Sample Paper

MATHEMATICS

DATE : **Time: 3 Hours**

CLASS : XI **M.M: 100**

General Instructions:

(i) All questions are compulsory.

(ii) The question paper contains 29 questions divided into three sections A,B, C and D. Section A consists of 4 questions of one mark each, Section B consists of 8 questions of two marks each Section C consists of 11 questions of four marks each, and Section D consists of six questions of six marks each.

(iii) All questions in section A are to be answered in one word, one sentence or as per the exact requirement of question.

(iv) There is no overall choice. However, internal choice has been provided in three questions of four marks each and three questions of six mark each. You have to attempt only one of the alternatives in all such questions

(v)Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION-A

1. If R= $\{(x, y) : x + 2y = 8\}$ is a relation on N, write the range of R
2. Find the ratio in which the line joining the points (2, 4, 5) and (3, 5,-4) is divided by XY-plane
3. Find the modulus and principle argument of $-2i$
4. By giving a counter example show that the statement *"If n is an odd integer then n is prime'* is false.

SECTION-B

5. Represent the Complex number $Z = -1 - i\sqrt{3}$ in the polar form
6. Find the equation of a line perpendicular to the line $x - 2y + 3 = 0$ and passing through the point $(1, -2)$
7. Find the coordinates of the foci, and the length of the latus rectum of the ellipse: $16x^2 + y^2 = 16$
8. Find the derivative of $\sin\sqrt{x}$ using first principle
9. Evaluate $\lim_{x \to 0} \frac{\log(1+x)}{3^x - 1}$
10. Write contra positive and converse of the given statement: x is an even number implies that x is divisible by 4
11. Find $\frac{dy}{dx}$ if $y = |\sin x + 2x|$
12. If A and B are two mutually exclusive events associated to a random experiment P(A)= 0.4 and P(B)= 0.5 find (i)$P(A \cup B)(ii)P(A' \cap B')$.

SECTION-C

13. For any two sets A and B, prove by using properties of sets that $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$.
14. Find domain and range of the function $f(x) = \frac{x}{1+x^2}$.
15. Prove that:- $\cos 6x = 32\, cos^6 x - 48\, cos^4 x + 18\, cos^2 x$.
16. Solve for x:- $4 \sin x . \cos x + 2 \sin x + 2 \cos x + 1 = 0$.
17. Prove by principle of mathematical induction that, $(1 + x)^n \geq (1 + nx)$, for all natural numbern, where $x > -1$.
18. If $a + ib = \frac{x+i}{x-i}$, where x is a real number, then prove that $a^2 + b^2 = 1$and $\frac{b}{a} = \frac{2x}{x^2-1}$.
19. Solve the following system of inequality :- $\frac{x}{2x+1} \geq \frac{1}{4}$ and $\frac{6x}{4x-1} < \frac{1}{2}$.

20. Find the number of 5-digit telephone numbers, having at least one of their digits repeated.

OR

Find the coefficients of x^5 and in the product $(1+2x)^6(1-x)^7$ using binomial theorem.

21. Find the equations of the line which passes through the point (3,-2) and are inclined at 60° to the line $\sqrt{3}x + y = 1$.

OR

Find the equation of the set of points P, the sum of whose distances from A(4,0,0) and B (-4 ,0,0) is equal to 10.

22. Find the equation of the circle which touches both the axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant.

23. While shuffling a pack of 52 cards, 2 are accidentally dropped. Find the probability that the missing cards are of different colours.

OR

Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

SECTION-D

24. In a group of children, 35 play football out of which 20 play football only, 22 play hockey, 25 play cricket out of which 11 play cricket only. Out of these 7 play cricket and football but not hockey, 3 play football and hockey but not cricket and 12 play football and cricket both. How many of them play cricket and hockey but not football, how many play hockey only? What is the total number of children in the group? What is the importance of sports in human's life?

25. In a Δ ABC, Prove that:

(i) $a \cos A + b \cos B + c \cos C = 2a \sin B \sin C$

(ii) $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2+b^{2+}c^2}{2abc}$

26. If $S_1, S_2, S_3, S_4 \ldots\ldots S_p$ denote the sum of infinite GP whose first terms are 1,2,3,4....p respectively and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots \ldots \frac{1}{p+1}$, respectively.

Show that $S_1 + S_2 + S_3 + S_4 \ldots\ldots\ldots S_p = \frac{p(p+3)}{2}$.

OR,

Find the sum of the following series (up to n terms):-

(i) $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) \ldots\ldots\ldots$ (ii) $5 + 7 + 13 + 31 + 85 \ldots\ldots\ldots$

27. Prove by mathematical induction that:-

$$sin\theta + sin2\theta + sin3\theta + \cdots \ldots\ldots sin\, n\theta = \frac{sin\frac{n\theta}{2}\, sin\left(\frac{n+1}{2}\right)}{\sin\frac{\theta}{2}}$$

OR,

Show that $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is a natural number $\forall\, n \in N$.

28. The sum of the coefficients of the first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^m, x \neq 0, m$ being a natural is 559. Find the term containing x^3 in the given expansion.

OR,

Solve the following inequalities graphically:-

$3x + 2y \leq 12,\ x + y \geq 8,\ -x + y \geq 4, 5x \leq 10$ and $x, y \geq 0$.

29. The mean and standard deviation 100 items were recorded as 40 and 5.1, respectively. Later on, it was found that one observation 40 was wrongly copied down as 50. Find the correct S.D.

DELHI PUBLIC SCHOOL, BHILAI

SAMPLE PAPER

SUB- MATHEMATICS **FM-100** **TIME-3Hours** **CLASS-XI**

General instructions-

a) All questions are compulsory irrespective of internal choices.
b) Q.NO. 1 to 4 carry 1 mark each.
c) Q.NO. 5 to 12 carry 2 marks each.
d) Q.NO. 13 to 23 carry 4 marks each.
e) Q.NO. 24 to 29 carry 6 marks each
f) Use graph paper for sketching curves and lines.

1 If R is the set of real numbers and Q is the set of rational numbers then find R – Q.

2 Determine the domain and range of the relation $R = \{(x : 2x+3) : x \in \{1,2,3,4,5\}\}$

3 Write the period of $\sin\left(\frac{\pi}{4} + 3x\right)$

4 Draw the graph for $\{x\}$ in the interval (-2, 2) and write periodicity also.

5 Which T- function increases from $-\infty$ to -1 in 2^{nd} quadrant?

6 If $z_1 = 2 - I$, $z_2 = -2 + I$, find $Re(z_1 z_2)$

7 Prove that $C_0 + C_1 + C_2 + \ldots + C_n = 2^n$

8 Find centre and radius of circle $3x^2 + 3y^2 + 18x + 15y - 9 = 0$.

9 Three vertices of a parallelogram ABCD are $A(3,-1,2), B(1,2,-4), C(-1,1,2)$ and fourth vertex is $D(x,-2,8)$. Find the value of x.

10 What is Sandwich Theorem? Write one application of Sandwich Theorem.

11 Sketch the plane for i) x = 1 and ii) y = 3

12 The lines ax + 2y + 1 = 0, bx + 3y + 1 = 0 and cx + 4y + 1 = 0 are concurrent if a, b and c are in G.P.(T/F)

13 Find the domain and range of $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - 8x + 12}$

OR

Find domain of the function f given by i) $f(x) = \dfrac{1}{\sqrt{[x]^2 - [x] - 6}}$ ii) $f(x) = \dfrac{1}{\sqrt{x - [x]}}$

14 Prove by Induction, for n ≥ 1, $\dfrac{1}{1.2} + \dfrac{1}{2.3} + \dfrac{1}{3.4} + \ldots + \dfrac{1}{n(n+1)} = \dfrac{n}{n+1}$

OR

Prove by Induction $10^{2n-1} + 1$ is divisible by 11

15 A group consists of 4 girls and seven boys. In how many ways can a team of 5 members be selected if the team has

i) no girl

ii) at least one boy and one girl

iii) at least 3 girls.

16 If p^{th}, q^{th}, r^{th} and s^{th} terms of an A.P. are in G.P. then show that *(p- q), (q-r), (r-s)* are also in G.P.

OR

If f is a function satisfying

$$f(x+y) = f(x)f(y) \text{ for all } x, y \in N \text{ such that } f(1) - 3 \text{ and } \sum_{x=1}^{n} f(x) = 120, \text{ find } n.$$

17 Find the distance of the line 4x – y = 0 from the point P(4, 1) measured along the line making an angle of 135 degrees with the positive x- axis.

18 Find the coordinates of foci, the length of major axis, eccentricity, vertices and latus rectum for the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$.

19 What is the geometrical significance of first derivative? Using first principle, find $\frac{dy}{dx}$ of $y = x\sin x$.

OR

Suppose $f(x)=\begin{cases} a+bx, x<1 \\ 4, x=1 \\ b-ax, x>1 \end{cases}$ *and if* $\lim_{x\to 1} f(x)=f(1)$. What are possible values of a and b?

20 Find mean deviation about median for the following frequency distribution

CI	0-10	10-20	20-30	30-40	40-50	50-60
f	6	8	14	16	4	2

21 Using section formula, prove that the 3 points (-4, 6, 10), (2, 4, 6) and (14, 0, -2)are collinear.

22 Prove that $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}=\frac{3}{2}$

23 Express in polar form $\frac{2+6\sqrt{3}\,i}{5+\sqrt{3}\,i}$. Also convert i in polar form.

24 Solve the following systems graphically:

$2x+y\geq 4; x+y\leq 3; 2x-3y\leq 6$

OR

How many liters of water will have to be added to 1125 liters of the 45 % solution of acid so that the resulting mixture will contain more than 25 % but less than 30 % acid content?

25 A Vertex of an equilateral triangle is (2, 3) and the opposite side is x + y = 2. Find the equations of other sides.

26 Calculate mean, variance and standard deviation for the following frequency distribution table:

CLASSES	33-36	37-40	41-44	45-48	49-52
FREQUENCY	15	17	21	22	25

27 If α *and* β are the roots of the equation $x^2 - px + q = 0$. Prove that

$$\log_e(1+px+qx^2)=(\alpha+\beta)x-(\alpha^2+\beta^2)\frac{x^2}{2}+(\alpha^3+\beta^3)\frac{x^3}{3}-\ldots$$

OR

Find the sum to n terms of the series 11 + 103 + 1005 + ...

28 If $0\leq x\leq 2\pi$, *find* $\sin\frac{x}{2}, \cos\frac{x}{2}$ *and* $\tan\frac{x}{2}$, when $\cos x=-\frac{1}{3}$, x lies in 3rd quadrant.

OR

If $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right)=z\cos\left(\theta+\frac{4\pi}{3}\right)$, then find the value of $xy + yz + zx$.

29 I) Find a positive value of m for which the coefficients of x^2 in the expansion of $(1 + x)^m$ is 6.

Ii) Find the coefficients of x^6y^3 in the expansion of $(x + 2y)^9$.

General instructions:-

a) The question paper consists of four parts A, B, C and D. Each question of each part is compulsory
b) Part A consists of 4 questions of 1 mark each.
c) Part B consists of 8 questions of 2 mark each
d) Part C consists of 11 questions of 4 mark each
e) Part D consists of 6 questions of 6 mark each

1 If tan θ = ½ and tan ∅ = 1/3, then what is the value of (θ + ∅)?

2 For a complex number z, what is the value of arg z + arg $\bar{z}$, z ≠ 0?

3 $\underset{x \to 0}{lt} \dfrac{1-\cos x}{x^2}$ is equal to ___________.

4 $If\ y = 1 + x + \dfrac{x^2}{2!} + \ldots + \dfrac{x^n}{n!}, then\ find \dfrac{dy}{dx}.$

5 $If\ {}^nC_{r-1} = 36,\ {}^nC_r = 84\ and\ {}^nC_{r+1} = 126, then\ find\ {}^rC_2.$

6 Solve $1 \leq |x-2| \leq 3$.

7 If $z_1 = \sqrt{3} + i\sqrt{3}\ and\ z_2 = \sqrt{3} + i, then\ find\ the\ quadrant\ in\ which \left(\dfrac{z_1}{z_2}\right) lies.$

8 Find the value of $\cos^2 48^0 - \sin^2 12^o$.

9 Find the middle term in the expansion of $\left(2ax - \dfrac{b}{x^2}\right)^{12}$

10 Show that $x^2 + 2y^2 = 100$ represents equations of an ellipse.

11 Sketch the plane i) x = 2 ii) y = 3.

12 The lines ax + 2y + 1 = 0, bx + 3y + 1 = 0 and cx + 4y + 1 = 0 are concurrent if a, b and c are in GP(T/F)

13 From 50 students taking examinations in mathematics, physics and chemistry each of the student has passed in at least one of the subject, 37 students passed mathematics, 24 physics and 43 chemistry. At most 19 passed mathematics and physics, at most 29 mathematics and chemistry and at most 20 physics and chemistry. What is the largest possible number that could have passed all the three examination?

14 Find the domain of the function f(x) given by $i) f(x) = \frac{1}{\sqrt{[x]^2 - [x] - 6}}$ $ii) f(x) = \frac{1}{\sqrt{x - |x|}}$

OR

Is g = {(1, 1), (2, 3), (3, 5), (4,7)} a function? If this is described by the relation g(x) = αx + β,then what values should be assigned to α and β?

15 If α and β are the solutions of the equations atan θ + bsec θ = c, then show that $\tan(\alpha + \beta) = \frac{2ac}{a^2 - c^2}$

OR

If $x \cos\theta = y \cos\left(\theta + \frac{2\pi}{3}\right) = z \cos\left(\theta + \frac{4\pi}{3}\right)$ then find the value of xy + yz + zx.

16 Prove by Induction $\cos\theta \cos 2\theta \cos 2^2\theta \ldots \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n \sin\theta}$ for all $n \geq 1$

OR Prove by Induction $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\ldots\left(1 - \frac{1}{n^2}\right) = \frac{n+1}{n}$ for all natural number $n \geq 2$.

17 Show that $\left|\frac{z-2}{z-3}\right| = 2$ represents a circle. Find the center and radius of the circle obtained.

18 A boy has three library tickets and 8 books of his interest in the library, of these 8, he doesn't want to borrow mathematics part II unless mathematics part I is also borrowed. In how many ways can he choose the three books to be borrowed?

19 If n is a positive integer, find the coefficient of x^{-1} in the expansion of $(1+x)\left(1+\frac{1}{x}\right)^n$

OR

Find numerically greatest term in the expansion of $(2+3x)^9$, $where\ x = \frac{3}{2}$

20 If the slope of a line passing through the point A(3, 2) is ¾, then find points on the line which are 5 units away from the point A.

21 The mid-point of the sides of a triangle are (1, 5, -1) , (0, 4, -2) and (2, 3, 4). Find its vertices. Also find the centroid of the triangle.

22 Translate the following in symbolic form

a) 2 and 3 are prime numbers

b) Tigers are found in Gir forest or Raja Ji national park.

23 Three of the six vertices of a regular hexagon are chosen at random. What is the probability that the triangle with these vertices is equilateral?

24 Life of bulbs produced by two factories A and B are given below

Length of life	Factory A Number of bulbs	Factory B Number of bulbs
550-650	10	8
650-750	22	60
750-850	52	24
850-950	20	16
950-1050	16	12
	120	120

The bulbs of which factory are more consistent from point of view of length of life?

25 a) Using first principle find $\frac{dy}{dx}$ for y = log(sin x)

b) Evaluate $\lim\limits_{x\to\frac{\pi}{6}} \frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$

26 If the lines 2x – 3y = 5 and 3x – 4y =7 are the diameters of a circle of area 154 square units, then obtain the equation of the circle.

OR

The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest wire being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

27 If $a_1, a_2, a_3, \ldots, a_n$ are in AP with cd d not equal to zero then sum of the series $\sin d(\operatorname{cosec} a_1 \operatorname{cosec} a_2 + \operatorname{cosec} a_2 \operatorname{cosec} a_3 + \ldots + \operatorname{cosec} a_{n-1} \operatorname{cosec} a_n$ *is equal to* $\cot a_1 - \cot a_n$

OR

Find the natural number 'a' for which $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$ where the function f satisfies f(x + y) = f(x)f(y) for all natural numbers x, y and further f(1) = 2

28 A solution of 9 % acid is to be diluted by adding 3 % acid solution to it. The resulting mixture is to be more than 5 % and less than 7 % acid. If there is 460 liters of the 9 % solution, how many liters of 3 % solution will have to be added?

OR

Show that the following system of linear in equations has no solution

$x + 2y \leq 3$, $3x + 4y \geq 12$, $x \geq 0$, $y \geq 1$.

29 i) Find the general solution of the equation $\left(\sqrt{3}-1\right)\cos\theta+\left(\sqrt{3}+1\right)\sin\theta=2$

ii) Find the value of $\sin 9^\circ \text{ and } \cos 9^\circ$.

DELHI PUBLIC SCHOOL, BHILAI.

Time : 3 hrs Max. Marks : 100

General Instructions

1. All questions are compulsory
2. Please check that this question paper contains 29 questions.
3. Questions 1- 4 in Section A are very short answer type questions carrying 1 mark each.
4. Questions 5 - 12 in Section B are short answer type questions carrying 2 marks each.
5. Questions 13 - 23 in Section C are long answer type questions carrying 4 marks each.
6. Questions 24 - 29 in Section D are long answer type questions carrying 6 marks each.
7. Please write down the serial number of the question before attempting it.

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SECTION - A

1. If $f(x) = 3x^4 - 5x^2 + 9, find\ f(x-1)$.
2. Write the component statements of the compound statement and check whether the compound statement is true or false . 'A line is straight and extends indefinitely in both directions' .
3. A line passes through P (1, 2) such that its intercept between the axes is bisected at P. Find the equation of line.
4. Find the total number of terms in the expansion of $(x+a)^{100} + (x-a)^{100}$.

SECTION - B

5. Write the contrapositive and converse of the following statement : If x is a prime number , then x is odd.
6. Find the equation of line passing through (2 , 3) and perpendicular to the line 3x + 4y – 5 =0.
7. If A, B, C are any three sets such that A $\subseteq$ C, then prove that A × B $\subseteq$ C × B.
8. Is the given relation a function? justify four answer. $R_3 = \{(4,6),(3,9),(-11,6),(3,11)\}$
9. Let A = { x, y, z } & B = { 1, 2 } . Find the number of relations from A into B.
10. Find the least positive integral value of n such that $\left(\frac{1+i}{1-i}\right)^n$ = 1.
11. If $15_{C_{3r}} = 15_{C_{r+3}}$, find r .
12. $Solve\ the\ inequality:\ |1-2x| \leq 11$

SECTION - C

13. If for non – zero $x, af\,(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5, where\ a \neq b\,, find\ f\,(x)$.
14. Show that $\left|\frac{z-2}{z-3}\right| = 2$ represents a circle . Also , find its centre and radius .

OR

If $arg(z-1) = \arg(z+3i), find\ (x-1){:}\,y, where\ z = x+iy$.

15. How many words with or without meaning , each 2 of vowels and 3 consonants can be formed from the letters of the word DAUGHTER ? In the today's society , we see that many parents don't want girl child and get the abortion before the birth . What values is violated by the parents ?
16. Let $A = \{1,2,3,4\} and\ B = \{1,4,9,16,25\} and\ R\ be\ a$ relation defined from $A\ to\ B\ as$ $R = \{(x,y){:}\, x \in A, y \in B\ and\ y = x^2\}$.
(i) Depict this relation using arrow diagram.(ii) Find domain of R.(iii) Find range of $R(iv)\ Write\ codomain\ of R$
17. A solution of 9% acid is to be diluted by adding 3% acid solution to it . The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 L of the 9% solution , how many liters of 3% solution will have to be added ?

OR

Solve the following system of linear inequalities graphically . $3x + 2y \geq 24, 3x + y \leq 15, x \geq 4$

18. If p and q are the lengths of perpendicular from the origin to the lines $xcos\theta - y\,sin\theta = k\,cos2\theta\ and\ x\,sec\theta + y\,cosec\theta = k,$ respectively . prove that $p^2 + 4q^2 = k^2$.
19. 150 workers were engaged to finish a piece of work in a certain number of days. Four workers dropped the second day , four more workers dropped the third day and so on . If takes 8 more days to finish the work now. Find the number of days in which the work was completed .
20. If x lies in third quadrant and $5sinx + 3 = 0$, find the value of $\frac{2\,tanx - 5\,sinx + cotx}{2sin\frac{x}{2}\cos\frac{x}{2}}$
21. If $y = 2x$ is a chord of the circle $x^2 + y^2 - 10x = 0$, find the equation of a circle with this chord as diameter.

OR

Find the equation of ellipse whose foci are (2, 3) and (- 2 , 3) and whose semi – minor axis is $\sqrt{5}$

22. Show that the coordinates of the centroid of the triangle with vertices $A(x_1,y_1,z_1), B(x_2,y_2,z_2)\ and\ C(x_3,y_3,z_3)\ is\ \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$.

23. For what value of $n, \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the arithmetic mean of $and\ b$?

SECTION - D

24. Prove by the principle of mathematical induction that $\frac{n^5}{5}+\frac{n^3}{3}+\frac{7n}{15}$ is a natural number for all $n \in N$.

25. There are 200 individuals with a skin disorder , 120 has been exposed to chemical C_1, 50 to chemical C_2 and 30 to both the chemicals C_1 and C_2, find the number of individuals exposed to
(i)chemical C_1 or chemical C_2. (ii) chemical C_1 but not chemical C_2. (iii) chemical C_2 but not chemical C_1

26. Determine the smallest positive value of x (in degrees) for which
$\tan(x+100°) = \tan(x+50°)\tan x \tan(x-50°)$.

OR

Prove that $\frac{cos7x-cos8x}{1+\ 2\cos 5x} = \cos 2x - cos3x$.

27. Let f(x) = $\begin{cases} 3-x^2, & x \leq \quad -2 \\ ax+b, & -2 \quad < x < 2 \\ \frac{x^2}{2}, & x \quad \geq 2 \end{cases}$ Find a and b so that $\lim\limits_{x\to 2}$ f (x) and $\lim\limits_{x\to -2}$ f (x) exist.

28. If the 6^{th} , 7^{th} and 8^{th} terms in the expansion of $(x+a)^n$ are respectively 112 , 7 and $\frac{1}{4}$, find $x, a\ and\ n$.

29. The measurements of the diameters (in mm) of the heads of 107 screws are given below

Diameter (in mm)	33 - 35	36 - 38	39 - 41	42 - 44	45 - 47
Number of screws	17	19	23	21	27

Calculate the mean and the standard deviation .

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