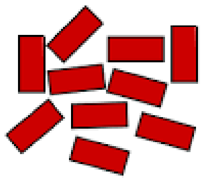
Concept of Entropy



If a point represents a gas molecule, then which system has the more entropy?

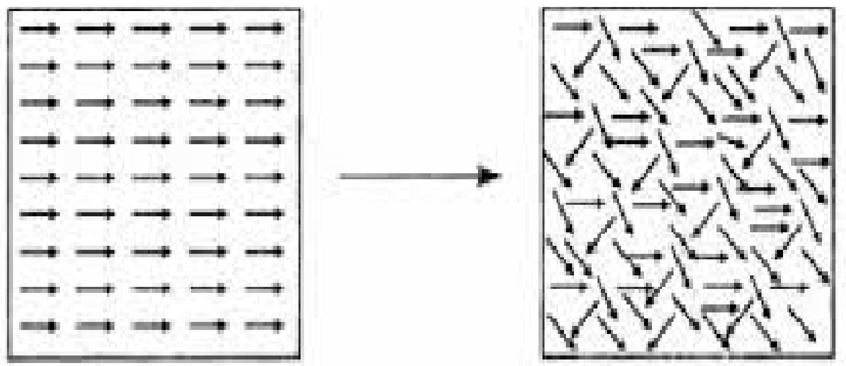
How to measure? $\Delta S = \frac{\Delta Q}{T}$?





More **ordered**
less **entropy**

Less ordered
higher entropy



More organized or
ordered (less **probable**)

Less organized or
disordered (**more** probable)

An Open Challenge!

Roll No.	Assignment	Project	Mid-Sem	End-Sem
12BT3FP06	89	99	56	91
10IM30013	95	98	55	93
12CE31005	98	96	58	97
12EC35015	93	95	54	99
12GG2005	90	91	53	98
12MI33006	91	93	57	97
13AG36001	96	94	58	95
13EE10009	92	96	56	96
13MA20012	88	98	59	96
14CS30017	94	90	60	94
14ME10067	90	92	58	95
14MT10038	99	89	55	93

Roll No.	Assignment	Project	Mid-Sem	End-Sem
12BT3FP06	19	59	16	71
10IM30013	37	38	25	83
12CE31005	38	16	48	97
12EC35015	23	95	54	19
12GG2005	40	71	43	28
12MI33006	61	93	47	97
13AG36001	26	64	48	75
13EE10009	92	46	56	56
13MA20012	88	58	59	66
14CS30017	74	20	60	44
14ME10067	50	42	38	35
14MT10038	29	69	25	33

Two sheets showing the tabulation of marks obtained in a course are shown.

Which tabulation of marks shows the “good” performance of the class?
How you can measure the same?

Entropy and its Meaning

- Entropy is an important concept used in Physics in the context of heat and thereby uncertainty of the states of a matter.

- At a later stage, with the growth of Information Technology, entropy becomes an important concept in Information Theory.

- To deal with the classification job, entropy is an important concept, which is considered as

  - an information-theoretic measure of the "uncertainty" contained in a training data

    - due to the presence of more than one classes.

Entropy in Information Theory

- The entropy concept in information theory first time coined by Claude Shannon (1850).
- The first time it was used to measure the "information content" in messages.
- According to his concept of entropy, presently entropy is widely being used as a way of representing messages for efficient transmission by Telecommunication Systems.

Measure of Information Content

Example 1

a) Guessing a birthday of your classmate

It is with uncertainty $\sim \frac{1}{365}$

Whereas guessing the day of his/her birthday is $\frac{1}{7}$.

This uncertainty, we may say varies between 0 to 1, both inclusive.

b) As another example, a question related to event with eventuality (or impossibility) will be answered with 0 or 1 uncertainty.

- Does sun rises in the East? (answer is with 0 uncertainty)
- Will mother give birth to male baby? (answer is with ½ uncertainty)
- Is there a planet like earth in the galaxy? (answer is with an extreme uncertainty)

Entropy Calculations

- Consider a dataset $\mathcal{D}$, which has various samples as $s^{(1)}, s^{(2)}, \ldots \ldots \ldots s^{(N)}$.
- The classes of dataset is y_1, y_2 (Binary Classification). It can be taken k classes too. (in general)
- Selecting a random sample from the dataset, which may belongs to class y_q.
- The probability of this selection is $P_q = \frac{freq(y_q, \mathcal{D})}{|\mathcal{D}|}$
- Where $freq(y_q, \mathcal{D})$ is the number of patterns in $\mathcal{D}$ that belongs to y_q, $|\mathcal{D}|$ is the total number of samples in $\mathcal{D}$.

Entropy Calculations…

- The expected information required to classify a pattern in $\mathcal{D}$ is $Info(\mathcal{D}) = -\sum_{q=1}^{k} p_q log_2^{(p_q)}$. M=2 for binary classification.
- A log of base 2 is used because information is encoded into bits.
- $Info(\mathcal{D})$ is average amount of information required to identify the class label of a sample in $\mathcal{D}$.
- $Info(\mathcal{D})$ can also be expressed as entropy as $E(\mathcal{D}) = -\sum_{q=1}^{k} p_q log_2^{(p_q)}$.
- Here, $E(\mathcal{D})$ is measured in "bits" of information.
- **Note:**
  - The above formula should be summed over the non-empty classes only, that is, classes for which $p_i \neq 0$
  - E is always a positive quantity
  - E takes it's minimum value (zero) if and only if all the instances have the same class (i.e., the training set with only **one** non-empty class, for which the probability 1).
  - Entropy takes its maximum value when the instances are equally distributed among k possible classes. In this case, the maximum value of E is $log_2 k$.

Entropy of a Training Set

Consider a dataset of OTPH as shown in the following table with total 24 instances in it.

Age	Eye sight	Astigmatic	Use Type	Class
1	1	1	1	3
1	1	1	2	2
1	1	2	1	3
1	1	2	2	1
1	2	1	1	3
1	2	1	2	2
1	2	2	1	3
1	2	2	2	1
2	1	1	1	3
2	1	1	2	2
2	1	2	1	3
2	1	2	2	1
2	2	1	1	3
2	2	1	2	2
2	2	2	1	3
2	2	2	2	3
3	1	1	1	3
3	1	1	2	3
3	1	2	1	3
3	1	2	2	1
3	2	1	1	3
3	2	1	2	2
3	2	2	1	3
3	2	2	2	3

A coded forms for all values of attributes are used to avoid the cluttering in the table.

Entropy of a training set…

Specification of the attributes are as follows.

Age	Eye Sight	Astigmatic	Use Type
1: Young	1: Myopia	1: No	1: Frequent
2: Middle-aged	2: Hypermetropia	2: Yes	2: Less
3: Old			

Class: 1: Contact Lens 2:Normal glass 3: Nothing

In the OPTH database, there are 3 classes and 4 instances with class 1, 5 instances with class 2 and 15 instances with class 3. Hence, entropy E of the database is:

$$E = -\frac{4}{24}\log_2\frac{4}{24} - \frac{5}{24}\log_2\frac{5}{24} - \frac{15}{24}\log_2\frac{15}{24} = 1.3261$$

Decision Tree Induction Techniques

- Decision tree induction is a top-down, recursive and divide-and-conquer approach.
- The procedure is to choose an attribute and split it into from a larger training set into smaller training sets.
- Different algorithms have been proposed to take a good control over
  1. Choosing the best attribute to be splitted, and
  2. Splitting criteria
- Several algorithms have been proposed for the above tasks. In this lecture, we shall limit our discussions into three important of them
  - **ID3**
  - **C 4.5**
  - **CART**

Algorithm ID3

ID3: Decision Tree Induction Algorithms

- Quinlan [1986] introduced the ID3, a popular short form of **Iterative Dichotomizer 3** for decision trees from a set of training data.

- In ID3, each node corresponds to a splitting attribute and each arc is a possible value of that attribute.

- At each node, the splitting attribute is selected to be the most informative among the attributes not yet considered in the path starting from the root.

Algorithm ID3

- In ID3, entropy is used to measure how informative a node is.
  - It is observed that splitting on any attribute has the property that average entropy of the resulting training subsets will be less than or equal to that of the previous training set.
- ID3 algorithm defines a measurement of a splitting called **Information Gain** to determine the goodness of a split.
  - The attribute with the largest value of information gain is chosen as the splitting attribute and
  - it partitions into a number of smaller training sets based on the distinct values of attribute under split.

Defining Information Gain

- We consider the following symbols and terminologies to define information gain, which is denoted as α.
- $D \equiv$ denotes the training set at any instant
- $|D| \equiv$ denotes the size of the training set D
- $E(D) \equiv$ denotes the entropy of the training set D
- The entropy of the training set D

$$E(D) = -\sum_{i=1}^{k} p_i \, log_2(p_i)$$

  - where the training set D has $c_1, c_2, \dots, c_k$, the k number of distinct classes and
  - p_i, $0 < p_i \leq 1$ is the probability that an arbitrary tuple in D belongs to class c_i (i $= 1, 2, \dots, k$).
  - p_i can be calculated as

$$p_i = \frac{|C_{i,D}|}{|D|}$$

  - where $C_{i,D}$ is the set of tuples of class c_i in D.

Defining Information Gain…

- Suppose, we want to partition D on some attribute A having m distinct values $\{a_1, a_2, \ldots, a_m\}$.
- Attribute A can be considered to split D into m partitions $\{D_1, D_2, \ldots, D_m\}$, where D_j (j = 1,2, … ,m) contains those tuples in D that have outcome a_j of A.
- The **weighted entropy** denoted as $E_A(D)$ for all partitions of D with respect to A is given by:
  - $E_A(D) = \sum_{j=1}^{m} \frac{|D_j|}{|D|} E(D_j)$
- Here, the term $\frac{|D_j|}{|D|}$ denotes the weight of the j-th training set.
- More meaningfully, $E_A(D)$ is the expected information required to classify a tuple from D based on the splitting of A.

Defining Information Gain…

- Our objective is to take A on splitting to produce an exact classification (also called pure), that is, all tuples belong to one class.
- However, it is quite likely that the partitions is impure, that is, they contain tuples from two or more classes.
- In that sense, $E_A(D)$ is a measure of impurities (or purity). A lesser value of $E_A(D)$ implying more power the partitions are.
- **Information gain, $\boldsymbol{\alpha(A, D)}$** of the training set D splitting on the attribute A is given by
  - $\boldsymbol{\alpha(A, D) = E(D) - E_A(\mathrm{D})}$
- In other words, $\alpha(A, D)$ gives us an estimation how much would be gained by splitting on A. The attribute A with the highest value of α should be chosen as the splitting attribute for D.

Compute Information Gain

Tid	Refund	Marital Status	Taxable Income	Class
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	?	Single	90K	Yes

Missing value

Before Splitting:

$$E(\mathcal{D}) = -\frac{3}{10}\log\left(\frac{3}{10}\right) - \frac{7}{10}\log\left(\frac{7}{10}\right) = 0.8813$$

	Class = Yes	Class = No
Refund=Yes	0	3
Refund=No	2	4
Refund=?	1	0

Split on Refund (A):

$$E(Refund = Yes) = 0$$

$$E(Refund = No) = \frac{2}{6}\log\left(\frac{2}{6}\right) - \frac{4}{6}\log\left(\frac{4}{6}\right) = 0.9183$$

$$E_A(\mathcal{D}) = \frac{3}{10} \times 0 + \frac{6}{10} \times 0.9183 = .5509$$

$$\alpha(A, \mathcal{D}) = .8813 - 0.5509 = .33032$$

Example

- **Information gain on splitting OPTH**
- Training set: D_1(Age = 1)

Age (x1)	Eye-sight (x2)	Astigmatism (x3)	Use type (x4)	Class (y)
1	1	1	1	3
1	1	1	2	2
1	1	2	1	3
1	1	2	2	1
1	2	1	1	3
1	2	1	2	2
1	2	2	1	3
1	2	2	2	1

$$E(D_1) = -\frac{2}{8} log_2\left(\frac{2}{8}\right) - \frac{2}{8} log_2\left(\frac{2}{8}\right) - \frac{4}{8} log_2\left(\frac{4}{8}\right) = \mathbf{1.5}$$

$$E_{Age}(D_1) = \frac{8}{24} \times 1.5 = \mathbf{0.5000}$$

Example…

Training set: D_2(Age = 2)

Age	Eye-sight	Astigmatism	Use type	Class
2	1	1	1	3
2	1	1	2	2
2	1	2	1	3
2	1	2	2	1
2	2	1	1	3
2	2	1	2	2
2	2	2	1	3
2	2	2	2	3

$$E(D_2) = -\frac{1}{8}log_2(\frac{1}{8}) - \frac{2}{8}log_2(\frac{2}{8}) - \frac{5}{8}log_2(\frac{5}{8}) = \mathbf{1.2988}$$

$$E_{Age}(D_2) = \frac{8}{24} \times 1.2988 = \mathbf{0.4329}$$

Example...

Training set: D_3(Age = 3)

Age	Eye-sight	Astigmatism	Use type	Class
3	1	1	1	3
3	1	1	2	3
3	1	2	1	3
3	1	2	2	1
3	2	1	1	3
3	2	1	2	2
3	2	2	1	3
3	2	2	2	3

$$E(D_3) = -\frac{1}{8}\log\left(\frac{1}{8}\right) - \frac{1}{8}\log\left(\frac{1}{8}\right) - \frac{6}{8}\log\left(\frac{6}{8}\right) = 1.0613$$

$$E_{Age}(D_3) = \frac{8}{24} \times 1.0613 = 0.3504$$

$\alpha\ (Age, D)$ = 1.3261 – (0.5000 + 0.4329 + 0.3504) = **0.0394**

Information Gains for Different Attributes

- In the same way, we can calculate the information gains, when splitting the OPTH database on Eye-sight, Astigmatic and Use Type. The results are summarized below.
- Splitting attribute: Age

$$\alpha(Age, OPTH) = 0.0394$$

- Splitting attribute: Eye-sight

$$\alpha(Eye - sight, OPTH) = 0.0395$$

- Splitting attribute: Astigmatic

$$\alpha(\text{Astigmatic}, OPTH) = 0.3770$$

- Splitting attribute: Use Type

$$\alpha(\text{Use Type}, OPTH) = 0.5488$$

Decision Tree Induction : ID3 Way

- The ID3 strategy of attribute selection is to choose to split on the attribute that gives the greatest reduction in the weighted average entropy
  - The one that maximizes the value of information gain

- In the example with OPTH database, the larger values of information gain is $\alpha(\text{Use Type}\,, OPTH) = 0.5488$
  - Hence, the attribute should be chosen for splitting is "Use Type".

- The process of splitting on nodes is repeated for each branch of the evolving decision tree, and the final tree, which would look like is shown in the following slide and calculation is left for practice.

Decision Tree Induction : ID3 Way

$OPTH$ $\quad$ $\boldsymbol{E(OPTH) = 1.3261}$

Age $\alpha = 0.0394$

Eye-sight $\alpha = 0.395$

Use Type $\alpha = 0.05488$

Astigmatic $\alpha = 0.3770$

$D1$ $\quad$ $Frequent(1)$

$E(D_1)$ =?

Age	Eye	Ast	Use	Class
1	1	1	1	3
1	1	2	1	3
1	2	1	1	3
1	2	2	1	3
2	1	1	1	3
2	2	1	1	3
2	2	2	1	3
3	1	1	1	3
3	1	2	1	3
3	2	1	1	3
3	2	2	1	3

? Age α =?

? Eye-sight α =?

? Astigmatic α =?

$D2$ $\quad$ $Less(2)$

$E(D_2)$ =?

Age	Eye	Ast	Use	Class
1	1	1	2	2
1	1	2	2	1
1	2	1	2	2
1	2	2	2	1
2	1	1	2	2
2	1	2	2	1
2	2	1	2	2
3	1	1	2	3
3	1	2	2	3
3	2	1	2	2
3	2	2	2	3

? Age α =?

? Eye-sight α =?

? Astigmatic α =?

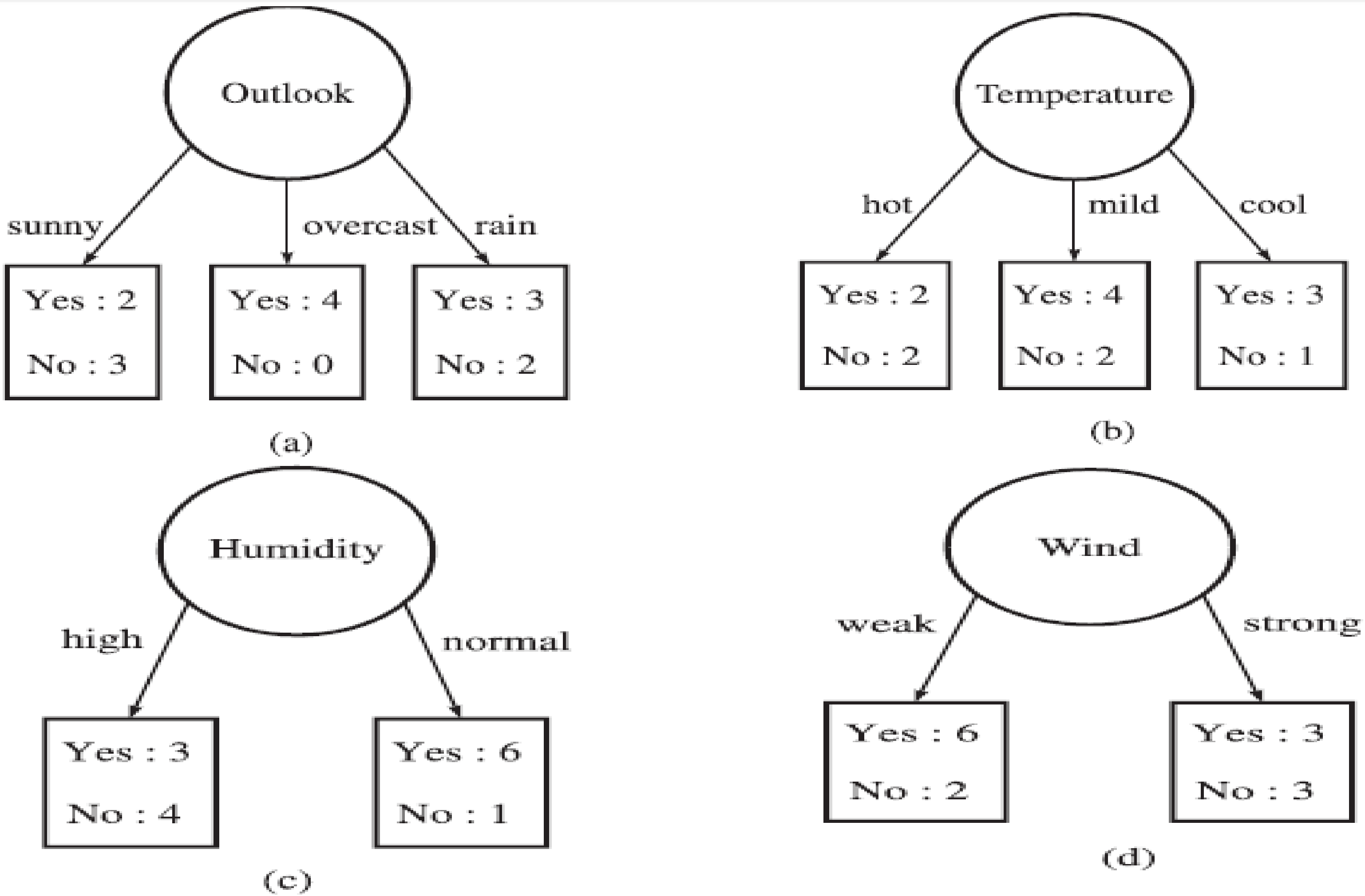
Example 3

- The input variables are:
  - $x_1 =$ *outlook* ; $x_2 =$ *Temperature*
  - $x_3 =$ *humidity*; $x_4 =$ *wind*
  - Target variable $y =$ *Play Tennis*.
  - Target function to be learnt as:

    $\hat{y}: S \rightarrow [0, 1]$

Instances	Outlook x_1	Temperature x_2	Humidity x_3	Wind x_4	Play Tennis y
$s^{(1)}$	Sunny	Hot	High	Weak	No
$s^{(2)}$	Sunny	Hot	High	Strong	No
$s^{(3)}$	Overcast	Hot	High	Weak	Yes
$s^{(4)}$	Rain	Mild	High	Weak	Yes
$s^{(5)}$	Rain	Cool	Normal	Weak	Yes
$s^{(6)}$	Rain	Cool	Normal	Strong	No
$s^{(7)}$	Overcast	Cool	Normal	Strong	Yes
$s^{(8)}$	Sunny	Mild	High	Weak	No
$s^{(9)}$	Sunny	Cool	Normal	Weak	Yes
$s^{(10)}$	Rain	Mild	Normal	Weak	Yes
$s^{(11)}$	Sunny	Mild	Normal	Strong	Yes
$s^{(12)}$	Overcast	Mild	High	Strong	Yes
$s^{(13)}$	Overcast	Hot	Normal	Weak	Yes
$s^{(14)}$	Rain	Mild	High	Strong	No

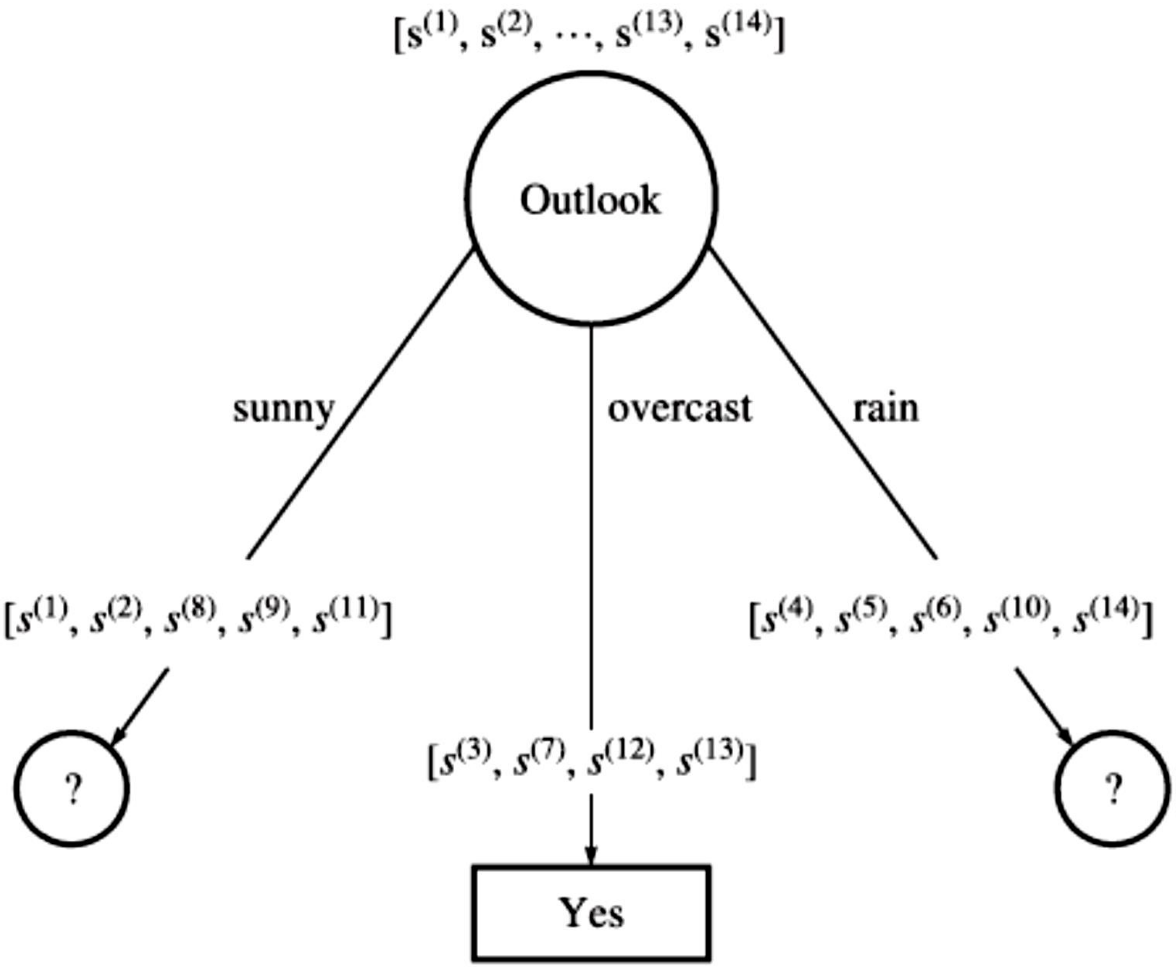
Example 3…



Outlook
sunny
overcast
rain
Yes : 2
No : 3
Yes : 4
No : 0
Yes : 3
No : 2
(a)
Temperature
hot
mild
cool
Yes : 2
No : 2
Yes : 4
No : 2
Yes : 3
No : 1
(b)
Humidity
high
normal
Yes : 3
No : 4
Yes : 6
No : 1
(c)
Wind
weak
strong
Yes : 6
No : 2
Yes : 3
No : 3
(d)

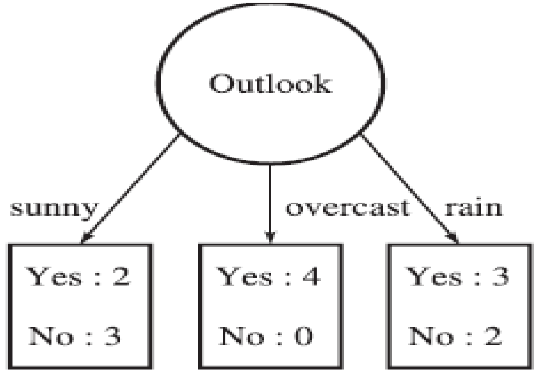
Example 3…

- A partially learned decision tree
- There are three possibility to select further attribute **Temperature, Humidity, and Wind.**
- **Intuitively, "Humidity" is the best to choose**



Example 3: Information Gain

- Information of dataset is computed as:
- $Info(D) = E(D) = -\frac{9}{14}\log\left(\frac{9}{14}\right) - \frac{5}{14}\log\left(\frac{5}{14}\right) = 0.94\ bits.$



(a)

- $E(D_1) = -\frac{2}{5}\log\left(\frac{2}{5}\right) - \frac{3}{5}\log\left(\frac{3}{5}\right) = 0.97$
- $E(D_2) = 0$
- $E(D_3) = -\frac{3}{5}\log\left(\frac{3}{5}\right) - \frac{2}{5}\log\left(\frac{2}{5}\right) = 0.97$
- $E_{outlook}(D) = \frac{5}{14} \times 0.97 + \frac{5}{14} \times .97 = 0.693 = \mathbf{Info(D, \boldsymbol{x_1})}$
- **Similarly,** $E_{Temperature}(D) = 0.911, E_{Humidity}(D) = 0.788$**,** $E_{Wind}(D) = 0.892$

Example 3: Information Gain…

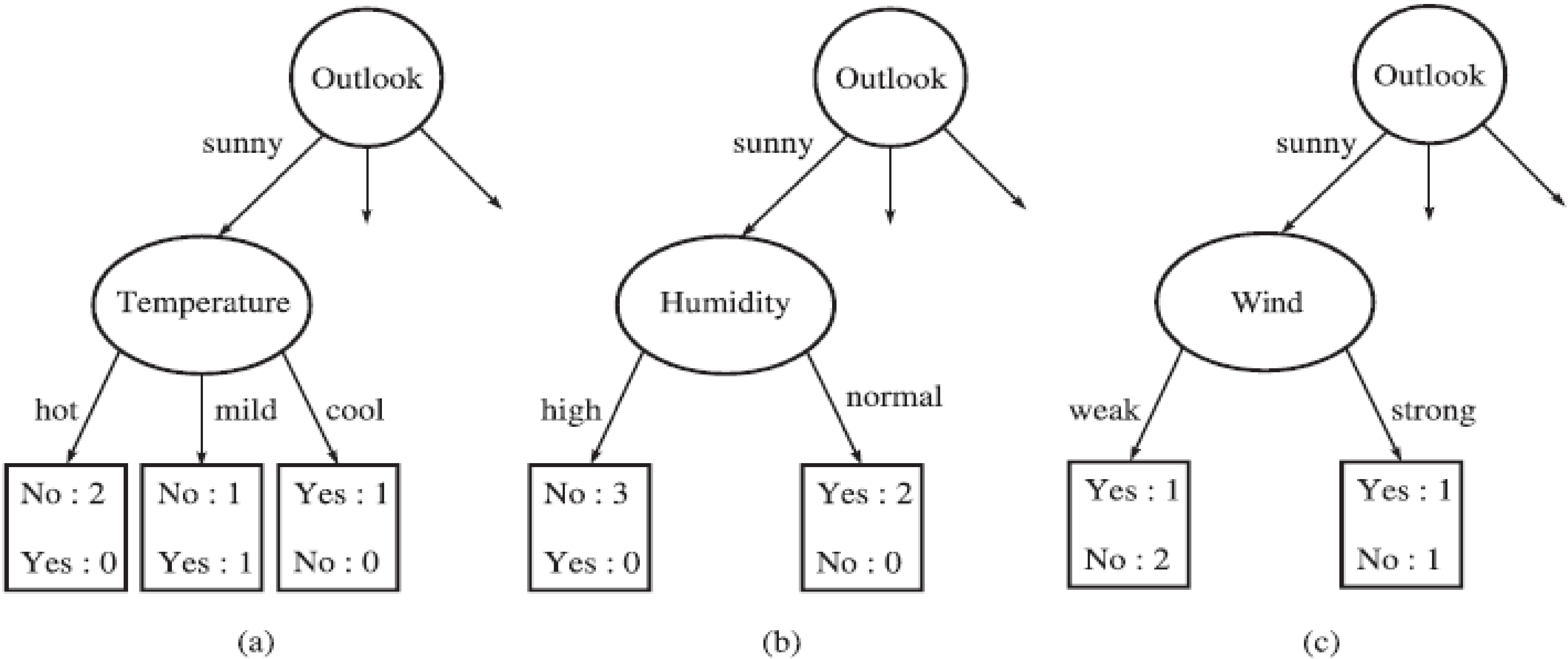
- $Gain(D, x_j) = E(D) - E(D, x_j)$
- $Gain(D, x_1) = .94 - 0.693 = 0.247$: **Outlook**
- $Gain(D, x_2) = .029$: **Temperature**
- $Gain(D, x_3) = .152$: **Humidity**
- $Gain(D, x_4) = .048$: **Wind**

Outlook has maximum gain

Root Node

Example 3: Information Gain…

- Same strategy is applied recursively to each subset of training instances.



Example 3: Information Gain…

- Further branching at the node reached when outlook is sunny.
- The information gain at daughter node are:
- ***Temperature: Hot-02, Mild-02, Cool-01***

$$Info(Hot) = -\frac{2}{2}\log\left(\frac{2}{2}\right) - \frac{0}{2}\log\left(\frac{0}{2}\right) = 0.0$$

$$Info(Mild) = -\frac{1}{2}\log\left(\frac{1}{2}\right) - \frac{1}{2}\log\left(\frac{1}{2}\right) = 0.5$$

$$Info(Cool) = -\frac{1}{1}\log\left(\frac{1}{1}\right) - \frac{0}{1}\log\left(\frac{0}{1}\right) = 0.0$$

$$Info(Ds, Temp) = \frac{2}{5} * 0.0 + \frac{2}{5} * 0.5 + \frac{1}{5} * 0.0 = 0.4$$

$$Info(Ds) = -\frac{3}{5}\log\left(\frac{3}{5}\right) - \frac{2}{5}\log\left(\frac{2}{5}\right) = 0.97095$$

$$Gain(Ds, Temp) = Info(Ds) - Info(Ds, Temp) = 0.97095 - 0.4 = 0.571$$

Example 3: Information Gain…

- Similarly, for Humidity and Wind
- ***Humidity: High-03 (N-3, Y-0), Normal-02(N-0, Y-2)***

$$Info(High) = -\frac{3}{3}\log\left(\frac{3}{3}\right) - \frac{0}{3}\log\left(\frac{0}{3}\right) = 0.0$$

$$Info(Normal) = -\frac{2}{2}\log\left(\frac{2}{2}\right) - \frac{0}{2}\log\left(\frac{0}{2}\right) = 0.0$$

$$Info(Ds, Humidity) = \frac{3}{5} * 0.0 + \frac{2}{5} * 0.0 = 0.0$$

$$Info(Ds) = -\frac{3}{5}\log\left(\frac{3}{5}\right) - \frac{2}{5}\log\left(\frac{2}{5}\right) = 0.97095$$

$$Gain(Ds, Humidity) = .97095 - 0.0 = 0.971$$

- ***Wind: Weak-03(Y-1, N-2), Strong-02 (Y-1, N-1)***

$$Info(Week) = -\frac{1}{3}\log\left(\frac{1}{3}\right) - \frac{2}{3}\log\left(\frac{2}{3}\right) = .9183$$

$$Info(Strong) = -\frac{1}{2}\log\left(\frac{1}{2}\right) - \frac{1}{2}\log\left(\frac{1}{2}\right) = 0.5$$

$$Info(Ds, Wind) = \frac{3}{5} * 0.9183 + \frac{2}{5} * 0.5 = 0.74978$$

$$Gain(Ds, Wind) = .97095 - 0.74978 = 0.221$$

We choose “Humidity”, since it has highest gain. No further splitting, reached as leaf node

Facts of Information Gain

- Information gain $G(\mathcal{D}, x_i)$ measures the expected reduction in entropy, caused by portioning the patterns in dataset $\mathcal{D}$.
- It gave good result and there are several data mining software's where it is used.
- It has problem when attribute has large possible values, which give rise in multiway splitting with daughter node.
- It has strong bias in favour of attributes with large number of values. The attribute with large number of values will get selected at root itself and may lead to all leaf nodes, resulting in to a too simple hypothesis model unable to capture the structure of the data.

C4.5
Gain Ratio

Gain Ratio

- It is successor of ID3, uses an extension of information gain, known as gain ratio, which attempts to overcome the bias in information gain.
- it uses a normalization of information gain using "split information" and analogous term of information gain $Info(\mathcal{D}, x_j)$ is used to defined as $SplitInfo(\mathcal{D}, x_j)$.
- $SplitInfo(\mathcal{D}, x_j) = -\sum_{l=1}^{d_j} \frac{|\mathcal{D}_l|}{|\mathcal{D}|} log_2^{\frac{|\mathcal{D}_l|}{|\mathcal{D}|}}$
- The gain ratio is defined as $GainRatio(\mathcal{D}, x_j) = \frac{Gain(\mathcal{D}, x_j)}{SplitInfo(\mathcal{D}, x_j)}$
- The attributes with maximum gain is selected as splitting node
  - Designed to overcome the disadvantage of Information Gain

Example :Gain Ratio

- Considering the same example of weather data, $x_i = Outlook$ splits the dataset into three subsets of size 5, 4, and 5.
- The *SplitInfo* is given by

$$SplitInfo(\mathcal{D}, x_1) = -\sum_{l=1}^{3} \frac{|\mathcal{D}_l|}{|\mathcal{D}|} \log_2 \frac{|\mathcal{D}_l|}{|\mathcal{D}|}$$

$$= -\tfrac{5}{14} \log_2 \tfrac{5}{14} - \tfrac{4}{14} \log_2 \tfrac{4}{14} - \tfrac{5}{14} \log_2 \tfrac{5}{14}$$

$$= 1.577$$

Example :Gain Ratio…

- Normalize the information gain by dividing by the ***split info*** value to get *gain ratio*.

$$GainRatio(\mathcal{D}, x_1) = \frac{Gain(\mathcal{D}, x_1)}{SplitInfo(\mathcal{D}, x_1)}$$

$$= \frac{0.247}{1.577} = 0.156$$

Example :Gain Ratio...

- Similarly, it can be calculated for others attributes such as:

$$\begin{aligned}
&Outlook &&: Gain(\mathcal{D}, x_1) = 0.247, SplitInfo(\mathcal{D}, x_1) = 1.577, GainRatio(\mathcal{D}, x_1) = 0.156 \\
&Temperature &&: Gain(\mathcal{D}, x_2) = 0.029, SplitInfo(\mathcal{D}, x_2) = 1.362, GainRatio(\mathcal{D}, x_2) = 0.019 \\
&Humidity &&: Gain(\mathcal{D}, x_3) = 0.152, SplitInfo(\mathcal{D}, x_3) = 1.000, GainRatio(\mathcal{D}, x_3) = 0.152 \\
&Wind &&: Gain(\mathcal{D}, x_4) = 0.048, SplitInfo(\mathcal{D}, x_4) = 0.985, GainRatio(\mathcal{D}, x_4) = 0.049
\end{aligned}$$

- Outlook still comes out on top but Humidity is much closers contender because it split the data into two subsets instead three.

CART
GINI INDEX

CLASSIFICATION AND REGRESSION TREE

GINI Index

- It is also popular splitting criterion, which is named as Gini in the name of Italian statistician and economist Corrado Gini.
- GINI index is defined as
  - $GINI(\mathcal{D}) = 1 - \sum_{q=1}^{M} P_q^2$
- Where P_q is the probability that a tuple in $\mathcal{D}$ belongs to class y_q, and is estimated by
  - $P_q = \frac{freq(y_d, \mathcal{D})}{|\mathcal{D}|}$
- Gini index considers a binary split for each attribute.

GINI Index...

- Let us first consider the case where x_j is continuous-valued attribute having d_j distinct values$v_{lx_j}; l = 1, 2 \ldots . d_j$.
- It is common to take mid-point between each pair of (sorted) adjacent values as a possible split-point.
- The point giving the **minimum Gini index** for the attribute x_j is taken as its split-point.

GINI Index...

- For a possible split-point of $x_j, \mathcal{D}_1$ is the number of tuples in $\mathcal{D}$ satisfying $x_j \leq split - point$, and $\mathcal{D}_2$ is the set of tuples satisfying $x_1 > split - point.$
- The reduction in impurity that would be incurred by a binary split on x_j is:
- $\Delta Gini(x_j) = Gini(\mathcal{D}) - Gini(\mathcal{D}, x_j)$
- $Gini(\mathcal{D}, x_j) = \frac{|\mathcal{D}_1|}{\mathcal{D}} Gini(\mathcal{D}_1) + \frac{|\mathcal{D}_2|}{\mathcal{D}} Gini(\mathcal{D}_2)$
- The attribute that maximizes the reduction in impurity is selected as the splitting attribute.
- The attribute that has Minimum GINI index.

Measure of Impurity: GINI

- Gini Index for a given node t :
  - Maximum (1 - 1/n_c) when records are equally distributed among all classes, implying least interesting information.
  - Minimum (0.0) when all records belong to one class, implying most interesting information.

C1	**0**
C2	**6**
Gini=0.000	

C1	**1**
C2	**5**
Gini=0.278	

C1	**2**
C2	**4**
Gini=0.444	

C1	**3**
C2	**3**
Gini=0.500	

$$GINI(\mathcal{D}) = 1 - \sum_{q=1}^{M} P_q^2$$

Examples for computing GINI

$$GINI(\mathcal{D}) = 1 - \sum_{q=1}^{M} P_q^2$$

C1	**0**
C2	**6**

P(C1) = 0/6 = 0 P(C2) = 6/6 = 1

Gini = 1 – $P(C1)^2$ – $P(C2)^2$ = 1 – 0 – 1 = 0

C1	**1**
C2	**5**

P(C1) = 1/6 P(C2) = 5/6

Gini = 1 – $(1/6)^2$ – $(5/6)^2$ = 0.278

C1	**2**
C2	**4**

P(C1) = 2/6 P(C2) = 4/6

Gini = 1 – $(2/6)^2$ – $(4/6)^2$ = 0.444

Splitting Based on GINI

- Used in CART: *Classification And Regression Trees*
- When a node p is split into k partitions (children), the quality of split is computed as,

$$GINI_{split} = \sum_{i=1}^{k} \frac{n_i}{n} GINI(i)$$

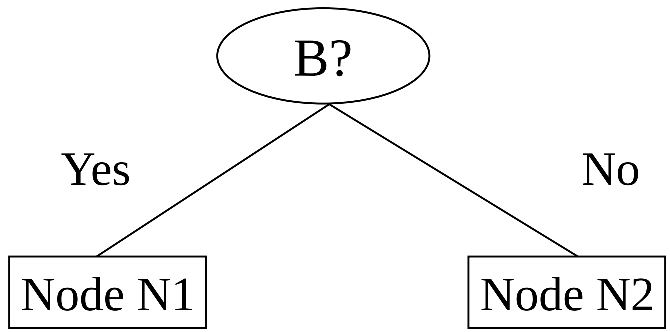
where, n_i = number of records at child i,

n = number of records at node p.

Binary Attributes: Computing GINI Index

- Splits into two partitions
- Effect of Weighing partitions:
  - Larger and Purer Partitions are sought for.

	Parent
C1	**6**
C2	**6**
Gini = 0.500	



Gini(N1)
$= 1 - (5/6)^2 - (2/6)^2$
$= 0.194$

Gini(N2)
$= 1 - (1/6)^2 - (4/6)^2$
$= 0.528$

	N1	**N2**
C1	**5**	**1**
C2	**2**	**4**
Gini=0.333		

Gini(Children)
$= 7/12 * 0.194 + 5/12 * 0.528$
$= 0.333$

Categorical Attributes: Computing Gini Index

- For each distinct value, gather counts for each class in the dataset
- Use the count matrix to make decisions

Multi-way split

	CarType		
	Family	Sports	Luxury
C1	1	2	1
C2	4	1	1
Gini	0.393		

Two-way split
(find best partition of values)

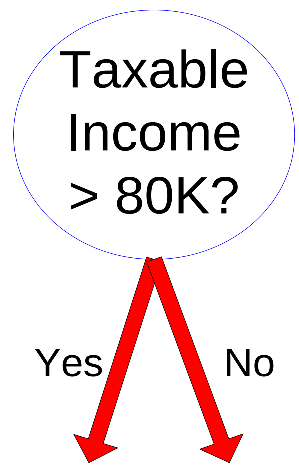
	CarType	
	{Sports, Luxury}	{Family}
C1	3	1
C2	2	4
Gini	0.400	

	CarType	
	{Sports}	{Family, Luxury}
C1	2	2
C2	1	5
Gini	0.419	

Continuous Attributes: Computing Gini Index

- Use Binary Decisions based on one value
- Several Choices for the splitting value
  - Number of possible splitting values = Number of distinct values
- Each splitting value has a count matrix associated with it
  - Class counts in each of the partitions, A < v and A ≥ v
- Simple method to choose best v
  - For each v, scan the database to gather count matrix and compute its Gini index
  - Computationally Inefficient! Repetition of work.

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



Continuous Attributes: Computing Gini Index...

- For efficient computation: for each attribute,
  - Sort the attribute on values
  - Linearly scan these values, each time updating the count matrix and computing gini index
  - Choose the split position that has the least gini index

Sorted Values →

Cheat	No	No	No	Yes	Yes	Yes	No	No	No	No
Taxable Income										
	60	70	75	85	90	95	100	120	125	220

Split Positions →

	55		65		72		80		87		92		97		110		122		172		230	
	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>
Yes	0	3	0	3	0	3	0	3	1	2	2	1	3	0	3	0	3	0	3	0	3	0
No	0	7	1	6	2	5	3	4	3	4	3	4	3	4	4	3	5	2	6	1	7	0
Gini	0.420		0.400		0.375		0.343		0.417		0.400		*0.300*		0.343		0.375		0.400		0.420	

Splitting Criteria based on Classification Error

- Classification error at a node t :

$$Error(t) = 1 - \max_i P(i \mid t)$$

- Measures misclassification error made by a node.
  - Maximum ($1 - 1/n_c$) when records are equally distributed among all classes, implying least interesting information
  - Minimum (0.0) when all records belong to one class, implying most interesting information

Examples for Computing Error

$$Error(t) = 1 - \max_i P(i \mid t)$$

C1	**0**
C2	**6**

P(C1) = 0/6 = 0   P(C2) = 6/6 = 1

Error = 1 – max (0, 1) = 1 – 1 = 0

C1	**1**
C2	**5**

P(C1) = 1/6   P(C2) = 5/6

Error = 1 – max (1/6, 5/6) = 1 – 5/6 = 1/6

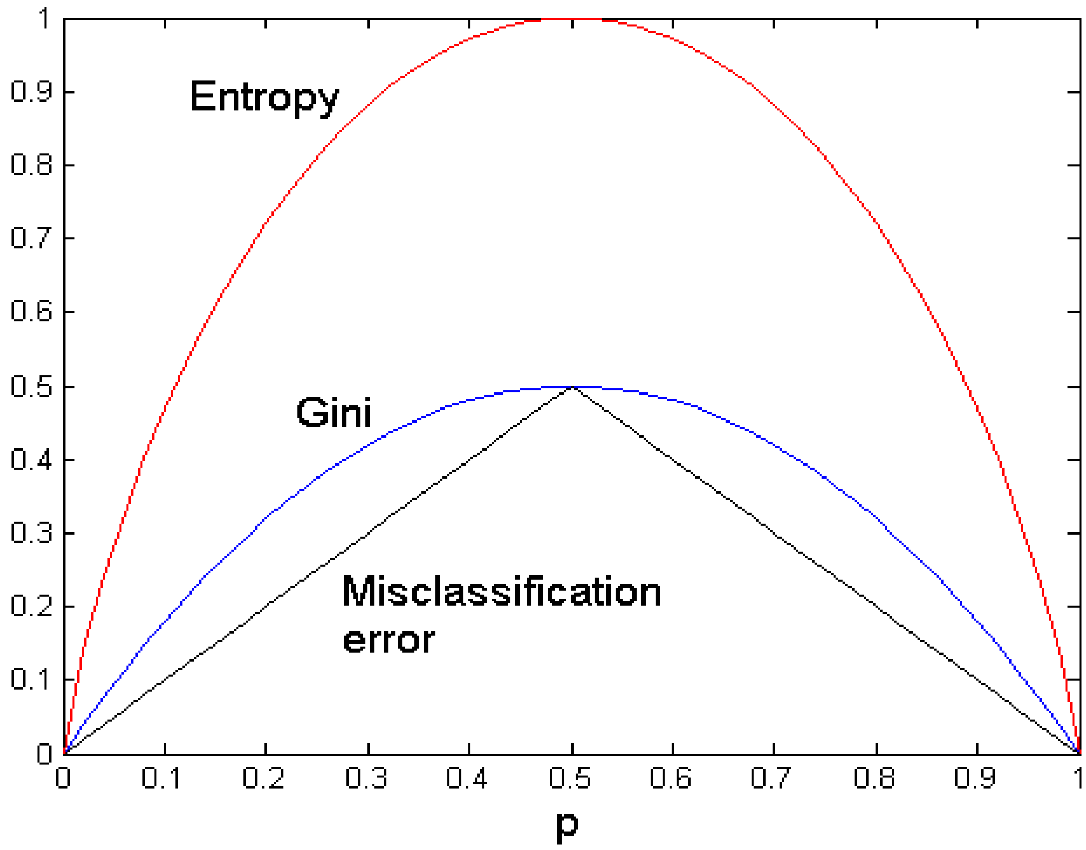
C1	**2**
C2	**4**

P(C1) = 2/6   P(C2) = 4/6

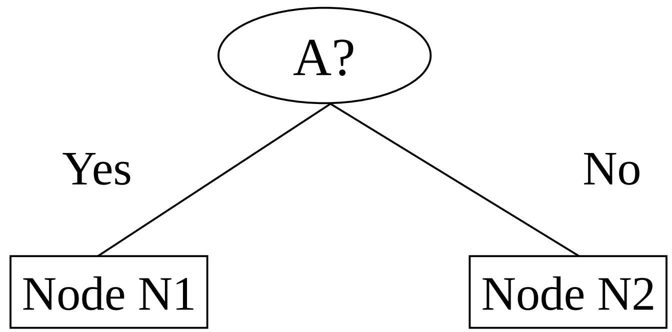
Error = 1 – max (2/6, 4/6) = 1 – 4/6 = 1/3

Comparison among Splitting Criteria

For a 2-class problem:



Misclassification Error vs Gini



	Parent
C1	**7**
C2	**3**
Gini = 0.42	

Gini(N1)
= 1 – $(3/3)^2$ – $(0/3)^2$
= 0

Gini(N2)
= 1 – $(4/7)^2$ – $(3/7)^2$
= 0.489

	N1	N2
C1	**3**	**4**
C2	**0**	**3**

Gini(Children)
= 3/10 * 0
+ 7/10 * 0.489
= 0.342

Example: GINI Index

$$GINI(\mathcal{D}) = 1 - \sum_{q=1}^{M} P_q^2$$

Instances	Outlook x_1	Temperature x_2	Humidity x_3	Wind x_4	Play Tennis y
$s^{(1)}$	Sunny	Hot	High	Weak	No
$s^{(2)}$	Sunny	Hot	High	Strong	No
$s^{(3)}$	Overcast	Hot	High	Weak	Yes
$s^{(4)}$	Rain	Mild	High	Weak	Yes
$s^{(5)}$	Rain	Cool	Normal	Weak	Yes
$s^{(6)}$	Rain	Cool	Normal	Strong	No
$s^{(7)}$	Overcast	Cool	Normal	Strong	Yes
$s^{(8)}$	Sunny	Mild	High	Weak	No
$s^{(9)}$	Sunny	Cool	Normal	Weak	Yes
$s^{(10)}$	Rain	Mild	Normal	Weak	Yes
$s^{(11)}$	Sunny	Mild	Normal	Strong	Yes
$s^{(12)}$	Overcast	Mild	High	Strong	Yes
$s^{(13)}$	Overcast	Hot	Normal	Weak	Yes
$s^{(14)}$	Rain	Mild	High	Strong	No

Example: GINI Index…

- First attribute is “Outlook”

Outlook	*Yes*	*No*	*Number of instances*
Sunny	2	3	5
Overcast	4	0	4
Rain	3	2	5

Gini(Outlook=Sunny) = $1 - (2/5)^2 - (3/5)^2 = 1 - 0.16 - 0.36 = 0.48$
Gini(Outlook=Overcast) = $1 - (4/4)^2 - (0/4)^2 = 0$
Gini(Outlook=Rain) = $1 - (3/5)^2 - (2/5)^2 = 1 - 0.36 - 0.16 = 0.48$

Then, we will calculate weighted sum of Gini indexes for outlook feature.
Gini(Outlook) = (5/14) x 0.48 + (4/14) x 0 + (5/14) x 0.48 = 0.171 + 0 + 0.171 = 0.342

Example: GINI Index…

- Second attribute is "Temperature"

Temperature	*Yes*	*No*	Number of instances
Hot	2	2	4
Cool	3	1	4
Mild	4	2	6

Gini(Temp=Hot) = $1 – (2/4)^2 – (2/4)^2 = 0.5$
Gini(Temp=Cool) = $1 – (3/4)^2 – (1/4)^2 = 1 – 0.5625 – 0.0625 = 0.375$
Gini(Temp=Mild) = $1 – (4/6)^2 – (2/6)^2 = 1 – 0.444 – 0.111 = 0.445$

We'll calculate weighted sum of gini index for temperature feature
Gini(Temp) = (4/14) x 0.5 + (4/14) x 0.375 + (6/14) x 0.445 = 0.142 + 0.107 + 0.190 = 0.439

Example: GINI Index…

- Third attribute is “Humidity”

Humidity	*Yes*	*No*	Number of instances
High	3	4	7
Normal	6	1	7

Gini(Humidity=High) = 1 – $(3/7)^2$ – $(4/7)^2$ = 1 – 0.183 – 0.326 = 0.489
Gini(Humidity=Normal) = 1 – $(6/7)^2$ – $(1/7)^2$ = 1 – 0.734 – 0.02 = 0.244

Weighted sum for humidity feature will be calculated next
Gini(Humidity) = (7/14) x 0.489 + (7/14) x 0.244 = 0.367

Example: GINI Index…

- Fourth attribute is "**Wind**"

Wind	*Yes*	*No*	Number of instances
Weak	6	2	8
Strong	3	3	6

Gini(Wind=Weak) = $1 – (6/8)^2 – (2/8)^2 = 1 – 0.5625 – 0.062 = 0.375$
Gini(Wind=Strong) = $1 – (3/6)^2 – (3/6)^2 = 1 – 0.25 – 0.25 = 0.5$

Gini(Wind) = (8/14) x 0.375 + (6/14) x 0.5 = 0.428

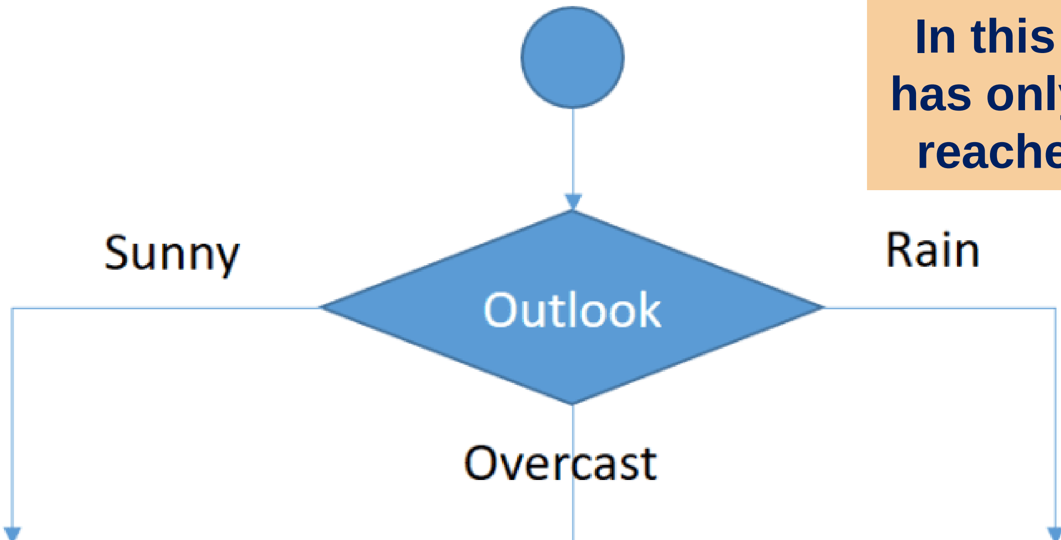
Example: GINI Index…

- Gini Index of all attributes are as:

Feature	Gini index
Outlook	**0.342**
Temperature	0.439
Humidity	**0.367**
Wind	0.428

Decision Tree at First Split

In this tree, overcast has only yes. Hence, it reaches at leaf node



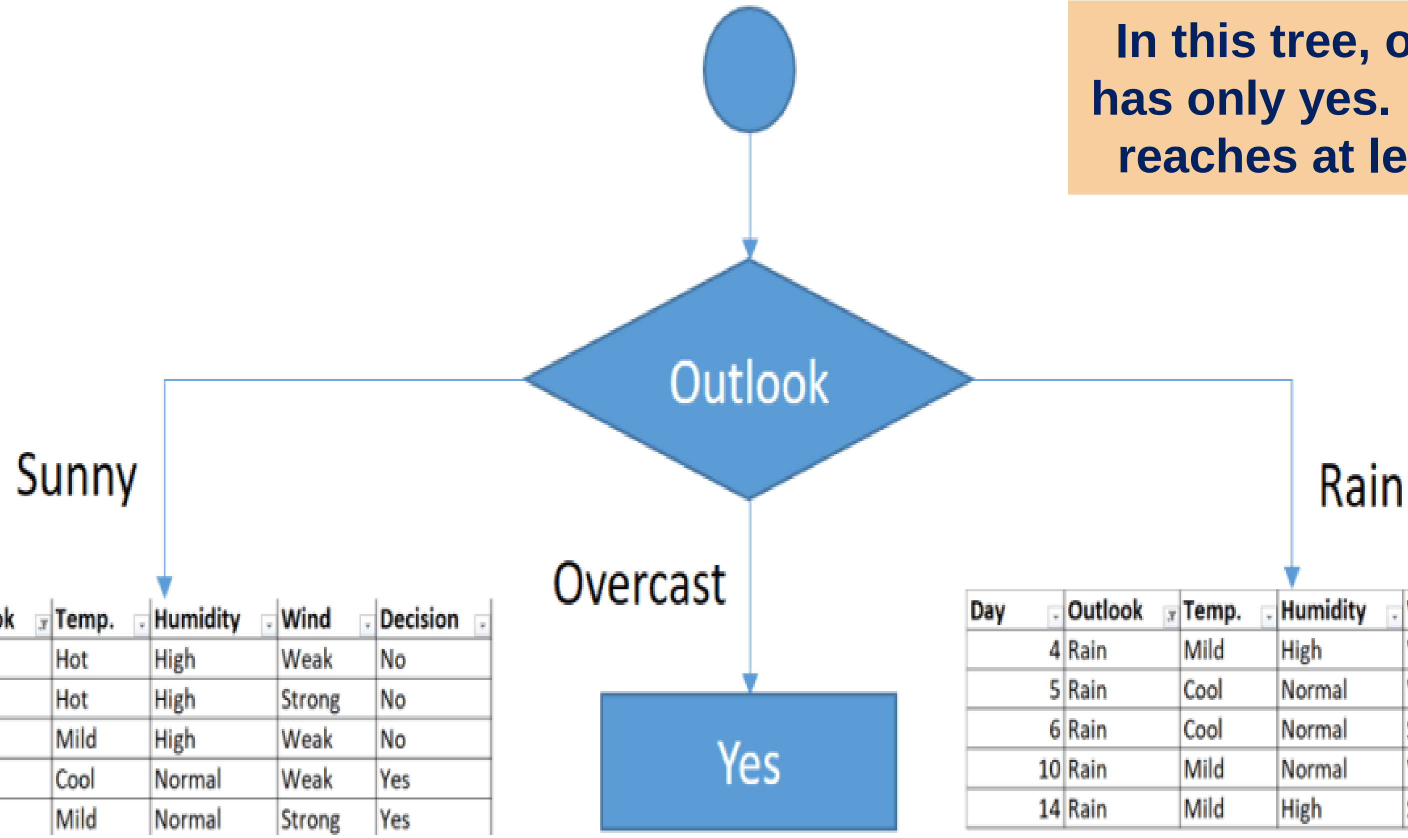
Day	Outlook	Temp.	Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes

Day	Outlook	Temp.	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
10	Rain	Mild	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

Day	Outlook	Temp.	Humidity	Wind	Decision
3	Overcast	Hot	High	Weak	Yes
7	Overcast	Cool	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Hot	Normal	Weak	Yes

Decision Tree at First Split

In this tree, overcast has only yes. Hence, it reaches at leaf node



Day	Outlook	Temp.	Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes

Day	Outlook	Temp.	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
10	Rain	Mild	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

DT: GINI Index for sub dataset

- Same procedure is applied for the sub datasets.
- The sub dataset for “**sunny**” outlook. We need to find the **GINI index** scores for temperature, humidity and wind features, respectively.

Day	Outlook	**Temp.**	Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes

DT: GINI Index for sub dataset

- Gini of temperature for “**sunny**” outlook.

Temperature	Yes	No	Number of instances
Hot	0	2	2
Cool	1	0	1
Mild	1	1	2

Gini(Outlook=Sunny and Temp.=Hot) = $1 – (0/2)^2 – (2/2)^2 = 0$
Gini(Outlook=Sunny and Temp.=Cool) = $1 – (1/1)^2 – (0/1)^2 = 0$
Gini(Outlook=Sunny and Temp.=Mild) = $1 – (1/2)^2 – (1/2)^2 = 1 – 0.25 – 0.25 = 0.5$

Gini(Outlook=Sunny and Temp.) = (2/5)x0 + (1/5)x0 + (2/5)x0.5 = 0.2

DT: GINI Index for sub dataset

- Gini of **humidity** for sunny outlook.

Humidity	Yes	No	Number of instances
High	0	3	3
Normal	2	0	2

Gini(Outlook=Sunny and Humidity=High) = $1 – (0/3)^2 – (3/3)^2 = 0$
Gini(Outlook=Sunny and Humidity=Normal) = $1 – (2/2)^2 – (0/2)^2 = 0$

Gini(Outlook=Sunny and Humidity) = (3/5)x0 + (2/5)x0 = 0

DT: GINI Index for sub dataset

- Gini of **wind** for sunny outlook.

Wind	Yes	No	Number of instances
Weak	1	2	3
Strong	1	1	2

Gini(Outlook=Sunny and Wind=Weak) = $1 - (1/3)^2 - (2/3)^2 = 0.266$
Gini(Outlook=Sunny and Wind=Strong) = $1 - (1/2)^2 - (1/2)^2 = 0.2$

Gini(Outlook=Sunny and Wind) = (3/5)x0.266 + (2/5)x0.2 = 0.466

DT: GINI Index for sub dataset

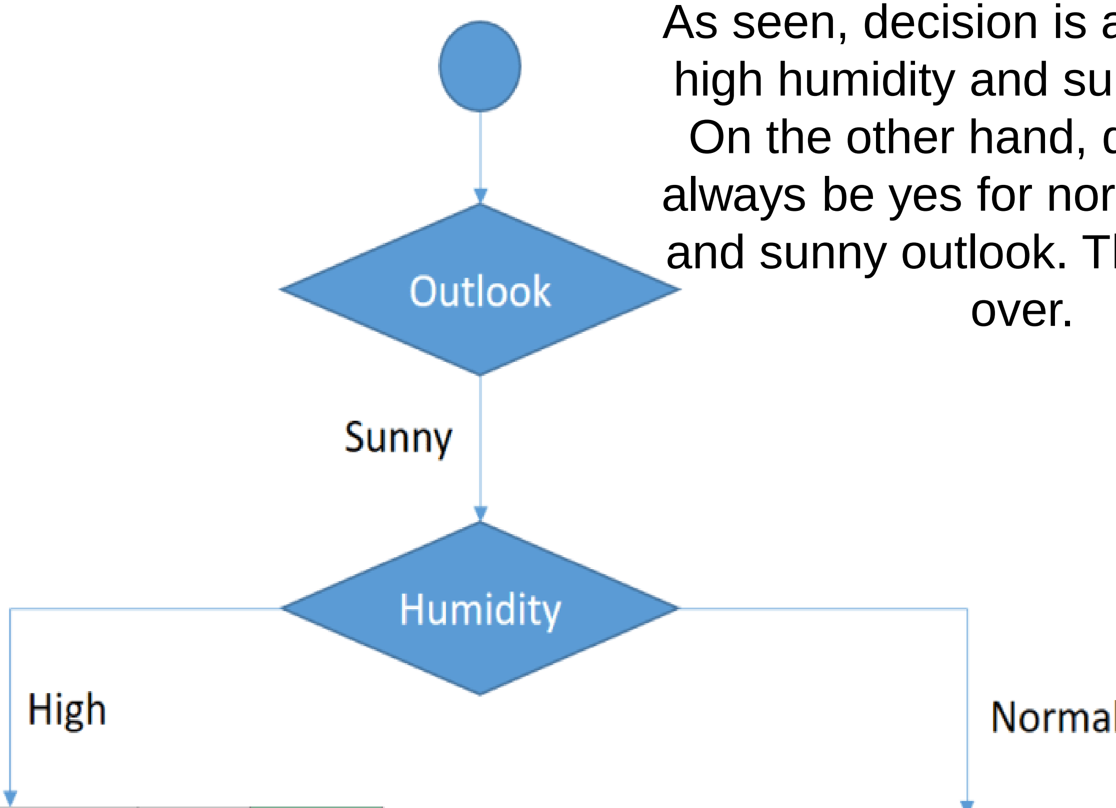
- Decision for **sunny** outlook.

Feature	Gini index
Temperature	0.2
Humidity	0
Wind	0.466

We'll put humidity check at the extension of sunny outlook.

DT: Second Split

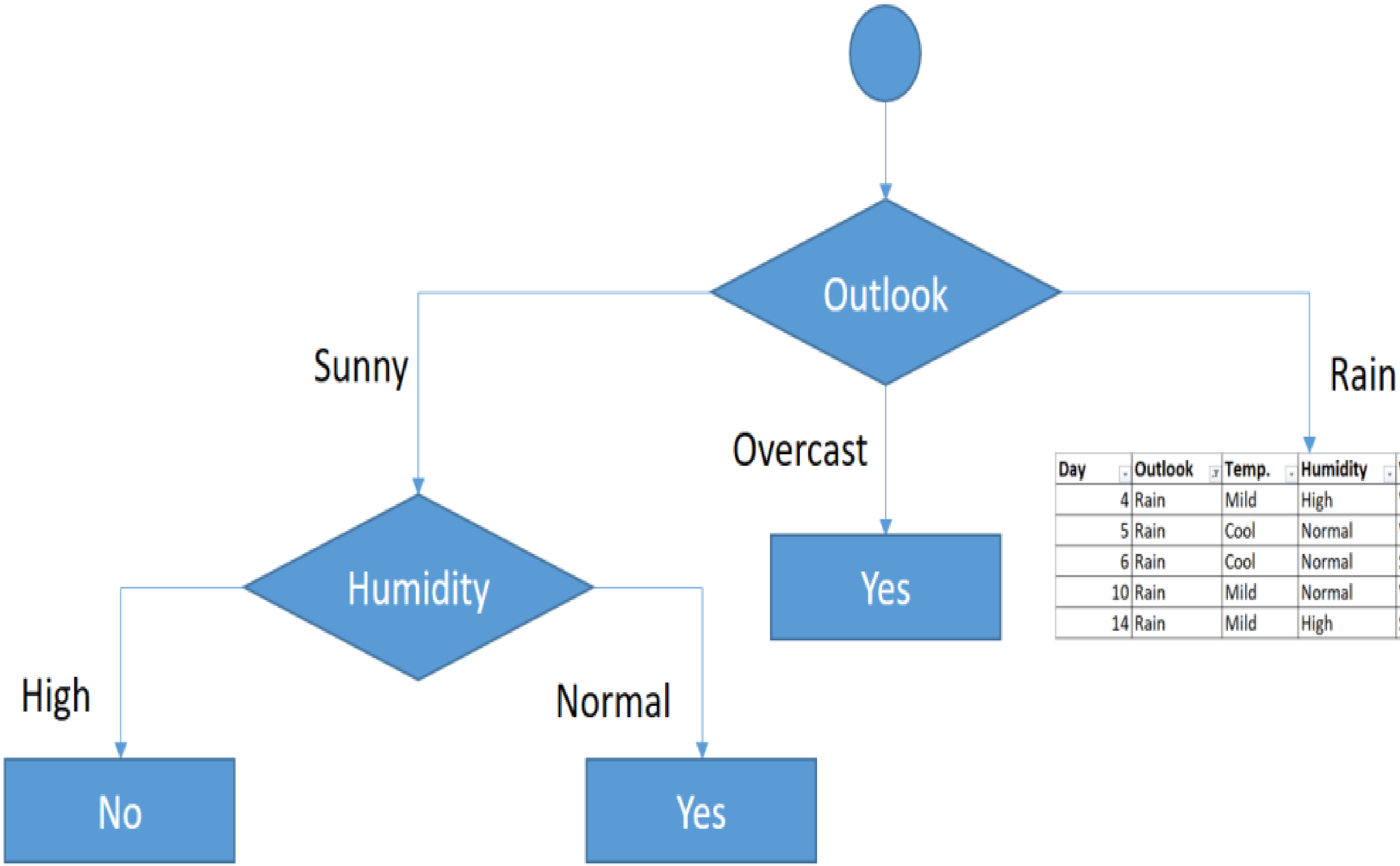
As seen, decision is always no for high humidity and sunny outlook. On the other hand, decision will always be yes for normal humidity and sunny outlook. This branch is over.



Day	Outlook	Temp.	Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
8	Sunny	Mild	High	Weak	No

Day	Outlook	Temp.	Humidity	Wind	Decision
9	Sunny	Cool	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes

Decision Tree: Second Split



Day	Outlook	Temp.	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
10	Rain	Mild	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

DT: Rain_Outlook

- **Rain outlook**

Day	Outlook	Temp.	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
10	Rain	Mild	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

We'll calculate Gini index scores for temperature, humidity and wind features when outlook is rain.

DT: Rain_Outlook…

- Gini of **temperature** for ***rain*** **outlook**

Temperature	Yes	No	Number of instances
Cool	1	1	2
Mild	2	1	3

Gini(Outlook=Rain and Temp.=Cool) = $1 – (1/2)^2 – (1/2)^2 = 0.5$
Gini(Outlook=Rain and Temp.=Mild) = $1 – (2/3)^2 – (1/3)^2 = 0.444$

Gini(Outlook=Rain and Temp.) = (2/5)x0.5 + (3/5)x0.444 = 0.466

DT: Rain_Outlook…

- Gini of **humidity** for ***rain* outlook**

Humidity	Yes	No	Number of instances
High	1	1	2
Normal	2	1	3

Gini(Outlook=Rain and Humidity=High) = $1 - (1/2)^2 - (1/2)^2 = 0.5$
Gini(Outlook=Rain and Humidity=Normal) = $1 - (2/3)^2 - (1/3)^2 = 0.444$

Gini(Outlook=Rain and Humidity) = (2/5)x0.5 + (3/5)x0.444 = 0.466

DT: Rain_Outlook…

- Gini of **wind** for ***rain*** **outlook**

Wind	Yes	No	Number of instances
Weak	3	0	3
Strong	0	2	2

Gini(Outlook=Rain and Wind=Weak) = 1 – $(3/3)^2$ – $(0/3)^2$ = 0
Gini(Outlook=Rain and Wind=Strong) = 1 – $(0/2)^2$ – $(2/2)^2$ = 0

Gini(Outlook=Rain and Wind) = (3/5)x0 + (2/5)x0 = 0

DT: Rain_Outlook...

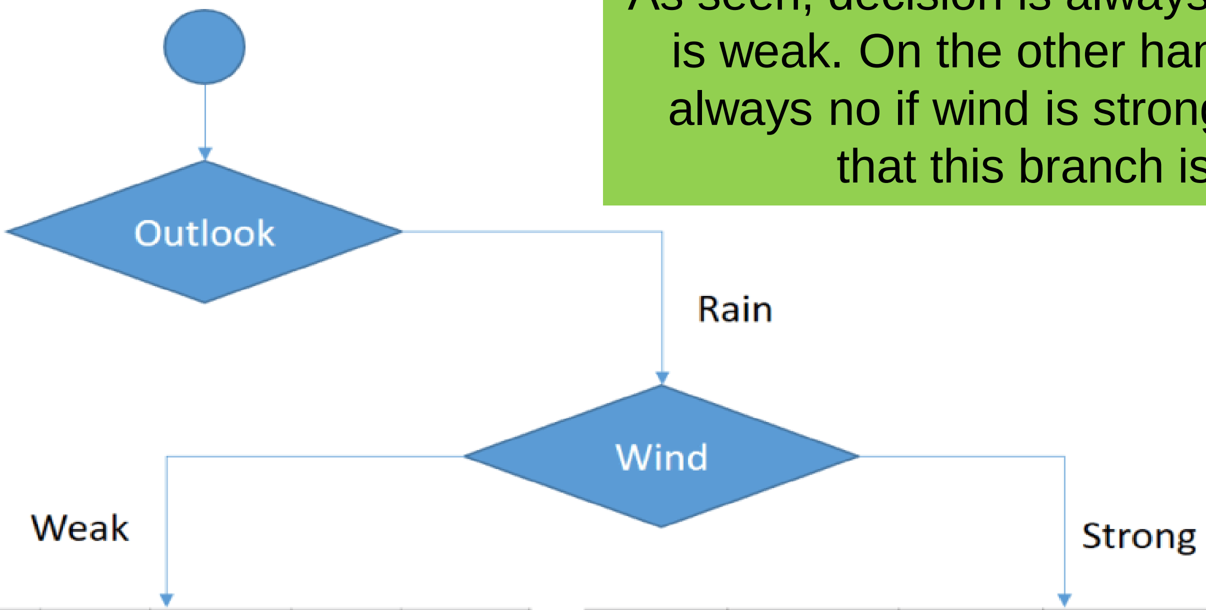
- **Decision for rain outlook**
  - The winner is wind feature for rain outlook because it has the minimum Gini index score in features.

Feature	Gini index
Temperature	0.466
Humidity	0.466
Wind	0

DT: at Third Split

- Put the wind feature for rain outlook branch and monitor the new sub data sets.

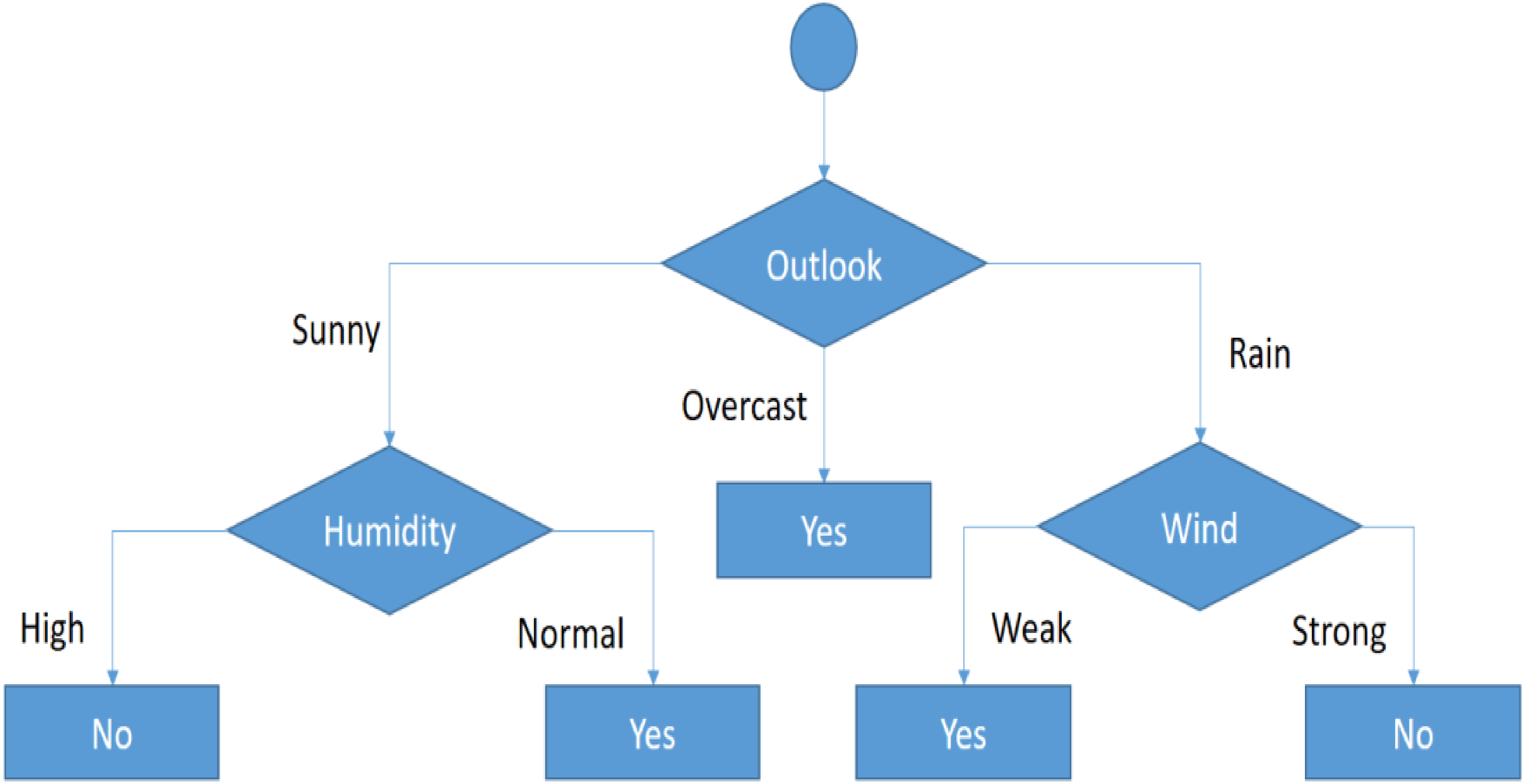
As seen, decision is always yes when wind is weak. On the other hand, decision is always no if wind is strong. This means that this branch is over.



Day	Outlook	Temp.	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
10	Rain	Mild	Normal	Weak	Yes

Day	Outlook	Temp.	Humidity	Wind	Decision
6	Rain	Cool	Normal	Strong	No
14	Rain	Mild	High	Strong	No

Decision Tree: Final



Tree Induction

- Greedy strategy.
  - Split the records based on an attribute test that optimizes certain criterion.

- Issues
  - Determine how to split the records
    - How to specify the attribute test condition?
    - How to determine the best split?
  - Determine when to stop splitting

Stopping Criteria for Tree Induction

- Stop expanding a node when all the records belong to the same class
- Stop expanding a node when all the records have similar attribute values.

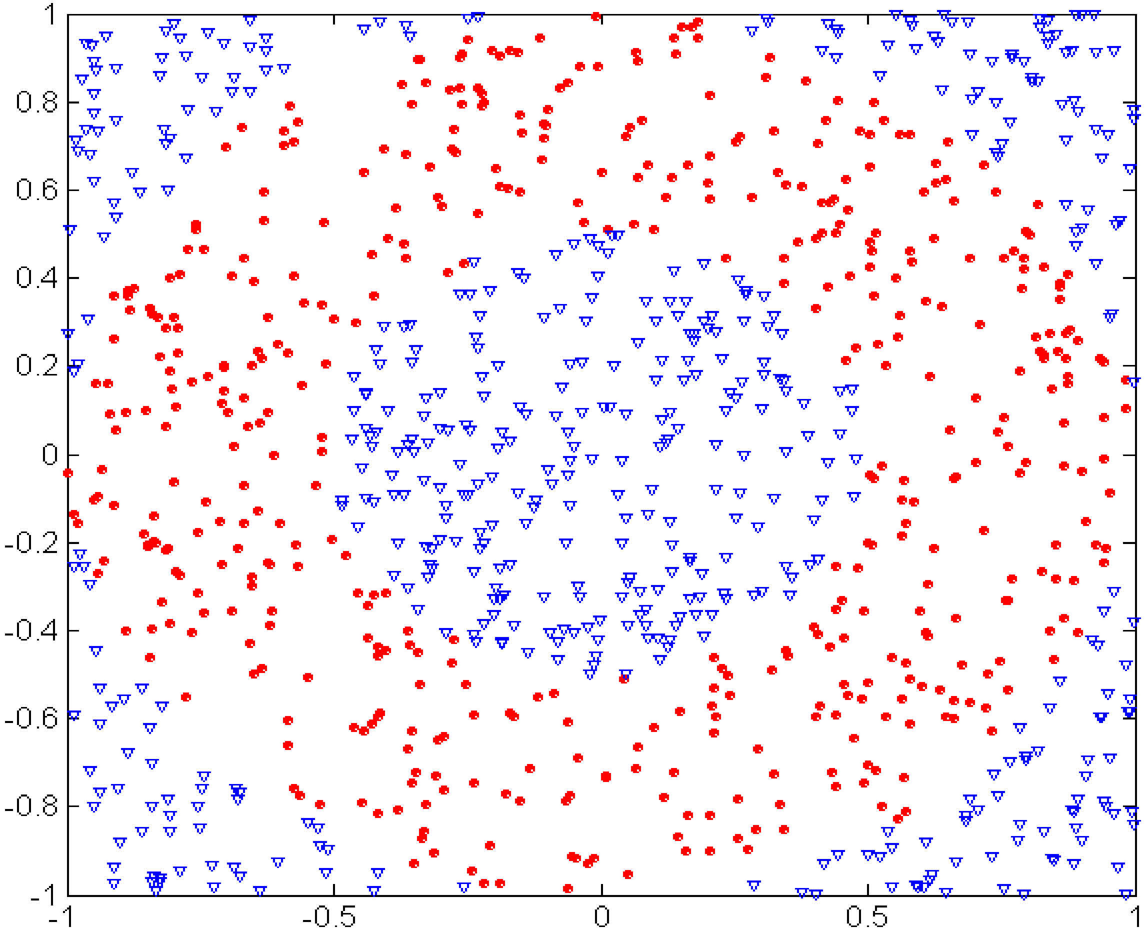
Decision Tree Based Classification

- **Advantages:**
  - Inexpensive to construct
  - Extremely fast at classifying unknown records
  - Easy to interpret for small-sized trees
  - Accuracy is comparable to other classification techniques for many simple data sets

Practical Issues of Classification

- Underfitting and Overfitting
- Missing Values
- Costs of Classification

Underfitting and Overfitting (Example)



500 circular and 500 triangular data points.

Circular points:

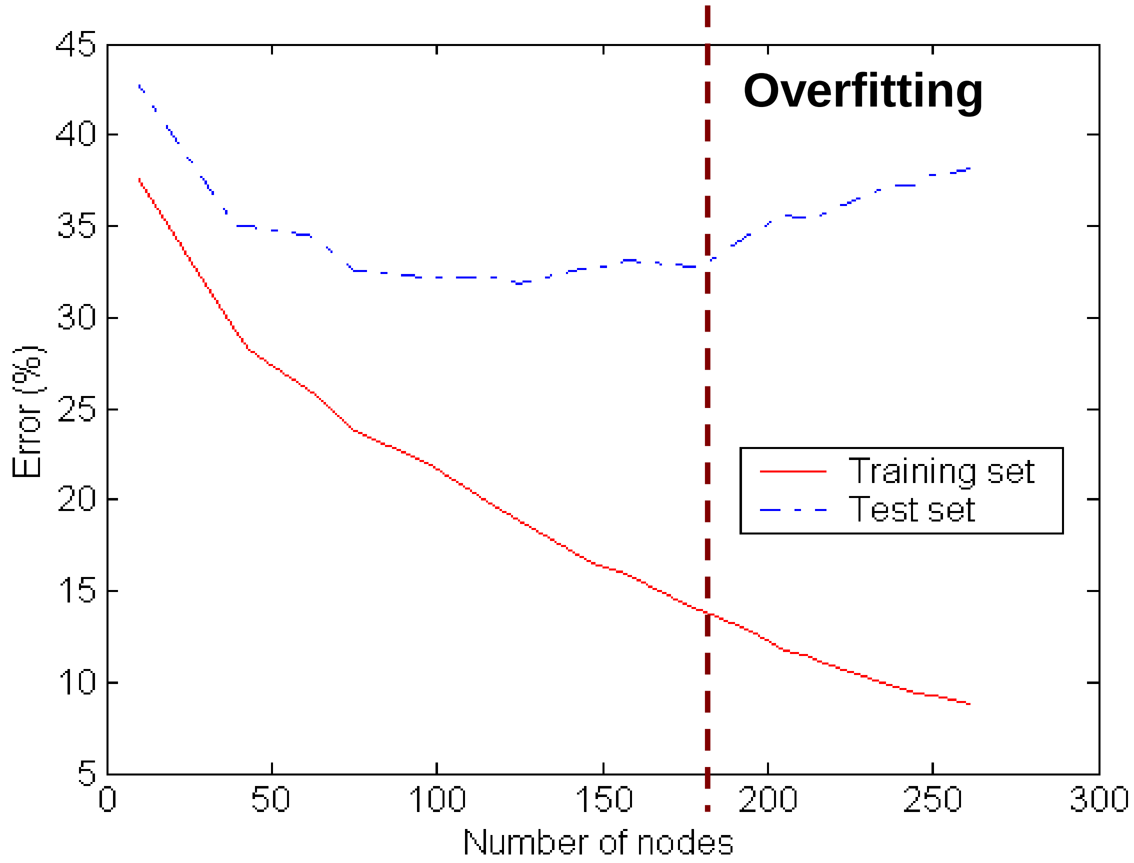
$0.5 \leq \text{sqrt}(x_1^2+x_2^2) \leq 1$

Triangular points:

$\text{sqrt}(x_1^2+x_2^2) > 0.5$ or

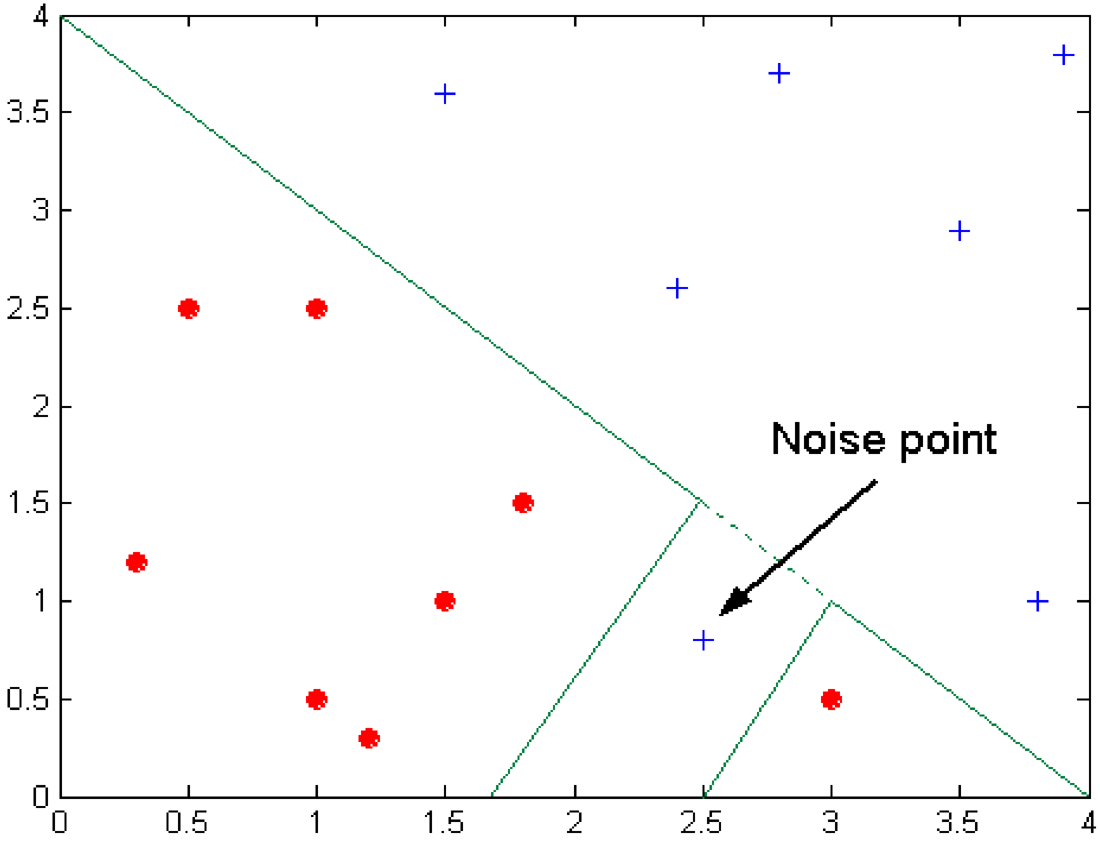
$\text{sqrt}(x_1^2+x_2^2) < 1$

Underfitting and Overfitting



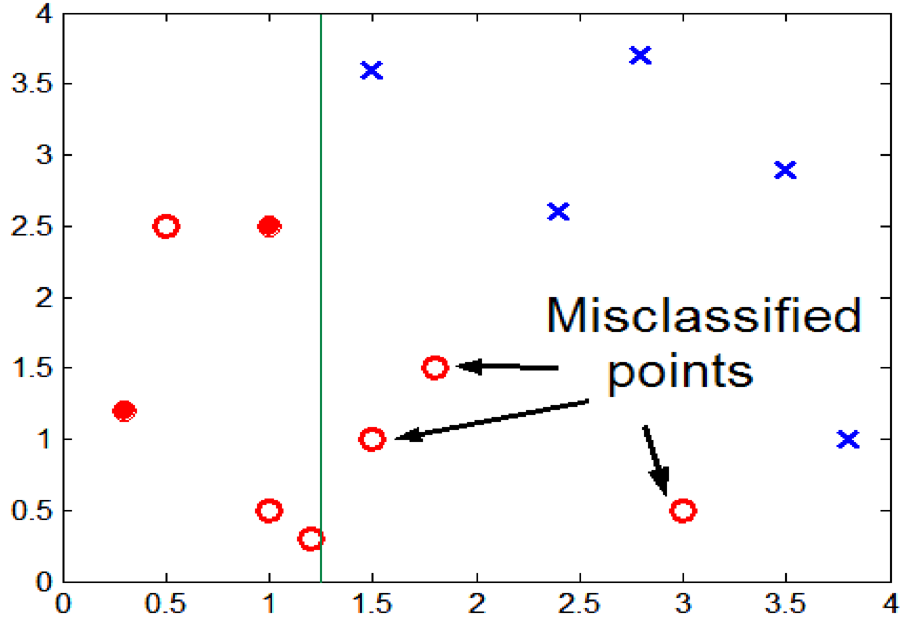
Underfitting: when model is too simple, both training and test errors are large

Overfitting due to Noise



Decision boundary is distorted by noise point

Overfitting due to Insufficient Examples



Lack of data points in the lower half of the diagram makes it difficult to predict correctly the class labels of that region

- Insufficient number of training records in the region causes the decision tree to predict the test examples using other training records that are irrelevant to the classification task

Notes on Overfitting

- Overfitting results in decision trees that are more complex than necessary.

- Training error no longer provides a good estimate of how well the tree will perform on previously unseen records

- Need new ways for estimating errors

Occam's Razor

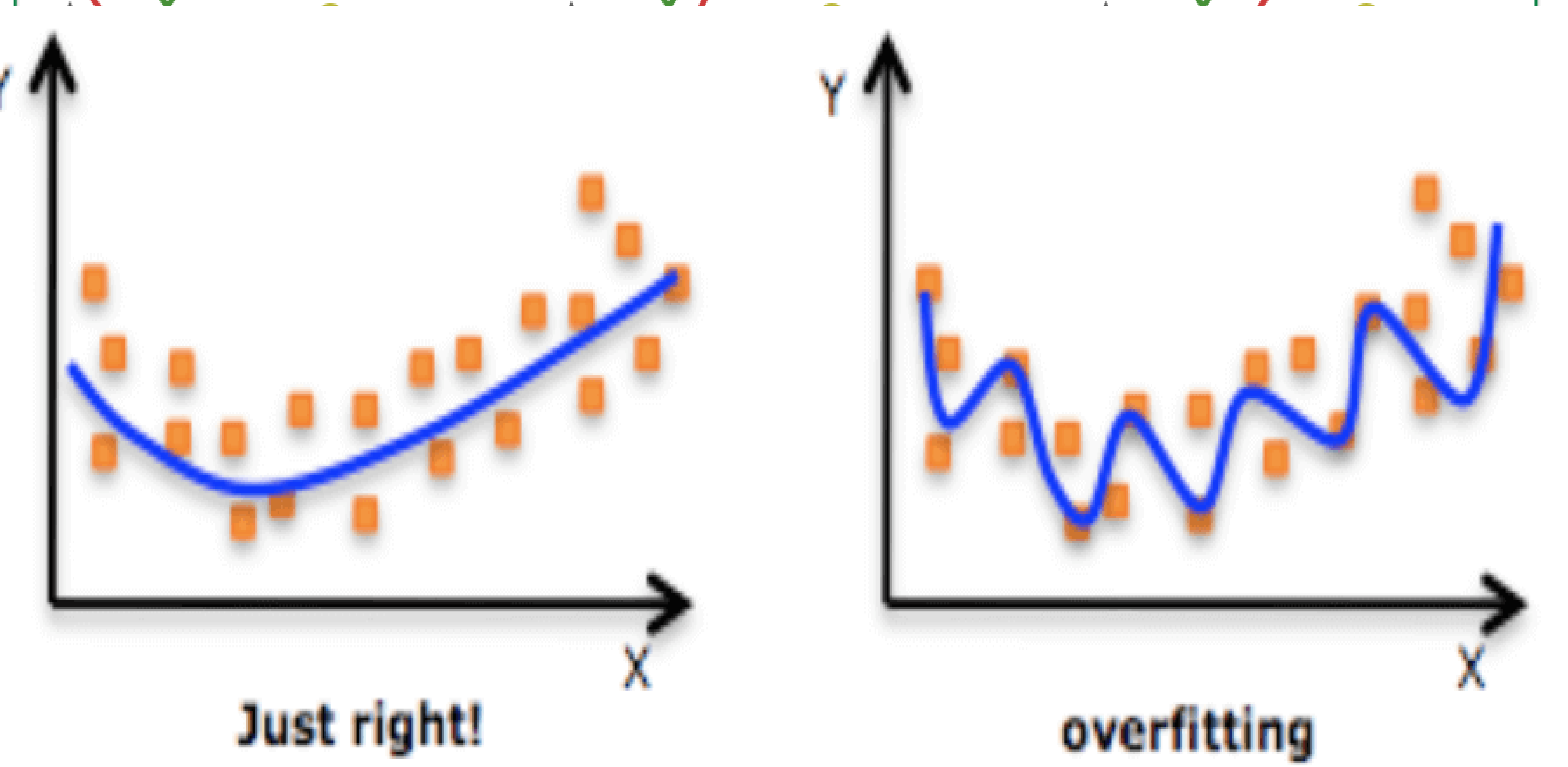
- Given two models of similar generalization errors, one should prefer the simpler model over the more complex model.

- For complex models, there is a greater chance that it was fitted accidentally by errors in data.

- Therefore, one should include model complexity when evaluating a model.

How to Address Overfitting

- Pre-Pruning (Early Stopping Rule)
  - Stop the algorithm before it becomes a fully-grown tree
  - Typical stopping conditions for a node:
    - Stop if all instances belong to the same class
    - Stop if all the attribute values are the same
  - More restrictive conditions:
    - Stop if number of instances is less than some user-specified threshold
    - Stop if class distribution of instances are independent of the available features (e.g., using χ^2 test)
    - Stop if expanding the current node does not improve impurity measures (e.g., Gini or information gain).



explain the variance)

good to be true)