Spiraling into Precision - An analysis of the Helical Method for determining E/M

Diego Aguirre
Cornell University, College of Engineering
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By enclosing a CRT within a solenoid, one is able to generate a magnetic field that forces electrons into a helical path. The accelerating voltage can then be oscillated and current finetuned in order to find various helical modes; from which a value for E/M can be extracted. 23 of these values were recorded and averaged yielding a value of 1.64 x 10^{11} C/kg +/- 7.6 x 10^{9} C/kg.

I. Introduction

Each step we take involves a nearly uncountable amount of electrons. As such, the behavior of matter and the state of our very existence depend heavily on the characteristics of these sub-atomic particles. Their electric charge dictates how they interact with electromagnetic fields, while their mass determines their inertia - influencing how electrons interact with these fields. Thus, while their charge-to-mass ratio may be purely empirical - its value is still extremely important.

Using a CRT, we are able to accelerate a stream of electrons through 2 sets of capacitor plates with a voltage V. This CRT is enclosed within a solenoid through which we run a current I, generating a magnetic field of strength B. This in turn forces the electrons to follow a helical path. By iterating through different voltages, and adjusting the current accordingly; we can find various helical modes where the electron has completed exactly 2pi of orbital motion. At each mode, we record the I & V values to extract a value for e/m.

II. Theory

Electrons are accelerated through a CRT by a voltage following a roughly linear path. Ampere's Law states that running a current I through a solenoid generates a magnetic field with a strength B. Following Lorentz, we note that electrons in the CRT will follow a helical path after being subjected to B. After completing 2pi of orbital motion electrons have returned to their undeflected position. We denote the condition for this first focus as:

$$2\pi R = v_{\perp} t \qquad (1)$$

Where R is the radius of the helical path, v is transverse electron velocity, and t is the transit time of electrons in the CRT. Then, by following the cyclotron radius condition we observe that the focal condition is the same for all values of deflection. We then consider the acceleration voltage V, which enables us to derive the following equation:

$$\frac{e}{m} = \frac{8\pi^2 V}{B^2 l^2} \qquad (2)$$

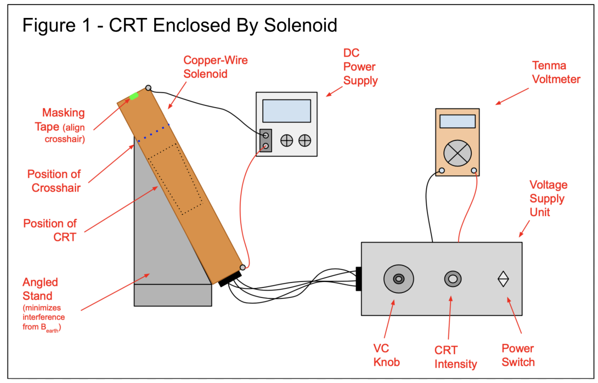

FIG 1 The apparatus consists of a CRT enclosed within a solenoid. A Voltage Supply provides the accelerating voltage V, which is measured by a Tenma Unit. A DC Power Supply runs a current through the solenoid, generating the magnetic field. Green tape is used to in order to reduce variance.

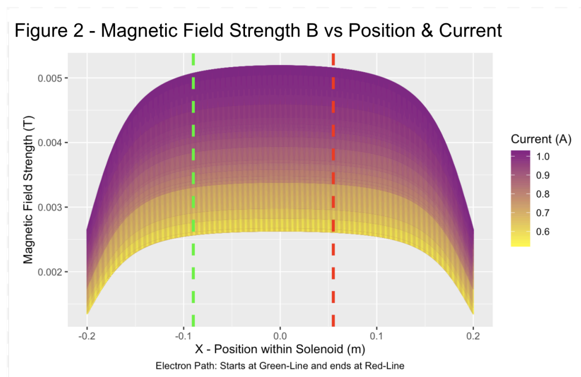

FIG 2 Graphical Representation of B as a function of Current and Position within the solenoid. The red and green dashed lines represent the portion of the solenoid that incident electrons travel through.

Where B is the strength of the magnetic field and l is the distance between the capacitor plates and the phosphor screen. To find B, we rely the universal finite solenoid field equation:

$$B(x) = \frac{\mu n I}{2}\left(\frac{x-x1}{\sqrt{(x-x1)^2+R^2}} - \frac{x-x2}{\sqrt{(x-x2)^2+R^2}}\right) \qquad (3)$$

Where n is the number of turns per unit length in the solenoid, x is the position within the solenoid, $x1$ and $x2$ represent the ends of the solenoid, I is the current, μ is the permeability of free space.

Figure 3 - Ratios with their Residuals, Voltage, and Current

Accelerating Voltage (V)	PS - Current (A)	Residual (V)	Ratio
400	0.52	-31.383642	164091734636
451	0.53	5.234961	178097656422
504	0.56	13.447182	178273948844
552	0.59	14.194021	175901059934
607	0.67	-68.867390	149993516722
649	0.68	-45.357757	155689840369
707	0.69	-6.122056	164723148326
757	0.71	5.527552	166576059410
797	0.73	6.081435	165899853731
853	0.74	41.947479	172790154784
904	0.78	9.672343	164821031234
954	0.81	-5.660289	161291595845
1011	0.83	6.414967	162790243508
1055	0.84	27.541698	165854511491
1104	0.85	53.394497	169498017890
1157	0.87	59.278301	169561892713
1205	0.91	9.758734	161412685907
1258	0.94	-13.257221	157927709222
1303	0.96	-20.304182	156832242227
1353	0.97	3.261441	159509932051
1402	1.00	-28.685278	155518269685
1463	1.02	-23.019415	155983041575
1501	1.03	-13.097380	156942166965

FIG 3 Table of the 23 computed Charge-To-Mass values, their Residuals, Voltages, and Currents.

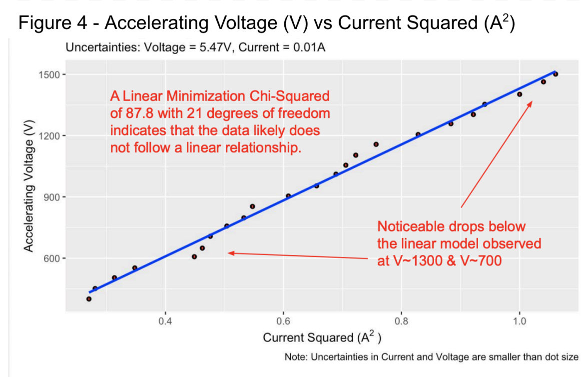

FIG 4 Plot of the Accelerating Voltages and Current Squared for each Helical Mode. A blue linear line of best fit has been overlaid.

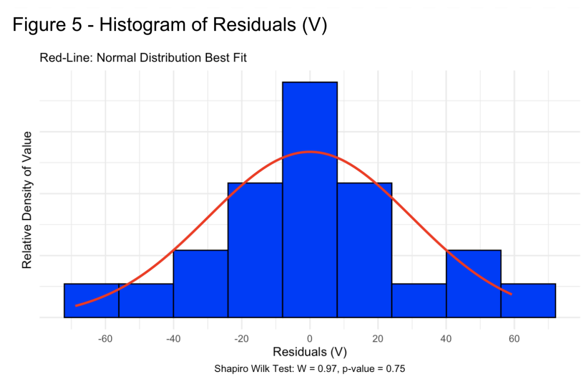

FIG 5 Histogram of the Residual Values (in volts) for each helical mode and the linear model in Figure 4. A Shapiro-Wilk and Normal Line of best fit indicate that the residuals are normally distributed.

III. Apparatus

Our apparatus and its general structure are outlined in *Figure 1*. It consists of a Sylvania 3BP1A Cathode Ray Tube enclosed within a Copper-Wire Solenoid. The CRT is connected to an analog controlled voltage supply which provides the accelerating voltage, and a Tenma Voltmeter to measure the amount. A Lavolta DC Power Supply is wired to the solenoid and provides the current that generates the Magnetic Field.

In order to minimize interference from Earth's magnetic field, the device is aimed at a ~20 degree polar angle and shoots electrons outwards from Earth's core towards the sky. Positioned directly above the CRT within the Solenoid is a cross-hair that helps quantify electron orbital path lengths. A piece of green masking tape attached to the top of the solenoid helps provide the user with a uniform vantage point of the crosshair.

IV. Experiment

Before compiling a set of helical modes, it was extremely important to find a single helical mode and minimize its uncertainty. As such, visual discrepancies were mitigated by placing a piece of masking tape on the right-side of the top of the Solenoid. By carefully positioning my nose so that the tape was barely visible, I was able to ensure that my dominant left eye was consistently centered directly above the crosshair. The voltage was then set to 400 V, and the solenoid current was varied until the incident electrons were maximally centered upon the crosshair. At this point, both values were then recorded. This process was then repeated 22 times at ~50 volt increments with the final recorded accelerating voltage being 1501V.

The solenoid's length was measured using a yard stick, and its radius was found via a caliper. The solenoid was observed to have 3 layers of wires, and the # coils per layer were measured via smartphone camera. This value was multiplied by 3 to find the aggregate coil count. The CRT's position within the solenoid was determined by first measuring the size of a spare CRT, and then recording the distance between the top of the Cross-Hair and the top of the Solenoid.

Uncertainty analysis was conducted on the Current and Voltage for Helical Modes as well as the Voltage Supply itself. First, to find the current uncertainty, helical modes were found at 12 fixed voltages ranging from 405V to 1509V. At each voltage, the Min and Max currents that constituted a maximally centered dot were recorded. Next, in

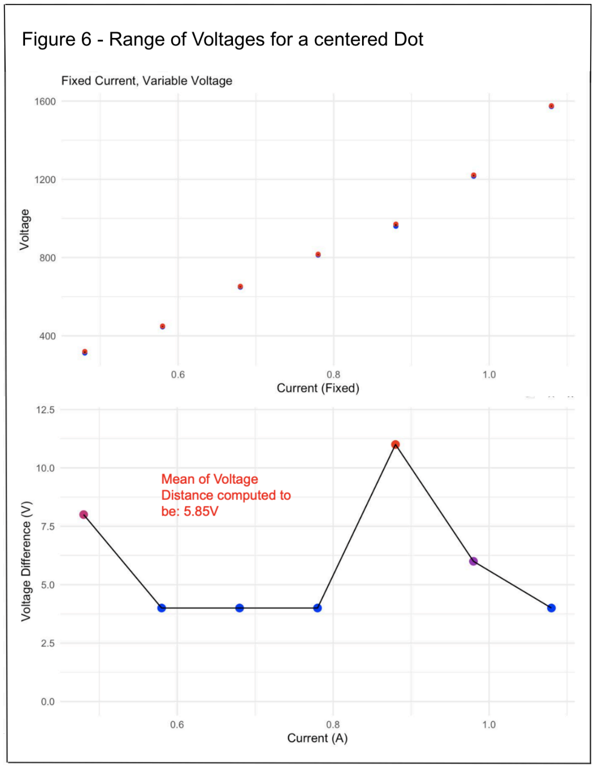

FIG 6 Display of the ranges of Voltages that yielded a centered dot. The upper facet plots the min and max voltage values for each fixed current. The lower facet plots the voltage differences vs current.

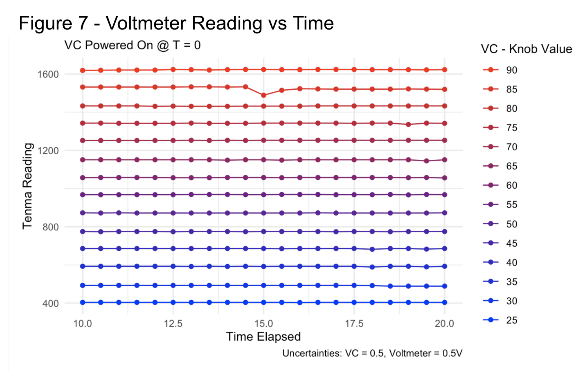

FIG 7 Plot of the range of voltmeter values for a given VC Knob Position recorded from t=10 to t=20. With the exception of the VC - Knob Value = 85 line, the variance is relatively uniform.

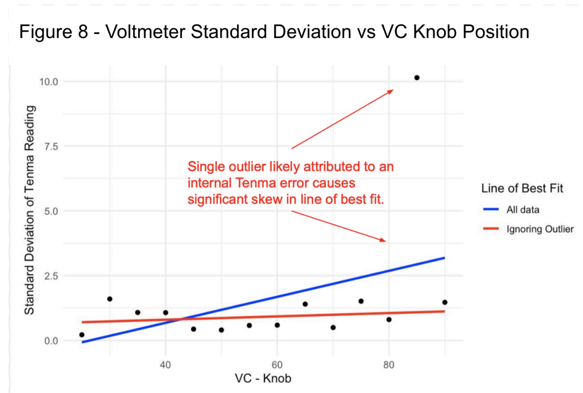

FIG 8 Displays the standard deviations as a function of thenVC Knob Value. With the exception of the single outlier, the Standard deviations are relatively uniform.

order to find the voltage uncertainty, helical modes were found at 7 fixed currents ranging from 0.48A to 1.08A. At each current, the Min and Max voltages that yielded a maximally centered dot were recorded. For the voltage supply, the following procedure was used. First, the VC knob was set to 25, and a timer was started as the device was powered on. From t~10s to t~20s, the voltmeter's reading was recorded at half second intervals; after which the device was powered down. This was repeated 13 times at VC knob increments of 5 all the way until the VC knob was at 90.

V. Results

The central result of this experiment was a set of 23 different values for the charge-to-mass ratio that were computed by finding helical modes at varying voltages. At each voltage, the current was finetuned until the incident electrons were "maximally" centered. This current was the input into a computer script that computed 1000 equally spaced B(x) values across the region of the solenoid where the electrons traveled using *equation 3* where R was measured to be 5.08 cm ∓ 0.1 cm using a caliper and x_1, x_2 represented the start and end of the solenoid. For more insight, reference *Figure 2* which plots B vs Current (A) and Position (m) for within the solenoid. The program then returned the mean B value, which was input into *equation 2* for each mode. The solenoid's length l was measured to be 40 cm ∓ 0.05cm, and the number of coils, N was determined to be 1655 by using a smartphone camera. To find n, these values were plugged into:

$$n = \frac{N}{l} \tag{5}$$

This yielded the values contained in *Figure 3*. More importantly, the mean was 1.64×10^{11} C/kg ∓ 7.6×10^{9} C/kg.

Figure 3 contains the 23 values that were derived for the charge-to-mass ratio, as well as the voltage and current for the corresponding helical mode. Additionally for each ratio, *Figure 3* also contains the residual (in volts) from the linear model contained in *Figure 4*. *Figure 4* plots the Accelerating Voltage vs Current Squared for each of the 23 helical modes. We note relatively large negative residuals at V~700 & V~1300. Deviations from the line of best fit can largely be attributed to the subjectivity regarding when the incident electrons are maximally centered; as well as discrepancies in the Voltage.

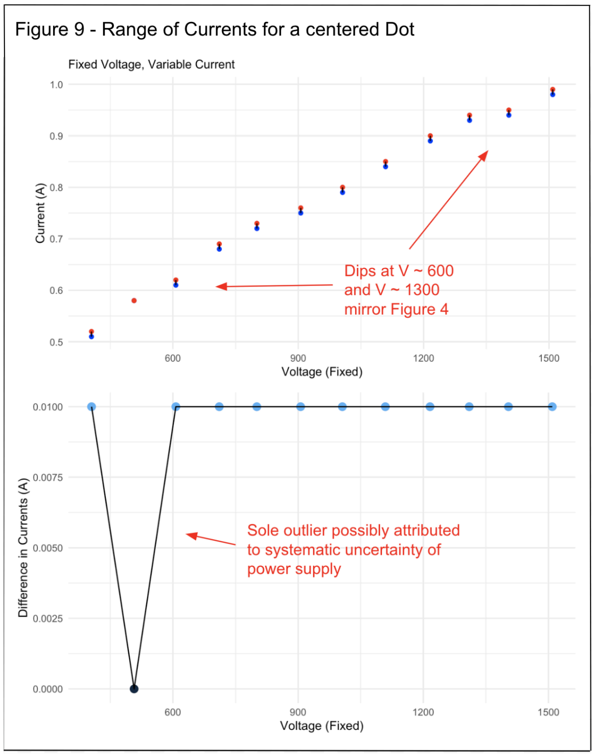

FIG 9 Displays the range of Current Values that yielded a centered dot. The upper facet plots the min and max current values for each fixed voltage. The lower facet plots the current difference for each fixed voltage.

Figure 5 shows a histogram of the voltage residuals. In order to evaluate the Normal Fit, A Shapiro-Wilk test was conducted yielding a W-Value of 0.97 and a P-Value of 0.75. The W-Value of 0.97 is extremely close to 1, suggesting that the residuals closely follow a normal distribution, and the P-Value of 0.75 is significantly higher than alpha = 0.05, indicating that there is insufficient evidence to reject the hypothesis that the data is normally distributed. Despite this, and even though F*igure 4* suggests a strong linear fit, a chi-squared minimization with 21 degrees of freedom yielded a value of 87.8, indicating that the linear model was not a good representation of the data. This minimization used a voltage uncertainty of ∓ 5.94V and a current uncertainty of ∓ 0.01A.

The voltage uncertainty was derived by combining the systematic and random uncertainties via Quadrature Summation which has the general form:

$$U = \sqrt{u_1^{\ 2} + u_2^{\ 2} + \ ... \ + u_n^{\ 2}} \qquad (4)$$

Where U is the combined uncertainty and u_i's are the uncertainties that are being combined. There were two sources of systematic uncertainty: the range of voltages that constituted a dot and the ∓0.5V of the Tenma read out.

Figure 6 displays the range of voltages that constituted a dot at a set of fixed currents. The observed

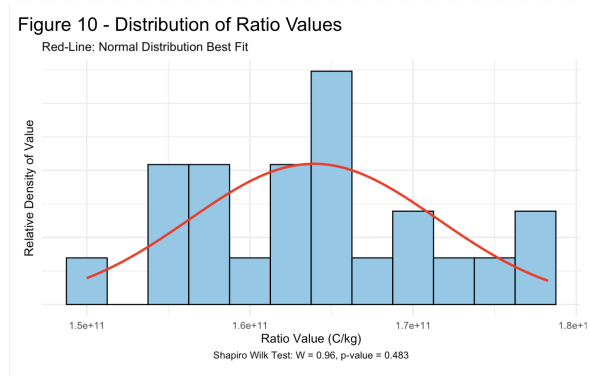

FIG 10 A Histogram of the resultant charge-to-mass ratios (see Fig 3 for actual values). A Shapiro-Wilk Test and the red line of best fit indicate that the ratios follow a normal distribution.

fluctuations are logical, as determining when the dot is centered is a relatively subjective process. Their mean was found to be ∓5.85V, and was combined with the ∓0.5V from the Tenma using Quadrature Summation yielding the overall Voltage Systematic Uncertainty of ∓5.87V. The random uncertainty was derived by recording Tenma readouts at varying VC Knob Positions over a uniform period of time. For each VC Knob position, the standard deviation of the voltmeter outputs was computed. The average was then computed and denoted as the random uncertainty. *Figure 7* displays these samples as a function of time. For the most part, the voltmeter is relatively consistent with the exception of a single value that was observed at t~15s when VC Knob was set to ~85. The source of this outlier is unknown, but most likely attributed to the Tenma unit itself.

Figure 8 plots the Standard Deviations vs the VC Knob Position, and the red line of best fit suggests that there is no relationship between the VC Knob's position and the Standard Deviation. Using the average standard deviation of 0.9V, and ∓5.87V as u_1 and u_2 , the voltage uncertainty of ∓5.94V was derived.

The current uncertainty was computed by combining the power supply's systematic uncertainty of ∓0.005A with the range of Current Values that constituted a maximally centered dot. *Figure 9* plots the size of each difference vs the Voltage and Current. Interestingly, large negative residuals are observed at V~700 & V~1300, mirroring what was observed in *Figure 4*'s data. In comparison to voltage, the ranges of plausible current values were significantly smaller. However, it is still important to note the subjective nature of determining when the incident electrons are maximally centered. The mean current range was found to be 0.009A. This was combined with the ∓0.005A using Quadrature Summation, yielding the Current Uncertainty of ∓0.01A.

Figure 10 shows the distribution of the recorded charge-to-mass ratios. We observe a peak roughly centered around the median. The red line shows a normal distribution line of best fit. A Shapiro Wilks test conducted in order to determine the degree of fit yielded a W-Value of 0.96, and a p-value of 0.483. Due to how close 0.96 is to 1, the W-value suggests that our data does follow a normal distribution. This is further supported by the p-value of 0.483, which is significantly higher than $\alpha = 0.05$, indicating that there is not enough evidence to reject our hypothesis that our ratio values follow a normal distribution. As such, the standard deviation of these 23 values was measured and denoted as the uncertainty in our mean charge-to-mass value.

VI. Conclusion

With the exception of the high Chi-Squared value for the linear minimization and the large negative residuals that were observed at V~700 & V~1300 in *Figures 3 & 7*; our observations aligned with what was expected. In fact, our mean charge-to-mass ratio value of 1.64×10^{11} C/kg $\mp$ 7.6×10^{9} C/kg is within 10% of the actual value of 1.76×10^{11} C/kg. As such, these results imply that the helical method is in fact a valid method for deriving the charge to mass ratio.

As previously outlined, the main source of error in this experiment was determining when the incident electrons were actually centered. This problem is further exacerbated by the age and dimness of the CRT. To mitigate this, one could conduct the experiment in a low-light environment. This would allow the CRT intensity to be extremely low, decreasing the amount of incident electrons and thus the dot size. This smaller dot would be easier to center under the crosshair, increasing the precision of the data for each mode. It is important to note that a more meticulous execution of this experiment would allocate more time to analyzing the large voltage residuals. While the large voltage range for a centered dot definitely contributes to these residuals, *Figure 7* highlights how the voltmeter itself is also riddled with variance.

If one wanted to further maximize the precision of this experiment, they could attach a camera to the top of the solenoid that was aimed down at the crosshair. Using a microcontroller and basic computer vision, they could then define a small region consisting of a few pixels on the camera's view that encompassed the true center. The intensity of the CRT and the size of this region could then be meticulously adjusted to maximize the congruency between the dot and the central region. A high precision servo could then be attached to the fine knob of the power supply, and wired into the microcontroller. A program could then be executed that used the servo to adjust the current until the dot occupied the pre-defined central region. Ensuring that the central region was securely centered and matched the dot size; this device would mechanically limit the variance between each helical mode. One could then repeat the same general process outlined under Experiment, but with minimal human error and subjectivity.