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To cite this article: J Y Tsao *et al* 2010 *J. Phys. D: Appl. Phys.* **43** 354001

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# Solid-state lighting: an energy-economics perspective

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Received 12 March 2010, in final form 10 June 2010

Published 19 August 2010

Online at [stacks.iop.org/JPhysD/43/354001](http://stacks.iop.org/JPhysD/43/354001)

## Abstract

Artificial light has long been a significant factor contributing to the quality and productivity of human life. As a consequence, we are willing to use huge amounts of energy to produce it. Solid-state lighting (SSL) is an emerging technology that promises performance features and efficiencies well beyond those of traditional artificial lighting, accompanied by potentially massive shifts in (a) the consumption of light, (b) the human productivity and energy use associated with that consumption and (c) the semiconductor chip area inventory and turnover required to support that consumption. In this paper, we provide estimates of the baseline magnitudes of these shifts using simple extrapolations of past behaviour into the future. For past behaviour, we use recent studies of historical and contemporary consumption patterns analysed within a simple energy-economics framework (a Cobb–Douglas production function and profit maximization). For extrapolations into the future, we use recent reviews of believed-achievable long-term performance targets for SSL. We also discuss ways in which the actual magnitudes could differ from the baseline magnitudes of these shifts. These include: changes in human societal demand for light; possible demand for features beyond lumens; and guidelines and regulations aimed at economizing on consumption of light and associated energy.

(Some figures in this article are in colour only in the electronic version)

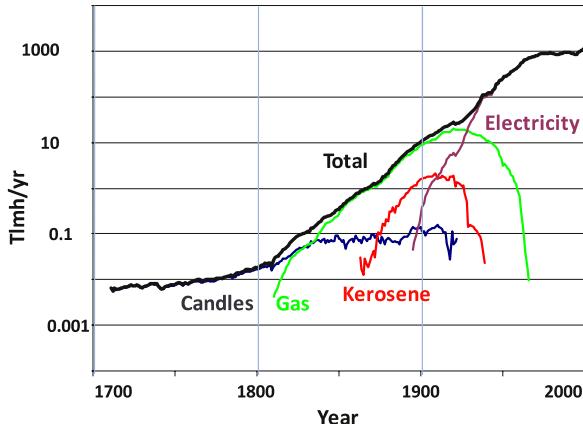
## 1. Introduction

The importance of artificial light to humans and human society has long been recognized. Though fire may have been used by our primate ancestors as far back as 2–6 million years ago (Burton 2009), it is still thought of as the quintessential human invention. Indeed, artificial light is so integrated into the human lifestyle as to be barely noticeable. As is sometimes said, artificial light ‘extends the day’ so that humans may be nearly as productive at night as during the day (Bowers 1998) and ‘opens up the indoors’ so that humans may be as productive indoors as outdoors, while enjoying the benefits of shelter from the vagaries of the environment (Schivelbusch 1988).

Indeed, the importance of (and demand for) artificial light is such that technologies for its more-efficient production

evolved spectacularly during the 18th, 19th and 20th centuries. This evolution is illustrated in figure 1, which shows a 5-order-of-magnitude increase in the consumption of artificial light over the past three centuries in the UK (Fouquet and Pearson 2006).

At this point in human history, artificial lighting consumes an estimated 0.72% of world gross domestic product and, because of its high energy intensity relative to that of other goods and services, an estimated 6.5% of world primary energy (Tsao and Waide 2010). These percentages are large and, coupled with increasing concern over world energy consumption, have inspired a number of projections of the consumption of light and associated energy into the future (Kendall and Scholand 2001, Tsao 2002, Navigant 2003, 2010, Schubert *et al* 2006). Such projections are of special



**Figure 1.** Three centuries of light consumption in the UK, adapted from Fouquet and Pearson (2006). The left axis has the units Tlm h/yr (teralumen-hours per year). The coloured lines represent consumption of light produced by technologies powered by particular fuels; the black line represents total consumption of light produced by all technologies. (Colour online.)

interest at this point in history when lighting technologies are evolving rapidly. Filament-based incandescent technology is giving way to gas-plasma-based fluorescent and high-intensity-discharge (HID) technology; and over the coming 10–20 years both may give way to solid-state lighting (SSL) technology (Schubert and Kim 2005, Shur and Zukauskas 2005, Krames *et al* 2007, Tsao *et al* 2010).

All these projections, however, have shared a common assumption—that consumption of light is relatively insensitive to the cost of light, and that evolution of lighting technology resulting in an increase in efficiency and a decrease in cost leads to a decrease in the consumption of energy rather than an increase in the consumption of light.

In this paper, we provide new projections of the consumption of light and associated energy. Rather than assuming that consumption of light is insensitive to the cost of light, we assume a sensitivity consistent with simple extrapolations of past behaviour into the future. In addition, we analyse the interplay between lighting, human productivity and energy consumption. After all, lighting is consumed not to waste energy, but to increase human productivity—energy consumption is simply the cost of that increased productivity. That this has been so in the past is self-evident; that it will be so in the future is not unlikely.

The rest of this paper is organized as follows<sup>3</sup>.

In section 2, we discuss recent studies of historical and contemporary consumption patterns and analyse them within the energy-economics framework of a simple Cobb–Douglas production function and profit maximization. In this framework, lighting is considered a factor affecting both production and energy consumption.

<sup>3</sup> Because this paper lies at the intersection between physical and social science, different sections of the paper may be difficult for the two communities. For the physical-science community, we recommend (Sorrell 2007) for an introduction to the economic concepts discussed in this paper. For the social-science community, we recommend (EERE 2010) and (Tsao 2002) for an introduction to the physics and engineering concepts discussed in this paper.

In section 3, we discuss recent estimates of the performance potential of SSL. Aside from its many other unique and beneficial attributes, SSL has the potential to increase the efficiency, and decrease the cost, of light factors 2–5× beyond that of current lighting technologies, including modern compact fluorescent lighting.

In section 4, we build on sections 2 and 3 to estimate the impact of SSL on human productivity (gdp) and the rate of energy consumption ( $\dot{e}$ ). We discuss how, within the Cobb–Douglas framework, human productivity and energy consumption are affected differently by improvements in the efficiency of lighting and by the cost of the energy that is converted into light. We also discuss the semiconductor chip area inventory and turnover associated with the lamps and luminaires that would be necessary in a world in which artificial lighting is dominated by SSL.

Finally, in section 5, we discuss alternative possible futures that deviate from the baseline future predicted from a simple extrapolation from the past. These include possible saturation in societal demand for lighting; possible demand for features beyond lumens; and government policies and regulations aimed at economizing on consumption of light.

## 2. Lighting, human productivity and energy

In this section, we discuss recent studies of historical and contemporary consumption patterns, and analyse them within the simple energy-economics framework of a Cobb–Douglas (Cobb and Douglas 1928) production function and profit maximization.

### 2.1. Consumption of light and associated energy

We begin with a discussion of a recent comprehensive study of historical and contemporary consumption of light (Tsao and Waide 2010). In that study, it was found that empirical data, drawn from a wide range of sources (Min *et al* 1997, Navigant 2002, Mills 2005, Fouquet and Pearson 2006, IEA 2006, Li 2007a, 2007b), were consistent with a per-capita consumption of light that is proportional to the ratio between per-capita gross domestic product and cost of light:

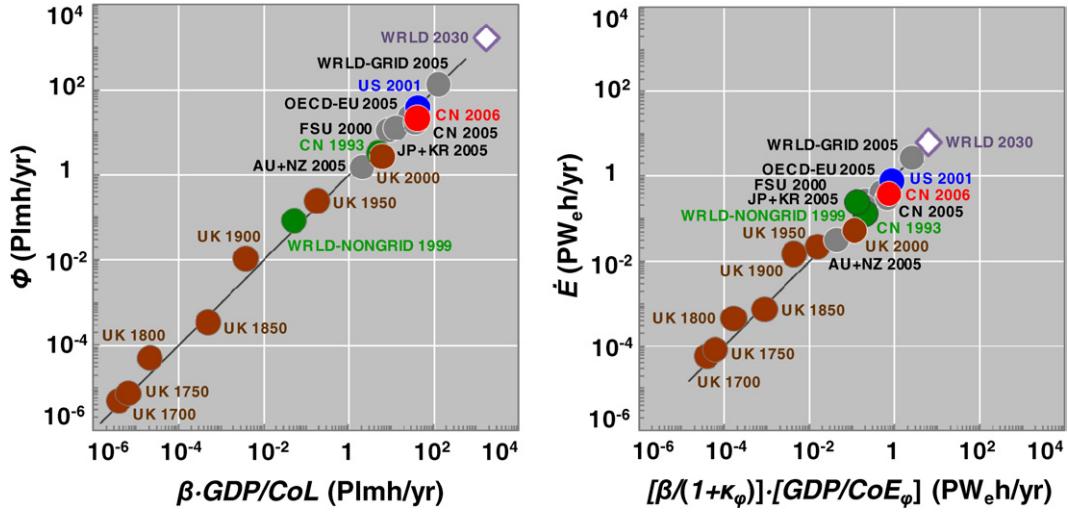
$$\varphi = \beta \cdot \frac{\text{gdp}}{\text{CoL}}. \quad (1)$$

In this equation:  $\varphi$  is per-capita consumption of light, in Mlmh/(per-yr) (megalumen-hours per person-year); gdp is per-capita gross domestic product, in \$(per-yr) (dollars per person-year); CoL is the ownership cost of light in \$/Mlmh (dollars per megalumen-hour) and  $\beta = 0.0072$  is a fixed constant<sup>4</sup>.

The consistency of this proportionality with empirical data is illustrated by the filled circles in the left panel<sup>5</sup> of figure 2. Each filled circle corresponds to independent empirical data

<sup>4</sup> Monetary units here and throughout this paper are year 2005 US\$.

<sup>5</sup> Note that, for our later purpose of comparing past and future total world consumption of light, in the left panel of figure 2 we plot total rather than per-capita quantities against each other. In other words, we plot  $\Phi = \beta \cdot \text{GDP}/\text{CoL}$ , which differs from equation (1) in that total quantities, denoted by uppercase symbols, are the per-capita quantities, denoted by lowercase symbols, multiplied by an appropriate population  $N$ .



**Figure 2.** The left panel shows consumption of light ( $\Phi$ ) plotted against  $\beta \cdot \text{GDP/CoL}$ , both in units of petalumen-hours per year. The right panel shows consumption of associated energy ( $\dot{E}_\varphi$ ) plotted against  $[\beta/(1 + \kappa_\varphi)] \cdot [\text{GDP/CoE}_\varphi]$ , both in units of petawatt-hours per year. The two relationships are the same as those of equations (1) and (4), except total rather than per-capita units are used (i.e. the left and right sides of those equations have been multiplied by population,  $N$ , as discussed in footnotes 2 and 5). The filled circles are data points (Tsao and Waide 2010) that illustrate the relationships represented by equations (1) and (4). The diagonal black lines have unity slope and zero offset. The various nation abbreviations are US = United States; CN = China; UK = United Kingdom; FSU = Former Soviet Union; OECD-EU = Organization for Economic Cooperation and Development Europe = Austria, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Sweden, Switzerland, UK, Ireland, Greece, Portugal, Spain, Hungary, Poland, Czech Republic, Slovak Republic, Turkey, Iceland, Luxembourg; JP + KR = Japan + South Korea; AU + NZ = Australia + New Zealand; WRLD = World; WRLD-NONGRID = World not on grid electricity; WRLD-GRID = World on grid electricity. The diamonds represent the consumptions of light and energy projected for the world in 2030, assuming the evolution and market penetration of SSL discussed in section 4, and a business-as-usual cost of energy.

for the three quantities in equation (1) for a nation or group of nations at a particular historical time. The filled circles fall very closely along a line of slope unity with zero offset.

The implication of the proportionality represented by equation (1) is that, over the past three centuries, and even now, the world spends about 0.72% of its GDP on light. This was the case in the UK in 1700 (UK 1700), is the case in the undeveloped world not on grid electricity (WRLD-NONGRID 1999) in modern times, and is the case for the developed world in modern times using the most advanced lighting technologies (WRLD-GRID 2005). For a 2005 world GDP of about 61 T\$/yr (Maddison 2007) this represents an expenditure of about 440 B\$/yr =  $\beta \cdot \text{GDP}$  and on a 2005 aggregate world CoL of about 3.35 \$/Mlmh this represents a world consumption of light of about 131 Plm h/yr =  $\beta \cdot \text{GDP/CoL}$  (Tsao and Waide 2010).

Note that, from equation (1) and a knowledge of the luminous efficacies ( $\eta_\varphi$ , in units of  $\text{Im W}^{-1}$ ) associated with each data point, we can also estimate the consumption of energy associated with the consumption of light<sup>6</sup>. The reason is that luminous efficacy connects two pairs of quantities.

The first pair is per-capita consumption of light ( $\varphi$ ) and per-capita rate of consumption of associated energy ( $\dot{e}_\varphi$ ) to produce the light:

$$\varphi = \dot{e}_\varphi \cdot \eta_\varphi. \quad (2)$$

<sup>6</sup> Note that we will often refer to these quantities simply as consumption of light or energy, though precisely speaking they are *rates* of consumption of light or energy.

Consumption of light is simply the product of the rate of consumption of energy and luminous efficacy.

The second pair is the cost of light (CoL, in units of \$/Mlmh) and the cost of the associated energy (CoE<sub>φ</sub>, in units of \$/MW<sub>e</sub>h)<sup>7</sup>:

$$\text{CoL} = \frac{\text{CoE}_\varphi}{\eta_\varphi} \cdot (1 + \kappa_\varphi). \quad (3)$$

Cost of light is basically the cost of the associated energy divided by luminous efficacy, within a correction factor,  $\kappa_\varphi \approx 1/3$ , which takes into account the approximately fixed ratio of the capital cost (lamp and luminaire) to operating cost (fuel) of light (Tsao and Waide 2010).

Thus, we can rewrite equation (1) as

$$\dot{e}_\varphi = \frac{\beta}{(1 + \kappa_\varphi)} \cdot \frac{\text{gdp}}{\text{CoE}_\varphi}, \quad (4)$$

and can replot the data in the left panel of figure 2 using the modified axes in the right panel. Because equation (4) is essentially equivalent to equation (1), the data points in the right panel<sup>8</sup> of figure 2 fall on a line of slope unity with zero

<sup>7</sup> For energy units here and throughout this paper we use W<sub>c</sub>h, Btu or Quads ( $10^{15}$  or a quadrillion BTUs) for primary chemical energy, W<sub>e</sub>h for equivalent electrical energy and a factor 0.316 W<sub>e</sub>/W<sub>c</sub> to convert between them.

<sup>8</sup> Note that, for our later purpose of comparing past and future total world consumption of energy, in the right panel of figure 2 we plot total rather than per-capita quantities against each other. In other words, we plot  $E_\varphi = [\beta/(1 + \kappa_\varphi)] \cdot \text{GDP/CoE}_\varphi$ , which differs from equation (4) in that total quantities, denoted by upper-case symbols, are the per-capita quantities, denoted by lower-case symbols, multiplied by an appropriate population  $N$ .

offset, just as did those in the left panel. However, because the data points correspond to different luminous efficacies, depending on time period and prevailing technology, the relative placements of the data points in the two figures are not the same.

The analogous implication of the proportionality of equation (4) is that, over the past three centuries, and even now, the world spends about  $0.54\% = \beta/(1 + \kappa_\varphi)$  of its GDP on the consumption of energy associated with light. For a 2005 world GDP of about 61 T\$/yr this represents an expenditure of about 330 B\$/yr =  $[\beta/(1 + \kappa_\varphi)] \cdot \text{GDP}$  and on a 2005 aggregate world CoE $_\varphi$  of about 119 \$/MW<sub>eh</sub>, this represents a world consumption of energy of about 2.7 PW<sub>eh</sub>/yr =  $[\beta/(1 + \kappa_\varphi)] \cdot \text{GDP}/\text{CoE}_\varphi$ .

Note that this consumption of energy represents about 16% of the world's total electrical energy generation of about 16.9 PW<sub>eh</sub>/yr in 2005 (EIA 2009c). And, since 2.7 PW<sub>eh</sub>/yr of electrical energy is equivalent to roughly 8.5 PW<sub>eh</sub>/yr and 29.5 Quads/yr = 1 TW<sub>c</sub> of primary chemical energy, this represents about 6.5% of the world's consumption of 460 Quads/yr = 16 TW<sub>c</sub> of primary energy in 2005 (EIA 2009c). That lighting represents a larger percentage (6.5%) of energy consumption than it does of GDP (0.72%) is a reflection of the extreme energy intensity of lighting (as of other energy services such as heating, cooling and transportation) compared with other goods and services in the economy.

Also note that the consumption of associated energy shown in the right panel of figure 2 does not span as wide a dynamic range (4.8 orders of magnitude) as the consumption of light (7.5 orders of magnitude) shown in the left panel. The reason is that cost of energy does not span as wide a range as cost of light, which has also benefited from the steady advancement, over the centuries, in luminous efficacy<sup>9</sup>.

## 2.2. Human productivity: a simple Cobb–Douglas model

In section 2.1, we discussed the empirical result that per-capita consumption of light is proportional to the ratio between per-capita gross domestic product and cost of light. Per-capita consumption of light thus superficially appears to be determined by an independently determined per-capita gross domestic product. In fact, per-capita gross domestic product is itself influenced by the cost and consumption of light—as discussed in section 1, light enables us to do useful work and enhances our productivity. In other words, there must be an *interplay* between consumption of light and economic productivity that self-consistently determines both.

In this section, we model this interplay using a simple Cobb–Douglas production function (Cobb and Douglas 1928). Although many other production functions have been studied (Saunders 2008), particularly in connection with the rebound effect<sup>10</sup> in energy economics, we choose Cobb–Douglas

<sup>9</sup> This steady advancement was first made quantitative in W D Nordhaus' classic study of the luminous efficacies of lighting technologies throughout history (Nordhaus 1997).

<sup>10</sup> The degree to which reductions in energy consumption are less than would be expected from improvements in the engineering efficiencies of energy technologies (Alcott 2005, Saunders 2000, Sorrell 2007).

for a number of reasons. First, as discussed later in this section, it is consistent with the empirical relationship of equation (1). Second, it is compact, relatively easy to manipulate analytically, and widely (perhaps the most widely) used in neoclassical economics. Third, it predicts a relatively large rebound effect in consumption of light and energy, and it is partly our purpose to show that, *even if* such a large rebound occurs, there would be significant benefits to SSL when the inter-relationships between consumption of light, gross domestic product and energy consumption are taken into account. Nevertheless, the Cobb–Douglas production function is not, as discussed by Saunders (Saunders 2008), ‘rebound-flexible’, and thus cannot represent the full range of possible rebound effects.

We begin by writing the Cobb–Douglas per-capita production function as

$$\text{gdp}(\chi, \varphi) = A \cdot \chi^\alpha \varphi^\beta, \quad (5)$$

along with a linear per-capita cost function as

$$\text{cost}(\chi, \varphi) = \chi \cdot \text{CoX} + \varphi \cdot \text{CoL}. \quad (6)$$

The per-capita production function,  $\text{gdp}(\chi, \varphi)$ , contains two factors of production:  $\varphi$  (per-capita consumption of light), which is the factor of production we wish to focus on here; and  $\chi$ , which represents all other factors of production (including capital, materials, other energy services, etc) except labour. It also contains a proportionality constant,  $A$ , and two exponents,  $\beta$  and  $\alpha$ , which from inspection of equation (5) can be seen to quantify the relative importance of the two production factors to gdp. Note that this per-capita production function is derived by normalizing the constant-returns-to-scale non-per-capita production function,  $\text{GDP}(X, \Phi, N) = AX^\alpha \Phi^\beta N^{1-\alpha-\beta}$ , by population,  $N = N^\alpha N^\beta N^{1-\alpha-\beta}$ . The three exponents  $\alpha$ ,  $\beta$  and  $1 - \alpha - \beta$  quantify the relative importance of the three production factors  $X$ ,  $\Phi$  and  $N$  to GDP. If we estimate the population (or labour) portion of production to be  $1 - \alpha - \beta = 0.7$  (Jones 2002), then we can also estimate that  $\alpha + \beta = 0.3$ .

The cost function,  $\text{cost}(\chi, \varphi)$  in equation (6), is the sum of the same two production factors,  $\varphi$  and  $\chi$  (which are of course also cost factors), weighted by their unit costs, CoX and CoL.

These two functions,  $\text{gdp}(\chi, \varphi)$  and  $\text{cost}(\chi, \varphi)$ , can be thought of as surfaces above a two-dimensional  $(\chi, \varphi)$  plane, with the shape of the gdp surface defined by the parameters  $A$ ,  $\alpha$  and  $\beta$  and the shape of the cost surface defined by the parameters CoX and CoL. Profit is the difference between the two surfaces,

$$\pi = \text{gdp}(\chi, \varphi) - \text{cost}(\chi, \varphi), \quad (7)$$

and is a quantity that we assume the economy maximizes at particular values of  $\chi$  and  $\varphi$ .

The profit maximizing conditions (often called the ‘first-order conditions’) that solve this problem are  $\partial\pi/\partial\chi = 0$  and  $\partial\pi/\partial\varphi = 0$ . These conditions equate the marginal productivity of each input to its marginal cost and reflect parallel tangency conditions on the two surfaces. Substituting

the per-capita production and cost functions of equations (5) and (6) then gives

$$\frac{\partial \text{gdp}(\chi, \varphi)}{\partial \varphi} - \text{CoL} = \frac{\beta}{\varphi} \cdot \text{gdp}(\chi, \varphi) - \text{CoL} = 0, \quad (8a)$$

$$\frac{\partial \text{gdp}(\chi, \varphi)}{\partial \chi} - \text{CoX} = \frac{\alpha}{\chi} \cdot \text{gdp}(\chi, \varphi) - \text{CoX} = 0. \quad (8b)$$

Solving these yields

$$\varphi = \beta \frac{\text{gdp}}{\text{CoL}}, \quad (9a)$$

$$\chi = \alpha \frac{\text{gdp}}{\text{CoX}}. \quad (9b)$$

Re-substituting these two equations back into equation (5) then leads to an expression for gdp at the profit maximization point:

$$\text{gdp} = A^{1/(1-\alpha-\beta)} \cdot \left( \frac{\alpha}{\text{CoX}} \right)^{\alpha/(1-\alpha-\beta)} \cdot \left( \frac{\beta}{\text{CoL}} \right)^{\beta/(1-\alpha-\beta)}. \quad (10)$$

Finally, substituting equation (10) back into equations (9a) and (9b) enables the point  $(\chi, \varphi)$  where profit is maximized to be defined exactly:

$$\chi = A^{1/(1-\alpha-\beta)} \left( \frac{\alpha}{\text{CoX}} \right)^{(1-\beta)/(1-\alpha-\beta)} \cdot \left( \frac{\beta}{\text{CoL}} \right)^{\beta/(1-\alpha-\beta)}, \quad (11a)$$

$$\varphi = A^{1/(1-\alpha-\beta)} \cdot \left( \frac{\alpha}{\text{CoX}} \right)^{\alpha/(1-\alpha-\beta)} \cdot \left( \frac{\beta}{\text{CoL}} \right)^{(1-\alpha)/(1-\alpha-\beta)}. \quad (11b)$$

Interestingly, the model equation (9a) is identical to the empirical equation (1). The Cobb–Douglas production function, as mentioned earlier, is therefore consistent with empirical findings with respect to  $\varphi$ . Given a similar empirical finding (analogous to equation (1)) for  $\chi$ , it could be shown that the two factors of production must then have unity elasticity of substitution between them<sup>11</sup>, and that *only* a Cobb–Douglas production would be consistent with our empirical findings (Saunders 2009). To our knowledge, however, such a similar empirical finding for  $\chi$  does not exist. Nevertheless, it is not implausible, for complex and nested multi-stage production systems (Lowe 2003), and over very long (decades to centuries) historical time periods, that elasticity of substitution tends towards unity and that Cobb–Douglas is a reasonable baseline model for our current purpose.

Also, since the model equation (9a) is identical to the empirical equation (1), we can equate the two  $\beta$ 's, with  $\beta = 0.0072$ . Using  $\alpha + \beta = 0.3$ , we then have  $\alpha = 0.2928$ . In other words, lighting, though important, is nonetheless a small fraction of a large world economy, with  $\beta \ll \alpha$ .

With these relative magnitudes of  $\alpha$  and  $\beta$  in mind, one can now see from equation (9a) the effect on  $\varphi$  of a unit decrease in CoL. The larger effect is a direct unit increase

<sup>11</sup> Elasticity of substitution is a measure of the ease with which various factors of production may be substituted for each other: formally (Varian 1984), the elasticity of the ratio of two inputs to a production function with respect to the ratio of their marginal rates of substitution.

in  $\varphi$  for a unit decrease in CoL. The smaller effect is an indirect subunit increase in  $\varphi$  due, from equation (10), to a small ( $0.01 = \beta/[1 - \alpha - \beta]$ ) subunit increase in gdp for a unit decrease in CoL. Put another way, consumption of light increases as cost of light decreases. But consumption of light, as a production factor, also mediates a small increase in gdp, and this causes consumption of light to increase very slightly more. Thus, in total,  $\varphi$  increases by 1.01 units for a 1 unit decrease in CoL.

### 2.3. Energy intensity: the cost of human productivity

As mentioned in section 1, just as the two factors of production enable production, they also consume energy. If we write their per-capita energy-consumption rates as

$$\dot{e}_\chi = \frac{\chi}{\eta_\chi}, \quad (12)$$

$$\dot{e}_\varphi = \frac{\varphi}{\eta_\varphi}, \quad (13)$$

where  $\eta_\chi$  and  $\eta_\varphi$  are the efficacies with which energy is used to produce  $\chi$  and  $\varphi$ , then we can write for total per-capita energy-consumption rate:

$$\dot{e} = \dot{e}_\chi + \dot{e}_\varphi = \frac{\chi}{\eta_\chi} + \frac{\varphi}{\eta_\varphi}. \quad (14)$$

The energy intensity<sup>12</sup> is this total per-capita energy-consumption rate (equation (14)) divided by gdp (equation (10)). This gives, substituting equations (9a) and (9b) into equation (14):

$$\frac{\dot{e}}{\text{gdp}} = \frac{\alpha}{\eta_\chi \cdot \text{CoX}} + \frac{\beta}{\eta_\varphi \cdot \text{CoL}}. \quad (15)$$

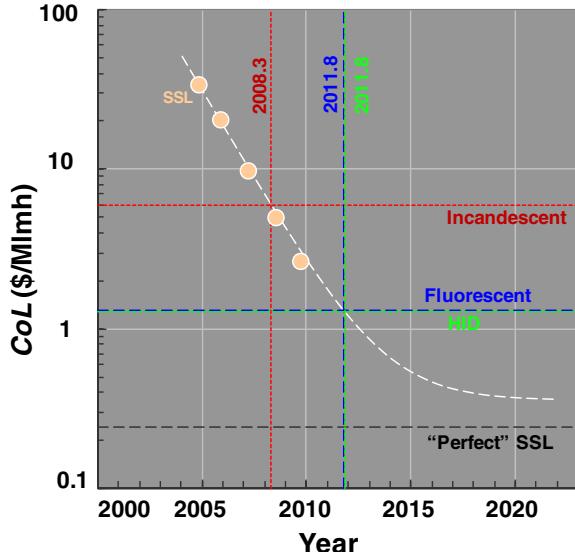
From this equation, it appears that energy intensity decreases with increases in the energy efficacies  $\eta_\chi$  and  $\eta_\varphi$ . For an energy service such as lighting, whose dominant cost is the cost of energy, however, this is not the case. Because the cost of light (CoL) is dominated by the cost of energy (up to the correction factor  $1 + \kappa_\varphi$ ), and therefore decreases as the luminous efficacy  $\eta_\varphi$  with which it is produced increases, energy intensity is actually independent of that luminous efficacy (Saunders 2008). This can be seen by substituting equation (3) into equation (15) to get

$$\frac{\dot{e}}{\text{gdp}} = \frac{\alpha}{\eta_\chi \cdot \text{CoX}} + \frac{\beta}{\text{CoE}_\varphi \cdot (1 + \kappa_\varphi)}. \quad (16)$$

Note that this independence of energy intensity on energy efficiency will not be the case for  $\chi$ , the other factor of production, as the cost of  $\chi$  (CoX) will contain additional significant costs that are independent of (do not decrease with) the energy efficacy with which  $\chi$  is produced.

The independence of energy intensity on luminous efficacy might seem paradoxical, but can be understood by substituting equation (3) into equations (9a) and (10) to

<sup>12</sup> As indicated in table 1, we use for energy intensity the units Btu/\$, or British Thermal Units per US dollar.



**Figure 3.** Evolution of SSL cost of light (CoL). Solid tan-coloured data points are from 2004–2009 state-of-the-art commercial devices described in (Tsao *et al* 2010), with CoL calculated assuming equivalent  $R_a = 85$  and CCT = 3800 K. The white curved line through the data points is an exponential fit with a 1.95-year time constant and a saturation CoL corresponding to an RGB light source, also described in (Tsao *et al* 2010). The coloured horizontal lines represent the 6.0, 1.3 and 1.3 \$/Mlmh costs of light for incandescent, fluorescent and HID lamps in 2001. The coloured vertical dashed lines represent the years at which SSL lamps achieved, or might be projected to achieve, parity with traditional lamps. Note that, unlike in the rest of this paper, the CoL plotted here includes only the operating cost of light and the capital cost of the lamp. Luminaire costs are not included, as these are evolving rapidly and are difficult to quantify.

separate the dependence of per-capita light consumption and gdp on luminous efficacy and cost of energy:

$$\varphi = \frac{\beta}{(1 + \kappa_\varphi)} \cdot \frac{gdp \cdot \eta_\varphi}{CoE_\varphi}, \quad (17)$$

$$gdp = A^{\frac{1}{1-\alpha-\beta}} \cdot \left( \frac{\alpha}{CoX} \right)^{\alpha/(1-\alpha-\beta)} \cdot \left( \frac{\beta \cdot \eta_\varphi}{CoE_\varphi \cdot (1 + \kappa_\varphi)} \right)^{\beta/(1-\alpha-\beta)}. \quad (18)$$

From equation (17), as luminous efficacy increases, per-capita consumption of light increases. This increase exactly cancels the reduction in per-capita energy consumption that would otherwise have occurred, and hence does not alter energy intensity. From equation (18), as luminous efficacy increases, there is also a smaller increase in per-capita gdp. But this increase is exactly matched by a concomitant and proportional small increase in per-capita energy consumption, and hence also does not alter energy intensity.

### 3. SSL: performance and cost projections

In this section, we recapitulate recent estimates of the performance potential of SSL (Tsao *et al* 2010). We choose a year, 2030, distant enough for the performance of SSL to be nearly saturated and the transition to SSL nearly complete. We choose a performance metric, cost of light (CoL), which is the

key parameter that enters into consumption of light, and which then couples to gdp, energy consumption and energy intensity. We discuss first the operating cost of light, then the capital cost of light, then the cost of light, which is sum of the two.

#### 3.1. Operating cost of light

The operating cost of light ( $CoL_{ope}$ , in units of \$/Mlmh) is simply the cost of electricity divided by luminous efficacy:

$$CoL_{ope} = \frac{CoE_\varphi}{\eta_\varphi}. \quad (19)$$

For the cost of electricity, we assume the estimated world aggregate of 119 \$/MW<sub>e</sub>h for 2005 (Tsao *et al* 2010, Tsao and Waide 2010). One might anticipate that this cost will have increased by 2030. Here, however, we assume a business-as-usual scenario in which this increase is small, as has been projected for the US (EIA 2009b). In section 4, though, we relax this assumption, and allow the cost of electricity to increase.

For luminous efficacy, we note that, as has been discussed recently (Phillips *et al* 2007), there is a limiting luminous efficacy for the production of high quality white light which renders well the colours of typical environments. For a correlated colour temperature (CCT) of 3800 K and a colour rendering index (CRI) of 85 (market-weighted averages for the US in 2001), this limiting luminous efficacy is roughly 400 lm/W<sub>e</sub> (Tsao *et al* 2010). In practice, the present luminous efficacies of SSL technology are far less than this limiting value. However, they are improving rapidly, and might ultimately reach 65–70% of this limiting value or  $\eta_\varphi \approx 268 \text{ lm W}^{-1}$ . Indeed, one might anticipate many variants of SSL with different combinations of luminous efficacy, colour rendering and colour temperature, tailored to particular applications. Some might have efficiencies as high as 86%, the current best efficiency for any semiconductor light emitter (Peters *et al* 2007). Our assumption of 268 lm W<sup>-1</sup> is thus considered to be in the mid-range of these variants.

Putting these two projections together, we assume that the operating cost of light, in 2030, will be on the order of  $CoL_{ope} = (119 \text{ $}/\text{MW}_e\text{h})/(268 \text{ lm W}^{-1}) = 0.44 \text{ $}/\text{Mlmh}$ .<sup>13</sup>

#### 3.2. Capital cost of light

The capital cost of light ( $CoL_{cap}$ , in units of \$/Mlmh) is the cost of the lamps and luminaires that produce the light. As was mentioned earlier, for traditional lighting this cost is about 1/3 of the operating cost of light, with roughly 1/6 due to the lamps and 1/6 due to the luminaires. For SSL technology, the capital cost of light is currently significantly higher than the operating cost. This is true both for the lamps that produce the light as well as for the luminaires that house the lamps and direct the light.

For SSL luminaires, costs are decreasing rapidly. Moreover, because SSL lamps are small, SSL luminaires

<sup>13</sup> As a point of reference, this operating cost is equivalent to 0.44 ¢ (less than half a penny) to produce 1000 lm (that produced by a typical 75 W incandescent bulb) for 10 h. The operating cost of an actual 75 W incandescent bulb run for 10 h, in comparison, would be 9 ¢.

can also be small and therefore have more headroom than traditional-lamp luminaires for continued cost decrease. In addition, because SSL lamps and luminaires have comparable lifetimes, tighter integration of the two in design and manufacturing may provide even more headroom for cost decrease. We thus make a reasonable assumption here that SSL luminaire costs have the potential to decrease to 1/6 of SSL operating costs.

For SSL lamps, costs are also decreasing rapidly. In fact, it has been shown that believed-achievable increases in the density of current injected into the semiconductor chips at the heart of SSL can by themselves enable lamp costs of 1/6 or less than the operating cost (Tsao *et al* 2010). Anticipated manufacturing and yield improvements that will accompany large-scale production will decrease lamp cost further. Indeed, as with semiconductor technologies in general, the capital cost of SSL per unit performance has significant potential to continue its Haitz's Law decrease (Haitz *et al* 1999, Martin 2001, NP 2007). We conclude that SSL lamp costs also have the potential to decrease to 1/6 of SSL operating costs.

Taken together, then, we assume here that the capital-cost-to-operating-cost ratio of SSL will be the same, in 2030, as the 1/3 that characterizes current traditional lighting. In other words,

$$\text{CoL}_{\text{cap}} = \kappa_\varphi \cdot \text{CoL}_{\text{ope}}, \quad (20)$$

where  $\kappa_\varphi = 1/3$ . Indeed,  $\kappa_\varphi$  has the potential to be lower than this. However, if it were much lower than this, the lamp and luminaire would essentially be free, and manufacturers would have incentive to add features so as to add back cost.

Note that, in assuming that the capital cost of SSL lamps and luminaires will be about 1/3 of their operating cost, we have implicitly assumed that subcosts associated with this capital cost are even less than this. Among these subcosts is that for the energy required to manufacture the lamp and luminaire or the energy 'embodied' in the lamp and luminaire. Hence, we have basically assumed that the energy embodied in the lamp and luminaire is much lower than the energy consumed by the lamp during its life. In fact, it is *much* less (Tsao and Waide 2010). Recent estimates are 1/66 for a 2009 state-of-the-art SSL lamp (Siemens AG 2009), and 1/36 for a 2009 state-of-the-art SSL integrated lamp and luminaire combination (Navigant 2009b). Thus, the dominant costs to manufacture SSL lamps and luminaires are materials and labour rather than embodied energy.

### 3.3. Ownership cost of light

The ownership cost of light (CoL, in units of \$/Mlmh) is the sum (Rea 2000, Dowling 2003, Azevedo *et al* 2009) of the operating and capital costs of light we have just discussed:

$$\text{CoL} = \frac{\text{CoE}_\varphi}{\eta_\varphi} \cdot (1 + \kappa_\varphi). \quad (21)$$

This equation (21) is identical to (and forms the basis for) equation (3). The two terms summed on the right side of these equations are simply the operating and capital contributions to the cost of light, with the operating fraction

equal to  $1/(1 + \kappa_\varphi) \sim 3/4$  and the capital fraction equal to  $\kappa_\varphi/(1 + \kappa_\varphi) \sim 1/4$ .

Note that, in principle, ownership cost of light should include the entire life cycle cost of light—not only the operating and capital costs but also the environmental impacts of the materials, manufacturing and transport-to-point-of-use associated with the lamp and luminaire, the power consumed during operation of the lamp and the end-of-life recycling or disposal of the lamp and luminaire. In fact, it has been shown that the environmental impact of SSL is dominated by that associated with the power consumed during operation of the lamp (Navigant 2009b, Siemens AG 2009). Hence, we assume the environmental impact of SSL can be accounted for by a shift in (or tax on) the cost of energy (to account for that environmental impact). A business-as-usual scenario would be the absence, while an environmentally proactive scenario would be the presence, of such a shift (or tax).

Note also that the absence or presence of shifts in the cost of energy affects all lighting technologies. Hence, the dominant factor affecting the relative competitiveness of SSL relative to traditional lighting is not cost of energy, but technology improvement in manufacturing (basically  $\kappa_\varphi$  in equation (21)) and performance (basically  $\eta_\varphi$  in equation (21)) of SSL.

Recently, the prospects for such technology improvement (for the lamp, but not the luminaire) have been analysed (Tsao *et al* 2010), and the results are shown in figure 3. From this evolution, one sees that the cost of light for SSL lamps became less than that of incandescent lamps in ~2008 and, if it continues to decrease at its past-several-year rate, will reach parity with those of fluorescent and HID lamps in ~2012. Assuming progress at a similar rate beyond this, but an eventual saturation in luminous efficacy at 268 lm W<sup>-1</sup> (corresponding to a 67%-efficient all-LED approach), one can project that SSL will approach this saturation in ~2020.

We note that these projections do not include SSL luminaires, whose evolution will very likely trail that of solid-state lamps somewhat. Hence, including luminaires will shift all of these dates out somewhat, perhaps by several years.

## 4. SSL: baseline futures ( $\beta = 0.0072$ )

In section 2 we considered what is known empirically about consumption of light in the past: that it scales as gdp/CoL with a proportionality constant  $\beta = 0.0072$ . We also developed a simple Cobb–Douglas model within which one can view the interplay between consumption of light, human productivity and energy consumption.

In section 3 we reviewed recent projections of SSL luminous efficacies ( $\eta_\varphi$ ) into the future. Given various scenarios for the cost of energy for lighting (CoE <sub>$\varphi$</sub> ), one can then infer various scenarios for cost of light (CoL).

In this section, we describe 'baseline future' scenarios in which SSL is assumed to be the dominant lighting technology, and in which consumption of light in the future is characterized by the same proportionality constant  $\beta = 0.0072$  that has been characteristic of it in the past. Note that these baseline futures

**Table 1.** Macroeconomic 2030 scenarios for various assumptions on the luminous efficacy of lighting and on the cost of energy for lighting. As discussed in the text, in scenarios FLU-L and FLU-H fluorescent lamps are dominant. In scenarios SSL-L, SSL-M and SSL-H solid-state lamps are dominant. In scenarios FLU-L and SSL-L cost of energy is low and similar to that in 2005 (business as usual). In scenarios SSL-M, SSL-H and FLU-H cost of energy is medium or high, and increased over business-as-usual projections. As a point of comparison, the unlabelled scenario in the first row represents actual luminous efficacies of lighting, costs of energy for lighting, and gdp and energy consumptions for 2005.

Scenario		Lighting efficiency and cost				Consumption of light and associated energy				World economy						Chip area			
Scenario	Year	Predominant energy source or lighting technology	Luminous efficacy ( $\eta_\varphi$ )	Cost of electricity for lighting ( $\text{CoE}_\varphi$ )	Cost of primary energy for lighting ( $\text{CoE}_\varphi$ )	Cost of light (Col)	per capita consumption of light ( $\varphi = \beta \cdot \text{gdp}/\text{Col}$ )	per capita consumption of electricity associated with light ( $\dot{\epsilon}_\varphi = \beta \cdot \text{gdp}/[(1+\kappa_\varphi) \cdot \text{CoE}]$ )	Consumption of light ( $\Phi = \beta \cdot \text{GDP}/\text{Col}$ )	Consumption of electricity associated with light ( $\dot{\Phi}_\varphi = \beta \cdot \text{GDP}/[(1+\kappa_\varphi) \cdot \text{CoE}_\varphi]$ )	Population (N)	Gross domestic product (GDP)	per capita gross domestic product (gdp)	Energy intensity ( $\dot{E}/\text{GDP}$ )	Consumption of primary energy ( $\dot{E}$ )	per capita consumption of primary energy ( $\dot{\epsilon}$ )	Semiconductor chip area inventory	Semiconductor chip area annual production (transition period)	Semiconductor chip area annual production (steady state)
			Mlmh W <sub>e</sub>	1\$ MW <sub>e</sub> h	1\$ MBtu	1\$ Mlmh	Mlmh per-yr	MW <sub>e</sub> h per-yr	Plmh yr	PW <sub>e</sub> h yr	Gper	G\$ yr	1\$ per-yr	Btu \$	Quad yr	MBtu per-yr	km <sup>2</sup> yr	km <sup>2</sup>	km <sup>2</sup> yr
Ker+Inc+	2005 Flu+HID	48	119	11.0	3.35	20.0	0.42	131	2.7	6.5	60670	9317	8071	490	75.2				
FLU-L	2030 Flu	87	119	11.0	1.84	64.7	0.75	539	6.2	8.3	137484	16511	4934	678	81.5				
SSL-L	2030 SSL	268	119	11.0	0.59	202.7	0.76	1688	6.3	8.3	139091	16704	4934	686	82.4	1.31	0.174	0.076	
SSL-M	2030 SSL	268	133	12.3	0.66	181.2	0.68	1508	5.6	8.3	138932	16685	4882	678	81.5	1.17	0.156	0.068	
SSL-H	2030 SSL	268	369	34.2	1.84	64.8	0.24	539	2.0	8.3	137484	16511	4603	633	76.0	0.42	0.056	0.024	
FLU-H	2030 Flu	87	369	34.2	5.68	20.7	0.24	172	2.0	8.3	135896	16320	4603	625	75.1				

within this discussion may have very different costs of energy for lighting, and hence very different costs of light, but we nonetheless consider them baseline in that the historical pattern of consumption of light does not change.

We begin by discussing various luminous-efficacy and costs-of-energy-for-lighting scenarios, analysing these within the Cobb–Douglas framework. Then we discuss the semiconductor chip area inventory and turnover that would be necessary to sustain these baseline futures. Finally, we compare these industry-scale figures with those of other important semiconductor technologies.

Because we are interested in the long-term impact of SSL, we choose a year, 2030, distant enough into the future that its ultimate performance potential, at least in terms of raw cost, might be expected to have been achieved. Moreover, this is a year in which the transition to SSL, anticipated to be massive and worldwide in scope, might also be expected to be well underway.

#### 4.1. Human productivity and energy consumption

Our starting point for understanding the impact of SSL on human productivity and energy consumption is equations (18) and (16). From initial values of per-capita gross domestic product ( $\text{gdp}_0$ ) and energy intensity ( $\dot{\epsilon}_0/\text{gdp}_0$ ) given initial values of luminous efficacy ( $\eta_{\varphi 0}$ ) and cost of energy for lighting ( $\text{CoE}_{\varphi 0}$ ), these equations can be used to estimate modified values of per-capita gross domestic product ( $\text{gdp}$ ) and energy intensity ( $\dot{\epsilon}/\text{gdp}$ ) given modified values of luminous efficacy

( $\eta_\varphi$ ) and cost of energy for lighting ( $\text{CoE}_\varphi$ ):

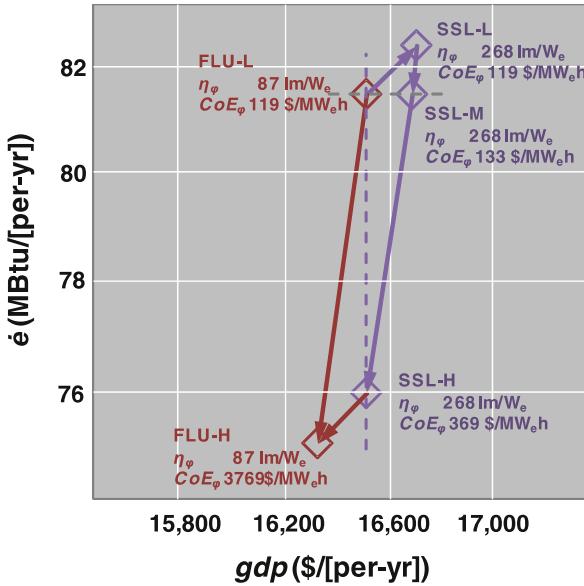
$$\frac{\text{gdp}}{\text{gdp}_0} = \left( \frac{\text{CoL}_0}{\text{CoL}} \right)^{\beta/(1-\alpha-\beta)} = \left( \frac{\text{CoE}_{\varphi 0}}{\text{CoE}_\varphi} \cdot \frac{\eta_\varphi}{\eta_{\varphi 0}} \right)^{\beta/(1-\alpha-\beta)}, \quad (22)$$

$$\frac{\dot{\epsilon}}{\dot{\epsilon}_0} = \frac{\beta}{\text{CoE}_\varphi \cdot (1 + \kappa_\varphi)} - \frac{\beta}{\text{CoE}_{\varphi 0} \cdot (1 + \kappa_{\varphi 0})}. \quad (23)$$

In this manner, we can essentially normalize the Cobb–Douglas model around initial projections for human productivity and energy consumption in the year 2030. Using these normalizations, various future scenarios are calculated in table 1 and illustrated in figure 4<sup>14</sup>.

Scenario FLU-L shows a hypothetical initial world in 2030 based on Energy Information Agency projections (EIA 2009a). In this world, per-capita world gross domestic product is  $\text{gdp} = 16511 \text{ } \$/\text{per-yr}$ , per-capita world primary energy consumption is  $\dot{\epsilon} = 81.5 \text{ } \text{MBtu}/\text{per-yr}$ , energy intensity is  $4934 \text{ } \text{Btu}/\text{$}$  and world population is  $N = 8.33$  billion. As these are business-as-usual projections, we assume they do not take into account breakthrough technologies such as SSL and hence use  $\eta_\varphi = 87 \text{ lm/W}_e$ , that anticipated for improved fluorescent lamps (Navigant 2009a), to be the initial world aggregate luminous efficacy. As mentioned in section 3.1, we also assume that the cost of electricity does not change much

<sup>14</sup> We note that, in deriving these equations, we have implicitly assumed that CoX is fixed. In practice, CoX cannot be fixed, as at minimum a shift in cost of energy is likely to apply to all energy consumption and not just to that for lighting. We have not modelled this, as economy-wide effects of increases in cost of energy are beyond the scope of this paper. However, the same qualitative behaviour discussed in this section would result, even if the quantitative results would be different.



**Figure 4.** Projected world 2030 per-capita primary energy consumption and per-capita gross domestic product scenarios for various assumptions on luminous efficacy ( $\eta_\varphi$ ) and cost of energy for lighting ( $\text{CoE}_\varphi$ ). Both axes are gridded so that equal vertical or horizontal grid spacings represent equal percentage changes. The point labelled FLU-L can be considered the ‘reference’ values for ( $\text{gdp}$ ,  $\dot{e}$ ) from which the other values are calculated from equations (22) and (23).

between now and the year 2030, an assumption similar to that made for the US (EIA 2009b). Hence, we assume the same,  $\text{CoE}_\varphi = 119 \$/\text{MW}_e\text{h}$ , as that estimated for 2005 (Tsao and Waide 2010).

Scenario SSL-L shows a hypothetical world similar to that of scenario FLU-L except with a luminous efficacy increased, due to a complete transition to SSL, to  $\eta_\varphi = 268 \text{ lm}/\text{W}_e$ . As anticipated from equations (22) and (23), the roughly  $102\% = 2 \cdot (268 - 87)/(268 + 87)$  increase in  $\eta_\varphi$  manifests itself as roughly  $102\% \cdot [\beta/(1 - \alpha - \beta)] \sim 1.02 \cdot 0.01 \sim 1\%$  increases in both  $\text{gdp}$  and  $\dot{e}$ , to  $\text{gdp} = 16\,704 \$/(\text{per-yr})$  and  $\dot{e} = 82.4 \text{ MBtu}/(\text{per-yr})$ . But because, as discussed above,  $\text{gdp}$  and  $\dot{e}$  both increase in the same proportion, energy intensity is constant at  $\dot{e}/\text{gdp} = 4934 \text{ Btu}/\$$ .

In other words, an increase in luminous efficacy, all other things held constant, has increased  $\text{gdp}$  with no penalty to energy intensity, though at the expense of an increase in absolute energy consumption.

Scenario SSL-M shows a hypothetical world similar to that of scenario SSL-L except with a cost of energy for lighting increased to  $\text{CoE}_\varphi = 133 \$/\text{MW}_e\text{h}$ . From equation (23), one sees that the result will be a decrease in energy intensity, so that  $\dot{e}$  makes a greater than proportional decrease than  $\text{gdp}$  does. It is then possible to return to exactly the same  $\dot{e}$  (81.5 MBtu/(per-yr)) as that of scenario FLU-L, while maintaining a large portion of the increase in  $\text{gdp}$  (now 16,685 \$/(per-yr)). Indeed, this modest increase in  $\text{CoE}_\varphi$  was chosen precisely to enable a return to the per-capita energy consumption of scenario FLU-L. The strikingly larger percentage decrease in  $\dot{e}$  than in  $\text{gdp}$  is due to the energy service nature of lighting and the larger percentage of  $\dot{e}$  (6.5%) than of  $\text{gdp}$  (0.72%) that it consumes.

Scenario SSL-H shows a hypothetical world similar to that of scenario SSL-M except with a cost of energy for lighting increased further to  $\text{CoE}_\varphi = 369 \$/\text{MW}_e\text{h}$ . This increase in cost of energy for lighting causes a further decrease in energy intensity, so that  $\dot{e}$  continues its greater than proportional decrease relative to  $\text{gdp}$ . It is then possible for  $\dot{e}$  to decrease below that of scenario FLU-L, to  $\dot{e} = 76 \text{ Btu}/(\text{per-yr})$ , while  $\text{gdp}$  has exactly returned to that of scenario FLU-L, to  $\text{gdp} = 16\,511 \$/(\text{per-yr})$ .

Scenarios SSL-L, SSL-M and SSL-H together illustrate the benefit of an increase in luminous efficacy. On the one hand, for a constant cost of energy for lighting, increased luminous efficacy enables per-capita  $\text{gdp}$  to increase, albeit at the expense of an increased per-capita energy consumption. In other words, increased luminous efficacy does not decrease per-capita energy consumption, but it *does* increase human productivity and standard of living.

On the other hand, for a non-constant (increased) cost of energy for lighting, increased luminous efficacy can offset the decrease in per-capita  $\text{gdp}$  that would otherwise occur. In other words, increased luminous efficacy allows human productivity and standard of living to be maintained even if policy or market forces cause increases in the cost of energy for lighting.

Indeed, scenario FLU-H shows a hypothetical world in which the cost of energy for lighting is as high as for scenario SSL-H, but luminous efficacy has returned to that ( $\eta_\varphi = 87 \text{ lm}/\text{W}_e$ ) of starting scenario FLU-L. Scenario FLU-H is thus the same as scenario FLU-L except with a cost of energy for lighting that has increased to  $\text{CoE}_\varphi = 369 \$/\text{MW}_e\text{h}$ . Without the benefit of an increase in luminous efficacy, the effect of this increase in the cost of energy for lighting is to decrease  $\text{gdp}$  by 1% and  $\dot{e}$  by 6% to  $\text{gdp} = 16\,320 \$/(\text{per-yr})$  and  $\dot{e} = 75.1 \text{ MBtu}/(\text{per-yr})$ .

#### 4.2. Semiconductor chip inventory and turnover

In this section, we discuss the semiconductor chip area inventory and turnover associated with SSL. We consider mainly scenario SSL-L outlined in section 4.1. In this scenario, luminous efficacy increases have *not* been ‘compensated’ by cost of energy increases. We do not discuss the other scenarios SSL-M and SSL-H explicitly, but do list their projected semiconductor chip area inventories and turnovers in table 1. These chip area inventories and turnovers are lower, of course, because, in these scenarios, luminous efficacies have been compensated to various degrees by increases in cost of energy for lighting, so that consumption of lighting is lower.

We begin by calculating the semiconductor chip area,  $A$ , that would be required to ‘light the world’ with SSL. From equation (17), scaled by population,  $N$ , the global consumption of light (in  $\text{Plm h}/\text{yr}$ ) is

$$\Phi = \frac{\beta}{(1 + \kappa_\varphi)} \cdot \frac{\text{GDP} \cdot \eta_\varphi}{\text{CoE}_\varphi}. \quad (24)$$

We also know that global consumption of light  $\Phi$  (again in  $\text{Plm h}/\text{yr}$ ) will depend on the total area  $A$  (in  $\text{km}^2$ ) of semiconductor chips used to produce light, the average density  $p$  (in  $\text{W cm}^{-2}$ ) of input electrical power that the semiconductor

is driven by, the duty cycle  $D$  (proportion of time a lamp is operated) and the luminous efficacy  $\eta_\varphi$  (in  $\text{lm W}^{-1}$ ):

$$\Phi = A \cdot p \cdot D \cdot \eta_\varphi \cdot \left\{ 10^{10} \frac{\text{cm}^2}{\text{km}^2} \cdot 10^{-15} \frac{\text{Plm}}{\text{lm}} \cdot 24 \cdot 365.25 \frac{\text{h}}{\text{yr}} \right\}. \quad (25)$$

Equating the two expressions for  $\Phi$  gives the following expression for  $A$ :

$$A = \frac{\beta \cdot \text{GDP}/[\text{CoE}_\varphi \cdot (1 + \kappa_\varphi)]}{p \cdot D \cdot \left\{ 10^{10} \frac{\text{cm}^2}{\text{km}^2} \cdot 10^{-15} \frac{\text{Plm}}{\text{lm}} \cdot 24 \cdot 365.25 \frac{\text{h}}{\text{yr}} \right\}}. \quad (26)$$

Note that equation (26) for  $A$  does not involve luminous efficacy. The reason is that increasing luminous efficacy decreases the cost of light and increases consumption of light, but also decreases the semiconductor area needed to produce a unit of light. This expression for  $A$  does involve cost of energy, however. The reason is that increasing cost of energy increases the cost of light and decreases consumption of light, but does not decrease the semiconductor area needed to produce a unit of light.

Equation (26) for  $A$  does involve a number of other quantities as well. Two of these quantities,  $\text{CoE}_\varphi$  and GDP, are macroeconomic, and for these we assume the business-as-usual estimates discussed for scenarios FLU-L and SSL-L in section 4.1: a 2030  $\text{CoE}_\varphi$  of 119 \$/MWh and a 2030 world GDP of 137 484 G\$/yr. Two of these quantities, duty cycle and average input power density, are microeconomic and depend on the details of how solid-state lamps and their use co-evolve in the future with residential, commercial, industrial and outdoor spaces and *their* use.

To estimate these latter two quantities, one can imagine two extremes of this co-evolution. At one extreme, control of the spatial distribution of illumination remains relatively coarse, as it is today. Then, fewer lamps would be necessary, but these few lamps would be on most of the time, and would be driven nearly to their maximum. At the other extreme, control of the spatial distribution of illumination becomes finer (Moreno *et al* 2006, Yang *et al* 2009), enabled by the ability of solid-state lamps to be fractionated into smaller units and to have their light output and spatial distribution manipulated without efficiency loss. Then, more lamps would be necessary, but these lamps might be off most of the time, and even when on would be driven less hard.

Here, we assume a co-evolution that is in between these two extremes. Rather than the duty cycle of roughly 1/2 characteristic of traditional lighting today (Navigant 2002), we assume a duty cycle of 1/4. Rather than assuming an average input power density of  $3 \text{ kW cm}^{-2}$ , the maximum one might imagine driving a solid-state lamp (Tsao 2004), we assume an average input power density of  $220 \text{ W cm}^{-2}$ , near that of a late-2009 state-of-the-art warm-white LED lamp (Tsao *et al* 2010).

The result is that the area of semiconductor required to ‘light the world’ would be  $A = 1.31 \text{ km}^2$  (roughly 250 American football fields). Note that this area scales inversely as the cost of energy for lighting: if the cost of energy for lighting (and therefore the cost of lighting) were to roughly

triple, to 369 \$/MW<sub>e</sub>h as in scenario SSL-H, then consumption of light and the area required to sustain that consumption of light would decrease to 1/3 of  $A = 1.3 \text{ km}^2$  or  $A = 0.42 \text{ km}^2$ . Also note that this area scales inversely as power density: if average input power density were to double, the area would decrease further from  $A = 0.42 \text{ km}^2$  to  $A = 0.21 \text{ km}^2$ .

**4.2.1. Chip area turnover: transition period.** To build up the steady-state semiconductor chip area inventory required to light the world, a yearly production  $dA_{\text{transition}}/dt$  (in  $\text{km}^2 \text{ yr}^{-1}$ ) will be necessary. This yearly production, during a transient period when traditional lighting is being displaced by SSL, depends on the area  $A$  (in  $\text{km}^2$ ) necessary to light the world, the duration  $T$  (in yrs) of the transition period and the manufacturing yield  $Y$ :

$$\frac{dA_{\text{transition}}}{dt} = \frac{A}{Y \cdot T}. \quad (27)$$

Assuming the transition period is  $T = 10$  years and the manufacturing yield is a relatively high  $Y = 3/4$ , one would need a  $dA_{\text{transition}}/dt = 0.17 \text{ km}^2 \text{ yr}^{-1}$ . Note that the manufacturing yield could easily be much lower than  $Y = 3/4$  during the transition period.

This simple estimate assumes that traditional lighting is displaced by SSL of near-saturated performance. Having done the displacement, subsequent turnover in SSL chip area would not occur until SSL lamp end-of-life.

It is entirely possible, however, that traditional lighting instead be displaced by less-mature generations of SSL with a lower performance, themselves to be displaced by more-mature generations with higher performance. Such higher performance might include the simple economic cost of lumens as well as new features beyond simple lumen output (as discussed later in section 5.2). Here, we note that the economic condition for obsolescence-induced turnover based simply on economic cost is that the total (operating plus capital) cost of light of the next generation be less than the operating cost of the previous generation. We can estimate the rate at which luminous efficacy would need to improve in order to satisfy this condition if we assume a cost structure (the ratio of capital to operating cost) of light similar to that of traditional lighting (Tsao and Waide 2010). Then,  $\kappa_\varphi \approx 1/3$ , as discussed in section 2.1, and it can be shown that  $d(\log \eta_\varphi)/dt > \kappa_\varphi/T$ , where  $T$  is the duration of one generation in SSL chip area turnover. For example, for a duration of  $T = 3.3$  years, the yearly percentage change in luminous efficacy would need to be  $\kappa_\varphi/T = (1/3)/(3.3 \text{ years}) = 10\%/\text{year}$ .

Because a 10%/year improvement in luminous efficacy is well within the realm of possibility, area turnovers with  $T \sim 3.3$  years are also within the realm of possibility, with proportional increases in  $dA_{\text{transition}}/dt$  in equation (27).

**4.2.2. Chip area turnover: steady-state.** After the steady-state semiconductor chip area inventory is in place (after traditional lighting has been fully displaced by SSL with near-saturated performance), a yearly production  $dA_{\text{steady-state}}/dt$  (in  $\text{km}^2 \text{ yr}^{-1}$ ) will still be necessary to maintain that

semiconductor chip area. This yearly production will depend on the area  $A$  (in  $\text{km}^2$ ), the manufacturing yield  $Y$ , the operating lifetime,  $\tau$  (in h) of the semiconductor lamps and the duty cycle  $D$ :

$$\frac{dA_{\text{steady-state}}}{dt} = \frac{A \cdot D \cdot \{24 \cdot 365.25 \text{ h yr}^{-1}\}}{Y \cdot \tau}. \quad (28)$$

Here, we assume an operating lifetime  $\tau = 50000 \text{ h}$ , similar to that of current state-of-the-art LEDs; a manufacturing yield  $Y = 3/4$  and the same duty cycle  $D = 1/4$  assumed above. The result is a steady-state annual production of semiconductor area of  $dA_{\text{steady-state}}/dt = 0.076 \text{ km}^2 \text{ yr}^{-1}$ .<sup>15</sup>

Note that this steady-state yearly production scales inversely with lamp lifetime. One might easily imagine the effective lifetime being smaller by more than a factor of two (to 20000 h) for a number of reasons, including: a lack of demand for lifetimes that significantly exceed typical occupancy periods of residents in buildings or continuing evolution of SSL performance or features that effectively make SSL lamps obsolete before burn-out. Then, the yearly turnover would increase to  $dA_{\text{steady-state}}/dt = 0.19 \text{ km}^2 \text{ yr}^{-1}$ .

#### 4.3. Semiconductor chip epitaxy

These semiconductor chip area turnovers are not insignificant, and it is of interest to assess the manufacturing infrastructure that would be required to sustain them. Central to this infrastructure is epitaxial growth by metalorganic vapour phase epitaxy (MOVPE), for which it is of interest to estimate both the numbers of necessary MOVPE tools as well as the throughput of raw materials necessary for operation of the MOVPE tools.

With respect to the numbers of MOVPE tools, we assume, similar to the current state-of-the-art, that these tools are capable of roughly twelve 4 inch wafers per growth run, three growth runs per day, and an up-time of 330 days/yr. Growth of  $0.076 \text{ km}^2 \text{ yr}^{-1}$ , the estimated semiconductor chip area turnover required after the transition to SSL has reached steady-state, would thus require the equivalent of roughly 800 such MOVPE tools. Growth of  $0.174 \text{ km}^2 \text{ yr}^{-1}$ , the estimated semiconductor chip area turnover required during the transition to SSL, would require the equivalent of roughly 1800 such MOVPE tools. Interestingly, these projected numbers are in the range of the numbers of GaN-based MOVPE tools currently in place for applications other than general illumination (Hackenberg 2010). If such applications continue to require similar numbers of MOVPE tools in the future, it may be necessary to double the number of such MOVPE tools in order to satisfy anticipated needs for SSL chip epitaxy.

With respect to the throughput of raw materials necessary for operation of the MOVPE tools, we consider here only the group-III elements, Ga and In, as these are relatively rare and produced primarily as minor byproducts of other base metal mining operations (USGS 2007a, 2007b).

<sup>15</sup> This is roughly (and perhaps surprisingly) consistent with a separate estimate (<http://compoundsemiconductor.net/csc/features-details.php?cat=features&id=20012>) of 25 million 2 inch wafers per year or  $25 \times 10^6 \cdot \pi \cdot [2 \text{ inch} \cdot (2.54 \text{ cm inch}^{-1}) \cdot (10^{-5} \text{ km cm}^{-1})/2]^2 = 0.05 \text{ km}^2 \text{ yr}^{-1}$ .

To calculate the Ga consumption rate ( $dM_{\text{Ga}}/dt$ ), we use

$$\frac{dM_{\text{Ga}}}{dt} \cong \frac{h_{\text{GaN}}}{\varepsilon_{\text{Ga}}} \cdot \rho_{\text{GaN}} \cdot (70/84) \cdot \frac{dA}{dt}, \quad (29)$$

where  $h_{\text{GaN}}$  is the thickness of the relevant epitaxial layers,  $dA/dt$  is the chip area turnover per year,  $\varepsilon_{\text{Ga}}$  is the utilization efficiency of the Ga-containing metalorganic precursor,  $\rho_{\text{GaN}}$  is the density of GaN and 70/84 is the Ga/GaN weight ratio. In other words, the Ga consumption rate is just the chip area turnover multiplied by the Ga content of that chip area. Using an efficiency of  $\varepsilon = 0.10$ , an epitaxial thickness of  $h_{\text{GaN}} = 2 \mu\text{m}$  and a  $dA/dt = dA_{\text{transition}}/dt = 0.17 \text{ km}^2 \text{ yr}^{-1}$ , we deduce  $dM_{\text{Ga}}/dt = 20000 \text{ kg yr}^{-1}$  or 20 metric tons of Ga per year. This Ga consumption rate is large, but only about 20% of current Ga refinery production<sup>16</sup>. It can be viewed as driving a moderate increase in demand for Ga over its current dominant use in compound semiconductor electronics and optoelectronics (USGS 2007a).

To calculate the In consumption rate ( $dM_{\text{In}}/dt$ ), we use

$$\frac{dM_{\text{In}}}{dt} \cong \frac{h_{\text{InN}}}{\varepsilon_{\text{In}}} \cdot \rho_{\text{InN}} \cdot (115/129) \cdot \frac{dA}{dt}, \quad (30)$$

where the various quantities have the same meanings as in equation (29) but have substantially different values. In particular, the MOVPE utilization efficiency of the In-containing metalorganic precursor is considerably lower than that of the Ga-containing metalorganic precursor: on the order  $\varepsilon_{\text{In}} = 0.005$ . Also, InN is typically present only in five or so thin (3 nm) quantum wells and even then only as a small (20%) percentage of the overall composition: the effective epitaxial thickness of InN is thus on the order  $h = 5 \cdot (3 \text{ nm}) \cdot 0.2 = 10 \text{ nm}$ . We deduce  $dM_{\text{In}}/dt = 600 \text{ kg yr}^{-1}$  or 0.6 metric tons of In per year.

This In consumption rate is 0.1% of current In refinery production<sup>17</sup>, a much smaller percentage than the Ga consumption rate was of current Ga refinery production. The reason is twofold. First, the In consumption rate is much lower than the Ga consumption rate, because current SSL device structures use much less In than they do Ga. Second, current In refinery production is much larger than Ga refinery production, because In is required in several very large applications unrelated to SSL or even to semiconductor applications. Specifically, 97% of all In is used for non-semiconductor applications (O'Neill 2004), and of this 75% is consumed as indium tin oxide (ITO) for transparent conducting oxide applications (TCO) (O'Neill 2004, USGS 2007b). We conclude that depletion of In reserves, if it were to become an issue (Cohen 2007), is not likely to be due to SSL, but to the other much larger applications for In.

#### 4.4. Comparison with other electronic thin-film technologies

It is also of interest to assess the magnitude of the semiconductor chip area turnover required for SSL relative

<sup>16</sup> Worldwide Ga primary production was 111 metric tons in 2008 and 78 metric tons in 2009 (USGS 2010a).

<sup>17</sup> Worldwide In refinery production was 570 metric tons in 2008 and 600 metric tons in 2009 (USGS 2010b).

to the current manufactured areas of other electronic thin-film technologies.

The electronic thin-film technology with perhaps the largest current manufactured area is that for liquid crystal displays. An estimate<sup>18</sup> for 2009 of  $\sim 200 \text{ km}^2 \text{ yr}^{-1}$  is  $\sim 3000 \times$  larger than that projected in section 4.2 for SSL.

The electronic thin-film technology with perhaps the largest future potential<sup>19</sup> is semiconductor photovoltaics for solar electricity generation. Currently, though, the manufactured area is modest. An estimate<sup>20</sup> for the peak watts shipped in 2009 is  $\sim 9.6 \text{ GWp/yr}$ . Assuming an average efficiency of  $\sim 15\%$  (or  $\sim 150 \text{ Wp/m}^{-2}$ ) gives a manufactured area of  $(9.6 \text{ GWp/yr})/(150 \text{ Wp/m}^{-2}) \sim 60 \text{ km}^2 \text{ yr}^{-1}$ . This is significantly smaller than the manufactured area of liquid crystal displays, though still  $\sim 800 \times$  larger than that projected in section 4.2 for SSL.

A third electronic thin-film technology with a fairly large manufactured area is silicon electronics<sup>21</sup>:  $\sim 4 \text{ km}^2 \text{ yr}^{-1}$ . This figure is smaller than the potential discussed above for solar photovoltaics. However, it is itself larger, by two orders of magnitude, than the steady-state chip area turnover required for SSL.

Finally, the highest area-turnover compound semiconductor substrate at present is GaAs (both electronics and optoelectronics), which in 2009 had an estimated semiconductor chip area turnover of<sup>22</sup>  $\sim 0.03 \text{ km}^2 \text{ yr}^{-1}$ . This figure is much smaller than the chip area turnover discussed above for silicon electronics. However, it is on the order of the steady-state chip area turnover required for SSL.

We conclude that SSL will likely be one of the largest of the compound semiconductor applications (assuming that in the future, as now, the vast majority of solar cells continue to be made entirely from Si), but will also likely be dwarfed by other electronic thin-film applications such as liquid crystal displays, solar cells and silicon electronics (in terms of electronic thin-film area turnover).

## 5. SSL: alternative futures ( $\beta \neq 0.0072$ )

In section 4, we considered only ‘baseline future’ scenarios in which the historical pattern of consumption of light was preserved in the future—scenarios, in other words, in which

<sup>18</sup> Rosenblum S S, Corning, private communication.

<sup>19</sup> The most extreme scenario for semiconductor photovoltaics is one in which it generates all the world’s energy. If we assume an average solar flux striking the earth of  $174.7 \text{ W m}^{-2}$  and a photovoltaic efficiency of 20%, then the area required to generate a projected world primary energy consumption in 2030 of 20 TW would be  $575\,000 \text{ km}^2$ . Assuming an operational lifetime of 10 years, the semiconductor chip area turnover required for such massive solar electricity generation would be  $115\,000 \text{ km}^2 \text{ yr}^{-1}$ , a figure which dwarfs by more than six orders of magnitude the steady-state chip area turnover required for solid-state lighting. The primary reason is power density: the  $174.7 \text{ W m}^{-2}$  power density associated with the average solar flux is over four orders of magnitude lower than the  $220 \text{ W cm}^{-2}$  power density we have assumed for solid-state lighting. The secondary reason is that world primary energy consumption is roughly 1.5 orders of magnitude higher than primary energy consumption for solid-state lighting would be.

<sup>20</sup> See, e.g., <http://www.displaybank.com/eng/info/sread.php?id=5730>

<sup>21</sup> See, e.g., <http://www.yole.fr/pagesAn/products/csmat.asp> or [http://www.fabtech.org/news/\\_a/gartner\\_revises\\_silicon\\_wafer\\_demand\\_forecast/](http://www.fabtech.org/news/_a/gartner_revises_silicon_wafer_demand_forecast/)

<sup>22</sup> Anwar A, Strategy Analytics, private communication.

the proportionality constant  $\beta = 0.0072$  is preserved in the future. We call these baseline scenarios because they represent the simplest extrapolations of the past into the future.

That such a simple extrapolation from the past will be predictive of the future, however, cannot be known, and it may very well be that  $\beta \neq 0.0072$  in the future. Indeed there are at least three ways in which the historical variation might be modified in the future.

First, we may be approaching a saturation in the demand for light. Though this is difficult to imagine for the developing world, whose per-capita consumption of light is much lower than for the developed world, perhaps at least the developed world is approaching a saturation. Second, countering the first, new features associated with SSL may actually cause an increase in the demand for light. Third, apart from how the intrinsic demand for light is increased or decreased through SSL technology, guidelines and regulations aimed at efficient lighting system designs and light usage may nonetheless bring about a saturation.

In this section we discuss these three possibilities in turn.

### 5.1. Saturation in demand for raw lumens

As has been discussed, at current levels of per-capita consumption of light, there is no evidence that we have reached a saturation in the demand for light. It is nevertheless an open question whether we will approach such a saturation in the future or whether per-capita consumption of light will continue to scale linearly with gdp/CoL. To understand this question more quantitatively, we decompose per-capita consumption of light into the product of three factors:  $I_N$ , the average illuminance<sup>23</sup> (or light per unit area, in units of  $\text{lm m}^{-2}$ ) that a person is surrounded by during his or her waking hours;  $\tau_{\text{on}}/(\tau_{\text{on}} + \tau_{\text{off}})$ , a dimensionless illumination duty factor that accounts for how many hours per year the area around a person is actually illuminated and  $a_N/(1 + a_N \rho_N)$ , the average unshared illuminated area (in units of  $\text{m}^2$ ) that a person is surrounded by<sup>24</sup>. In other words,

$$\varphi = I_N \cdot \left( \frac{\tau_{\text{on}}}{\tau_{\text{on}} + \tau_{\text{off}}} \right) \cdot \left( \frac{a_N}{1 + a_N \rho_N} \right). \quad (31)$$

It is possible that, in developed countries, each of these factors is nearing saturation. As we discuss in the rest of this section, however, plausible arguments can be made that, even in developed countries, the first and third factors may each yet be a factor of 10 or more below saturation.

#### 5.1.1. Illuminance: $I_N$ .

The first term in equation (31) is  $I_N$ , the average illuminance (or light per unit area, in units of lux or  $\text{lm m}^{-2}$ ) that a person is surrounded by during his or her waking hours.

<sup>23</sup> We use the symbol  $I$  rather than the usual symbol for illuminance,  $E$ , as in this paper  $E$  refers to energy. The subscript ‘ $N$ ’ refers to local illuminance from the perspective of an average person, as opposed to global illuminance from the perspective of an average area of land.

<sup>24</sup> Just as for the symbol  $I$ , for the symbols  $a$  and  $\rho$  the subscript ‘ $N$ ’ refers to local illuminated area and local population density from the perspective of an average person, as opposed to from the perspective of an average area of land.

Illuminance levels have gradually increased over the centuries and, for modern indoor office or living spaces, are now on the order of  $I_N \approx 500 \text{ lm m}^{-2}$ . Such illuminances are, from a purely visual acuity point of view, clearly enough for most people for most tasks, and might be anticipated to be near a saturation level.

Moreover, we do not always wish to be surrounded by illuminances suitable for tasks requiring high visual acuity. Ambient illuminances for enhancing particular moods or emotional states of mind can be much lower than  $500 \text{ lm m}^{-2}$ . And even when high visual acuity is desired, not all illuminance must be supplied artificially—artful use of sunlight can be an important supplement.

Nevertheless, arguments can be made that we have not yet approached saturation levels for illuminance. Considerable uncertainty exists regarding what constitutes optimal lighting—despite over a century of research, recommended levels for comparable spaces still vary by a factor of up to 20. It is now recognized that optimal lighting conditions are contingent on numerous factors other than just average horizontal illuminance levels and include visual contrast and light distribution parameters.

And, even if one considers only horizontal illuminance, the evidence regarding the levels that people would choose were affordability not a factor is far from complete. People might well choose higher illuminances than they do today, particularly to help mitigate losses in visual acuity in an ageing world population, but perhaps also to function as neuropsychological modifiers helping, e.g., to synchronize circadian rhythms (Arendt 2006, Figueiro *et al* 2009), to reduce seasonal affective disorder or to enhance mood.

Indeed, the generally comfortable outdoor illuminance characteristic of an overcast or cloudy day is of the order  $5000 \text{ lm m}^{-2}$ – $10 \times$  higher than the  $500 \text{ lm m}^{-2}$  mentioned above as typical of modern indoor office or living spaces. And the outdoor illuminance characteristic of a bright sunny day is of the order  $30000 \text{ lm m}^{-2}$ ,  $60 \times$  higher than today's  $500 \text{ lm m}^{-2}$ . Though this latter illuminance is uncomfortable viewed from a close distance (requiring the use of sunglasses), it may well be desirable viewed from a farther distance (e.g. viewing scenery).

We conclude that it is possible that the developed countries are nearing a saturation point in average illuminance, but plausible arguments can be made that the saturation point may yet be a factor of 10 or more higher<sup>25</sup>.

### 5.1.2. Illumination duty factor: $\tau_{\text{on}}/(\tau_{\text{on}} + \tau_{\text{off}})$ .

The second term in equation (31) is  $\tau_{\text{on}}/(\tau_{\text{on}} + \tau_{\text{off}})$ , a dimensionless

<sup>25</sup> We note in passing that, whatever its saturation value, average illuminance might be expected to vary with geography. In countries further from the equator, illuminance from the sun is lower and illuminance from artificial sources might be expected to increase to compensate. The limited data which is available appears not to support this, however. For example, Japan consumes significantly more light per-capita than Northern Europe despite being nearer the equator and despite a similar standard of living. A proximate explanation for this is the greater penetration of higher luminous efficacy fluorescence lighting technology, hence lower cost of light, in Japan than in Northern Europe. But an ultimate explanation for the greater penetration of fluorescence technology itself may be a desire for higher artificial illuminance levels so as not to provide too stark a contrast with outdoor illuminance levels.

illumination duty factor that accounts for how many hours per year the area around a person is actually illuminated. The duty factor for a person who spends most of his or her time indoors, either at work or at home, is roughly the number of waking hours per day or about  $\tau_{\text{on}}/(\tau_{\text{on}} + \tau_{\text{off}}) \approx (16 \text{ h/day})/(24 \text{ h/day}) = 2/3$ .

This is the term that is most clearly nearing saturation. Most people need on the order of 8 h of sleep each day. And most people need darkness to sleep, and even apart from sleep, to reset their human circadian rhythms (IEA 2006).

**5.1.3. Unshared illuminated area:  $a_N/(1 + a_N \rho_N)$ .** The third term in equation (31) is  $a_N/(1 + a_N \rho_N)$ , the average unshared illuminated area (in units of  $\text{m}^2$ ) that a person is surrounded by. This area  $a_N$  is the average illuminated area (in units of  $\text{m}^2$ ) that a person is surrounded by (regardless of how many other persons share that area), divided by  $1 + a_N \rho_N$ , the number of persons that share that area. Here,  $\rho_N$  (in units of  $\text{per}/\text{m}^2$ ) is the density of people within the illuminated area that a person is surrounded by. When  $\rho_N \ll 1/a_N$ , light is not shared, and  $a_N/(1 + a_N \rho_N)$  approaches  $a_N$ ; when  $\rho_N \gg 1/a_N$ , light is shared amongst many people, and  $a_N/(1 + a_N \rho_N)$  approaches  $1/\rho_N$ .

The order of magnitude of  $a_N$  can be estimated as follows. Per-capita consumption of light in the US, representative of the high end in the world, was about  $\varphi \approx 136 \text{ Mlm h/(per yr)}$  in 2001 (Tsao and Waide 2010). As discussed above, the average illuminance in modern indoor office or living spaces is roughly  $I_N \approx 500 \text{ lm m}^{-2}$ , and let us assume the illumination duty factor of  $\tau_{\text{on}}/(\tau_{\text{on}} + \tau_{\text{off}}) \approx 2/3$  estimated above. In the absence of light sharing ( $\rho_N \ll a_N$ ), the illuminated area that the average person in the US is surrounded by is thus, using equation (31), roughly  $a_N \approx \varphi \cdot (\tau_{\text{on}} + \tau_{\text{off}})/(I_N \cdot \tau_{\text{on}}) \approx 41 \text{ m}^2$ . This area is plausible: larger than a typical one-person office area, but smaller than a typical one-person residential area.

Regarding how the unshared illuminated area  $a_N/(1 + a_N \rho_N)$  might evolve in the future, plausible arguments can be made in both directions: that it is either saturated and might even decrease or that it has considerable room for growth.

On the one hand, humans, often characterized as den animals, find comfort in enclosed areas, and to be surrounded by a  $(41 \text{ m}^2/\pi)^{1/2} = 3.6 \text{ m} \approx 11.8 \text{ ft}$  radius of illuminated area is surely sufficient for most people most of the time. Indeed, an increasing trend in modern buildings is the use of motion sensors to turn lights on and off when a person enters or exits a space, with typical coverage areas comparable to  $41 \text{ m}^2$ . With new technologies such as SSL, such opportunities for sensor-based intelligent control will only increase in the future, potentially decreasing  $a_N$ .

Moreover, humans are not only den animals, they are social animals and tend to cluster in groups. Indeed, local population density<sup>26</sup> can, in a typical office building or urban public space, easily be on the order of  $\rho_N \approx 0.1 \text{ m}^{-2}$ . Hence,

<sup>26</sup> By local population density, we mean as seen from the perspective of a person, which includes the tendency towards clustering. As seen from the perspective of the land, median world population density is much lower, on the order of  $4 \times 10^{-6} \text{ m}^{-2}$  (Cohen and Small 1998).

for  $a_N \approx 41 \text{ m}^2$  and  $\rho_N \approx 0.1 \text{ m}^{-2}$ , we have  $1/(1 + a_N \rho_N) \approx 1/5$ , and for these environments the number of people that share an illuminated area might be as high as 5. And, as the world continues to urbanize, the number of people that share illuminated areas might increase.

On the other hand, humans, den and social animals though they may be, also like space. Environments in which local population density is so high, and space is shared so heavily, are not necessarily the desired norm. Even the most densely populated city in the US (New York) only has an average population density of about  $0.009 \text{ m}^{-2}$  (Gibson 1998), implying that its average resident has plenty of less-dense areas to ‘escape’ to. Moreover, as nations develop, the densities of their cities tend to decrease, as transportation costs decrease relative to income (Tobler 1969, Stephan and Tedrow 1977). Clearly, humans do not *prefer* to share space to an extreme. Indeed, if the average size of residences is an indication of people’s preferred size of spaces, it is clear that these can be rather large. The average area per person in new single-family homes in the US increased from  $27 \text{ m}^2$  in 1950 to  $45 \text{ m}^2$  in 1970 to  $78 \text{ m}^2$  in 2000, and can easily be  $2\text{--}5\times$  larger still in ‘upper-end’ homes. Hence, the saturation illuminated area surrounding each person could be more than  $5\text{--}10\times$  larger than the current  $46 \text{ m}^2$  estimated above.

Moreover, even if the enclosed indoor areas in which we work and live might ultimately saturate, the unenclosed outdoor areas which we either occupy for short periods during the day or evening, or which are visible from enclosed indoor areas, may be less prone to saturation. Such unenclosed outdoor areas (e.g. streets, parks and other recreation and public spaces) could all be rendered more useful if better illuminated in the evening hours (albeit at the cost of reducing the contrast of the night sky due to light pollution (Boyce 2003)). And there is a natural human tendency to gaze out (we value windows, not just because they are a portal for incoming light, but because of the view they afford (Boyce 2003)) into faraway spaces, *even if we do not directly occupy those spaces*.

We conclude that it is possible that the average unshared illuminated area is nearing a saturation point, but plausible arguments can be made that the saturation point may yet be a factor  $10\times$  or greater away.

### 5.2. Demand for features beyond lumens

Although the primary demand for light is to illuminate our environment, there are many other features of light that are important to the consumer. Arguably, these features were just as important as decreased cost in the historical increases in consumption of light as one lighting technology gave way to the next. They therefore counter the possible saturations in consumption of light that were discussed in section 5.1.

For example, the transitions within chemical-fuel-based lighting (e.g. from candles to oil lamps to gas lamps), and the final transition from chemical-fuel-based to electricity-based lighting, brimmed over with new performance attributes (Schivelbusch 1988): increased cleanliness, faster turn-on and turn-off, greatly decreased concomitant room heating and reduced fire hazard. These new performance attributes had

tremendous potential, ultimately realized, to unleash new and unforeseen ways of consuming light. Note that they also had a similar potential to unleash new ways of consuming less light, through instant turn-on and turn-off and through the increased ability to focus light sources with smaller spatial extent. This potential for reduced consumption of light, however, was ultimately dwarfed by the potential for increased consumption of light.

It is difficult to guess whether the coming transition to SSL will be quantitatively similar, but plausible arguments can be made that, in the long run, it will be (Kim and Schubert 2008). We mention here in particular two potential features of SSL.

A first potential feature is real-time control of its precise mix of wavelengths and intensities (Schubert and Kim 2005). SSL is, after all, *based* on the mixing of light of various wavelengths and intensities. If real-time control were built into this mixing technology, it would be possible to tailor in real-time the rendering of the colours of natural objects in the environment, either to be as accurate as possible (as measured, e.g., by the CRI) or to deliberately create subjective emotional responses (by mimicking, e.g., daylight, moonlight, candlelight). It would also be possible to tailor the spectral match to those (non-retinal) components of the human photoreceptor system responsible for circadian, seasonal and neuroendocrine regulation (Thapan *et al* 2001, Rea *et al* 2010).

A second potential feature is real-time control of the *placement* of light. SSL is, after all, a semiconductor technology with all of the potential micro-optoelectromechanical (MOEMs) functionality for focusing and directing light associated with such technologies. It would in principle be possible to combine a distributed array of SSL lamps with such functionality; a distributed array of sensors and control systems. Then, it might be possible to tailor local illuminances according to how people are interacting at that instant with their environment.

Combined with other features such as compactness and ruggedness, these two potential performance features have the potential to unleash new and unforeseen ways of consuming light. Again, they also have the potential to unleash new ways of consuming *less* light, through sensor-based control of light flux and directionality. However, if history is a guide, the potential for reduced consumption of light may ultimately be dwarfed by the potential for increased consumption of light.

### 5.3. Guidelines and regulations

Even if we are not approaching saturation in the intrinsic demand for light, guidelines and regulations aimed at efficient lighting system designs and light usage may nonetheless bring about such a saturation. Here, we discuss, in turn, non-legislated guidelines and legislated regulations.

**5.3.1. Non-legislated recommendations and guidelines.** Non-legislated guidelines (standards and recommendations) on lighting and lighting system design have long been published by professional organizations in many industrial and post-industrial countries (Boyce 2003). Examples are those of the Chartered Institution of Building Services Engineers in

the UK (CIBSE 2002) and of the Illumination Engineering Society of North America (Rea 2000). Such guidelines cover recommended illuminance levels for spaces with various functional purposes and, though without formal legal status, could be a mechanism by which light usage might be curtailed or even reduced.

Note, though, that the intent of these guidelines is not so much to reduce light usage but to enhance the ways in which humans experience and use light. Such guidelines would not necessarily limit the expansion of light usage into spaces that are currently only poorly lit (e.g. the outdoor spaces discussed at the end of section 5.1), but rather might guide such usage after such an expansion has begun.

Moreover, even in spaces, such as indoor offices, which are currently well lit, the effectiveness of guidelines is limited by a lack of consensus on how much light is needed to satisfy human needs. In many OECD (Organization for Economic Cooperation and Development) countries recommended interior lighting levels peaked in the late 1970s and then declined as performance on visual reading and writing tasks, previously the main consideration, was extended to include more holistic criteria. It is now recognized that optimal lighting conditions are contingent on numerous factors other than just average horizontal illuminance levels and include visual contrast and light distribution parameters. Glare, e.g., caused by a sharp imbalance in the brightness of adjacent areas in the field of view, is an important consideration. As mentioned above, the result is that, despite over a century of research into the topic, recommended levels for comparable spaces in different countries still vary by a factor of 10× or more: typical office spaces in most countries are illuminated to 500 lux but national recommendations can be as high as 2000 lux and as low as 250 lux (IEA 2006).

We conclude that non-legislated recommendations and guidelines may have a limited ability to cause a saturation in the consumption of light, if such a saturation would not already have naturally occurred.

**5.3.2. Legislated regulations.** Legislated regulations have more recently (over the last decade) been introduced in many countries.

A first type of regulation limits maximum permissible energy consumption, either within a building on a per-unit-floor-area basis or for whole buildings. The limits can either be on energy consumption as a whole (of which lighting is a key piece) or on energy consumption for lighting as a separate piece. These limits are derived by assuming a blend of reasonably efficient lighting technologies inserted into lighting designs sufficient to provide recommended lighting requirements.

If luminous efficacies are constant, such regulations effectively constrain the degree to which interior light levels can be increased (IEA 2006). They also provide incentives to optimize light usage, and in particular to limit illumination of unoccupied or daylit spaces. If, however, luminous efficacies are not constant, but increase with new lighting technologies, light usage within a building can increase even while satisfying an older generation of energy-consumption regulations. It

would then be necessary for such regulations to be continually updated to keep pace with new lighting technologies. In addition, new lighting technologies can enable new uses not associated with buildings and hence not covered by such regulations.

A second type of regulation governs the luminous efficacies of lamps that may be sold. These regulations effectively force consumers to upgrade their lamp stock to technologies with ever increasing efficiencies. Incandescent bulbs, e.g., have now been effectively banned in many jurisdictions (Simmons *et al* 2007). However, these regulations do nothing to limit the use of a given stock of lamps, and hence to limit overall consumption of light.

We conclude that legislated regulations may also have a limited ability to cause a saturation in the consumption of light, if such a saturation would not already have naturally occurred.

## 6. Summary

In this paper we have provided baseline projections of the consumption of light in a world in which SSL is the dominant artificial lighting technology. Our projections are based on simple extrapolations of past behaviour into the future, and hence represent the historically consistent baseline assumption for constructing future scenarios for consumption of light and associated energy.

For past behaviour, we have used recent studies of historical and contemporary consumption patterns, analysed within a simple energy-economics framework of a Cobb-Douglas production function and profit maximization. For extrapolations into the future, we have used recent reviews of believed-achievable long-term performance targets for SSL.

A principal conclusion is that there is a massive potential for growth in the consumption of light if new lighting technologies are developed with higher luminous efficacies and lower cost of light. A secondary conclusion is that this increased consumption of light has the potential to increase both human productivity and the consumption of energy associated with that productivity. These conclusions suggest a subtle but important shift in how one views the baseline consequence of the increased energy efficiency associated with SSL. The consequence is not a simple ‘engineering’ decrease in energy consumption with consumption of light fixed, but rather an increase in human productivity and quality of life due to an increase in consumption of light.

Moreover, if one views the efficiency of lighting and the cost of energy associated with lighting as being two independent variables, one can see that these have different and complementary effects on human productivity and energy consumption. Changes in the efficiency of lighting affect both the cost of light and the amount of light that can be consumed per unit energy; while changes in the cost of energy associated with lighting affect only the cost of light. Thus, an increase in the cost of energy associated with lighting, which would normally reduce both human productivity and energy consumption, can be mitigated by an increase in the efficiency of lighting: energy consumption can be held constant while maintaining some human productivity increase

or energy consumption can be reduced without a decrease in human productivity.

Whether history can be used in this case to predict the future cannot be known, of course, and there are at least two ways in which this growth might be moderated. First, as discussed in section 5.1, demand for light may be nearing saturation, both in terms of illuminances ( $\text{lm m}^{-2}$ ) as well as in terms of the per-capita illuminated areas surrounding people. Second, as discussed in section 5.3, even if intrinsic demand for light is not nearing saturation, non-legislated guidelines and legislated regulations aimed at efficient lighting system designs and light usage may nonetheless bring about a saturation.

However, reasonable arguments can also be made that as a human society we are not near a saturation in consumption of light. As discussed in section 5.1, the average illuminance that a person is surrounded by during his or her waking hours and the average unshared illuminated area that a person is surrounded by may each yet be  $10\times$  or more from saturation. As discussed in section 5.2, new performance attributes associated with SSL have the potential to unleash new and unforeseen ways of consuming light. As discussed in section 5.3, guidelines and regulations may also have limited influence on light consumption.

## Acknowledgments

The authors acknowledge helpful input and comments from Eric Shaner, Paul Waide and a very thorough anonymous referee; and permission from Roger Fouquet and Peter Pearson for our adaptation of figure 1 from their monumental work on the consumption of light in the UK. Work at Sandia National Laboratories was supported by Sandia's Solid-State-Lighting Science Energy Frontier Research Center, funded by the US Department of Energy, Office of Basic Energy Sciences. Sandia is a multi-program laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the US Department of Energy's National Nuclear Security Administration under Contract No DE-AC04-94AL85000.

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