

REVIEW:

RANDOM VARIABLES AND DISTRIBUTION FUNCTIONS

- **Review Random Variables**

- Discrete Random Variables
- Binomial Distribution
- Poisson Distribution
- Continuous Random Variables
- Uniform Distribution
- Normal Distribution

RANDOM VARIABLES

A random variable is function that maps the sample space to real value.

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DISCRETE RANDOM VARIABLES

A random variable is considered to be discrete if it can only map to a finite or countably infinite number of distinct values.

PMF

The probability mass function of discrete variable can be represented by a formula, table, or a graph. The Probability of a random variable Y can be expressed as $P(Y = y)$ for all values of y .

CDF

The cumulative distribution function provides the $P(Y \leq y)$ for a random variable Y .

EXPECTED VALUE

The *expected value* is the value we expect when we randomly sample from population that follows a specific distribution. The expected value of Y is

$$E(Y) = \sum_y yP(y)$$

VARIANCE

The *variance* is the expected squared difference between the random variable and expected value.

$$\text{Var}(Y) = \sum_y \{y - E(Y)\}^2 P(y)$$

$$\text{Var}(Y) = E(X^2) - E(X)^2$$

KNOWN DISTRIBUTIONS

Distribution	Parameter(s)	PMF $P(Y = y)$
Bernoulli	p	p
Binomial	n and p	$\binom{n}{y} p^y (1 - p)^{n-y}$
Geometric	p	$(1 - p)^{y-1} p$
Negative Binomial	r and p	$\binom{y-1}{r-1} p^r (1 - p)^{y-r}$
Hypergeometric	N , n , and r	$\frac{\binom{r}{y} \binom{N-r}{n-y}}{\binom{N}{n}}$
Poisson	λ	$\frac{\lambda^y}{y!} e^{-\lambda}$

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BINOMIAL DISTRIBUTION

An experiment is said to follow a binomial distribution if

1. Fixed n
2. Each trial has 2 outcomes
3. The probability of success is a constant p
4. The trials are independent of each

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

EXPECTED VALUE OF A BINOMIAL DISTRIBUTION

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POISSON DISTRIBUTION

The poisson distribution describes an experiment that measures that occurrence of an event at specific point and/or time period.

$$P(X = x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

EXPECTED VALUE OF A POISSON DISTRIBUTION

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CONTINUOUS RANDOM VARIABLES

A random variable X is considered continuous if the $P(X = x)$ does not exist.

CDF

The cumulative distribution function of X provides the $P(X \leq x)$, denoted by $F(x)$, for the domain of X .

Properties of the CDF of X :

1. $F(-\infty) \equiv \lim_{y \rightarrow -\infty} F(y) = 0$
2. $F(\infty) \equiv \lim_{y \rightarrow \infty} F(y) = 1$
3. $F(x)$ is a nondecreasing function

PDF

The probability density function of the random variable X is given by

$$f(x) = \frac{dF(x)}{d(x)} = F'(x)$$

wherever the derivative exists.

Properties of pdfs:

1. $f(x) \geq 0$
2. $\int_{-\infty}^{\infty} f(x)dx = 1$
3. $P(a \leq X \leq b) = P(a < X < b) = \int_a^b f(x)dx$

EXPECTED VALUE

The expected value for a continuous distribution is defined as

$$E(X) = \int x f(x) dx$$

The expectation of a function $g(X)$ is defined as

$$E\{g(X)\} = \int g(x) f(x) dx$$

EXPECTED VALUE PROPERTIES

1. $E(c) = c$, where c is constant
2. $E\{cg(X)\} = cE\{g(X)\}$
3. $E\{g_1(X) + g_2(X) + \cdots + g_n(X)\} = E\{g_1(X)\} + E\{g_2(X)\} + \cdots$

VARIANCE

The variance of continuous variable is defined as

$$Var(X) = E[\{X - E(X)\}^2] = \int \{X - E(X)\}^2 f(x) dx$$

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UNIFORM DISTRIBUTION

A random variable is said to follow uniform distribution if the density function is constant between two parameters.

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}$$

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NORMAL DISTRIBUTION

A random variable is said to follow a normal distribution if the frequency of occurrence follow a Gaussian function.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left\{ -\frac{(x - \mu)^2}{2\sigma^2} \right\}$$

EXPECTED VALUE