

# Point Estimates

Guesses                      Parameters

$$\bar{X} \rightarrow \mu$$

$$T(X) \rightarrow \theta$$

$$PE \pm \underbrace{\text{Margin of Error}}$$

## Statistical Intervals

A set of values that may  
contain/capture the true parameter  
(Bayesian) (Frequentist)

$$P(L \leq \theta \leq U) = 1 - \alpha$$

$L?$        $U?$

$\alpha$ : Significance level

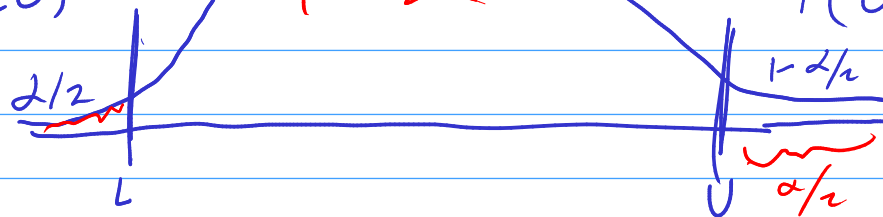
This is the probability that your  
interval does not contain/capture your  
true parameter.

$$\alpha = 0.05$$

$L, U$  are constructed from  
a mathematical model such as  
 $N(\mu, \sigma^2), \dots$

$$P(L \leq \theta \leq U) = 1 - \alpha$$

$$P(\theta \leq L) = \alpha/2$$



$$P(\theta \leq U) = 1 - \alpha/2$$

Mathematical Model

$$N(\mu, \sigma^2/n)$$

What is  $\mu$ ?

$$X = \{X_1, \dots, X_n\} : E(X) = \mu \quad \text{Var}(X) = \sigma^2$$

$n \rightarrow \infty$

$$\bar{X} \sim N(\mu, \sigma^2/n)$$

Via Central Limit Theorem

$$P(L \leq \mu \leq U) = 1 - \alpha$$

$$\bar{X} \sim N(\mu, \sigma^2/n)$$

$$X \sim N(\mu, \sigma^2)$$

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

$$Z_{\bar{X}} = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$

$$P\left(L \leq \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq U\right) = 1 - \alpha$$

$$P\left(Z_{\alpha/2} \leq \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq Z_{1-\alpha/2}\right)$$

$$P(Z \leq z_{\alpha/2}) = \alpha/2 \quad P(Z \leq z_{1-\alpha/2}) = 1 - \alpha/2$$

$$\alpha = 0.05$$

$$z_{\alpha/2} = -1.96$$

$$z_{1-\alpha/2} = 1.96$$

$$P\left(-1.96 \leq \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq 1.96\right)$$

$$P\left(-1.96 \frac{\sigma}{\sqrt{n}} \leq \bar{X} - \mu \leq 1.96 \frac{\sigma}{\sqrt{n}}\right) = .95$$

$$P\left(-\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}} \leq -\mu \leq -\bar{X} + 1.96 \frac{\sigma}{\sqrt{n}}\right) = .95$$

$$P\left(\bar{X} + 1.96 \frac{\sigma}{\sqrt{n}} \geq \mu \geq \bar{X} - 1.96 \frac{\sigma}{\sqrt{n}}\right) = .95$$

$$P\left(\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}}\right) = .95$$

95% CI of  $\mu$

$$\bar{X} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

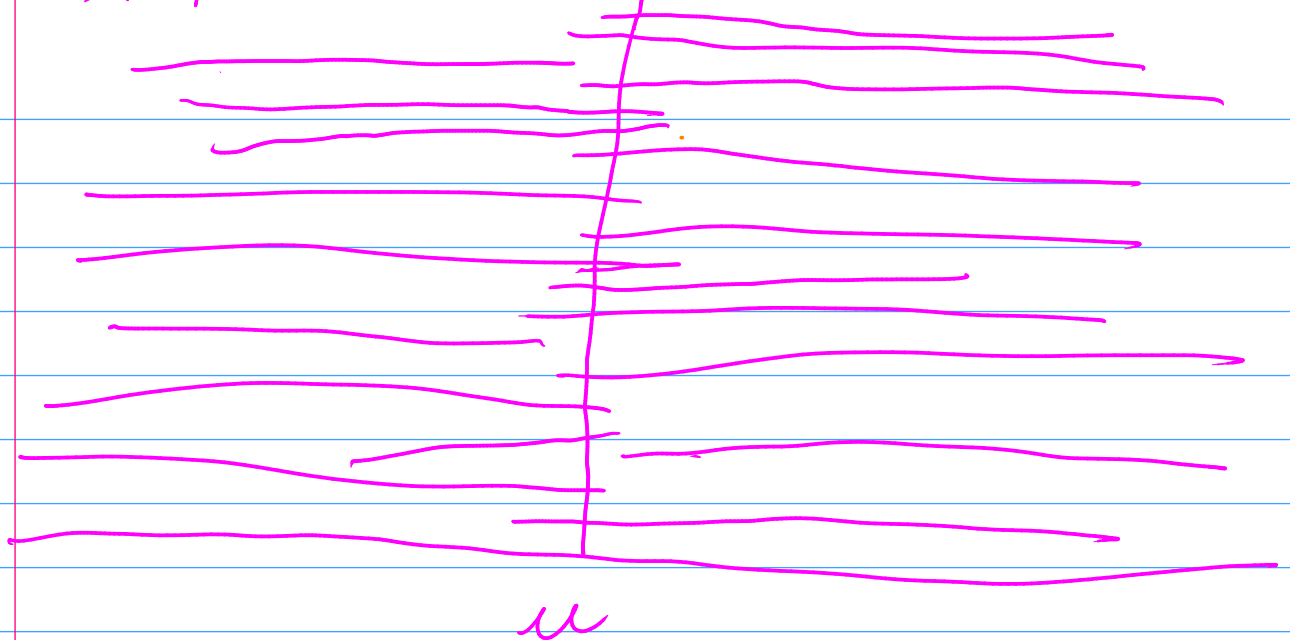
$$n \rightarrow \infty$$

$$S^2 \rightarrow \sigma^2$$

Interpretation

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$(1-\alpha)\% \mid 95\%$



Width of the interval

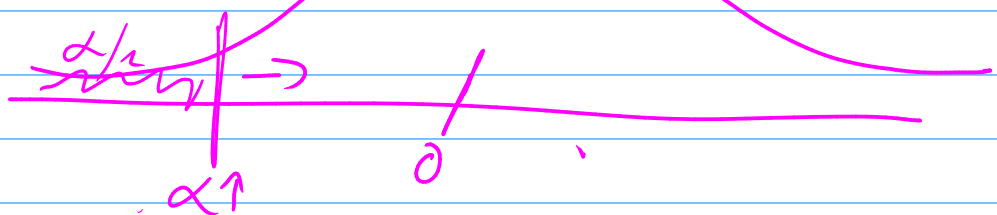
$\alpha$  value  $\uparrow$   
 $n$   $\uparrow$

width  $\downarrow$   
width  $\downarrow$

$U-L$

$$\bar{x} \pm z_{\alpha/2} \sigma/\sqrt{n}$$

$$P(Z \leq z_{\alpha/2})$$



$$\text{Confidence} = 1 - \alpha$$

$$(1-\alpha)100\%$$

$$MOE = |z_{\alpha/2}| \sigma/\sqrt{n}$$

Margin of Error