The Visual Computer

Singularity theoretical modeling and animation of garment wrinkle formation processes

Tosiyasu L. Kunii
and Hironobu Gotoda

Department of Information Science,
Faculty of Science, University of Tokyo,
7-3-1 Hongo, Bunkyo-ku, Tokyo 113, Japan;
e-mail: kunii @ is.s.u.-tokyo.ac.jp

To understand the nature of garments as worn, it is essential to model and animate the formation process of garment wrinkles. Because the number of components making up a garment is extremely high, simulating its behavior under dynamic constraints requires a very large amount of computation, and the result is difficult to analyze and understand. We show that exploiting geometric features of wrinkles can greatly increase the understandability of the computed result, while not much increasing the amount of computation needed. We present the modeling primitives of garment wrinkles, which can suitably represent geometric features of wrinkles under the dynamic constraints. Extracting geometric features based on singularity theory enables us to model the qualitative shape change of wrinkles. The formation process of wrinkles is animated by using these primitives.

Key words: Soft objects – Garment wrinkle formation – Singularity theory – Metric invariant deformation – Approximation

1 Introduction

1.1 Modeling of soft objects

Since the term *soft objects* was introduced by Wyvill et al. (1986), modeling of soft objects has been intensively studied and has grown to be one of the most attractive fields in computer graphics and animation. Because soft objects are structurally unstable and often consist of many components, it is extremely difficult to analyze their behavior under various constraints. In the case of hard or rigid objects, shape change occurs on a comparatively small scale and only a few factors, such as Young's modulus, Poisson's ratio, and Lamé's constant, are sufficient to determine their behavior. In the case of soft objects, however, many elements interact with each other in a very complicated manner and sometimes result in a drastic shape change. Due to these difficulties in modeling soft objects, it was not until recently that the problem has become fully addressed.
Recent developments in computer animation seem to have completely changed that situation. Two major modeling techniques, usually called physically based modeling and geometric modeling, have been devised to tackle these problems and have attained some successes in modeling soft objects. Although current trends suggest that physically based modeling is superior to geometric modeling, geometric modeling has its own advantages. Therefore, it is not easy to determine which technique is the best candidate for modeling soft objects in general.
The primary purpose of this study is to model garment wrinkles and to animate their formation processes. A garment is a family of soft objects composed of many threads in various woven patterns. Physical properties of a garment depend both on the physical properties of the threads and on the patterns in which they are woven. Furthermore, the threads are composed of smaller units, called fibers. The physical properties of the threads can be determined by the properties of the fibers and the ways they are bundled. Friction, anisotropy, and viscoelasticity, inherent in such complex structures, make a garment completely different from a thin metallic film. It seems that simulation of garment wrinkle formation processes with due consideration for all these factors is almost impossible.
To handle such a complicated object, we need to develop a new approach, incorporating both the physical and the geometric modeling techniques. Before we explain this approach in more detail in

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Sect. 1.3, we first compare the physically based and geometric modeling techniques in Sect. 1.2. Section 2 describes how geometric features are extracted by applying singularity theory, Sect. 3 presents the modeling primitives of garment wrinkles, Sect. 4 shows the animation of the wrinkle formation processes and Sect. 5 concludes this paper.

1.2 Physically based modeling vs geometric modeling

Earlier models of soft objects were usually based on geometry. Barr (1984) employed Jacobians to deform a solid object locally and globally. Wyvill et al. (1986) used a scalar field around a key point to represent a soft object. Such geometric models are simple and easy to program. However, because dynamic constraints are relatively difficult to incorporate into these models, the resulting objects sometimes behave unrealistically.

Physically based models, while computationally more complex than geometric models, offer that realism. A set of differential equations derived from physical laws plays a central role here. Platt and Barr (1988) discussed the methodology for combining constraint mathematics with finite element methods. The models developed by Terzopoulos and Fleisher (1988) and Terzopoulos et al. (1987) were based on the finite difference method. At first glance, these physically based models, with the advent of high-power and low-cost workstations, look superior to geometric models, because they seem to be able to simulate what is really occurring. However, for the following reasons, simulating the behavior of soft objects by solving differential equations is not a simple task.

The first problem with physically based models is that they tend to require too much computation. Because the number of elements making up a soft object is very large (usually over 10^8), real-time animation becomes impossible. This problem may be solved in the future by using more powerful computers and by developing more efficient algorithms. A more serious problem, however, is that physically based models are described only by differential equations. While differential equations are no more than local descriptions of an object, numerical simulation is necessary to determine the global structure. The resulting global structure depends on the parameter values of the simulation, but not always in a continuous manner. Sometimes it is drastically affected by a small change in the parameter values. To avoid such a drastic change, we have to determine the parameter values more accurately, which is very difficult because of the complex structures of soft objects.

From the above discussion, it follows that physically based models can certainly represent local structures of soft objects better than geometric models, but are not always suitable to represent the global structure. In contrast, geometric models can represent the global structure, but are not always suitable to represent local structures accurately. If we succeed in integrating both modeling techniques, we obtain a model that behaves well locally as well as globally. Our approach, outlined in the next section, is such an attempt to achieve a better modeling of soft objects.

1.3 Overview of our approach

As stated earlier, our major objective is to model and animate the formation processes of garment wrinkles. Because a garment is an object composed of an extremely large number of components, it is not practical to simulate its behavior by directly solving differential equations. We prefer to use some global information to reduce the amount of computation.

The existence of wrinkles can itself serve as such global information. When a garment is deformed, wrinkles are formed or extinguished. Shape changes are mainly observed around wrinkles, while the other parts of the garment remain unchanged. In fact, the geometry of the wrinkles and of the other parts seems to be different: for wrinkles, both metric and curvature parameters change; whereas for other parts, metric parameters are preserved and only curvature parameters change. This difference may be best utilized by employing a mathematical method known as singularity theory (Arnold 1986). This theory, which was recently found to be useful also in medical applications (Kergosien 1983), provides a mathematical foundation for dealing with qualitative geometric changes of a given system.

The discussions in the previous section suggest that we need a model, which can directly represent global geometric information without violating dynamic constraints. Our approach to this goal is to represent local information by dynamics and to represent global information by the singularity theory.

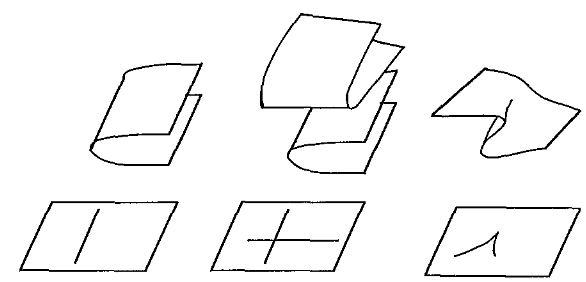
More specifically, we first produce various kinds of wrinkles by performing numerical simulation and then analyze their behavior by applying singularity theory. As a result of this step, we can extract geometric features of wrinkles, which can be used to roughly approximate the shapes of wrinkles. We employ these geometric features as our modeling primitives. We can refine the approximated shapes of the wrinkles by using dynamics again to any precision as needed. The last step can be considered to be an optimization procedure under dynamic constraints. An important point here is that the better the approximation is, the less time-consuming the refinement procedure becomes. This means that extracting well-suited geometric features is a very essential step, and that is why we adopt the singularity theory.

2 Geometric feature extraction

2.1 Singularity theory

In mathematics, two different methods are provided for studying a given surface: local and global. Locally, a surface can be described in terms of curvature. Although local descriptions themselves contain certain implicit global information, they are not usually well recognized until they are explicitly globalized. For example, a surface with positive curvature everywhere represents a completely different figure than a surface with negative curvature everywhere. However, it is not so simple to determine the difference if we only examine the surfaces locally. Also, a surface containing two regions, one with positive curvature and the other with negative curvature, together with a boundary of null curvature separating these two regions, is certainly different from a surface with positive curvature everywhere. Yet it is difficult to distinguish them when we compare them locally in the positive curvature regions. Thus, when we talk about a distinctive property of a given surface, it is not the local structure but the global structure that is understood.

Singularity theory, which has been developed by Morse, Whitney, Thom, and others, provides a method for globalization: a passage from local descriptions to global structures (Thom 1972). It first studies a surface locally and then examines its meaning (particularly, a sign) from a global viewpoint. More specifically, a series of projections of a given surface are taken and analyzed. Figure 1 shows three typical types of projections. In the framework of singularity theory, we are interested in contours only. The theory shows that the signs depicted in Fig. 1 are the only stable patterns in general. The other types of signs are unstable, i.e., if we take a projection from a slightly different direction, the pattern is decomposed into some combinations of the signs in Fig. 1. The basic idea of singularity theory is that one can distinguish general types of stable signs from special types of unstable signs. If no a priori knowledge is assumed for the surface to be analyzed, and if a projection is taken in an arbitrary direction, then the sign to be observed is almost always one of those in Fig. 1. The other types of signs are too rare to be observed. However, if the surface does have a special structure, one can expect that the rare types of signs are observed.



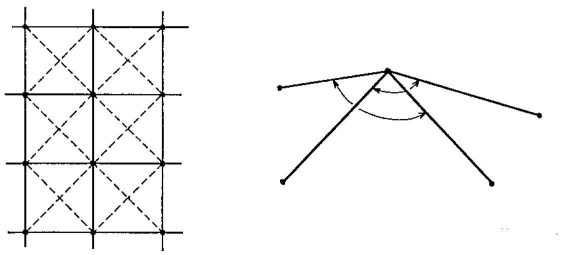
a Fold **b** Crossing lines **c** Cusp

Fig 1a–c. Typical signs

Although the kinds of observable signs and their rarity were extensively studied and classified by the 1960s, it was not until recently that the relationship between the signs and the global structure of a surface was determined. What was remarkable in Kergosien's work (1981) was that the signs were shown to be related to some concepts of classic differential geometry, such as curvature and geodesics. By using his theory, local and global structures can be studied simultaneously. In Sect. 2.3, we apply the method for analyzing the shape of garment wrinkles.

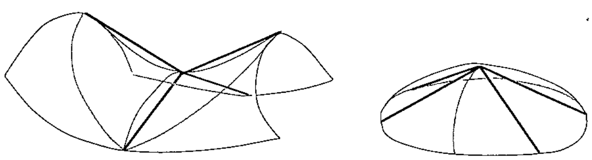
2.2 Dynamic analysis

Because garment wrinkles are formed by some dynamic constraints, it is necessary to study the dy-



a b

Fig. 2a, b. Dynamic model: **a** metric energy; **b** curvature energy



a b

Fig. 3. Local energy is different at **a** a peak point; **b** a saddle point

namic properties of a garment. Although the dynamic constraints can be described by a set of differential equations, and the local structures of wrinkles may be deduced from these equations, their global structures cannot be immediately known. Numerical simulation is necessary for determining the global structures.
Before performing simulation, a dynamic model representing outstanding properties of a garment should be prepared. Figure 2 briefly explains our dynamic model. Our model can be described as a collection of a finite number of lines corresponding to the threads in a real garment and a finite number of nodes corresponding to the crossings of these threads. In Fig. 2, each line works as a spring that reacts to the change of the length of the line, and each arc also works as a spring that reacts to the change of the arc's angle. Here we note that the dotted lines, which seem to have no physical counterparts, also work as springs. These lines are necessary for distinguishing the different configurations, as shown in Fig. 3. If no dotted lines work as springs, configurations (a) and (b) may have the same local energy, although in reality, it will not happen and (a) will have less local energy than (b). Such a difference comes from the complex structure each pair of threads form at their crossing point. Thus, the dotted lines do not directly correspond to physical objects, but represent some factors in thread interactions to form varieties of the weave patterns.
We call the energy stored at the places represented in solid or dotted lines *metric energy*, and the energy stored at the places represented in arcs *curvature energy*. The interaction between metric and curvature energy, is a characteristic phenomenon, which will not be observed in the deformation of rigid objects, where the metric will not change so much in comparison with the curvature. In our simulation, parameters are carefully chosen not to suppress either one of the two types of energy.
The most interesting aspects of the shapes of garment wrinkles are *branching* and *vanishing*. *Branching* is the phenomenon where one wrinkle splits into two wrinkles, and *vanishing* is the phenomenon where a wrinkle disappears. To observe these phenomena, a piece of cloth is prepared and deformed by picking up two sides and gradually moving them inwards. Because the interaction between curvature and metric energy is nonlinear, numerical algorithms, such as the finite-element method (Zienkiewicz 1977), cannot be efficiently applied. As a compromise, the following procedure is used to propagate the changes on the boundary into the interior region.

For each node in the piece of cloth, repeat the following steps until the maximum displacement falls below a certain threshold:

1) Compute the gradient vector of the energy function.
2) Move the node in the opposite direction of the gradient (in the direction of maximum decrease in energy) by a small distance. The distance is determined by the length of the gradient vector.

By the minimal-energy principle, this gradient-descent procedure can correctly compute the shape of garment wrinkles.

2.3 Characteristic points

The results obtained by simulation are analyzed as follows. To apply the method of singularity theory, a series of projections in horizontal directions are taken as the first step. The next step is to compare the signs of these projections with the signs that are theoretically known. Theoretically, the signs appearing in these projections are almost always the following ones: cusps, folds, or crossing

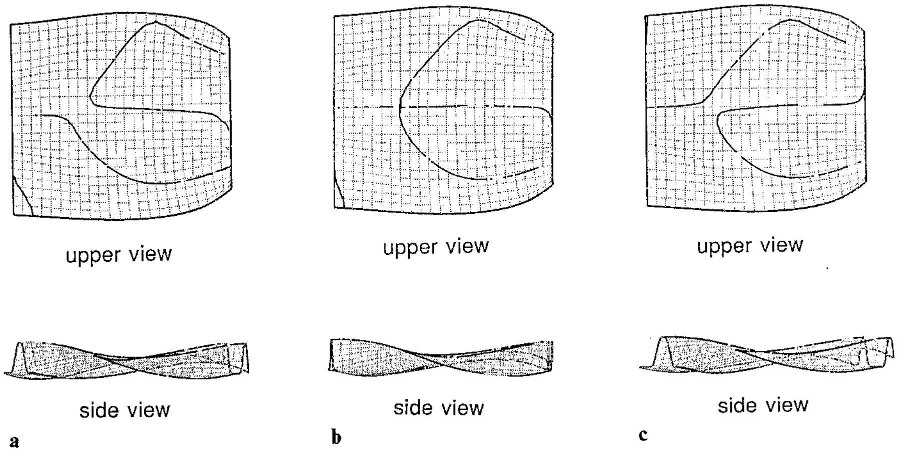


Fig. 4a–c. A case where a $p++c$ singularity emerges: **a** fold and cusp; **b** $p++c$ singularity; **c** fold and cusp

lines. However, there are special instances where the other types of signs emerge by changing the directions of viewing a given shape. Figure 4 shows a situation where a cusp and a fold approach each other (Fig. 4a), then merge (Fig. 4b), and finally separate from each other (Fig. 4c). Different configurations are seen in Fig. 4a and c: the lines that form a cusp and a fold are exchanged in the process of merging. The merged state (Fig. 4b) is classified as the $p++c$ singularity (Arnold 1986; Kergosien 1981), which describes the phenomenon of branching.

The same $p++c$ singularity also describes the phenomenon of vanishing. (Branching and vanishing are complimentary to each other. A point on a surface can be a branching point when it is looked at from one side, and can be a vanishing point when it is looked at from the other side.) Because this singularity is very rare, the behavior of the points corresponding to this singularity will be far greater constraints than the behavior of the other points. Such points will be the characteristic points of garment wrinkles. By specifying the local structures around the characteristic points, more realistic garment-wrinkle images can be synthesized.

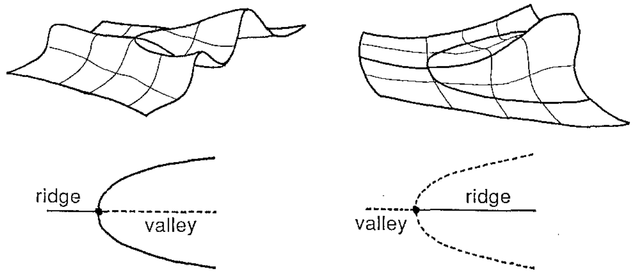
3 Modeling of garment wrinkles

3.1 Modeling primitives

As explained in the previous section, the local structures around the characteristic points remarkably influence the global structures of garment wrinkles. Because the number of characteristic points is small, we can achieve great data compression if the shape of wrinkles can be represented by a collection of characteristic points and the local structures around them. In the following discussion, we present modeling primitives of wrinkles and the method for reconstructing the shape from the primitives.

The basic sets of modeling primitives are the positions and the types (branching or vanishing) of characteristic points. These sets of primitives alone, however, are not sufficient to reconstruct the original shape in a satisfactory manner. Other primitives are necessary for specifying the local structures around the characteristic points more precisely. There is a trade-off between the simplicity of the modeling primitives and the exactness of representation. We have several choices.

Our choice is to add the contours that are associated with the special singular configurations (see Fig. 5) as another set of modeling primitives. This enables us to restrict the number of possible wrinkle shapes to be reconstructed. Later, in Sect. 4, we will see that the contours serve as markers in the process of forming wrinkles. For specifying the contours, we choose several sample points and interpolate the coordinate values between those points by a special kind of spline function, called "spline in tension" (Schweikert 1966). This function is known to be adequate in representing the effect of tension applied to both ends of a curve. Other choices are of course possible, but our method can produce a reasonably good appearance without requiring so much computation.



5a b

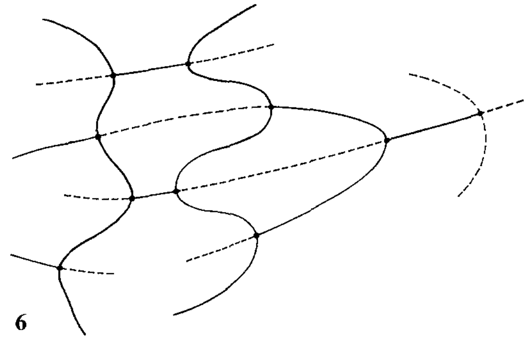


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Fig. 5a, b. Modeling primitives. **a** Branching. **b** Vanishing. · characteristic point; ---- contour

Fig. 6. Global arrangement of wrinkles

In summary, the modeling primitives of garment wrinkles are:

1) Positions and types of characteristic points.
2) Contours associated with characteristic points.

(To represent the contours, several sample points need to be specified.)

It is usually observed that several wrinkles appear on one garment cloth and become connected with each other or diminish at their ends. Our modeling primitives can describe such a complex situation: connections can be represented by branching points and diminishings by vanishing points. Such a description will yield a graph, as shown in Fig. 6, where vertices correspond to the characteristic points and edges to the associated contours. This graph adequately illustrates the "backbones" of wrinkles. In the next section, we discuss the method for reconstructing the wrinkle surface from this graph.

3.2 Surface reconstruction

We have proposed a method for reconstructing the surface cross-sectionally by using spline functions (Kunii and Gotoda 1990). This method works reasonably well when the wrinkles are small. However, when the depth of the wrinkles increases above a certain threshold, the area of the reconstructed surface also increases and sometimes gives an artificial impression, as seen in Fig. 7. This is mainly due to the fact that the curves generated by spline functions do not preserve their length.

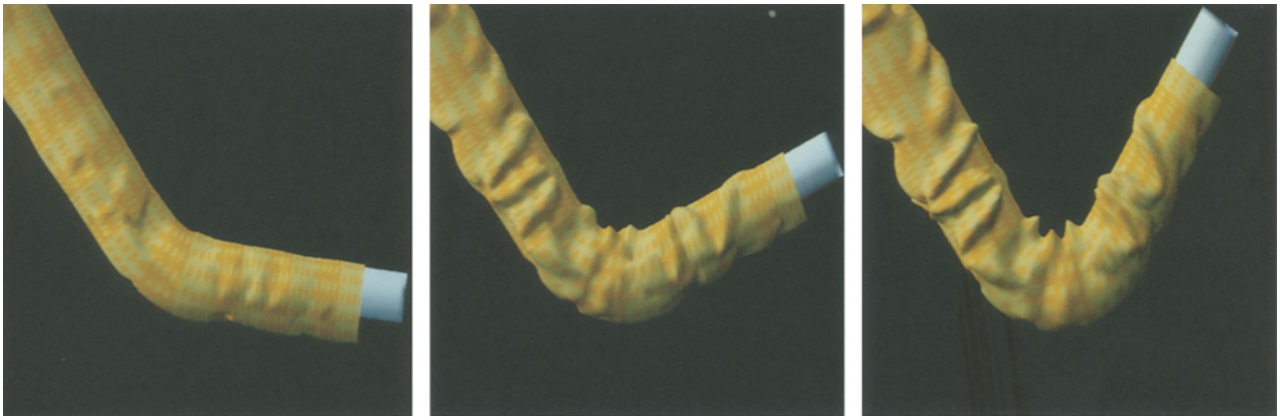
Here we present an alternative method. This takes two stages, approximation and relaxation (optimization), as in the method proposed by Weil (1986). The main difference is that we do not use catenary curves in the approximation stage, because the constraints we specify are not only positions but also tangent vectors at sampling points.

Several methods for approximating the wrinkle surface can be considered. Our approach is to partially solve the dynamic constraints by trying to achieve a metric invariant deformation, i.e., the length of the threads contained in a garment does not change. Because metric invariant deformations are difficult to simulate numerically, we aim at getting close to them theoretically. Of course, garments are elastic in reality and in some parts (particularly around characteristic points) the metric may not be preserved, but this effect can be incorporated in the relaxation stage.

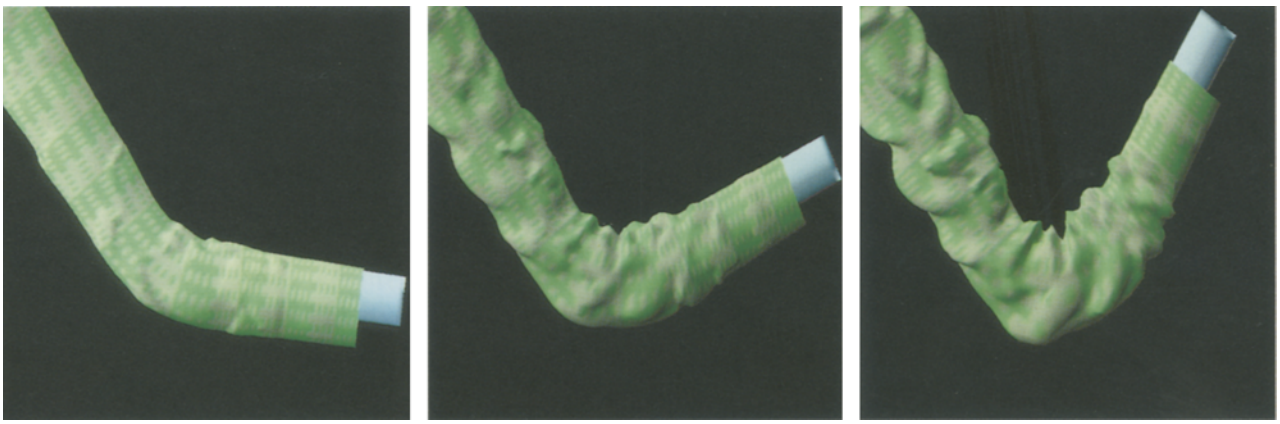
Metric invariant deformation is a classic problem in the calculus of variations, and it is well understood in the case of one-dimensional curves restricted to a plane. As well explained by Zeeman (1977), a deformed curve satisfies the form

$$f(s)=X \sin \frac{n\pi s}{\lambda},$$

where X is a small constant, n is a natural number, λ is the length of the curve, s is the parameter for arc-length ($0 \leqq s \leqq \lambda$), and $f(s)$ is the vertical displacement of the points s (Fig. 8). (Note: This form is not in fact an exact solution. It is correct only to the second order, but is sufficient for our purpose.) In the case of two-dimensional surfaces, however, metric invariant deformation still remains a difficult problem in general. Thus, as a comprom-



Case **a**



Case **b**

Fig. 7. Animation of wrinkle-formation processes without preserving metric invariance

ise, we use the above form in one direction only (Fig. 9).

In the relaxation stage, the gradient-descent method described in Sect. 2.2 is applied. This is really an optimization procedure minimizing the total energy contained in the garment under the constraint that the garment passes through the characteristic points and the associated contours. Environmental factors, such as gravity, can be incorporated at this stage.

This approximation/relaxation method is used to generate the animation of the process of forming wrinkles. While computationally more complex

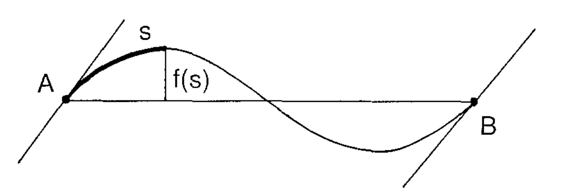


Fig. 8. Metric invariant deformation (s = parameter for arc-length; $f(s)$ = vertical displacement; A, B = fixed points)

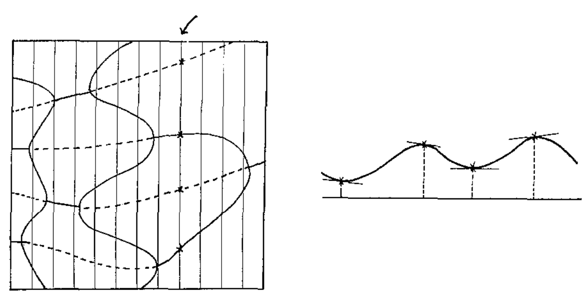


Fig. 9. a Global arrangement of wrinkles; **b** reconstructed cross section

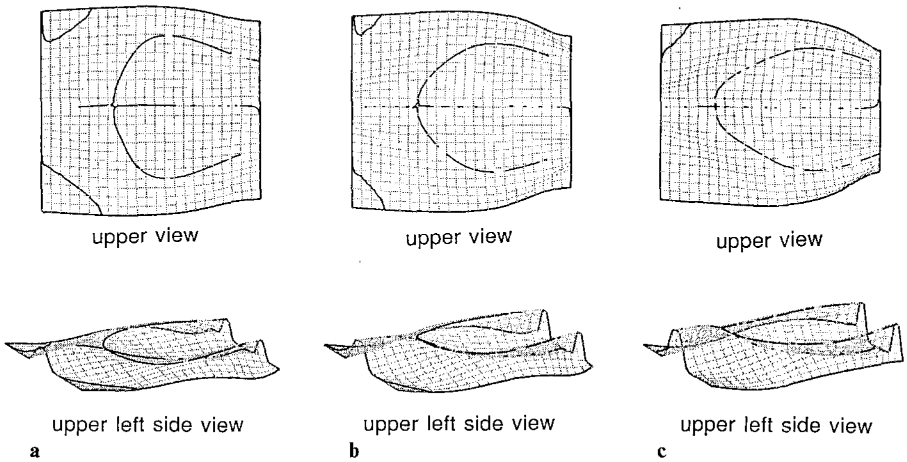


Fig. 10a–c. Wrinkle formation process: **a** after 8 steps; **b** after 12 steps; **c** after 16 steps

than the spline method, much better garment-wrinkle images are obtained, as seen in Fig. 11.

4 Animation of garment wrinkles

4.1 Process of forming garment wrinkles

In the previous chapters, we have shown that garment wrinkles are characterized by a set of characteristic points and the local structures around them. The process of forming garment wrinkles can be described as the movement of the characteristic points and as the structural change around these points. If the movement and the structural change are also characterized by some simple rules, more efficient data compression becomes possible. Such compression is practically important in performing animation.

From the observation of natural phenomena, wrinkles seem to be stable, i.e., they will not be easily extinguished once they are formed. The positions of the characteristic points will certainly move, but the depth of the wrinkles will continue to change, becoming more pronounced. The next step is to observe the phenomenon in full detail through simulation. However, in contrast to the analysis performed in Sect. 2, the task is not as simple as it seems at first glance. For several reasons, we should be careful in interpreting the results of simulation:

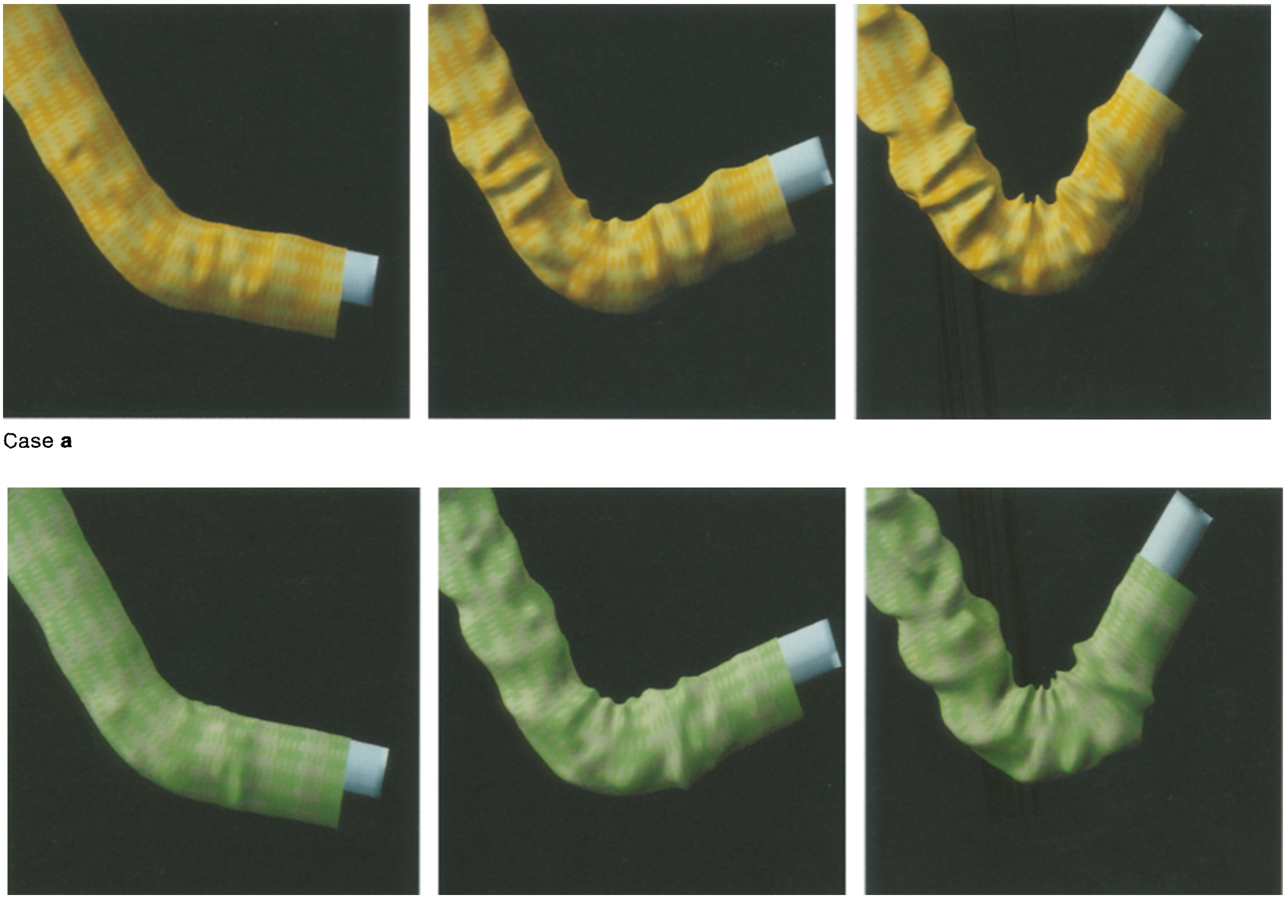
1) We do not have much control over the wrinkle formation processes. Although there are many possible formation processes, we cannot simulate all of them.

2) If the energy is lost due to the friction between the threads of the garment or due to the non-linear nature of the threads, the process becomes irreversible. This means that the shape of wrinkles should depend on the history of the formation process and that the nature of hysteresis needs to be studied.

Thus, form a limited number of simulations, it is difficult to extract the characteristic features that are generally applicable. Our preliminary simulation, however, seems to indicate that there are some common rules that govern the process of forming wrinkles.

Figure 10 shows a series of steps in wrinkle formation. The contours representing the local structures around the characteristic points become sharper as the wrinkle formation proceeds. The positions of the characteristic points move away from the sharpened contours as if they were pushed by the sharp change of the contours. This will be a result of the interaction between the metric and curvature energy (we have not yet achieved theoretical proof).

From now on, we hypothesize that the shape change, as stated above, occurs in general. When we animate the process of garment-wrinkle formation, specifiying both the positions and behavior of the characteristic points may become a great overhead. Therefore, we prefer to specify as few parameters as possible. In the next section, several assumptions are proposed for decreasing the number of parameters. Animation is also made on those assumptions.



Case **a**

Case **b**

Fig. 11. Animation of wrinkle formation processes partially preserving metric invariance

4.2 Animation of the formation process

As an example, we take the wrinkles formed around the arm of a jacket. The first basic parameter here is the angle between the human forearm and upper arm at the elbow. The more the arm is bent, the deeper the wrinkles become. In order to animate such a phenomenon, we assume:

1) The characteristic points will not be newly created or destroyed during the process of wrinkle formation.
2) The contours will get sharper during the process.
3) The wrinkles will become deeper during the process.

Based on these assumptions, animation is conducted by using the modeling primitives of wrinkles. Now, the initial configuration of characteristic points needs to be added to the angle at the elbow as the second parameter of this animation. Figure 11 shows two animation sequences of wrinkle formation. The difference in the two cases lies in the initial configuration data. Case (a) has 13 branching points and 23 vanishing points. Case (b) has 16 branching points and 24 vanishing points. The results are relatively good for such a small number of parameters.

4.3 Problems in deriving the initial configuration by simulation

Although the assumptions in the previous section greatly simplify the task of animation, the initial data for configuration of wrinkles are still required as the parameter values. This requirement, at first sight, seems to be unnecessary if the initial configuration can be gained by numerical simulation.

However, for several reasons, it is not practical to obtain the initial configuration by simulation.
First, a garment is usually inhomogeneous according to the material, wearing process, and history of its use. In fact, if we repeatly bend and stretch the arm, it can be observed that wrinkles tend to be formed at the places where they were once formed. Because the process of wrinkle formation is irreversible, the past history strongly influences the current state. In a short-term simulation, such a history cannot be correctly incorporated. The initial configuration can be considered to correspond to such inhomogeneity of the garment. Thus, requiring it as a part of the dynamic properties of the garment is not unreasonable.
Second, the simulation will be time-consuming, because the system contains both small- and large-scale factors. While a wrinkle has its own structure, such as the characteristic points (a small-scale factor), the problem to be answered is the total arrangement of the wrinkles (a large-scale factor). To solve such a problem correctly, we have to decompose it into the smallest unit, a polymer molecule, which means handling a very large amount of data at a time.
Third, it is theoretically difficult to predict the emergence of a new wrinkle. An emerging of a wrinkle is a qualitative shape change involving the creation of new characteristic points and the establishment of new connections to the other points. Singularity theory can deal with such changes, and there are several works that address the problem in the case of a rigid object. The so-called *Euler buckling* phenomenon (Chillingworth 1975; Zeeman 1977), in which a metallic beam bends drastically at some moment when increasing force is applied at its ends, can be explained in the framework of this theory. In the case of a two-dimensional plate, however, the type and mechanism of the shape change have not yet been sufficiently understood.
The problem we are looking at is still an open research area. The initial configuration data are, therefore, necessary for efficient execution of the animation. As our understanding of the nature of singularity for creating drastic shape changes advances, we intend to develop a different approach.

5 Concluding remarks

Modeling of soft objects is becoming popular in computer graphics and animation. This study selected garment wrinkles as class of soft objects, and attempted to model their static and dynamic structures.
First, the static structures were examined. We presented modeling primitives that sufficiently represent the static structures of wrinkles. Because the primitives were extracted from the geometric analysis of dynamic-simulation results, they successfully integrated both the dynamic and the geometric features in a unified manner.
Then the dynamic structures were studied. Although analysis of dynamic structures was far more difficult than for static structures, we were able to come up with a simple model with several assumptions for efficiently executing animation. The result of the animation showed the applicability of that model.
The modeling procedure we described in the case of garment wrinkles probably can be applied to the modeling of other soft objects. The combination of numerical simulation based on dynamics and mathematical analysis of the results by singularity theory will become a powerful tool for exploring the world of "softness."
Our future research directions include theoretical analysis of the wrinkle dynamic structures and representation of initially wrinkled (non-smooth) surfaces. Developing an efficient algorithm to simulate metric invariant deformations is also a practically important theme for achieving better approximation of wrinkle shapes from the modeling primitives.

Acknowledgments. We would like to thank Dr. Yannick L. Kergosien for his valuable comments at the early stage of this research. Thanks are also extended to the International Information Science Foundation for support in inviting Dr. Kergosien to Kunii Laboratory.

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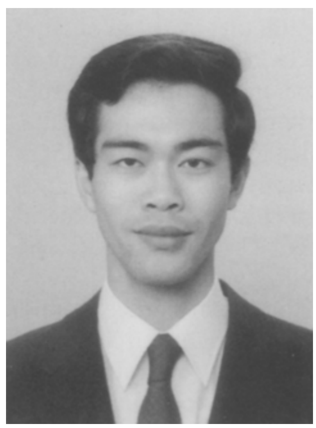
TOSIYASU L. KUNII is currently professor and chairman of the Information and Computer Science Department at the University of Tokyo. At the University of Tokyo, he started his work in raster computer graphics in 1968, which led to the Tokyo Raster Technology Project. His research interests include computer graphics, database systems, and software engineering. He has authored and edited more than 30 computer science books and published more than 100 refereed academic/technical papers in computer science and applications areas.
Dr. Kunii is honorary president and founder of the Computer Graphics Society, editor in chief of *The Visual Computer: An International Journal of Computer Graphics* (Springer-Verlag), associate editor in chief of *The Journal of Visualization and Computer Animation* (John Wiley & Sons), and on the editorial board of *IEEE Transactions on Knowledge and Data Engineering*, *VLDB Journal*, and *IEEE Computer Graphics and Applications*. He is a member of the IFIP Modeling and Simulation Working Group, the IFIP Data Base Working Group, and the IFIP Computer Graphics Working Group. He organized and chaired the Technical Committee on Software Engineering of the Information Processing Society of Japan from 1976 to 1981. He also organized and was president of the Japan Computer Graphics Association (JCGA) from 1981 to 1983. He served as general chairman of the 3rd International Conference on Very Large Data Bases (VLDB) in 1977, program chairman of InterGraphics in 1983, organizing committee chairman and program chairman of Computer Graphics Tokyo in 1984, program chairman of Computer Graphics Tokyo in 1985 and 1986, organizing committee chairperson and program chairperson of CG International '87, program co-chairman of IEEE COMPSAC '87, and honorary committee chairperson of CG International '88. He served as organizing committee chairperson and program chairperson of IFIP TC-2/WG 2.6 Working Conference on Visual Database Systems in 1989 and program chairperson of IFIP TC-5/WG 5.10 Working Conference on Modeling in Computer Graphics in 1991.
He received his BSc, MSc, and DSc degrees in chemistry all from the University of Tokyo in 1962, 1964, and 1967, respectively.



HIRONOBU GOTODA is currently a master-course graduate student of information science at the University of Tokyo. His research interests include computer graphics, computer animation, and database systems. He received his BSc degree in information science from the University of Tokyo in 1989. He is a student member of the IEEE Computer Society and ACM.