

# Functions of Random Variables

More Probability Theory

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# Using the Distribution Function

Let there be a random variable  $X$  with a known distribution function  $F_X(x)$ , the density function for the random variable  $Y = g(X)$  can be found with the following steps

1. Find the region of  $Y$  in the space of  $X$ , find  $g^{-1}(y)$
2. Find the region of  $Y \leq y$
3. Find  $F_Y(y) = P(Y \leq y)$  using the probability density function of  $X$  over region  $Y \leq y$
4. Find  $f_Y(y)$  by differentiating  $F_Y(y)$

## Example 1

Let  $X$  have the following probability density function:

$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability density function of  $Y = 3X - 1$ ?

## Using the PDF

Let there be a random variable  $X$  with a known distribution function  $F_X(x)$ , if the random variable  $Y = g(X)$  is either increasing or decreasing, then the probability density function can be found as

$$f_Y(y) = f_X\{g^{-1}(y)\} \left| \frac{dg^{-1}(y)}{dy} \right|$$

## Example 2

Let  $X$  have the following probability density function:

$$f_X(x) = \begin{cases} \frac{3}{2}x^2 + x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability density function of  
 $Y = 5 - (X/2)$ ?

# Using the MGF

Using the uniqueness property of Moment Generating Functions, for a random variable  $X$  with a known distribution function  $F_X(x)$  and random variable  $Y = g(X)$ , the distribution of  $Y$  can be found by:

1. Find the moment generating function of  $Y$ ,  $M_Y(t)$ .
2. Compare  $M_Y(t)$ , with known moment generating functions. If  $M_Y(t) = M_V(t)$ , for all values  $t$ , then  $Y$  and  $V$  have identical distributions.

## Example 3

Let  $X$  follow a normal distribution with mean  $\mu$  and variance  $\sigma^2$ . Find the distribution of  $Z = \frac{X - \mu}{\sigma}$ .

## Example 4

Let  $Z$  follow a standard normal distribution with mean 0 and variance 1. Find the distribution of  $Y = Z^2$