

National University of Computer & Emerging Sciences Karachi Campus



LINEAR ALGEBRA IN CHEMISTRY

EXAMPLES MANUAL

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EQUATION NO 1 :



Now the amount of each molecule needed is represented by x, y, z , and t .

The amount of each type of atom is written in parenthesis:

$$(2X)\text{Fe} + (3X)\text{S} + (12X)\text{O} + (Y)\text{K} + (Y)\text{O} + (Y)\text{H} = (2Z)\text{K} + (Z)\text{S} + (4Z)\text{O} + (T)\text{Fe} + (3T)\text{O} + (3T)\text{H}$$

We can break this down into equations by matching them up by the atom:

$$\text{Fe} \quad 2X=T$$

$$\text{S} \quad 3X=Z$$

$$\text{O} \quad 12X+Y=4Z+3T$$

$$\text{K} \quad Y=2Z$$

$$\text{H} \quad Y=3T$$

Now, rewriting the equations, we get:

$$\text{Fe} \quad 2X-T=0$$

$$\text{S} \quad 3X-Z=0$$

$$\text{O} \quad 12X+Y-4Z-3T=0$$

$$\text{K} \quad Y-2Z=0$$

$$\text{H} \quad Y-3T=0$$

Now, rewriting the equations as matrix, we get:

$$\begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 3 & 0 & -1 & 0 & 0 \\ 12 & 1 & -4 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 \end{bmatrix}$$

Now, step by step applying the row operations to get the row echelon form:

$$\begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 3 & 0 & -1 & 0 & 0 \\ 12 & 1 & -4 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 \end{bmatrix}$$

Multiply the 1st row by $1/2$

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 3 & 0 & -1 & 0 & 0 \\ 12 & 1 & -4 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 \end{bmatrix}$$

Add -3 times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 0 & -1 & 3/2 & 0 \\ 12 & 1 & -4 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 \end{bmatrix}$$

Add -12 times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 0 & -1 & 3/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 \end{bmatrix}$$

Interchange the 2nd row and the 3rd row

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & -1 & 3/2 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 \end{bmatrix}$$

Add -1 times the 2nd row to the 4th row

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & -1 & 3/2 & 0 \\ 0 & 0 & 1 & -3 & 0 \\ 0 & 1 & 0 & -3 & 0 \end{bmatrix}$$

Add -1 times the 2nd row to the 5th row

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & -1 & 3/2 & 0 \\ 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 4 & -6 & 0 \end{bmatrix}$$

Multiply the 3rd row by -1

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 4 & -6 & 0 \end{bmatrix}$$

Add -1 times the 3rd row to the 4th row

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 4 & -6 & 0 \end{bmatrix}$$

Add -4 times the 3rd row to the 5th row

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Multiply the 4th row by -2/3

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

T = Least common denominator

$$Z = \frac{3}{2} T$$

$$Y = 4Z - 3T$$

$$X = \frac{1}{2} T$$

here as least common denominator is 2 so:

$$T = 2$$

$$Z = \frac{3}{2} \times 2 = 3$$

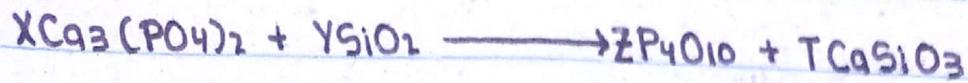
$$Y = 4(3) - (3)(2) = 6$$

$$X = \frac{1}{2} \times 2 = 1$$

Finally, the balanced chemical equation will be:

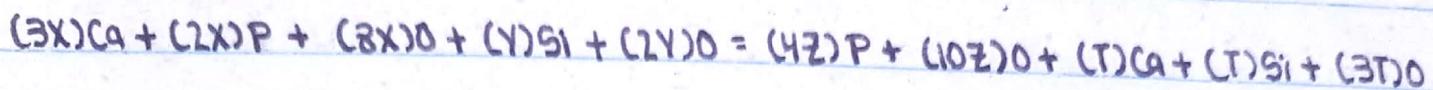


EQUATION NO 2:



Now the amount of each molecule needed is represented by X, Y, Z , and T .

The amount of each type of atom is written in parentheses.



We can break this down into equations by matching them up by the atoms:

$$\text{Ca} \quad 3X = T$$

$$\text{P} \quad 2X = 4Z$$

$$\text{O} \quad 8X + 2Y = 10Z + 3T$$

$$\text{Si} \quad Y = T$$

Now, rewriting the equations, we get:

$$\text{Ca} \quad 3X - T = 0$$

$$\text{P} \quad 2X - 4Z = 0$$

$$\text{O} \quad 8X + 2Y - 10Z - 3T = 0$$

$$\text{Si} \quad Y - T = 0$$

Now, rewriting the equations as matrix, we get:

$$\begin{bmatrix} 3 & 0 & 0 & -1 & 0 \\ 2 & 0 & -4 & 0 & 0 \\ 8 & 2 & -10 & -3 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Now step by step applying the row operations to get row echelon form:

$$\begin{bmatrix} 3 & 0 & 0 & -1 & 0 \\ 2 & 0 & -4 & 0 & 0 \\ 8 & 2 & -10 & -3 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Multiply 1st row by $1/3$

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 2 & 0 & -4 & 0 & 0 \\ 8 & 2 & -10 & -3 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Add -2 times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 0 & -4 & 2/3 & 0 \\ 8 & 2 & -10 & -3 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Add \rightarrow 3 times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -4 & \frac{2}{3} & 0 \\ 0 & 2 & -10 & -\frac{1}{3} & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Interchange the 2nd row and the 3rd row

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 2 & -10 & -\frac{1}{3} & 0 \\ 0 & 0 & -4 & \frac{2}{3} & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Multiply the 2nd row by $\frac{1}{2}$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & -5 & -\frac{1}{6} & 0 \\ 0 & 0 & -4 & \frac{2}{3} & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Add -1 times the 2nd row to the 4th row

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & -5 & -\frac{1}{6} & 0 \\ 0 & 0 & -4 & \frac{2}{3} & 0 \\ 0 & 0 & 5 & -\frac{5}{6} & 0 \end{bmatrix}$$

Multiply the 3rd row by $-\frac{1}{4}$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & -5 & -\frac{1}{6} & 0 \\ 0 & 0 & 1 & -\frac{1}{6} & 0 \\ 0 & 0 & 5 & -\frac{5}{6} & 0 \end{bmatrix}$$

Add -5 times the 3rd row to the 4th row

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & -5 & -\frac{1}{6} & 0 \\ 0 & 0 & 1 & -\frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

T = least common denominator

$$Z = \frac{1}{6}T$$

$$Y = 5Z + \frac{1}{6}T$$

$$X = \frac{1}{3}T$$

Here as least common denominator is 6 so:

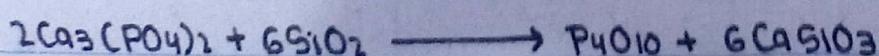
$$T = 6$$

$$Z = 1$$

$$Y = 5(1) + \frac{1}{6}(6) = 6$$

$$X = \frac{1}{3}(6) = 2$$

Finally the balanced chemical equation will be:



EQUATION NO 3:



Now the amount of each molecule reacted is represented by x, y, z , and t

The amount of each type of atom is written in parenthesis.

$$(x)\text{Sn} + (4x)\text{N} + (8x)\text{O} + (3y)\text{Pt} + (4y)\text{N} = (3z)\text{Sn} + (4z)\text{N} + (t)\text{Pt} + (4t)\text{N} + (8t)\text{O}$$

We can break this down into equations by matching them up by the atom.

$$\text{Sn} \quad x = 3z$$

$$\text{N} \quad 4x + 4y = 4z + 4t$$

$$\text{O} \quad 8x = 8t$$

$$\text{Pt} \quad 3y = t$$

Now, rewriting the equations, we get

$$\text{Sn} \quad x - 3z = 0$$

$$\text{N} \quad 4x + 4y - 4z - 4t = 0$$

$$\text{O} \quad 8x - 8t = 0$$

$$\text{Pt} \quad 3y - t = 0$$

Now, rewriting the equations as matrix we get:

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 4 & 4 & -4 & -4 & 0 \\ 8 & 0 & 0 & -8 & 0 \\ 0 & 3 & 0 & -1 & 0 \end{bmatrix}$$

Now step by step applying the row operations to get the reduced row echelon form.

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 4 & 4 & -4 & -4 & 0 \\ 8 & 0 & 0 & -8 & 0 \\ 0 & 3 & 0 & -1 & 0 \end{bmatrix}$$

add -4 times the 1st row to 2nd row

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 4 & 8 & -4 & 0 \\ 8 & 0 & 0 & -8 & 0 \\ 0 & 3 & 0 & -1 & 0 \end{bmatrix}$$

Add -8 times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 4 & 8 & -4 & 0 \\ 0 & 0 & 24 & -8 & 0 \\ 0 & 3 & 0 & -1 & 0 \end{bmatrix}$$

Multiply the 2nd row by $\frac{1}{4}$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 24 & -8 & 0 \\ 0 & 3 & 0 & -1 & 0 \end{bmatrix}$$

P

Add -3 times the 2nd row to the 4th row

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 24 & -8 & 0 \\ 0 & 0 & -6 & 2 & 0 \end{bmatrix}$$

Multiply the 3rd row by $\frac{1}{24}$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & -6 & 2 & 0 \end{bmatrix}$$

Add 6 times the 3rd row to the 4th row

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Add -2 times the 3rd row to the 2nd row

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Add 3 times the 3rd row to the 1st row

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now $T = \text{least common denominator}$

$$z = \frac{1}{3}T$$

$$y = \frac{1}{3}T$$

$$x = T$$

here as least common denominator is 3 so:

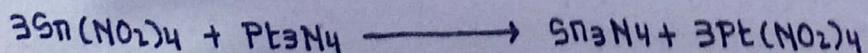
$$T = 3$$

$$z = \frac{3}{3} = 1$$

$$y = \frac{3}{3} = 1$$

$$x = 3$$

Finally, the balanced chemical equation will be:

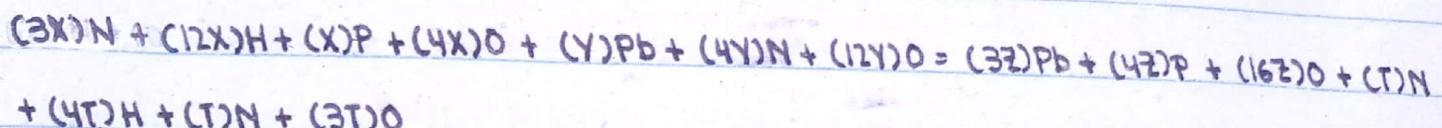


EQUATION NO 4:



Now the amount of each molecule needed is represented by x, y, z , and t .

The amount of each type of atom is written in parenthesis.



We can break this down into equations by matching them up by the atom.

$$\text{N} \quad 3x+4y=2t$$

$$\text{H} \quad 12x=4t$$

$$\text{P} \quad x=4z$$

$$\text{O} \quad 4x+12y=16z+3t$$

$$\text{B} \quad y=3z$$

Now, rewriting the equations, we get:

$$\text{N} \quad 3x+4y-2t=0$$

$$\text{H} \quad 12x-4t=0$$

$$\text{P} \quad x-4z=0$$

$$\text{O} \quad 4x+12y-16z-3t=0$$

$$\text{B} \quad y-3z=0$$

Now, rewriting the equations as matrix, we get:

$$\begin{bmatrix} 3 & 4 & 0 & -2 & 0 \\ 12 & 0 & 0 & -4 & 0 \\ 1 & 0 & -4 & 0 & 0 \\ 4 & 12 & -16 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Now step by step applying the row operations to get the reduced row echelon form:

$$\begin{bmatrix} 3 & 4 & 0 & -2 & 0 \\ 12 & 0 & 0 & -4 & 0 \\ 1 & 0 & -4 & 0 & 0 \\ 4 & 12 & -16 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Multiply the 1st row by $\frac{1}{3}$

$$\begin{bmatrix} 1 & \frac{4}{3} & 0 & -\frac{2}{3} & 0 \\ 12 & 0 & 0 & -4 & 0 \\ 1 & 0 & -4 & 0 & 0 \\ 4 & 12 & -16 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Add -12 times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & -16 & 0 & 4 & 0 \\ 1 & 0 & -4 & 0 & 0 \\ 4 & 12 & -16 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Add -1 times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & -16 & 0 & 4 & 0 \\ 0 & -4/3 & -4 & 2/3 & 0 \\ 4 & 12 & -16 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Add -4 times the 1st row to the 4th row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & -16 & 0 & 4 & 0 \\ 0 & -4/3 & -4 & 2/3 & 0 \\ 0 & 20/3 & -16 & -1/3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Multiply the 2nd row by $-1/16$

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & -4/3 & -4 & 2/3 & 0 \\ 0 & 20/3 & -16 & -1/3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Add $4/3$ times the 2nd row to the 3rd row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & -4 & 1/3 & 0 \\ 0 & 20/3 & -16 & -1/3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Add $-20/3$ times the 2nd row to the 4th row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & -4 & 1/3 & 0 \\ 0 & 0 & -16 & 4/3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Add -1 times the 2nd row to the 5th row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & -4 & 1/3 & 0 \\ 0 & 0 & -16 & 4/3 & 0 \\ 0 & 0 & -3 & 1/4 & 0 \end{bmatrix}$$

Multiply the 3rd row by $-1/4$

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & 1 & -1/12 & 0 \\ 0 & 0 & -16 & 4/3 & 0 \\ 0 & 0 & -3 & 1/4 & 0 \end{bmatrix}$$

Add 16 times the 3rd row to the 4th row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & 1 & -1/12 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 1/4 & 0 \end{bmatrix}$$

Add 3 times the 3rd row to the 5th row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & 1 & -1/12 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Add $-4/3$ times the 2nd row to the 1st row

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & 1 & -1/12 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now T = least common denominator

$$z = \frac{1}{12}T$$

$$y = \frac{1}{4}T$$

$$x = \frac{1}{3}T$$

here as least common denominator is 12 so:

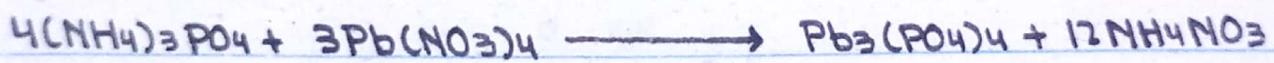
$$T = 12$$

$$z = \frac{12}{12} = 1$$

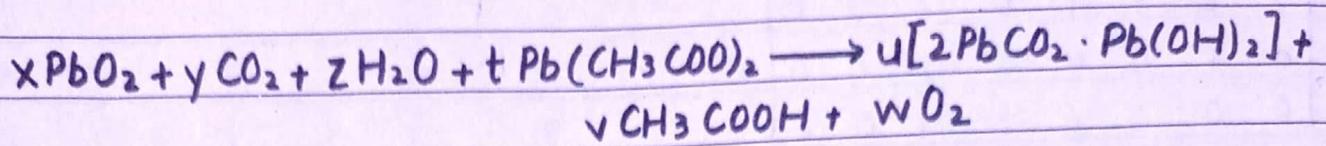
$$y = \frac{12}{4} = 3$$

$$x = \frac{12}{3} = 4$$

Finally, the balanced chemical equation will be:

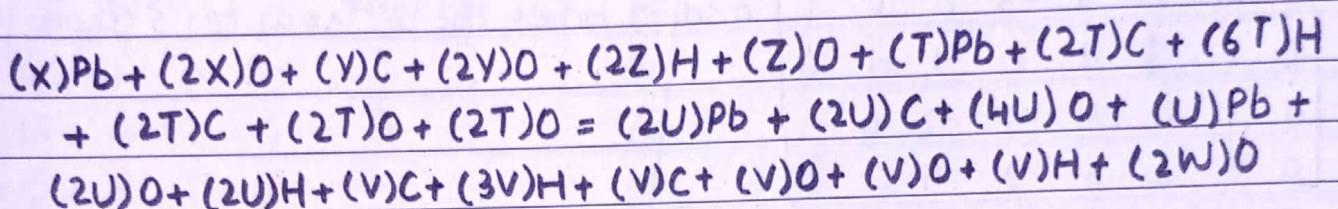


EQUATION NO 5



The amount of each molecule needed is represented by x, y, z, t, u, v and w .

The amount of each type of atom is written in parenthesis.



We can break this down into equations by matching them up by the atom:

$$\text{Pb: } x + t = 3u$$

$$\text{O: } 2x + 2y + z + 4t = 6u + 2v + 2w$$

$$\text{C: } y + 4t = 2u + 2v$$

$$\text{H: } 2z + 6t = 2u + 4v$$

Now, rewrite these equations as:-

$$x + t - 3u = 0$$

$$2x + 2y + z + 4t - 6u - 2v - 2w = 0$$

$$y + 4t - 2u - 2v = 0$$

$$2z + 6t - 2u - 4v = 0$$

Now, rewriting the equations as matrix:

$$\Rightarrow \left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & -3 & 0 & 0 & 0 \\ 2 & 2 & 1 & 4 & -6 & -2 & -2 & 0 \\ 0 & 1 & 0 & 4 & -2 & -2 & 0 & 0 \\ 0 & 0 & 2 & 6 & -2 & -4 & 0 & 0 \end{array} \right]$$

Now step by step applying the row operations to get the reduced echelon form:

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & -3 & 0 & 0 \\ 2 & 2 & 1 & 4 & -6 & -2 & -2 \\ 0 & 1 & 0 & 4 & -2 & -2 & 0 \\ 0 & 0 & 2 & 6 & -2 & -4 & 0 \end{array} \right] \text{ add } -2 \text{ times the 1st row to the 2nd row}$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & 2 & 0 & -2 & -2 & 0 \\ 0 & 1 & 0 & 4 & -2 & -2 & 0 \\ 0 & 0 & 2 & 6 & -2 & -4 & 0 \end{array} \right] \text{ multiply the 2nd row by } \frac{1}{2}$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 4 & -2 & -2 & 0 \\ 0 & 0 & 2 & 6 & -2 & -4 & 0 \end{array} \right] \text{ add } -1 \text{ times the 2nd row to the 3rd row}$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 1 & 0 & -1 & -1 \\ 0 & 0 & \frac{1}{2} & 3 & -2 & -1 & 1 \\ 0 & 0 & 2 & 6 & -2 & -4 & 0 \end{array} \right] \text{ multiply the 3rd row by } -2$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & -6 & 4 & 2 & -2 \\ 0 & 0 & 2 & 6 & -2 & -4 & 0 \end{array} \right] \text{ add } -2 \text{ times the 3rd row to the 4th row}$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & -6 & 4 & 2 & -2 \\ 0 & 0 & 0 & 18 & -10 & -8 & 4 \end{array} \right] \text{ multiply the 4th row by } \frac{1}{18}$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & -6 & 4 & 2 & -2 \\ 0 & 0 & 0 & 1 & -\frac{5}{9} & -\frac{4}{9} & \frac{2}{9} \end{array} \right] \text{ add 6 times the 4th row to the 3rd row}$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{-2}{3} & \frac{-2}{3} \\ 0 & 0 & 0 & 1 & -\frac{5}{9} & -\frac{4}{9} & \frac{2}{9} \end{array} \right] \text{ add } -1 \text{ times the 4th row to the 2nd row}$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{9} & \frac{-5}{9} & \frac{-11}{9} \\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{-2}{3} & \frac{-2}{3} \\ 0 & 0 & 0 & 1 & \frac{-5}{9} & \frac{-4}{9} & \frac{2}{9} \end{array} \right] \quad \text{add } -1 \text{ times the 4th row to the 1st row}$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 0 & \frac{22}{9} & \frac{4}{9} & \frac{-2}{9} \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{9} & \frac{-5}{9} & \frac{-11}{9} \\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{-2}{3} & \frac{-2}{3} \\ 0 & 0 & 0 & 1 & \frac{-5}{9} & \frac{-4}{9} & \frac{2}{9} \end{array} \right] \quad \text{add } -\frac{1}{2} \text{ times 3rd row to 2nd row}$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 0 & \frac{22}{9} & \frac{4}{9} & \frac{-2}{9} \\ 0 & 1 & 0 & 0 & \frac{2}{9} & \frac{-2}{9} & \frac{-8}{9} \\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{-2}{3} & \frac{-2}{3} \\ 0 & 0 & 0 & 1 & \frac{-5}{9} & \frac{-4}{9} & \frac{2}{9} \end{array} \right]$$

Now, u, v and w are least common denominator.

$$x = \frac{22}{9}u - \frac{4}{9}v + \frac{2}{9}w$$

$$y = -\frac{2}{9}u + \frac{2}{9}v + \frac{8}{9}w$$

$$z = -\frac{2}{3}u + \frac{2}{3}v + \frac{2}{3}w$$

$$t = \frac{5}{9}u + \frac{4}{9}v - \frac{2}{9}w$$

least common denominator of $u=9$, $v=9$ and $w=9$

Substituting these values in above equations:-

$$x = \frac{22}{9}(9) - \frac{4}{9}(9) + \frac{2}{9}(9) = 20$$

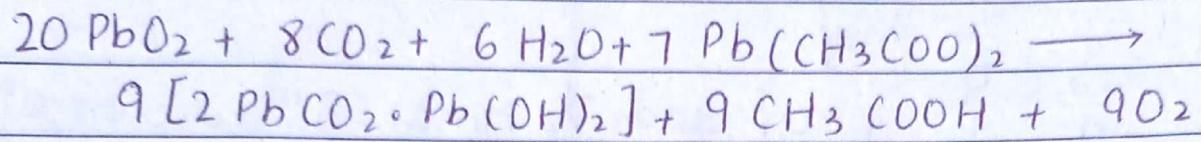
$$y = -\frac{2}{9}(9) + \frac{2}{9}(9) + \frac{8}{9}(9) = 8$$

$$z = -\frac{2}{3}(9) + \frac{2}{3}(9) + \frac{2}{3}(9) = 6$$

$$t = \frac{5}{9}(9) + \frac{4}{9}(9) - \frac{2}{9}(9) = 7$$

Date: _____

Balanced chemical equation is:

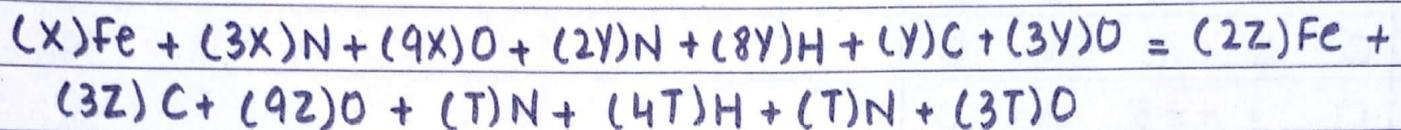


EQUATION NO 6



The amount of each molecule needed is represented by x, y, z and t .

The amount of each type of atom is written in parenthesis.



We can break this down into equations by matching them up by the atom:

$$\text{Fe : } x = 2z$$

$$\text{N : } 3x + 2y = 2t$$

$$\text{O : } 9x + 3y = 9z + 3t$$

$$\text{H : } 8y = 4t$$

$$\text{C : } y = 3z$$

Now, rewrite these equations as:

$$x - 2z = 0$$

$$3x + 2y - 2t = 0$$

$$9x + 3y - 9z - 3t = 0$$

$$8y - 4t = 0$$

$$y - 3z = 0$$

Now, rewriting the equations as matrix:

$$\Rightarrow \left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 3 & 2 & 0 & -2 & 0 \\ 9 & 3 & -9 & 3 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array} \right] \begin{array}{l} \text{add -3 times the 1st row to the} \\ \text{2nd row} \end{array}$$

$$\left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 2 & 6 & -2 & 0 \\ 9 & 3 & -9 & 3 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array} \right] \text{ add } -9 \text{ times } 1^{\text{st}} \text{ row to } 3^{\text{rd}} \text{ row}$$

$$\left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 2 & 6 & -2 & 0 \\ 0 & 3 & 9 & 3 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array} \right] \text{ multiply the } 2^{\text{nd}} \text{ row by } \frac{1}{2}$$

$$\left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 3 & 9 & 3 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array} \right] \text{ add } -3 \text{ times the } 2^{\text{nd}} \text{ row to the } 3^{\text{rd}} \text{ row}$$

$$\left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array} \right] \text{ add } -8 \text{ times } 2^{\text{nd}} \text{ row to } 4^{\text{th}} \text{ row}$$

$$\left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & -24 & 4 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array} \right] \text{ add } -1 \text{ times } 2^{\text{nd}} \text{ row to } 5^{\text{th}} \text{ row}$$

$$\left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & -24 & 4 & 0 \\ 0 & 0 & -6 & 1 & 0 \end{array} \right] \text{ interchange } 3^{\text{rd}} \text{ row and } 4^{\text{th}} \text{ row}$$

$$\left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & -24 & 4 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & -6 & 1 & 0 \end{array} \right] \text{ multiply 3rd row by } -1/24$$

$$\left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 1 & \frac{-1}{6} & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & -6 & 1 & 0 \end{array} \right] \text{ add 6 times 3rd row to 5th row}$$

$$\left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 1 & \frac{-1}{6} & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

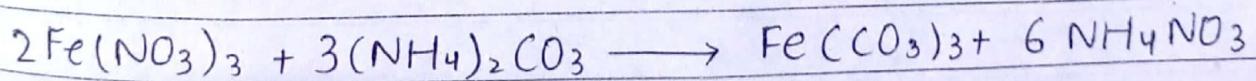
$$X = 2$$

$$Y = -3Z + T = -3(1) + (6) = -3 + 6 = 3$$

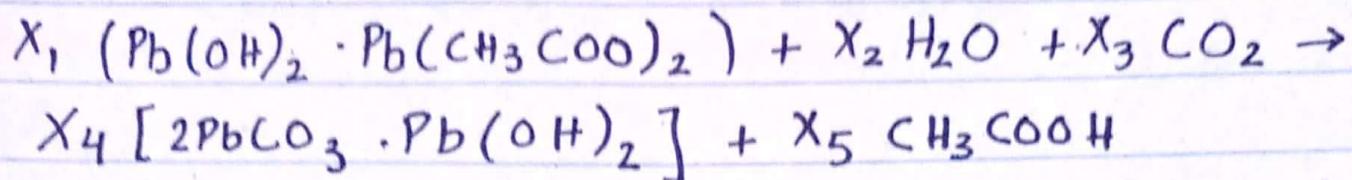
$$Z = \frac{1}{6}T = \frac{1}{6}(6) = 1$$

$$T = 6$$

Balanced Chemical Equation :-

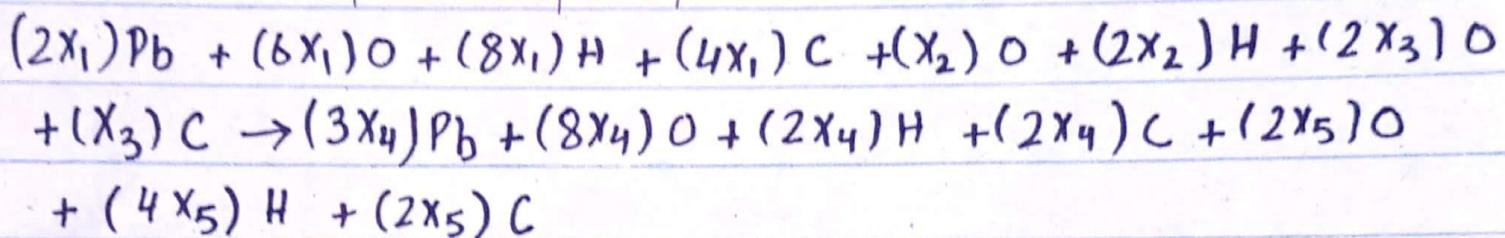


EQUATION NO 7



Now the amount of each molecule needed is represented by X_1, X_2, X_3, X_4 and X_5 .

The amount of each type of atom is written in parenthesis.



We can break this down into equations by matching them up by the atom.

$$Pb \quad 2X_1 = 3X_4$$

$$O \quad 6X_1 + X_2 + 2X_3 = 8X_4 + 2X_5$$

$$H \quad 8X_1 + 2X_2 = 2X_4 + 4X_5$$

$$C \quad 4X_1 + X_3 = 2X_4 + 2X_5$$

Now, rewriting the equations, we get

$$Pb \quad 2X_1 - 3X_4 = 0$$

$$O \quad 6X_1 + X_2 + 2X_3 - 8X_4 - 2X_5 = 0$$

$$H \quad 8X_1 + 2X_2 - 2X_4 - 4X_5 = 0$$

$$C \quad 4X_1 + X_3 - 2X_4 - 2X_5 = 0$$

Now, rewriting the equations as matrix, we get:

$$\begin{bmatrix} 2 & 0 & 0 & -3 & 0 & 0 \\ 6 & 1 & 2 & -8 & -2 & 0 \\ 8 & 2 & 0 & -2 & -4 & 0 \\ 4 & 0 & 1 & -2 & -2 & 0 \end{bmatrix}$$

Now, step by step applying the row operations to get the row echelon form

$$\left[\begin{array}{cccccc} 2 & 0 & 0 & -3 & 0 & 0 \\ 6 & 1 & 2 & -8 & -2 & 0 \\ 8 & 2 & 0 & -2 & -4 & 0 \\ 4 & 0 & 1 & -2 & -2 & 0 \end{array} \right]$$

Multiply the 1st row by $\frac{1}{2}$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 6 & 1 & 2 & -8 & -2 & 0 \\ 8 & 2 & 0 & -2 & -4 & 0 \\ 4 & 0 & 1 & -2 & -2 & 0 \end{array} \right]$$

Add -6 times the 1st row to the 2nd row

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 8 & 2 & 0 & -2 & -4 & 0 \\ 4 & 0 & 1 & -2 & -2 & 0 \end{array} \right]$$

Add -8 times the 1st row to the 3rd row

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 0 & 2 & 0 & 10 & -4 & 0 \\ 4 & 0 & 1 & -2 & -2 & 0 \end{array} \right]$$

Add -4 times the 1st row to the 4th row

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 0 & 2 & 0 & 10 & -4 & 0 \\ 0 & 0 & 1 & 4 & -2 & 0 \end{array} \right]$$

Add -2 times the 2^{nd} row to the 3^{rd} row

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 0 & 0 & -4 & 8 & 0 & 0 \\ 0 & 0 & 1 & 4 & -2 & 0 \end{array} \right]$$

Multiply the 3^{rd} row by $-\frac{1}{4}$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 4 & -2 & 0 \end{array} \right]$$

Add -1 times the 3^{rd} row to the 4^{th} row

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 6 & -2 & 0 \end{array} \right]$$

Multiply the 4^{th} row by $\frac{1}{6}$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 \end{array} \right]$$

Add 2 times the 4^{th} row to the 3^{rd} row

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 \end{array} \right]$$

Add -1 times the 4th row to the 2nd row

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 0 & -\frac{5}{3} & 0 \\ 0 & 0 & 1 & 0 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 \end{array} \right]$$

Add $\frac{3}{2}$ times the 4th row to the 1st row

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 2 & 0 & -\frac{5}{3} & 0 \\ 0 & 0 & 1 & 0 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 \end{array} \right]$$

Add -2 times the 3rd row to the 2nd row

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 \end{array} \right]$$

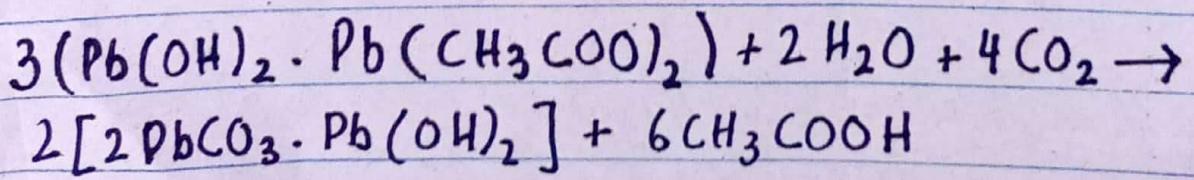
$$x_5 = 6$$

$$x_1 = \frac{1}{2} x_5 = \frac{1}{2}(6) = 3$$

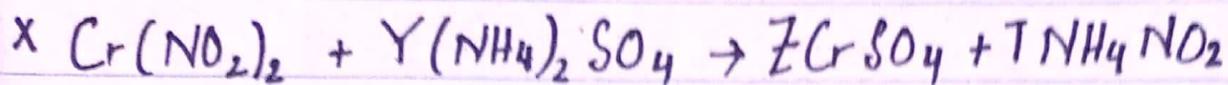
$$x_2 = \frac{1}{3} x_5 = \frac{1}{3}(6) = 2$$

$$x_3 = \frac{2}{3} x_5 = \frac{2}{3}(6) = 4$$

$$x_4 = \frac{1}{3} x_5 = \frac{1}{3}(6) = 2$$

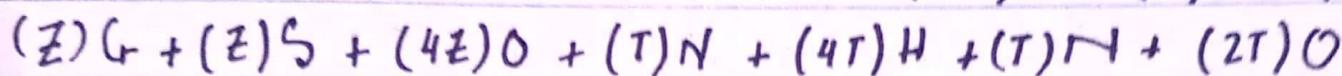
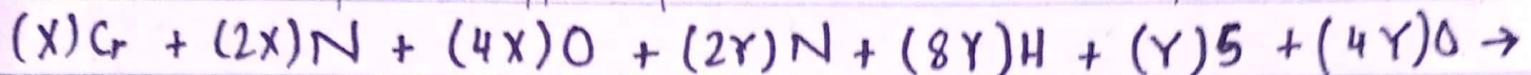


EQUATION NO 8



Now, the amount of each molecule needed is represented by X, Y, Z and T

The amount of each type of atom is written in parentheses



We can break this down into equations by matching them up by the atom

$$\text{Cr} \quad X = Z$$

$$\text{N} \quad 2X + 2Y = T + T$$

$$\text{O} \quad 4X + 4Y = 4Z + 2T$$

$$\text{H} \quad 8Y = 4T$$

$$\text{S} \quad Y = Z$$

Now, rewriting the equations we get;

$$\text{Cr} \quad X - Z = 0$$

$$\text{N} \quad 2X + 2Y - 2T = 0$$

$$\text{O} \quad 4X + 4Y - 4Z - 2T = 0$$

$$\text{H} \quad 8Y - 4T = 0$$

$$\text{S} \quad Y - Z = 0$$

Now, rewriting the equations as matrix we get;

$$\left[\begin{array}{ccccc} 1 & 0 & -1 & 0 & 0 \\ 2 & 2 & 0 & -2 & 0 \\ 4 & 4 & -4 & -2 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array} \right]$$

Now, step by step applying the row operations to get the row echelon form

$$\left[\begin{array}{ccccc} 1 & 0 & -1 & 0 & 0 \\ 2 & 2 & 0 & -2 & 0 \\ 4 & 4 & -4 & -2 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array} \right]$$

Add -2 times the 1^{st} row to the 2^{nd} row

$$\left[\begin{array}{ccccc} 1 & 0 & -1 & 0 & 0 \\ 0 & 2 & 2 & -2 & 0 \\ 4 & 4 & -4 & -2 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array} \right]$$

Add -4 times the 1^{st} row to the 3^{rd} row

$$\left[\begin{array}{ccccc} 1 & 0 & -1 & 0 & 0 \\ 0 & 2 & 2 & -2 & 0 \\ 0 & 4 & 0 & -2 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array} \right]$$

Multiply the 2^{nd} row by $\frac{1}{2}$

$$\left[\begin{array}{ccccc} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 4 & 0 & -2 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array} \right]$$

Add -4 times the 2nd row to the 3rd row

$$\left[\begin{array}{ccccc} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & -4 & 2 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array} \right]$$

Add -8 times the 2nd row to the 4th row

$$\left[\begin{array}{ccccc} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & -4 & 2 & 0 \\ 0 & 0 & -8 & 4 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array} \right]$$

Add -1 times the 2nd row to the 5th row

$$\left[\begin{array}{ccccc} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & -4 & 2 & 0 \\ 0 & 0 & -8 & 4 & 0 \\ 0 & 0 & -2 & 1 & 0 \end{array} \right]$$

Multiply the 3rd row by $-\frac{1}{4}$

$$\left[\begin{array}{ccccc} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & -8 & 4 & 0 \\ 0 & 0 & -2 & 1 & 0 \end{array} \right]$$

8 times the 3rd row to the 4th row

$$\left[\begin{array}{ccccc} 0 & -1 & 0 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \end{array} \right]$$

2 times the 3rd row to the 5th row

$$\left[\begin{array}{ccccc} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

-1 times the 3rd row to the 2nd row

$$\left[\begin{array}{ccccc} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

add 1 times the 3rd row to the 1st row

$$\left[\begin{array}{ccccc} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Date:

$$T = 2$$

$$X = \frac{1}{2} T = \frac{1}{2}(2) = 1$$

$$Y = \frac{1}{2} T = \frac{1}{2}(2) = 1$$

$$Z = \frac{1}{2} T = \frac{1}{2}(2) = 1$$

Balanced Chemical Equation.

