National University of Computer & Emerging Sciences Karachi Campus

LINEAR ALGEBRA IN CHEMISTRY

EXAMPLES MANUAL

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EQUATION NO 1 :

$$XFe_2(SO_4)_3 + YKOH \longrightarrow ZK_2SO_4 + TFe(OH)_3$$

Now the amount of each molecule needed is represented by x, y, z, and t.

The amount of each type of atom is written in paranthesis:

$$(2X)Fe + (3X)S + (12X)O + (Y)K + (Y)O + (Y)H = (2Z)K + (Z)S + (4Z)O + (T)Fe + (3T)O + (3T)H$$

We can break this down into equations by matching them up by the atom:

Fe $2X = T$

S $3X = Z$

O $12X + Y = 4Z + 3T$

K $Y = 2Z$

H $Y = 3T$

Now, rewriting the equations, we get:

Fe $2X - T = 0$

S $3X - Z = 0$

O $12X + Y - 4Z - 3T = 0$

K $Y - 2Z = 0$

H $Y - 3T = 0$

Now, rewriting the equations as matrix, we get:

$$\begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 3 & 0 & -1 & 0 & 0 \\ 12 & 1 & -4 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 \end{bmatrix}$$

Now, step by step applying the row operations to get the row echelon form:

$$\begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 3 & 0 & -1 & 0 & 0 \\ 12 & 1 & -4 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 \end{bmatrix} \quad \text{Multiply the 1st row by } 1/2 \quad \begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 3 & 0 & -1 & 0 & 0 \\ 12 & 1 & -4 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 \end{bmatrix}$$

Add -3 times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 0 & -1 & 3/2 & 0 \\ 12 & 1 & -4 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 \end{bmatrix}$$

Add -12 times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 0 & -1 & 3/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 \end{bmatrix}$$

Interchange the 2nd row and the 3rd row

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & -1 & 3/2 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 \end{bmatrix}$$

Add -1 times the 2nd row to the 4th row

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & -1 & 3/2 & 0 \\ 0 & 0 & 1 & -3 & 0 \\ 0 & 1 & 0 & -3 & 0 \end{bmatrix}$$

Add -1 times the 2nd row to the 5th row

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & -1 & 3/2 & 0 \\ 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 4 & -6 & 0 \end{bmatrix}$$

Multiply the 3rd row by -1

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 4 & -6 & 0 \end{bmatrix}$$

Add -1 times the 3rd row to the 4th row

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 4 & -6 & 0 \end{bmatrix}$$

Add -4 times the 3rd row to the 5th row

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Multiply the 4th row by -2/3

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

T= least com̂mon denominator

$Z = \frac{3}{2}T$

$Y = 4Z - 3T$

$X = \frac{1}{2}T$

here as least com̂mon denominator is 2 so:

$T = 2$

$Z = \frac{3}{2} \times 2 = 3$

$Y = 4(3) - (3)(2) = 6$

$X = \frac{1}{2} \times 2 = 1$

Finally, the balanced chemical equation will be:

$$Fe_2(SO_4)_3 + 6KOH \longrightarrow 3K_2SO_4 + 2Fe(OH)_3$$

EQUATION NO 2:

$$XCa_3(PO_4)_2 + YSiO_2 \longrightarrow ZP_4O_{10} + TCaSiO_3$$

Now the amount of each molecule needed is represented by x, y, z, and t.

The amount of each type of atom is written in paranthreses:

$$(3X)Ca + (2X)P + (8X)O + (Y)Si + (2Y)O = (4Z)P + (10Z)O + (T)Ca + (T)Si + (3T)O$$

We can break this down into equations by matching them up by the atoms:

Ca $3X = T$

P $2X = 4Z$

O $8X + 2Y = 10Z + 3T$

Si $Y = T$

Now, rewriting the equations, we get:

Ca $3X - T = 0$

P $2X - 4Z = 0$

O $8X + 2Y - 10Z - 3T = 0$

Si $Y - T = 0$

Now, rewriting the equations as matrix, we get:

$$\begin{bmatrix} 3 & 0 & 0 & -1 & 0 \\ 2 & 0 & -4 & 0 & 0 \\ 8 & 2 & -10 & -3 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Now step by step applying the row operations to get row echelon form:

$$\begin{bmatrix} 3 & 0 & 0 & -1 & 0 \\ 2 & 0 & -4 & 0 & 0 \\ 8 & 2 & -10 & -3 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix} \quad \text{Multiply 1st row by } 1/3 \quad \begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 2 & 0 & -4 & 0 & 0 \\ 8 & 2 & -10 & -3 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Add -2 times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 0 & -4 & 2/3 & 0 \\ 8 & 2 & -10 & -3 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Add -8 times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 0 & -4 & 2/3 & 0 \\ 0 & 2 & -10 & -1/3 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Interchange the 2nd row and the 3rd row

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 2 & -10 & -1/3 & 0 \\ 0 & 0 & -4 & 2/3 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Multiply the 2nd row by 1/2

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & -5 & -1/6 & 0 \\ 0 & 0 & -4 & 2/3 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Add -1 times the 2nd row to the 4th row

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & -5 & -1/6 & 0 \\ 0 & 0 & -4 & 2/3 & 0 \\ 0 & 0 & 5 & -5/6 & 0 \end{bmatrix}$$

Multiply the 3rd row by -1/4

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & -5 & -1/6 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & 5 & -5/6 & 0 \end{bmatrix}$$

Add -5 times the 3rd row to the 4th row

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & -5 & -1/6 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

T = least common denominator

$Z = \frac{1}{6}T$

$Y = 5Z + \frac{1}{6}T$

$X = \frac{1}{3}T$

Here as least common denominator is 6 so:

$T = 6$

$Z = 1$

$Y = 5(1) + \frac{1}{6}(6) = 6$

$X = \frac{1}{3}(6) = 2$

Finally the balanced chemical equation will be:

$$2Ca_3(PO_4)_2 + 6SiO_2 \longrightarrow P_4O_{10} + 6CaSiO_3$$

EQUATION NO 3:

$$XSn(NO_2)_4 + YPt_3N_4 \longrightarrow ZSn_3N_4 + TPt(NO_2)_4$$

Now the amount of each molecule needed is represented by x, y, z, and t

The amount of each type of atom is written in paranthesis.

$$(X)Sn + (4X)N + (8X)O + (3Y)Pt + (4Y)N = (3Z)Sn + (4Z)N + (T)Pt + (4T)N + (8T)O$$

We can break this down into equations by matching them up by the atom.

Sn $\quad X = 3Z$

N $\quad 4X + 4Y = 4Z + 4T$

O $\quad 8X = 8T$

Pt $\quad 3Y = T$

Now, rewriting the equations, we get

Sn $\quad X - 3Z = 0$

N $\quad 4X + 4Y - 4Z - 4T = 0$

O $\quad 8X - 8T = 0$

Pt $\quad 3Y - T = 0$

Now, rewriting the equations as matrix we get:

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 4 & 4 & -4 & -4 & 0 \\ 8 & 0 & 0 & -8 & 0 \\ 0 & 3 & 0 & -1 & 0 \end{bmatrix}$$

Now step by step applying the row operations to get the reduced row echelon form:

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 4 & 4 & -4 & -4 & 0 \\ 8 & 0 & 0 & -8 & 0 \\ 0 & 3 & 0 & -1 & 0 \end{bmatrix} \text{ add } -4 \text{ times the 1st row to 2nd row } \begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 4 & 8 & -4 & 0 \\ 8 & 0 & 0 & -8 & 0 \\ 0 & 3 & 0 & -1 & 0 \end{bmatrix}$$

Add -8 times the 1st row to the 3rd row
$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 4 & 8 & -4 & 0 \\ 0 & 0 & 24 & -8 & 0 \\ 0 & 3 & 0 & -1 & 0 \end{bmatrix}$$

Multiply the 2nd row by 1/4

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 24 & -8 & 0 \\ 0 & 3 & 0 & -1 & 0 \end{bmatrix}$$

Add -3 times the 2nd row to the 4th row

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 24 & -8 & 0 \\ 0 & 0 & -6 & 2 & 0 \end{bmatrix}$$

Multiply the 3rd row by 1/24

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -1/3 & 0 \\ 0 & 0 & -6 & 2 & 0 \end{bmatrix}$$

Add 6 times the 3rd row to the 4th row

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -1/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Add -2 times the 3rd row to the 2nd row

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & -1/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Add 3 times the 3rd row to the 1st row

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & -1/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now T= least common denominator

$Z = \frac{1}{3}T$

$Y = \frac{1}{3}T$

$X = T$

here as least common denominator is 3 so:

$T = 3$

$Z = \frac{3}{3} = 1$

$Y = \frac{3}{3} = 1$

$X = 3$

Finally, the balanced chemical equation will be:

$$3Sn(NO_2)_4 + Pt_3N_4 \longrightarrow Sn_3N_4 + 3Pt(NO_2)_4$$

EQUATION NO 4:

$X(NH_4)_3PO_4 + YPb(NO_3)_4 \longrightarrow ZPb_3(PO_4)_4 + TNH_4NO_3$

Now the amount of each molecule needed is represented by x, y, z, and t

The amount of each type of atom is written in paranthesis:

$(3X)N + (12X)H + (X)P + (4X)O + (Y)Pb + (4Y)N + (12Y)O = (3Z)Pb + (4Z)P + (16Z)O + (T)N$

$+ (4T)H + (T)N + (3T)O$

We can break this down into equations by matching them up by the atom.

N $3X+4Y=2T$

H $12X=4T$

P $X=4Z$

O $4X+12Y=16Z+3T$

B $Y=3Z$

Now, rewriting the equations, we get:

N $3X+4Y-2T=0$

H $12X-4T=0$

P $X-4Z=0$

O $4X+12Y-16Z-3T=0$

B $Y-3Z=0$

Now, rewriting the equations as matrix, we get:

$$\begin{bmatrix} 3 & 4 & 0 & -2 & 0 \\ 12 & 0 & 0 & -4 & 0 \\ 1 & 0 & -4 & 0 & 0 \\ 4 & 12 & -16 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Now step by step applying the row operations to get the reduced row echelon form:

$$\begin{bmatrix} 3 & 4 & 0 & -2 & 0 \\ 12 & 0 & 0 & -4 & 0 \\ 1 & 0 & -4 & 0 & 0 \\ 4 & 12 & -16 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix} \quad \text{Multiply the 1st row by } 1/3 \quad \begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 12 & 0 & 0 & -4 & 0 \\ 1 & 0 & -4 & 0 & 0 \\ 4 & 12 & -16 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Add -12 times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & -16 & 0 & 4 & 0 \\ 1 & 0 & -4 & 0 & 0 \\ 4 & 12 & -16 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Add -1 times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & -16 & 0 & 4 & 0 \\ 0 & -4/3 & -4 & 2/3 & 0 \\ 4 & 12 & -16 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Add -4 times the 1st row to the 4th row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & -16 & 0 & 4 & 0 \\ 0 & -4/3 & -4 & 2/3 & 0 \\ 0 & 20/3 & -16 & -1/3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Multiply the 2nd row by -1/16

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & -4/3 & -4 & 2/3 & 0 \\ 0 & 20/3 & -16 & -1/3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Add 4/3 times the 2nd row to the 3rd row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & -4 & 1/3 & 0 \\ 0 & 20/3 & -16 & -1/3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Add -20/3 times the 2nd row to the 4th row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & -4 & 1/3 & 0 \\ 0 & 0 & -16 & 4/3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Add -1 times the 2nd row to the 5th row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & -4 & 1/3 & 0 \\ 0 & 0 & -16 & 4/3 & 0 \\ 0 & 0 & -3 & 1/4 & 0 \end{bmatrix}$$

Multiply the 3rd row by -1/4

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & 1 & -1/12 & 0 \\ 0 & 0 & -16 & 4/3 & 0 \\ 0 & 0 & -3 & 1/4 & 0 \end{bmatrix}$$

Add 16 times the 3rd row to the 4th row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & 1 & -1/12 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 1/4 & 0 \end{bmatrix}$$

Add 3 times the 3rd row to the 5th row

$$\begin{bmatrix} 1 & 4/3 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & 1 & -1/12 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Add -4/3 times the 2nd row to the 1st row

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & 1 & -1/12 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now T= least common denominator

$Z = \frac{1}{12}T$

$Y = \frac{1}{4}T$

$X = \frac{1}{3}T$

here as least common denominator is 12 so:

$T = 12$

$Z = \frac{12}{12} = 1$

$Y = \frac{12}{4} = 3$

$X = \frac{12}{3} = 4$

Finally, the balanced chemical equation will be:

$$4(NH_4)_3PO_4 + 3Pb(NO_3)_4 \longrightarrow Pb_3(PO_4)_4 + 12NH_4NO_3$$

Date:

EQUATION No 5

$$xPbO_2 + yCO_2 + zH_2O + tPb(CH_3COO)_2 \longrightarrow u[2PbCO_2 \cdot Pb(OH)_2] + vCH_3COOH + wO_2$$

The amount of each molecule needed is represented by x, y, z, t, u, v and w.

The amount of each type of atom is written in paranthesis.

$$(x)Pb + (2x)O + (y)C + (2y)O + (2z)H + (z)O + (T)Pb + (2T)C + (6T)H + (2T)C + (2T)O + (2T)O = (2U)Pb + (2U)C + (4U)O + (U)Pb + (2U)O + (2U)H + (V)C + (3V)H + (V)C + (V)O + (V)O + (V)H + (2W)O$$

We can break this down into equations by matching them up by the atom:

Pb : $X + T = 3U$

O : $2X + 2Y + Z + 4T = 6U + 2V + 2W$

C : $Y + 4T = 2U + 2V$

H : $2Z + 6T = 2U + 4V$

Now, rewrite these equations as:-

$X + T - 3U = 0$

$2X + 2Y + Z + 4T - 6U - 2V - 2W = 0$

$Y + 4T - 2U - 2V = 0$

$2Z + 6T - 2U - 4V = 0$

Now, rewriting the equations as matrix:

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & -3 & 0 & 0 & 0 \\ 2 & 2 & 1 & 4 & -6 & -2 & -2 & 0 \\ 0 & 1 & 0 & 4 & -2 & -2 & 0 & 0 \\ 0 & 0 & 2 & 6 & -2 & -4 & 0 & 0 \end{bmatrix}$$

Now step by step applying the row operations to get the reduced echelon form:

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$$\begin{bmatrix} 1 & 0 & 0 & 1 & -3 & 0 & 0 & 0 \\ 2 & 2 & 1 & 4 & -6 & -2 & -2 & 0 \\ 0 & 1 & 0 & 4 & -2 & -2 & 0 & 0 \\ 0 & 0 & 2 & 6 & -2 & -4 & 0 & 0 \end{bmatrix}$$ add -2 times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -3 & 0 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 & -2 & -2 & 0 \\ 0 & 1 & 0 & 4 & -2 & -2 & 0 & 0 \\ 0 & 0 & 2 & 6 & -2 & -4 & 0 & 0 \end{bmatrix}$$ multiply the 2nd row by $1/2$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -3 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 4 & -2 & -2 & 0 & 0 \\ 0 & 0 & 2 & 6 & -2 & -4 & 0 & 0 \end{bmatrix}$$ add -1 times the 2nd row to 3rd row

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -3 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & -\frac{1}{2} & 3 & -2 & -1 & 1 & 0 \\ 0 & 0 & 2 & 6 & -2 & -4 & 0 & 0 \end{bmatrix}$$ multiply 3rd row by -2

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -3 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -6 & 4 & 2 & -2 & 0 \\ 0 & 0 & 2 & 6 & -2 & -4 & 0 & 0 \end{bmatrix}$$ add -2 times the 3rd row to the 4th row

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -3 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -6 & 4 & 2 & -2 & 0 \\ 0 & 0 & 0 & 18 & -10 & -8 & 4 & 0 \end{bmatrix}$$ multiply the 4th row by $1/18$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -3 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -6 & 4 & 2 & -2 & 0 \\ 0 & 0 & 0 & 1 & \frac{-5}{9} & \frac{-4}{9} & \frac{2}{9} & 0 \end{bmatrix}$$ add 6 times the 4th row to the 3rd row

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -3 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{-2}{3} & \frac{-2}{3} & 0 \\ 0 & 0 & 0 & 1 & \frac{-5}{9} & \frac{-4}{9} & \frac{2}{9} & 0 \end{bmatrix}$$ add -1 times the 4th row to 2nd row

Date:

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -3 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{9} & \frac{-5}{9} & \frac{-11}{9} & 0 \\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{-2}{3} & \frac{-2}{3} & 0 \\ 0 & 0 & 0 & 1 & \frac{-5}{9} & \frac{-4}{2} & \frac{2}{9} & 0 \end{bmatrix}$$

add -1 times the 4th row to the 1st row

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \frac{-22}{9} & \frac{4}{9} & \frac{-2}{9} & 0 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{9} & \frac{-5}{9} & \frac{-11}{9} & 0 \\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{-2}{3} & \frac{-2}{3} & 0 \\ 0 & 0 & 0 & 1 & \frac{-5}{9} & \frac{-4}{9} & \frac{2}{9} & 0 \end{bmatrix}$$

add $-\frac{1}{2}$ times 3rd row to 2nd row

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \frac{-22}{9} & \frac{4}{9} & \frac{-2}{9} & 0 \\ 0 & 1 & 0 & 0 & \frac{2}{9} & \frac{-2}{9} & \frac{-8}{9} & 0 \\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{-2}{3} & \frac{-2}{3} & 0 \\ 0 & 0 & 0 & 1 & \frac{-5}{9} & \frac{-4}{9} & \frac{2}{9} & 0 \end{bmatrix}$$

Now, u, v and w are least common denominator:

$$x = \frac{22}{9}u - \frac{4}{9}v + \frac{2}{9}w$$

$$y = \frac{-2}{9}u + \frac{2}{9}v + \frac{8}{9}w$$

$$z = \frac{-2}{3}u + \frac{2}{3}v + \frac{2}{3}w$$

$$t = \frac{5}{9}u + \frac{4}{9}v - \frac{2}{9}w$$

least common denominator of $u = 9$, $v = 9$ and $w = 9$

Substituting these values in above equations:-

$$x = \frac{22}{9}(9) - \frac{4}{9}(9) + \frac{2}{9}(9) = 20$$

$$y = \frac{-2}{9}(9) + \frac{2}{9}(9) + \frac{8}{9}(9) = 8$$

$$z = \frac{-2}{3}(9) + \frac{2}{3}(9) + \frac{2}{3}(9) = 6$$

$$t = \frac{5}{9}(9) + \frac{4}{9}(9) - \frac{2}{9}(9) = 7$$

Date:

Balanced chemical equation is:

$$20\,PbO_2 + 8\,CO_2 + 6\,H_2O + 7\,Pb(CH_3COO)_2 \longrightarrow 9\,[2\,PbCO_2 \cdot Pb(OH)_2] + 9\,CH_3COOH + 9\,O_2$$

Date:

EQUATION NO 6

$$x\,Fe(NO_3)_3 + y\,(NH_4)_2CO_3 \longrightarrow z\,Fe(CO_3)_3 + t\,NH_4NO_3$$

The amount of each molecule needed is represented by x, y, z and t.
The amount of each type of atom is written in paranthesis :-

$(X)Fe + (3X)N + (9X)O + (2Y)N + (8Y)H + (Y)C + (3Y)O = (2Z)Fe + (3Z)C + (9Z)O + (T)N + (4T)H + (T)N + (3T)O$

We can break this down into equations by matching them up by the atom:

Fe : $X = 2Z$

N : $3X + 2Y = 2T$

O : $9X + 3Y = 9Z + 3T$

H : $8Y = 4T$

C : $Y = 3Z$

Now, rewrite these equations as :

$X - 2Z = 0$

$3X + 2Y - 2T = 0$

$9X + 3Y - 9Z - 3T = 0$

$8Y - 4T = 0$

$Y - 3Z = 0$

Now, rewriting the equations as matrix:

$$\Rightarrow \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 3 & 2 & 0 & -2 & 0 \\ 9 & 3 & -9 & 3 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

add -3 times the 1st row to the 2nd row

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$$\begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 2 & 6 & -2 & 0 \\ 9 & 3 & -9 & 3 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$ add -9 times 1^{st} row to 3^{rd} row

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 2 & 6 & -2 & 0 \\ 0 & 3 & 9 & 3 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$ multiply the 2^{nd} row by $1/2$

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 3 & 9 & 3 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$ add -3 times the 2^{nd} row to the 3^{rd} row

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$ add -8 times 2^{nd} row to 4^{th} row

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & -24 & 4 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$ add -1 times 2^{nd} row to 5^{th} row

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & -24 & 4 & 0 \\ 0 & 0 & -6 & 1 & 0 \end{bmatrix}$$ interchange 3^{rd} row and 4^{th} row

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & -24 & 4 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & -6 & 1 & 0 \end{bmatrix}$$ multiply 3rd row by $-1/24$

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 1 & \frac{-1}{6} & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & -6 & 1 & 0 \end{bmatrix}$$ add 6 times 3rd row to 5th row

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 1 & \frac{-1}{6} & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$X = 2$

$Y = -3Z + T = -3(1) + (6) = -3 + 6 = 3$

$Z = \frac{1}{6}T = \frac{1}{6}(6) = 1$

$T = 6$

Balanced Chemical Equation:-

$$2Fe(NO_3)_3 + 3(NH_4)_2CO_3 \longrightarrow Fe(CO_3)_3 + 6NH_4NO_3$$

EQUATION NO 7

$$X_1\,(Pb(OH)_2 \cdot Pb(CH_3COO)_2) + X_2\,H_2O + X_3\,CO_2 \rightarrow$$
$$X_4\,[2PbCO_3 \cdot Pb(OH)_2] + X_5\,CH_3COOH$$

Now the amount of each molecule neededed is represented by X_1, X_2, X_3, X_4 and X_5.

The amount of each type of atom is written in parenthesis.

$$(2X_1)Pb + (6X_1)O + (8X_1)H + (4X_1)C + (X_2)O + (2X_2)H + (2X_3)O$$
$$+ (X_3)C \rightarrow (3X_4)Pb + (8X_4)O + (2X_4)H + (2X_4)C + (2X_5)O$$
$$+ (4X_5)H + (2X_5)C$$

We can break this down into equations by matching them up by the atom

Pb $\quad 2X_1 = 3X_4$

O $\quad 6X_1 + X_2 + 2X_3 = 8X_4 + 2X_5$

H $\quad 8X_1 + 2X_2 = 2X_4 + 4X_5$

C $\quad 4X_1 + X_3 = 2X_4 + 2X_5$

Now, rewriting the equations, we get

Pb $\quad 2X_1 - 3X_4 = 0$

O $\quad 6X_1 + X_2 + 2X_3 - 8X_4 - 2X_5 = 0$

H $\quad 8X_1 + 2X_2 - 2X_4 - 4X_5 = 0$

C $\quad 4X_1 + X_3 - 2X_4 - 2X_5 = 0$

Now, rewriting the equations as matrix, we get:

$$\begin{bmatrix} 2 & 0 & 0 & -3 & 0 & 0 \\ 6 & 1 & 2 & -8 & -2 & 0 \\ 8 & 2 & 0 & -2 & -4 & 0 \\ 4 & 0 & 1 & -2 & -2 & 0 \end{bmatrix}$$

Now, step by step applying the row operations to get the row echelon form

$$\begin{bmatrix} 2 & 0 & 0 & -3 & 0 & 0 \\ 6 & 1 & 2 & -8 & -2 & 0 \\ 8 & 2 & 0 & -2 & -4 & 0 \\ 4 & 0 & 1 & -2 & -2 & 0 \end{bmatrix}$$

Multiply the 1^{st} row by $\frac{1}{2}$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 6 & 1 & 2 & -8 & -2 & 0 \\ 8 & 2 & 0 & -2 & -4 & 0 \\ 4 & 0 & 1 & -2 & -2 & 0 \end{bmatrix}$$

Add -6 times the 1^{st} row to the 2^{nd} row

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 8 & 2 & 0 & -2 & -4 & 0 \\ 4 & 0 & 1 & -2 & -2 & 0 \end{bmatrix}$$

Add -8 times the 1^{st} row to the 3^{rd} row

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 0 & 2 & 0 & 10 & -4 & 0 \\ 4 & 0 & 1 & -2 & -2 & 0 \end{bmatrix}$$

Add -4 times the 1^{st} row to the 4^{th} row

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 0 & 2 & 0 & 10 & -4 & 0 \\ 0 & 0 & 1 & 4 & -2 & 0 \end{bmatrix}$$

Add -2 times the 2^{nd} row to the 3^{rd} row
$$\begin{bmatrix} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 0 & 0 & -4 & 8 & 0 & 0 \\ 0 & 0 & 1 & 4 & -2 & 0 \end{bmatrix}$$

Multiply the 3^{rd} row by $-\frac{1}{4}$
$$\begin{bmatrix} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 4 & -2 & 0 \end{bmatrix}$$

Add -1 times the 3^{rd} row to the 4^{th} row
$$\begin{bmatrix} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 6 & -2 & 0 \end{bmatrix}$$

Multiply the 4^{th} rows by $\frac{1}{6}$
$$\begin{bmatrix} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 \end{bmatrix}$$

Add 2 times the 4^{th} row to the 3^{rd} row
$$\begin{bmatrix} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 \end{bmatrix}$$

Add -1 times the 4^{th} row to the 2^{nd} row
$$\begin{bmatrix} 1 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 2 & 0 & \frac{-5}{3} & 0 \\ 0 & 0 & 1 & 0 & \frac{-2}{3} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 \end{bmatrix}$$

Add $\frac{3}{2}$ times the 4^{th} row to the 1^{st} row
$$\begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 2 & 0 & \frac{-5}{3} & 0 \\ 0 & 0 & 1 & 0 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 \end{bmatrix}$$

Add -2 times the 3^{rd} row to the 2^{nd} row
$$\begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 \end{bmatrix}$$

$X_5 = 6$
$X_1 = \frac{1}{2} X_5 = \frac{1}{2}(6) = 3$
$X_2 = \frac{1}{3} X_5 = \frac{1}{3}(6) = 2$
$X_3 = \frac{2}{3} X_5 = \frac{2}{3}(6) = 4$
$X_4 = \frac{1}{3} X_5 = \frac{1}{3}(6) = 2$

$3(Pb(OH)_2 \cdot Pb(CH_3COO)_2) + 2H_2O + 4CO_2 \rightarrow$
$2[2PbCO_3 \cdot Pb(OH)_2] + 6CH_3COOH$

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EQUATION NO 8

$x\, Cr(NO_2)_2 + Y(NH_4)_2SO_4 \rightarrow Z\,CrSO_4 + T\,NH_4NO_2$

Now, the amount of each molecule needed is represented by X, Y, Z and T

The amount of each type of atom is written in parenthesis

$(X)Cr + (2X)N + (4X)O + (2Y)N + (8Y)H + (Y)S + (4Y)O \rightarrow$
$(Z)Cr + (Z)S + (4Z)O + (T)N + (4T)H + (T)N + (2T)O$

We can break this down into equations by matching them up by the atom

Cr $X = Z$

N $2X + 2Y = T + T$

O $4X + 4Y = 4Z + 2T$

H $8Y = 4T$

S $Y = Z$

Now, rewritting the equations, we get;

Cr $X - Z = 0$

N $2X + 2Y - 2T = 0$

O $4X + 4Y - 4Z - 2T = 0$

H $8Y - 4T = 0$

S $Y - Z = 0$

Now, rewritting the equations as matrix we get;

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 2 & 2 & 0 & -2 & 0 \\ 4 & 4 & -4 & -2 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

Date:

Now, step by step applying the row operations to get the row echelon form

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 2 & 2 & 0 & -2 & 0 \\ 4 & 4 & -4 & -2 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

Add -2 times the 1^{st} row to the 2^{nd} row

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 2 & 2 & -2 & 0 \\ 4 & 4 & -4 & -2 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

Add -4 times the 1^{st} row to the 3^{rd} row

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 2 & 2 & -2 & 0 \\ 0 & 4 & 0 & -2 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

Multiply the 2^{nd} row by $\frac{1}{2}$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 4 & 0 & -2 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

Date:

Add -4 times the 2^{nd} row to the 3^{rd} row

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & -4 & 2 & 0 \\ 0 & 8 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

Add -8 times the 2^{nd} row to the 4^{th} row

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & -4 & 2 & 0 \\ 0 & 0 & -8 & 4 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

Add -1 times the 2^{nd} row to the 5^{th} row

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & -4 & 2 & 0 \\ 0 & 0 & -8 & 4 & 0 \\ 0 & 0 & -2 & 1 & 0 \end{bmatrix}$$

Multiply the 3^{rd} row by $-\frac{1}{4}$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & -8 & 4 & 0 \\ 0 & 0 & -2 & 1 & 0 \end{bmatrix}$$

Date:

8 times the 3^{rd} row to the 4^{th} row

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \end{bmatrix}$$

2 times the 3^{rd} row to the 5^{th} row

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

d -1 times the 3^{rd} row to the 2^{nd} row

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

dd 1 times the 3^{rd} row to the 1^{st} row

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Date:

$T = 2$

$X = \frac{1}{2} T = \frac{1}{2}(2) = 1$

$Y = \frac{1}{2} T = \frac{1}{2}(2) = 1$

$Z = \frac{1}{2} T = \frac{1}{2}(2) = 1$

Balanced Chemical Equation.

$$Cr(NO_2)_2 + (NH_4)_2SO_4 \rightarrow CrSO_4 + 2\,NH_4NO_2$$