

MONOTONICITY, CONVEXITY AND BOUNDS INVOLVING THE BETA AND RAMANUJAN R -FUNCTIONS

TI-REN HUANG, LU CHEN, SHEN-YANG TAN AND YU-MING CHU *

Abstract. In the article, we provide several new asymptotical sharp bounds for the functions involving the Beta function and Ramanujan R -functions via the monotonicity and convexity properties of certain combinations defined in terms of polynomials, Beta and Ramanujan R -functions.

Mathematics subject classification (2020): 33C05, 26D20.

Keywords and phrases: Ramanujan R -function, beta function, monotonicity, convexity.

REFERENCES

- [1] M. ABRAMOWITZ, I. A. STEGUN, *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, Dover, New York, 1965.
- [2] G. D. ANDERSON, M. K. VAMANAMURTHY, M. VUORINEN, *Conformal Invariants, Inequalities, and Quasiconformal Mappings*, John Wiley and Sons, New York, 1997.
- [3] G. D. ANDERSON, S.-L. QIU, M. K. VAMANAMURTHY, *Elliptic integral inequalities, with applications*, Constr Approx. 1998, **14** (2), 195–207.
- [4] G. D. ANDERSON, R. W. BARNARD, K. C. RICHARDS, M. K. VAMANAMURTHY, M. VUORINEN, *Inequalities for zero-balanced hypergeometric functions*, Trans. Amer. Math. Soc. 1995, **347**, 1713–1723.
- [5] G. D. ANDERSON, S.-L. QIU, M. K. VAMANAMURTHY, M. VUORINEN, *Generalized elliptic integrals and modular equations*, Pacific J. Math. 2000, **192** (1), 1–37.
- [6] G. E. ANDREWS, R. ASKEY, R. ROY, *Special Functions*, Cambridge Univiversity Press, Cambridge, 1999.
- [7] J. M. BORWEIN, P. B. BORWEIN, *Pi and the AGM*, John Wiley & Sons, New York, 1987.
- [8] B. C. BERNDT, *Ramanujan's Notebooks*, Part II, Springer-Verlag, New York, 1989.
- [9] B. C. BERNDT, *Ramanujan's Notebooks*, Part IV, Springer-Verlag, New York, 1994.
- [10] R. BALASUBRAMANIAN, S. PONNUSAMY, M. VUORINEN, *Functional inequalities for quotients of hypergeometric functions*, J. Math. Anal. Appl. 1998, **218**, 256–268.
- [11] W. A. DAY, *On monotonicity of the relaxation functions of viscoelastic materials*, Proc. Cambridge Philos. Soc. 1970, **67**, 503–508.
- [12] W. FELLER, *An Introduction to Probability Theory and Its Applications*, Vol. II, John Wiley & Sons, New York, 1966.
- [13] T.-R. HUANG, S.-L. QIU, X.-Y. MA, *Monotonicity properties and inequalities for the generalized elliptic integral of the first kind*, J. Math. Anal. Appl. 2019, **469** (1), 95–116.
- [14] Y.-X. LI, M. A. ALI, H. BUDAK, M. ABBAS, Y.-M. CHU, *A new generalization of some quantum integral inequalities for quantum differentiable convex functions*, Adv. Difference Equ. 2021, **2021**, Article 225, 15 pages.
- [15] Y.-X. LI, M. H. ALSHBOOL, Y.-P. LV, I. KHAN, M. RIZA KHAN, A. ISSAKHOV, *Heat and mass transfer in MHD Williamson nano uid ow over an exponentially porous stretching surface*, Case Stud. Therm. Eng. 2021, **26**, Article ID 100975, 10 pages.
- [16] Y.-X. LI, A. RAUF, M. NAEEM, M. A. BINAYMIN, A. ASLAM, *Valency-based topological properties of linear hexagonal chain and hammer-like benzenoid*, Complexity, 2021, **2021**, Article ID 9939469, 16 pages.

- [17] S.-L. QIU, B.-P. FENG, *Some properties of the Ramanujan constant*, J. Hangzhou Dianzi Univ. 2007, **27** (3), 88–91.
- [18] S.-L. QIU, X.-Y. MA, T.-R. HUANG, *Some properties of the difference between the Ramanujan constant and Beta function*, J. Math. Anal. Appl., 2017, **446**, 114–129.
- [19] S.-L. QIU, M.-K. VAMANAMURTHY, *Elliptic integrals and the modulus of Grötzsch ring*, Panamer. Math. J. 1995, **5**, 41–60.
- [20] M.-K. WANG, Y.-M. CHU, S.-L. QIU, *Sharp bounds for generalized elliptic integrals of the first kind*, J. Math. Anal. Appl. 2015, **429**, 744–757.
- [21] E. T. WHITTAKER, G. N. WASTON, *A course of Modern Analysis*, Cambridge University Press, London, 1958.
- [22] J. WIMP, *Sequence Transformations and Their Applications*, Academic Press, New York, 1981.
- [23] P.-G. ZHOU, S.-L. QIU, G.-Y. TU, Y.-L. LI, *Some properties of the Ramanujan constant*, J. Zhejiang Sci-Tech Univ. 2010, **27** (5), 835–841.