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The Pillow Investor: The Algorithm That Makes Profit While You Sleep

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Abstract

Investing in an index fund like the Nifty 50 offers broad market exposure, but sometimes it's preferable to hand-pick a smaller group of stocks from that fund. The goal? To match (or even outperform) the returns of the index without buying into all of its constituents. This project explores a way to do just that by using a tailored variation of Adaptive Elastic Net (AENet), designed specifically for selecting a smaller set of stocks that still aligns with the index's performance.My approach takes some of the best features of regression techniques—like Linear Regression, Lasso, Adaptive Lasso, and Elastic Net (Enet) regression—and rolls them into a single, adaptive method. I introduce a coordinate-wise closed-form update rule, allowing efficient solutions to the optimization problem and enabling a portfolio that keeps pace with the index, without mirroring every stock in it. This method opens up new possibilities for targeted, cost-effective investing that doesn't sacrifice performance.

1 Math

Under the full investment and no short selling constraint the most intuitive thing to do is to solve the problem using regression with constraints. We write the regression problem as follows

$$\min_{w \in \mathcal{S}} \frac{1}{N} ||y - Xw||_2^2$$

with the given constraints $\Sigma_{i=1}^{n} w_i = 1$ with $w_i \geq 0 \; \forall \; i \in 1, \cdots, n$
Here $y = [y_1, y_2, \cdots, y_n]'$ is a $n \times 1$ vector of index returns here $X = (x_{i,j})_{n \times p}$ is the $n \times p$ matrix of returns on the p constituents and n time periods here $w = [w_1, w_2, \cdots, w_n]'$ is a $p \times 1$ vector of weights to be determined to for minimizing the index tracking error
The cardinality constraint just changes the constraint equation which is $\sum_{j \in \mathcal{J}} w_j = 1$ where number of $\mathcal{J} \leq K_{\max}$ this restricts the number of active positions to no more than $K_{\max}$ that controls the sparsity of the problem

1.1 Alasso Estimator

$$\hat{\beta}(\text{Alasso}) = \arg\min_{\beta} ||y - X\beta||_2^2 + \lambda \sum_{j=1}^{p} \hat{v_j} |\beta_j|$$

here

$$v_j = (|\hat{\beta_j}|^{\text{init}})^{-\tau}$$

here $\tau > 0$ we set $\tau = 1$
Here we come with another issue that is high dimensional collinearity this makes collinearity makes lasso solution paths more unstable hence we come up with an extra l_2 penalty

1.2 Enet estimator

$$\hat{\beta}(\text{Enet}) = \underset{\beta}{\text{argmin}} ||y - X\beta||_2^2 + \lambda_1 \sum_{j=1}^{p} v_j |\beta_j| + \lambda_2 ||\beta||_2^2$$

Notice that the Alasso and Enet estimators improve Lasso in two different directions.The Alasso overcomes the inconsistency problem while the Enet improves the stability of the solution paths, compared to Lasso. It is also natural to combine the ideas of the Alasso and Enet in order to obtain an even better method

1.3 Math and idea behind Aenet Penalty

$$\hat{\beta}(\text{Aenet}) = (1 + \frac{\lambda_2}{n})(\underset{\beta}{\text{argmin}} ||y - X\beta||_2^2 + \lambda_1 \sum_{j=1}^{p} \hat{v_j} |\beta_j| + \lambda_2 ||\beta||^2)$$

The following formulation was first introduced by Zou and Zhang According to our problem we can restate every condition and define everything The sparse index tracking problem with the Aenet penalty can be formulated as:

$$\min_{w} \frac{1}{n} \sum_{i=1}^{n} \left(y_i - \sum_{j=1}^{p} x_{ij} w_j \right)^2 + \lambda_1 \sum_{j=1}^{p} \hat{v}_j \, |w_j| + \lambda_2 \|w\|_2^2 + \lambda_c \sum_{j=1}^{p} |w_j - \bar{w}_j|$$
$$\text{s.t.} \quad w' e = 1, w_j \geq 0$$

where e is a vector of ones. The term $\lambda_1 \sum_{j=1}^p \hat{v}_j |w_j|$ is the weighted ℓ_1 penalty, which controls the sparsity of the portfolio weights. In this paper, we set the adaptive weight as $\hat{v}_j = \left(\left|\hat{\beta}_j^{\text{init}}\right|\right)^{-\tau}$, where $\hat{\beta}_j^{\text{init}}$ is chosen as the solution to index tracking problems. The term $\lambda_2\|w\|_2^2$ is the ℓ_2 penalty, i.e. $\lambda_2\|w\|_2^2 = \lambda_2 \sum_{j=1}^p w_j^2$. The term $\lambda_c \sum_{j=1}^p |w_j - \bar{w}_j|$ is the turnover penalty, where $\bar{w}_j$ is portfolio weight of asset j in previous time period.

1.4 Effects of Turnover restriction

It is known that the transaction cost depends on the trading volume which is often a monotonic increasing function of the latter. That raises some interest to investigate the effect of the regularization parameter λ_c.In the previous analysis it was shown that the optimal values of $\lambda_1$and $\lambda_2$are found by cross validation. The parameter $\lambda_c$discourages the change in portfolio weights between two consecutive time periods. To examine the following we define the operator Δ as the number of stocks rebalanced in period t+1 expressed as $\Delta = ||w_{t+1} - w_{t+}||_0$ where $w_{t+}$is the weights before rebalancing. It is expected that the average number of stocks rebalanced across the rolling windows significantly decreases with λ_c

2 Constructing The algorithm

$$\begin{aligned}\min_w \frac{1}{2}\sum_{i=1}^n\left(y_i - \sum_{j=1}^p x_{ij}w_j\right)^2 &+ g_1(w) + \lambda_c \sum_{j=1}^p |w_j - \bar{w}_j| \\ + \lambda_1 \sum_{j=1}^p \hat{v}_j w_j &+ \lambda_2 \sum_{j=1}^p w_j^2 + \sum_{j=1}^p g_2(w_j)\end{aligned}$$

where

$$g_1(w) = \begin{cases} 0, & \text{if } w'e = 1, \\ \infty, & \text{otherwise} \end{cases}$$

and

$$g_2(w_j) = \begin{cases} 0, & \text{if } w_j \geq 0, \\ \infty, & \text{otherwise .} \end{cases}$$

Let $f_0(w) = \frac{1}{2}\sum_{i=1}^n \left(y_i - \sum_{j=1}^p x_{ij}w_j\right)^2 + g_1(w)$ and $f_j(w_j) = \lambda_c |w_j - \bar{w}_j| + \lambda_1 \hat{v}_j w_j + \lambda_2 w_j^2 + g_2(w_j)$. The objective function can then be rewritten as

$$\min_w f(w) = \min_w \left\{ f_0(w) + \sum_{j=1}^p f_j(w_j) \right\}.$$

Note: Denote $\tilde{g}$ as the sub-gradient of $\|w - \hat{w}\|_1$. When w_i is greater, smaller than, or equal to $\hat{w}_i$, the ith element in $\tilde{g}$ is 1, -1, or in the interval $[-1, 1]$, respectively.
Now this is of the form where we discussed about separability of functions case hence we can apply coordinate descent to our function
We consider the Lagrangian corresponding to the optimization problem

$$\begin{aligned} L(w,\gamma;\lambda_1,\lambda_2) = &\min_w \frac{1}{2}\sum_{i=1}^n\left(y_i - \sum_{j=1}^p x_{ij}w_j\right)^2 + \lambda_c \sum_{j=1}^p |w_j - \bar{w}_j| \\ &+ \lambda_1 \sum_{j=1}^p \hat{v}_j w_j + \lambda_2 \sum_{j=1}^p w_j^2 + -\gamma_0(w'e - 1) - \sum_{i=1}^p \gamma_i w_i \\ &= \sum_{i=1}^n \left(y_i - \sum_{j=1}^p x_{ij}w_j\right)^2 + \sum_{j=1}^p \left(\lambda_c|w_j - \bar{w}_j| + \lambda_1\hat{v}_j w_j + \lambda_2 w_j^2 - \gamma_0 w_j\right) + \gamma_0 - \sum_{j=1}^p \gamma_j w_j \end{aligned}$$

Let $\tilde{y}_i^{(j)} = \sum_{k \neq j} x_{ik} \tilde{w}_k$, $\quad d_j = \sum_{i=1}^n x_{ij} \left(y_i - \tilde{y}_i^{(j)} \right)$, and $c_j = \sum_{i=1}^n x_{ij}^2$. The Karush-Kuhn-Tucker (KKT) conditions for the above lagrangian are

$$
\begin{aligned}
& c_j w_j - d_j + 2\lambda_2 w_j - \gamma_0 - \gamma_j + \hat{v}_j \lambda_1 = -\lambda_c, \quad \text{if } w_j > \bar{w}_j, \\
& |c_j w_j - d_j + 2\lambda_2 w_j - \gamma_0 - \gamma_j + \hat{v}_j \lambda_1| \leq \lambda_c, \quad \text{if } w_j = \bar{w}_j, \\
& c_j w_j - d_j + 2\lambda_2 w_j - \gamma_0 - \gamma_j + \hat{v}_j \lambda_1 = \lambda_c, \quad \text{if } w_j < \bar{w}_j, \\
& w' e = 1, w_j \geq 0, \gamma_i \geq 0, \gamma_i w_i = 0.
\end{aligned}
$$

The condition $\gamma_i w_i = 0$ is the complementary slackness condition. For a fixed γ_0, we can use the following formula to update each weight w_j :

$$
w_j \leftarrow \begin{cases}
\frac{d_j + \gamma_0 - \hat{v}_j \lambda_1 - \lambda_c}{c_j + 2\lambda_2}, & \text{if } \bar{w}_j < \frac{d_j + \gamma_0 - \hat{v}_j \lambda_1 - \lambda_c}{c_j + 2\lambda_2}, \\
\bar{w}_j, & \text{if } \frac{d_j + \gamma_0 - \hat{v}_j \lambda_1 - \lambda_c}{c_j + 2\lambda_2} \leq \bar{w}_j \leq \frac{d_j + \gamma_0 - \hat{v}_j \lambda_1 + \lambda_c}{c_j + 2\lambda_2}, \\
\frac{d_j + \gamma_0 - \hat{v}_j \lambda_1 + \lambda_c}{c_j + 2\lambda_2}, & \text{if } 0 \leq \frac{d_j + \gamma_0 - \hat{v}_j \lambda_1 + \lambda_c}{c_j + 2\lambda_2} < \bar{w}_j, \\
0, & \text{if } \frac{d_j + \gamma_0 - \hat{v}_j \lambda_1 + \lambda_c}{c_j + 2\lambda_2} < 0.
\end{cases} \tag{1}
$$

However, the updated weights may not satisfy the full investment constraint. In order to meet this constraint, we need to update γ_0. Let $S_u = \{j : w_j > \bar{w}_j \geq 0\}, S_m = \{j : w_j = \bar{w}_j > 0\}$, and $S_l = \{j : 0 < w_j < \bar{w}_j\}$. Then

$$
w' e = \sum_{j \in S_u} \frac{d_j + \gamma_0 - \hat{v}_j \lambda_1 - \lambda_c}{c_j + 2\lambda_2} + \sum_{j \in S_m} \bar{w}_j + \sum_{j \in S_l} \frac{d_j + \gamma_0 - \hat{v}_j \lambda_1 + \lambda_c}{c_j + 2\lambda_2} = 1.
$$

Here γ_0 is the Lagrange multiplier associated with the full investment constraint.γ_i is the Lagrange multiplier associated with short selling.The above algorithm is just updating w keeping γ_0 fixed and then updating γ_0 via full investment constraint using the updated w.
Solving for γ_0 we get the following updating formula for γ_0 which is the following

$$
\gamma_0 \leftarrow \frac{1 - \sum_{j \in S_m} \bar{w}_j - \sum_{j \in S_u \cup S_l} \frac{d_j - \hat{v}_j \lambda_1}{c_j + 2\lambda_2} - \sum_{j \in S_l} \frac{\lambda_c}{c_j + 2\lambda_2} + \sum_{j \in S_u} \frac{\lambda_c}{c_j + 2\lambda_2}}{\sum_{j \in S_u \cup S_l} \frac{1}{c_j + 2\lambda_2}}
$$

Now to implement the algorithm the initial weight of each weight is set to be

$$
w_1^{(0)} = \cdots = w_p^{(0)} = \frac{1}{p}
$$

and the initial value of $\gamma_0^{(0)} > \max_{1 \leq j \leq p} \hat{v}_j \lambda_1 + \lambda_c$

3 Algorithm

Combining the above steps we get the following algorithm
Algorithm step by step
(1) Fix $\lambda_1, \lambda_2, \lambda_c$, and τ at some constant levels;
(2) Compute the values of $\hat{\beta}^{\text{init}}$ and $\bar{w}$ before rebalancing;
(3) Initialize $w^{(0)} = p^{-1} e$ and $\gamma_0^{(0)} > \max_{1 \leq j \leq p} \hat{v}_j \lambda_1 + \lambda_c$;
(4)

$$
w_j^{(m)} \leftarrow \begin{cases}
\frac{d_j^{(m)} + \gamma_0^{(m-1)} - \hat{v}_j \lambda_1 - \lambda_c}{c_j + 2\lambda_2}, & \text{if } \bar{w}_j < \frac{d_j^{(m)} + \gamma_0^{(m-1)} - \hat{v}_j \lambda_1 - \lambda_c}{c_j + 2\lambda_2}, \\
\bar{w}_j, & \text{if } \frac{d_j^{(m)} + \gamma_0^{(m-1)} - \hat{v}_j \lambda_1 - \lambda_c}{c_j + 2\lambda_2} \leq \bar{w}_j \leq \frac{d_j^{(m)} + \gamma_0^{(m-1)} - \hat{v}_j \lambda_1 + \lambda_c}{c_j + 2\lambda_2}, \\
\frac{d_j^{(m)} + \gamma_0^{(m-1)} - \hat{v}_j \lambda_1 + \lambda_c}{c_j + 2\lambda_2}, & \text{if } 0 \leq \frac{d_j^{(m)} + \gamma_0^{(m-1)} - \hat{v}_j \lambda_1 + \lambda_c}{c_j + 2\lambda_2} < \bar{w}_j, \\
0, & \text{if } \frac{d_j^{(m-1)} + \gamma_0^{(m-1)} - \hat{v}_j \lambda_1 + \lambda_c}{c_j + 2\lambda_2} < 0.
\end{cases} \tag{2}
$$

For $j = 1, \ldots, p$, and $m > 0$, update each weight w_j sequentially using the following form: where
$d_j^{(m)} = \sum_{i=1}^n x_{ij} \left(y_i - \sum_{k<j} x_{ik} w_k^{(m)} - \sum_{k>j} x_{ik} \ w_k^{(m-1)} \right)$.

(5) For $m > 0$, update γ_0 using the following formula

$$\gamma_0^{(m)} \leftarrow \left[\sum_{j \in S_u^{(m)} \cup S_l^{(m)}} \frac{1}{c_j + 2\lambda_2} \right]^{-1}$$

$$\times \left[1 - \sum_{i \in S_m^{(m)}} \bar{w}_j - \sum_{j \in S_u^{(m)} \cup S_l^{(m)}} \frac{d_j - \hat{v}_j \lambda_1}{c_j + 2\lambda_2} - \sum_{j \in S_j^{(m)}} \frac{\lambda_c}{c_j + 2\lambda_2} + \sum_{j \in S_u^{(m)}} \frac{\lambda_c}{c_j + 2\lambda_2} \right].$$

Repeat Steps 4 and 5 till both converge

4 Training

We compute the following daily log return

$$\log(\frac{P_{t,j}}{P_{t-1,j}}) = x_{t,j}$$

We define the following
(i)In sample and out of sample tracking error(TE)
(ii) in-sample and out-of-sample average active return (AR);
(iii) out-of-sample tracking portfolio turnover (TO); and
(iv) out-of-sample correlation with the index
The dependence between the tracking portfolio and the index, which ranges between -1 and 1.The closer to 1 the better.The turnover measures the stability of the tracking portfolio. Lower turnover means lower transaction cost. In all the experiments a moving time window procedure was employed to determine index tracking investment strategies.In particular a training window of size $T_{\text{Train}} < T$ it is selected to determine optimal portfolio.Then we evaluate the performance in the subsequent training days
Based on the portfolio weights determined in the first training period we set the weights to be w_1),The out of sample return r_t^{os} at time $(t = T_{\text{Train}} + 1, \cdots, T_{\text{Train}} + T_{\text{Test}})$ and the following is calculated as

$$r_t^{\text{os}} = \left[\sum_{j=1}^{p} w_{1,j} \Pi_{i=T_{\text{Train}+1}}^{t} (1 + x_{i,j}) \right] / \left[\sum_{j=1}^{p} w_{1,j} \Pi_{i=T_{\text{Train}+1}}^{t-1} (1 + x_{i,j}) \right] - 1$$

The out of sample average active return and tracking error for the first testing window are given by

$$AR_1^{\text{os}} = \frac{1}{T_{\text{test}}} \sum_{t=T_{\text{Train}}+1}^{T_{\text{Train}}+T_{\text{Test}}} (r_t^{\text{os}} - y_t)$$

and

$$TE_1^{\text{os}} = \sqrt{\frac{1}{T_{\text{test}}} \sum_{t=T_{\text{Train}}+1}^{T_{\text{Train}}+T_{\text{Test}}} (r_t^{\text{os}} - y_t)^2}$$

we define $N = (T - T_{\text{train}})/(T_{\text{test}})$ as the total number of rolling windows.Based on the sequence of w_i the turnover is computed by

$$TO = \frac{1}{N-1} \sum_{i=1}^{N-1} \sum_{j=1}^{p} (|w_{i+1,j} - w_{i+,j}|)$$

here $w_{i+1,j}$ is the desired weight of asset j at the (i+1)th window after rebalancing and $w_{i+,j}$ is the desired weight of the asset at (i+1)th window before rebalancing given by

$$w_{i+,j} = \prod_{m=i \times T_{\text{train}}+1}^{i \times T_{\text{train}}+T_{\text{test}}} (1 + x_{m,j}) w_{i,j} / \left(\sum_{j=1}^{p} \prod_{m=i \times T_{\text{train}}+1}^{i \times T_{\text{train}}+T_{\text{test}}} (1 + x_{m,j}) w_{i,j} \right)$$

In the first training window,the in sample returns $r_t^{is} = \sum_{j=1}^{p} w_{1,j} x_{t,j}$

the in sample average active return and tracking error for the first training window can be computed in the following manner

$$AR_1^{is} = \frac{1}{T_{\text{train}}} \sum_{t=1}^{T_{\text{train}}} \left(r_t^{\text{is}} - y_t\right)$$

and

$$TE_1^{is} = \sqrt{\frac{1}{T_{\text{train}}} \sum_{t=1}^{T_{\text{train}}} \left(r_t^{\text{is}} - y_t\right)^2}$$

5 Equivalent Conditions

Proposition 1. Effect of No short selling constraints.The solution to the following equations

$$\min_{w \in \mathcal{S}} \frac{1}{N} ||y - Xw||_2^2$$

with the given constraints $\Sigma_{i=1}^n w_i = 1$ with $w_i \geq 0 \forall i \in 1, \cdots, n$ is equivalent to solving

$$\min_{w \in \mathcal{S}} x'Ax - 2Bw$$

where A is replaced by $\tilde{A} = A - \frac{\gamma e' + e\gamma'}{2}$ and B remains unchanged where $A = \frac{X'X}{n}, B = \frac{y'X}{n}$

Proof. Under both no short selling and full investment conditions the KKT conditions for the problem

$$\min_{w \in \mathcal{S}} \frac{1}{N} ||y - Xw||_2^2$$

with the given constraints $\Sigma_{i=1}^n w_i = 1$ with $w_i \geq 0 \forall i \in 1, \cdots, n$ are the following

$$2Aw - 2B - \gamma - \gamma_0 e = 0$$

$$\gamma_j \geq 0, w_j \geq 0, w_j \gamma_j = 0$$

here $\gamma = (\gamma_1, \cdots, \gamma_p)'$ are the Lagrange Multipliers for the non negative constraints and γ_0 is the multiplier for full investment constraints.To show that w is the solution to the index tracking problem which is

$$\min_{w \in \mathcal{S}} x'Ax - 2Bw$$

when A is replaced by $\tilde{A}$ and B is unchanged is equivalent to showing the following The 1st condition can be written as following

$$2Aw - 2B - \gamma - \gamma_0 e = 2\tilde{A}w + (\gamma e' + e\gamma')w - 2B - \gamma - \gamma_0 e$$

Based on the slackness condition we get $w_i \gamma_i = 0 \ \forall i \implies e\gamma' w = 0$ and by the full investment condition it implies $\gamma e' w = \gamma$ which reduces the equation to the following

$$2Aw - 2B - \gamma - \gamma_0 w = 2\tilde{A}w - 2B - \gamma_0 e$$

.Now the following coincides with the first order condition for the index tracking problem.That implies that w solves the unconstrained index problem when A is replaced by $\tilde{A}$ and B is unchanged □

Proposition 2. Effect of AENET penalty: The solution to the problem

$$\min_{w} \frac{1}{n} \sum_{i=1}^{n} \left(y_i - \sum_{j=1}^{p} x_{ij} w_j \right)^2 + \lambda_1 \sum_{j=1}^{p} \hat{v}_j |w_j| + \lambda_2 \|w\|_2^2 + \lambda_c \sum_{j=1}^{p} |w_j - \bar{w}_j|$$
$$\text{s.t.} \quad w'e = 1, w_j \geq 0$$

is equivalent to solving

$$\min_{w \in \mathcal{S}} x'Ax - 2Bw$$

where A is replaced by $\tilde{A} = A - (\gamma e' + e\gamma')/2 + \lambda_2 I$ and B by $\tilde{B} = B - \frac{1}{2}\lambda_c \tilde{g} - \frac{1}{2}\lambda_1 \hat{v}_1$

Proof. We know that $\exists$ a Lagrange Multiplier λ_c such that the solution to the problem

$$\begin{aligned}&\min_w \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \sum_{j=1}^{p} x_{ij}w_j\right)^2 + \lambda_1 \sum_{j=1}^{p}\hat{v}_j\,|w_j| + \lambda_2\|w\|_2^2 + \lambda_c \sum_{j=1}^{p}|w_j - \bar{w}_j|\\ \text{s.t.}\quad & w'e = 1, w_j \geq 0\end{aligned}$$

coincides with the solution to

$$\min_w\ w'Aw - 2(B - \frac{1}{2}\lambda_c\tilde{g})w + \lambda_1\hat{v}'w + \lambda_2 w'w$$

s.t. $w'e = 1, w_i \geq 0$ The KKT conditions for the above equations are

$$2(A + \lambda_2 I)w - 2(B - \frac{1}{2}\lambda_c\tilde{g} - \frac{1}{2}\lambda_1\hat{v}') - \gamma - \gamma_0 e = 0$$

s.t. $\gamma_j \geq 0,\ w_j \geq 0, w_j\gamma_j = 0$ We need to show that w is a solution to the index tracking problem when A is replaced by $\tilde{A}$ and B is replaced by $\tilde{B}$ it suffices to verify the first order condition

$$\begin{aligned}2\tilde{A}w - 2B &= 2(A + \lambda_2 I)w - (\gamma e' + e\gamma')w - 2(B - \frac{1}{2}\lambda_c\tilde{g} - \frac{1}{2}\lambda_1\tilde{v})\\ &= 2(A + \lambda_2 I)w - \gamma e'w - 2(B - \frac{1}{2}\lambda_c\tilde{g} - \frac{1}{2}\lambda_1\hat{v}_1\\ &= 2(A + \lambda_2 I)w - \gamma - 2(B - \frac{1}{2}\lambda_c\tilde{g} - \frac{1}{2}\lambda_1\hat{v}) = \gamma_0 e\end{aligned}$$

The second equality follows from the slackness condition of $w_i\gamma_i = 0$ for all i.The third equality comes from the following $e'w = 1$ and the last equality comes from

$$2(A + \lambda_2 I)w - 2(B - \frac{1}{2}\lambda_c\tilde{g} - \frac{1}{2}\lambda_1\hat{v}') - \gamma - \gamma_0 e = 0$$

This proves our second proposition $\square$

6 Data and Results

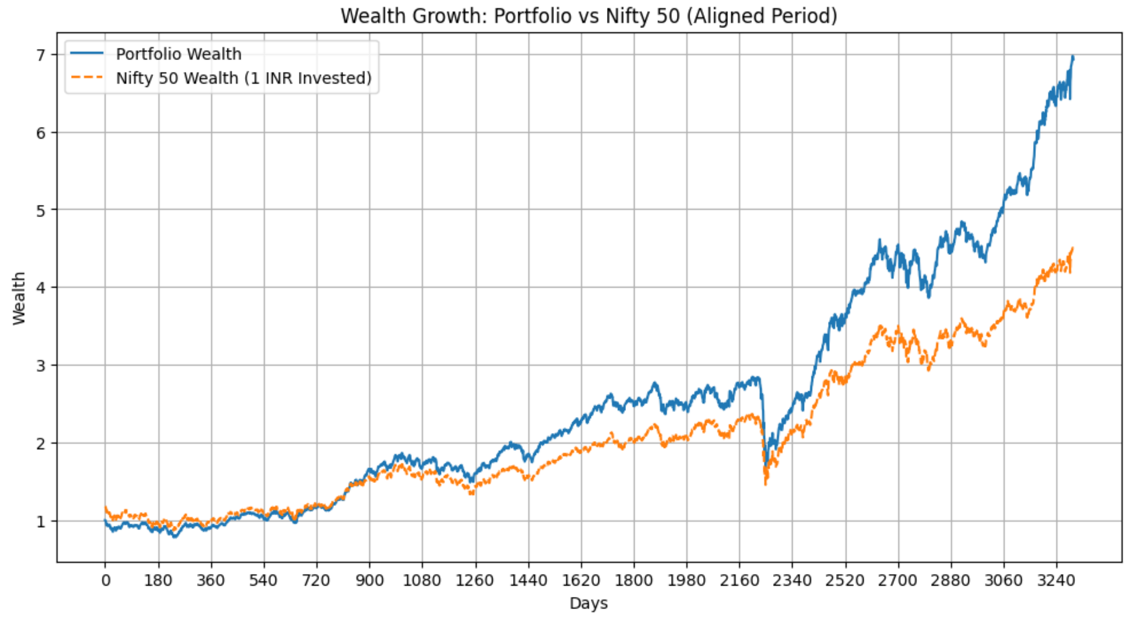

Figure 6.1: C

The figure above displays the wealth curve of my portfolio compared with that of the NIFTY 50 index. Both curves start with an initial investment of 1 INR. The visible section of the graph does not include the very beginning, as the total period considered by the algorithm is **3553** trading days, covering January 1, 2010, to July 1, 2024.
For this analysis, the training period $T_{\text{Train}} = 250$ days and testing period $T_{\text{Test}} = 21$ days were used, resulting in 157 sliding windows where the portfolio was rebalanced. The graph excludes the initial 250 trading days (during which only training occurred) and the last 6 days, which fall short of completing a full 21-day testing window after the final rebalancing.
Over this 14-year span, the wealth curve indicates that my strategy would grow 1 INR to 7 INR, compared to a growth to 4.5 INR for the NIFTY 50.
My portfolio has a constant list of 46 stocks which were present in Nifty 50 on 1st January 2010 where the rebalancing of weights is happening.

6.1 Data

yfinance library on Python