Numerical Methods Cheatsheet

Bisection Method

$$x_0 = \frac{x_1 + x_2}{2}$$

Update based on $f(x_0)$:

$$\text{If } f(x_0)f(x_1) < 0 : x_2 = x_0$$

$$\text{If } f(x_2)f(x_0) < 0 : x_1 = x_0$$

False Position Method

$$x_0 = x_1 - \frac{f(x_1)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

Update x_1 or x_2 based on $f(x_0)$.

$$\text{If } f(x_0)f(x_1) < 0 : x_2 = x_0$$

$$\text{else } x_1 = x_0$$

Newton-Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Secant Method

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

Fixed-Point Method

$$x_{n+1} = g(x_n)$$

Rewrite $f(x) = 0$ as $x = g(x)$.

Jacobi Iteration

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j \neq i} a_{ij} x_j^{(k)} \right)$$

Gauss-Seidel Method

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j<i} a_{ij} x_j^{(k+1)} - \sum_{j>i} a_{ij} x_j^{(k)} \right)$$

Gauss Elimination

Eliminate variables row by row to form an upper triangular matrix.

Gauss-Jordan Method

Reduce the system of equations to row-reduced echelon form.

Cramer's Rule

$$x_i = \frac{\det(A_i)}{\det(A)}$$

Replace i-th column of A with the solution vector.

Least Squares Regression

For $y = a + bx$:

$$b = \frac{n\sum(xy) - \sum x \sum y}{n\sum(x^2) - (\sum x)^2}$$

$$a = \frac{\sum y - b\sum x}{n}$$

Fitting Transcendental Eqns

For $y = a + bx$:

$$b = \frac{n\sum ln(x)ln(y) - \sum ln(x) \sum ln(y)}{n\sum(ln(x))^2 - \sum(ln(x))^2}$$

$$ln(a) = \frac{\sum ln(y) - b\sum ln(x)}{n}$$

Euler's Method

$$y_{n+1} = y_n + hf(x_n, y_n)$$

Trapezoidal Rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(a) + f(b) + 2\sum f(x_i)\right]$$

Simpson's 1/3 Rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + f(b) + 4\sum_{\text{odd}} f(x_i) + 2\sum_{\text{even}} f(x_i)\right]$$

Simpson's 3/8 Rule

$$\int_a^b f(x)dx \approx \frac{3h}{8}\left[f(a) + f(b) + 3\sum f(x_i) + 2\sum_{x3i} f(x_i)\right]$$

Heun's Method

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right]$$

Runge-Kutta (4th Order)

$$k_1 = f(x_n, y_n)$$

$$k_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right)$$

$$k_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right)$$

$$k_4 = f(x_n + h, y_n + hk_3)$$

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$