

Setting up your ML application

Basic “recipe”
for machine learning

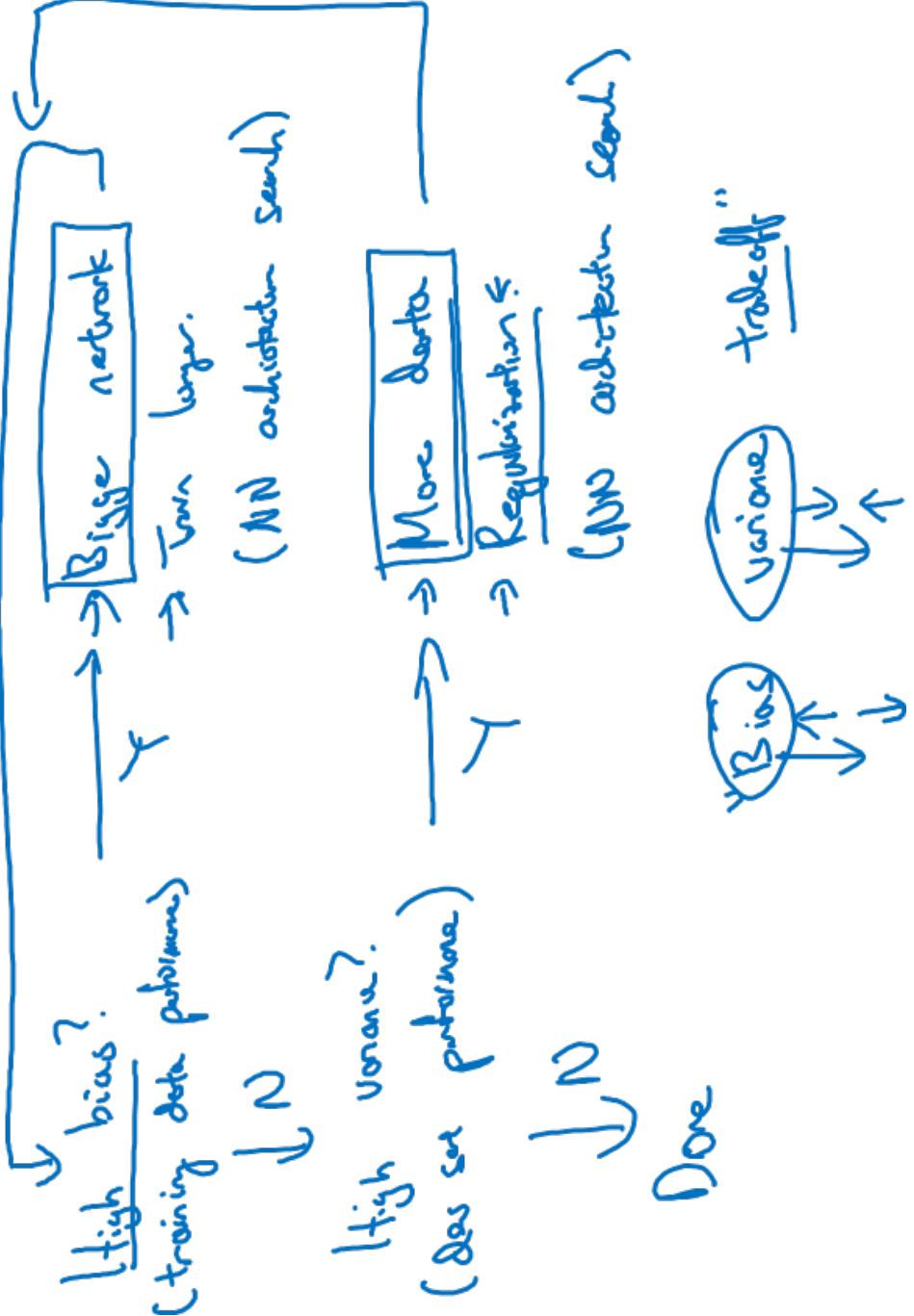
deeplearning.ai



Basic “recipe” for machine learning

Andrew

Basic recipe for machine learning



Orthogonalization

Orthogonalization or orthogonality is a system design property that assures that modifying an instruction or a component of an algorithm will not create or propagate side effects to other components of the system. It becomes easier to verify the algorithms independently from one another, it reduces testing and development time.

When a supervised learning system is design, these are the 4 assumptions that needs to be true and orthogonal.

1. Fit training set well in cost function
 - If it doesn't fit well, the use of a bigger neural network or switching to a better optimization algorithm might help.
2. Fit development set well on cost function
 - If it doesn't fit well, regularization or using bigger training set might help.
3. Fit test set well on cost function
 - If it doesn't fit well, the use of a bigger development set might help
4. Performs well in real world
 - If it doesn't perform well, the development test set is not set correctly or the cost function is not evaluating the right thing.

Single number evaluation metric

To choose a classifier, a well-defined development set and an evaluation metric speed up the iteration process.

Example : Cat vs Non- cat

$y = 1$, cat image detected

		Actual class y	
		1	0
Predict class \hat{y}	1	True positive	False positive
	0	False negative	True negative

Precision

Of all the images we predicted $y=1$, what fraction of it have cats?

$$\text{Precision (\%)} = \frac{\text{True positive}}{\text{Number of predicted positive}} \times 100 = \frac{\text{True positive}}{(\text{True positive} + \text{False positive})} \times 100$$

Recall

Of all the images that actually have cats, what fraction of it did we correctly identifying have cats?

$$\text{Recall (\%)} = \frac{\text{True positive}}{\text{Number of predicted actually positive}} \times 100 = \frac{\text{True positive}}{(\text{True positive} + \text{False negative})} \times 100$$

Let's compare 2 classifiers A and B used to evaluate if there are cat images:

Classifier	Precision (p)	Recall (r)
A	95%	90%
B	98%	85%

In this case the evaluation metrics are precision and recall.

For classifier A, there is a 95% chance that there is a cat in the image and a 90% chance that it has correctly detected a cat. Whereas for classifier B there is a 98% chance that there is a cat in the image and a 85% chance that it has correctly detected a cat.

The problem with using precision/recall as the evaluation metric is that you are not sure which one is better since in this case, both of them have a good precision et recall. F1-score, a harmonic mean, combine both precision and recall.

$$\text{F1-Score} = \frac{2}{\frac{1}{p} + \frac{1}{r}}$$

Classifier	Precision (p)	Recall (r)	F1-Score
A	95%	90%	92.4 %
B	98%	85%	91.0%

Classifier A is a better choice. F1-Score is not the only evaluation metric that can be use, the average, for example, could also be an indicator of which classifier to use.

Satisficing and optimizing metric

There are different metrics to evaluate the performance of a classifier, they are called evaluation matrices. They can be categorized as satisficing and optimizing matrices. It is important to note that these evaluation matrices must be evaluated on a training set, a development set or on the test set.

Example: Cat vs Non-cat

Classifier	Accuracy	Running time
A	90%	80 ms
B	92%	95 ms
C	95%	1 500 ms

In this case, accuracy and running time are the evaluation matrices. Accuracy is the optimizing metric, because you want the classifier to correctly detect a cat image as accurately as possible. The running time which is set to be under 100 ms in this example, is the satisficing metric which mean that the metric has to meet expectation set.

The general rule is:

$$N_{metric} : \begin{cases} 1 & \text{Optimizing metric} \\ N_{metric} - 1 & \text{Satisficing metric} \end{cases}$$

Training, development and test distributions

Setting up the training, development and test sets have a huge impact on productivity. It is important to choose the development and test sets from the same distribution and it must be taken randomly from all the data.

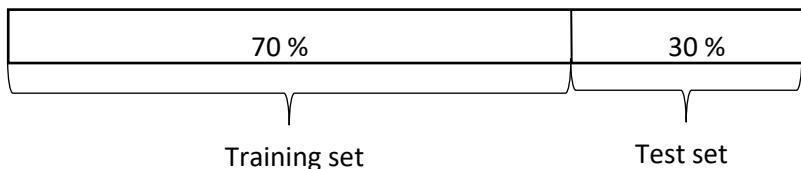
Guideline

Choose a development set and test set to reflect data you expect to get in the future and consider important to do well.

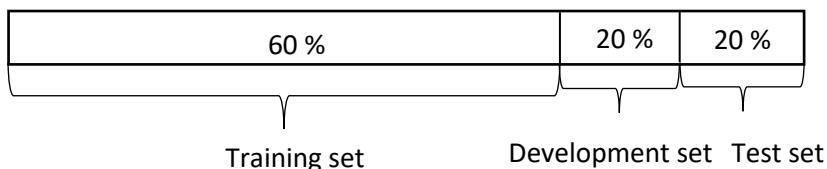
Size of the development and test sets

Old way of splitting data

We had smaller data set therefore we had to use a greater percentage of data to develop and test ideas and models.

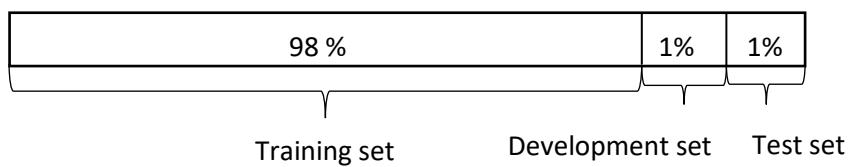


Or



Modern era – Big data

Now, because a large amount of data is available, we don't have to compromise as much and can use a greater portion to train the model.



Guidelines

- Set up the size of the test set to give a high confidence in the overall performance of the system.
- Test set helps evaluate the performance of the final classifier which could be less 30% of the whole data set.
- The development set has to be big enough to evaluate different ideas.

When to change development/test sets and metrics

Example: Cat vs Non-cat

A cat classifier tries to find a great amount of cat images to show to cat loving users. The evaluation metric used is a classification error.

Algorithm	Classification error [%]
A	3%
B	5%

It seems that Algorithm A is better than Algorithm B since there is only a 3% error, however for some reason, Algorithm A is letting through a lot of the pornographic images.

Algorithm B has 5% error thus it classifies fewer images but it doesn't have pornographic images. From a company's point of view, as well as from a user acceptance point of view, Algorithm B is actually a better algorithm. The evaluation metric fails to correctly rank order preferences between algorithms. The evaluation metric or the development set or test set should be changed.

The misclassification error metric can be written as a function as follow:

$$\text{Error} : \frac{1}{m_{dev}} \sum_{i=1}^{m_{dev}} \mathcal{L}\{\hat{y}^{(i)} \neq y^{(i)}\}$$

This function counts up the number of misclassified examples.

The problem with this evaluation metric is that it treats pornographic vs non-pornographic images equally. One way to change this evaluation metric is to add the weight term $w^{(i)}$.

$$w^{(i)} = \begin{cases} 1 & \text{if } x^{(i)} \text{ is non-pornographic} \\ 10 & \text{if } x^{(i)} \text{ is pornographic} \end{cases}$$

The function becomes:

$$\text{Error} : \frac{1}{\sum w^{(i)}} \sum_{i=1}^{m_{dev}} w^{(i)} \mathcal{L}\{\hat{y}^{(i)} \neq y^{(i)}\}$$

Guideline

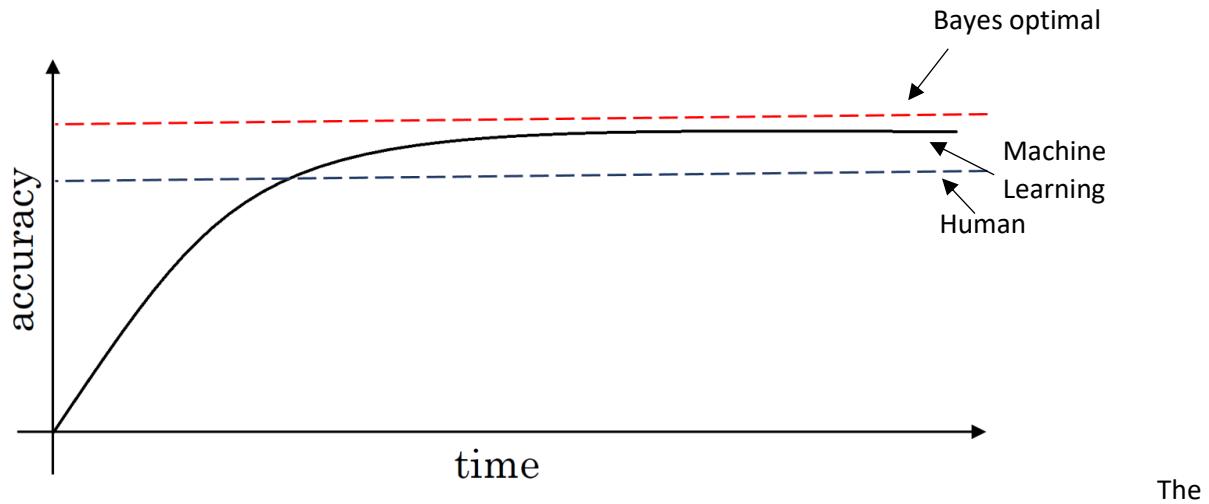
1. Define correctly an evaluation metric that helps better rank order classifiers
2. Optimize the evaluation metric

Why human-level performance?

Today, machine learning algorithms can compete with human-level performance since they are more productive and more feasible in a lot of application. Also, the workflow of designing and building a machine learning system, is much more efficient than before.

Moreover, some of the tasks that humans do are close to "perfection", which is why machine learning tries to mimic human-level performance.

The graph below shows the performance of humans and machine learning over time.



Machine learning progresses slowly when it surpasses human-level performance. One of the reason is that human-level performance can be close to Bayes optimal error, especially for natural perception problem.

Bayes optimal error is defined as the best possible error. In other words, it means that any functions mapping from x to y can't surpass a certain level of accuracy.

Also, when the performance of machine learning is worse than the performance of humans, you can improve it with different tools. They are harder to use once its surpasses human-level performance.

These tools are:

- Get labeled data from humans
- Gain insight from manual error analysis: Why did a person get this right?
- Better analysis of bias/variance.

Avoidable bias

By knowing what the human-level performance is, it is possible to tell when a training set is performing well or not.

Example: Cat vs Non-Cat

	Classification error (%)	
	Scenario A	Scenario B
Humans	1	7.5
Training error	8	8
Development error	10	10

In this case, the human level error as a proxy for Bayes error since humans are good to identify images. If you want to improve the performance of the training set but you can't do better than the Bayes error otherwise the training set is overfitting. By knowing the Bayes error, it is easier to focus on whether bias or variance avoidance tactics will improve the performance of the model.

Scenario A

There is a 7% gap between the performance of the training set and the human level error. It means that the algorithm isn't fitting well with the training set since the target is around 1%. To resolve the issue, we use bias reduction technique such as training a bigger neural network or running the training set longer.

Scenario B

The training set is doing good since there is only a 0.5% difference with the human level error. The difference between the training set and the human level error is called avoidable bias. The focus here is to reduce the variance since the difference between the training error and the development error is 2%. To resolve the issue, we use variance reduction technique such as regularization or have a bigger training set.

Understanding human-level performance

Human-level error gives an estimate of Bayes error.

Example 1: Medical image classification

This is an example of a medical image classification in which the input is a radiology image and the output is a diagnosis classification decision.

	Classification error (%)
Typical human	3.0
Typical doctor	1.0
Experienced doctor	0.7
Team of experienced doctors	0.5

The definition of human-level error depends on the purpose of the analysis, in this case, by definition the Bayes error is lower or equal to 0.5%.

Example 2: Error analysis

	Classification error (%)		
	Scenario A	Scenario B	Scenario C
Human (proxy for Bayes error)	1	1	0.5
	0.7	0.7	
	0.5	0.5	
Training error	5	1	0.7
Development error	6	5	0.8

Scenario A

In this case, the choice of human-level performance doesn't have an impact. The avoidable bias is between 4%-4.5% and the variance is 1%. Therefore, the focus should be on bias reduction technique.

Scenario B

In this case, the choice of human-level performance doesn't have an impact. The avoidable bias is between 0%-0.5% and the variance is 4%. Therefore, the focus should be on variance reduction technique.

Scenario C

In this case, the estimate for Bayes error has to be 0.5% since you can't go lower than the human-level performance otherwise the training set is overfitting. Also, the avoidable bias is 0.2% and the variance is 0.1%. Therefore, the focus should be on bias reduction technique.

Summary of bias/variance with human-level performance

- Human - level error – proxy for Bayes error
- If the difference between human-level error and the training error is bigger than the difference between the training error and the development error. The focus should be on bias reduction technique
- If the difference between training error and the development error is bigger than the difference between the human-level error and the training error. The focus should be on variance reduction technique

Surpassing human-level performance

Example1: Classification task

	Classification error (%)	
	Scenario A	Scenario B
Team of humans	0.5	0.5
One human	1.0	1
Training error	0.6	0.3
Development error	0.8	0.4

Scenario A

In this case, the Bayes error is 0.5%, therefore the available bias is 0.1% et the variance is 0.2%.

Scenario B

In this case, there is not enough information to know if bias reduction or variance reduction has to be done on the algorithm. It doesn't mean that the model cannot be improved, it means that the conventional ways to know if bias reduction or variance reduction are not working in this case.

There are many problems where machine learning significantly surpasses human-level performance, especially with structured data:

- Online advertising
- Product recommendations
- Logistics (predicting transit time)
- Loan approvals

Improving your model performance

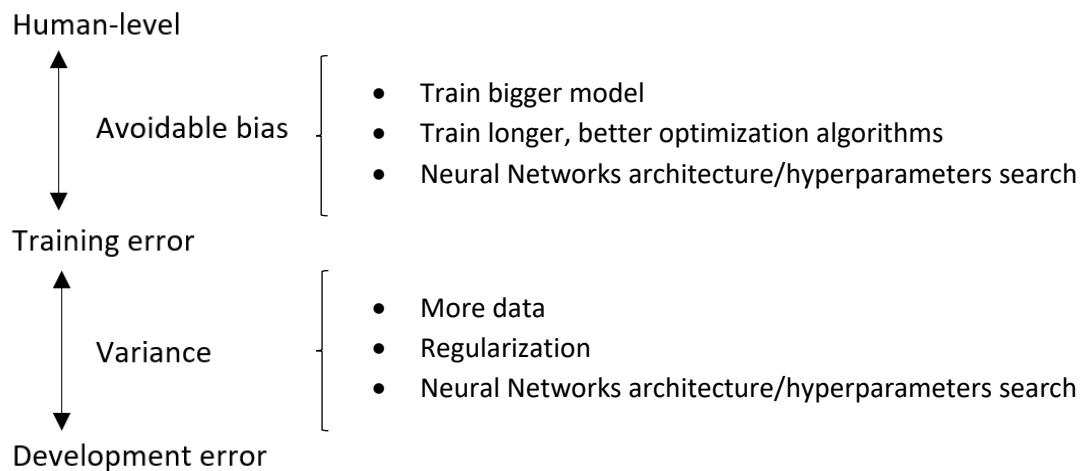
The two fundamental assumptions of supervised learning

There are 2 fundamental assumptions of supervised learning. The first one is to have a low avoidable bias which means that the training set fits well. The second one is to have a low or acceptable variance which means that the training set performance generalizes well to the development set and test set.

If the difference between human-level error and the training error is bigger than the difference between the training error and the development error, the focus should be on bias reduction technique which are training a bigger model, training longer or change the neural networks architecture or try various hyperparameters search.

If the difference between training error and the development error is bigger than the difference between the human-level error and the training error, the focus should be on variance reduction technique which are bigger data set, regularization or change the neural networks architecture or try various hyperparameters search.

Summary



Build system quickly, then iterate

Depending on the area of application, the guideline below will help you prioritize when you build your system.

Guideline

1. Set up development/ test set and metrics
 - Set up a target
2. Build an initial system quickly
 - Train training set quickly: Fit the parameters
 - Development set: Tune the parameters
 - Test set: Assess the performance
3. Use Bias/Variance analysis & Error analysis to prioritize next steps

Training and testing on different distributions

Example: Cat vs Non-cat

In this example, we want to create a mobile application that will classify and recognize pictures of cats taken and uploaded by users.

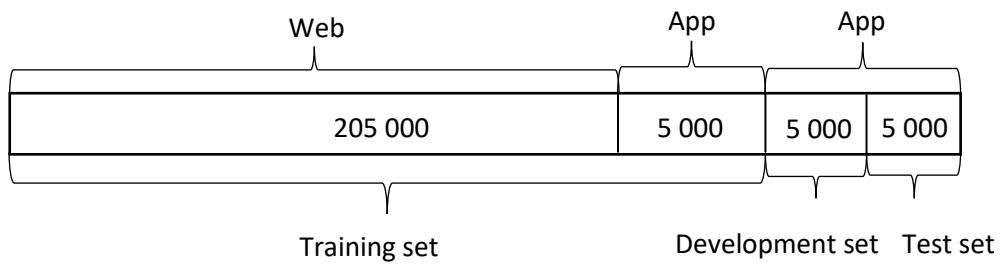
There are two sources of data used to develop the mobile app. The first data distribution is small, 10 000 pictures uploaded from the mobile application. Since they are from amateur users, the pictures are not professionally shot, not well framed and blurrier. The second source is from the web, you downloaded 200 000 pictures where cat's pictures are professionally framed and in high resolution.

The problem is that you have a different distribution:

- 1- small data set from pictures uploaded by users. This distribution is important for the mobile app.
- 2- bigger data set from the web.

The guideline used is that you have to choose a development set and test set to reflect data you expect to get in the future and consider important to do well.

The data is split as follow:



The advantage of this way of splitting up is that the target is well defined.

The disadvantage is that the training distribution is different from the development and test set distributions. However, this way of splitting the data has a better performance in long term.

Bias and variance with mismatched data distributions

Example: Cat classifier with mismatch data distribution

When the training set is from a different distribution than the development and test sets, the method to analyze bias and variance changes.

	Classification error (%)					
	Scenario A	Scenario B	Scenario C	Scenario D	Scenario E	Scenario F
Human (proxy for Bayes error)	0	0	0	0	0	4
Training error	1	1	1	10	10	7
Training-development error	-	9	1.5	11	11	10
Development error	10	10	10	12	20	6
Test error	-	-	-	-	-	6

Scenario A

If the development data comes from the same distribution as the training set, then there is a large variance problem and the algorithm is not generalizing well from the training set.

However, since the training data and the development data come from a different distribution, this conclusion cannot be drawn. There isn't necessarily a variance problem. The problem might be that the development set contains images that are more difficult to classify accurately.

When the training set, development and test sets distributions are different, two things change at the same time. First of all, the algorithm trained in the training set but not in the development set. Second of all, the distribution of data in the development set is different.

It's difficult to know which of these two changes what produces this 9% increase in error between the training set and the development set. To resolve this issue, we define a new subset called training-development set. This new subset has the same distribution as the training set, but it is not used for training the neural network.

Scenario B

The error between the training set and the training- development set is 8%. In this case, since the training set and training-development set come from the same distribution, the only difference between them is the neural network sorted the data in the training and not in the training development. The neural network is not generalizing well to data from the same distribution that it hadn't seen before

Therefore, we have really a variance problem.

Scenario C

In this case, we have a mismatch data problem since the 2 data sets come from different distribution.

Scenario D

In this case, the avoidable bias is high since the difference between Bayes error and training error is 10 %.

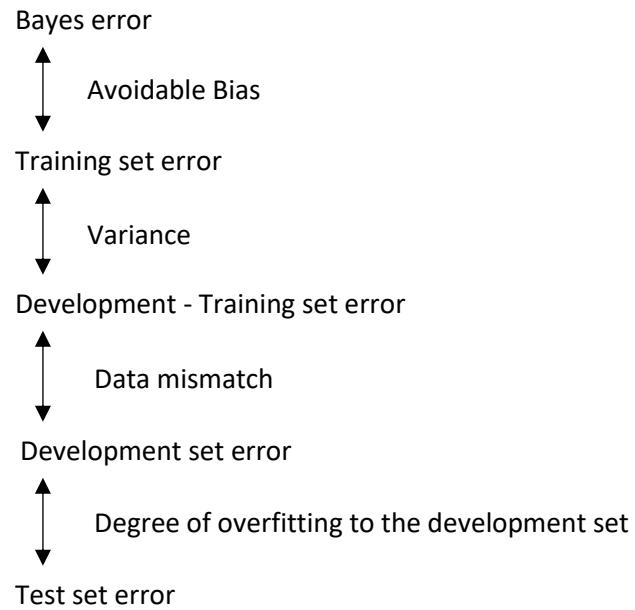
Scenario E

In this case, there are 2 problems. The first one is that the avoidable bias is high since the difference between Bayes error and training error is 10 % and the second one is a data mismatched problem.

Scenario F

Development should never be done on the test set. However, the difference between the development set and the test set gives the degree of overfitting to the development set.

General formulation



Addressing data mismatch

This is a general guideline to address data mismatch:

- Perform manual error analysis to understand the error differences between training, development/test sets. Development should never be done on test set to avoid overfitting.
- Make training data or collect data similar to development and test sets. To make the training data more similar to your development set, you can use is artificial data synthesis. However, it is possible that if you might be accidentally simulating data only from a tiny subset of the space of all possible examples.

Transfer Learning

Transfer learning refers to using the neural network knowledge for another application.

When to use transfer learning

- Task A and B have the same input x
- A lot more data for Task A than Task B
- Low level features from Task A could be helpful for Task B

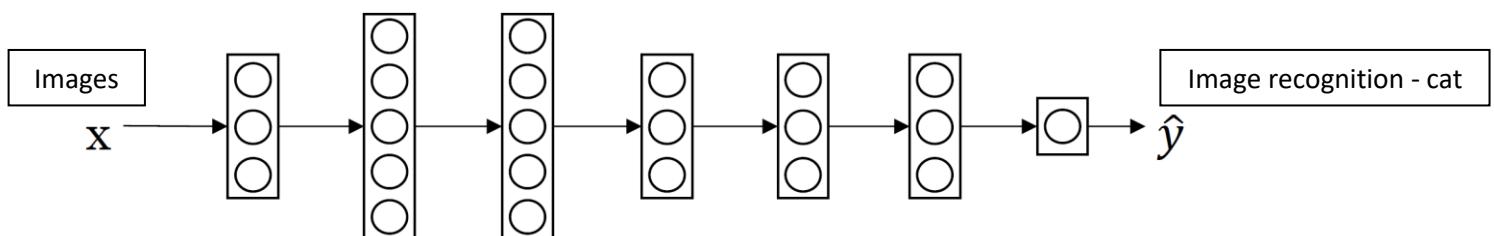
Example 1: Cat recognition - radiology diagnosis

The following neural network is trained for cat recognition, but we want to adapt it for radiology diagnosis. The neural network will learn about the structure and the nature of images. This initial phase of training on image recognition is called pre-training, since it will pre-initialize the weights of the neural network. Updating all the weights afterwards is called fine-tuning.

For cat recognition

Input x : image

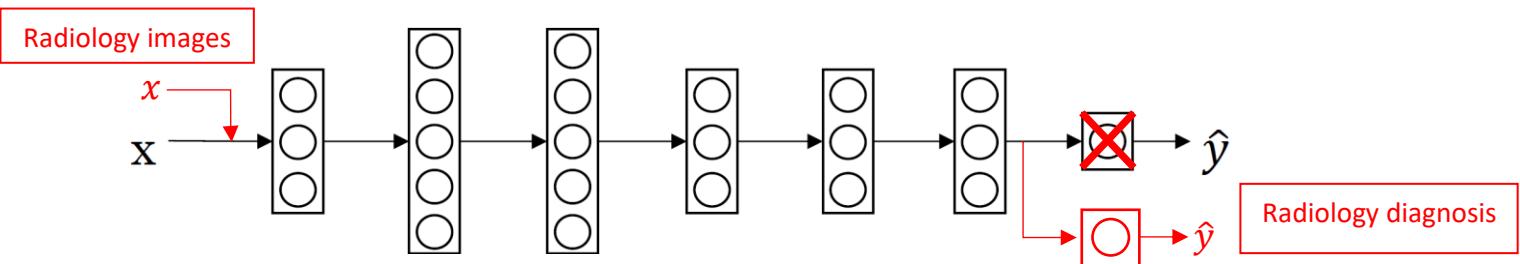
Output y – 1: cat, 0: no cat



Radiology diagnosis

Input x : Radiology images – CT Scan, X-rays

Output y :Radiology diagnosis – 1: tumor malign, 0: tumor benign



Guideline

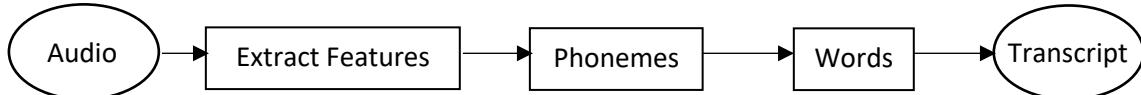
- Delete last layer of neural network
- Delete weights feeding into the last output layer of the neural network
- Create a new set of randomly initialized weights for the last layer only
- New data set (x, y)

What is end-to-end deep learning

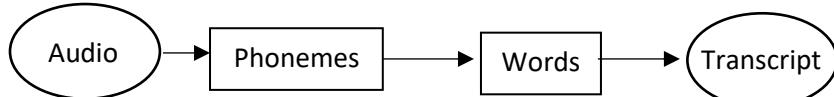
End-to-end deep learning is the simplification of a processing or learning systems into one neural network.

Example - Speech recognition model

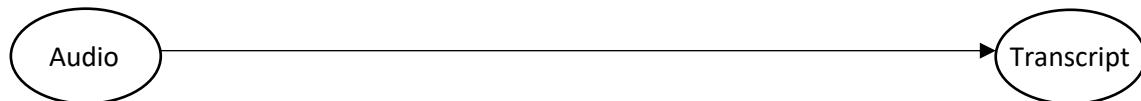
The traditional way - small data set



The hybrid way - medium data set



The End-to-End deep learning way – large data set



End-to-end deep learning cannot be used for every problem since it needs a lot of labeled data. It is used mainly in audio transcripts, image captures, image synthesis, machine translation, steering in self-driving cars, etc.

Whether to use end-to-end deep learning

Before applying end-to-end deep learning, you need to ask yourself the following question: Do you have enough data to learn a function of the complexity needed to map x and y?

Pro:

- Let the data speak
 - By having a pure machine learning approach, the neural network will learn from x to y. It will be able to find which statistics are in the data, rather than being forced to reflect human preconceptions.
- Less hand-designing of components needed
 - It simplifies the design work flow.

Cons:

- Large amount of labeled data
 - It cannot be used for every problem as it needs a lot of labeled data.
- Excludes potentially useful hand-designed component
 - Data and any hand-design's components or features are the 2 main sources of knowledge for a learning algorithm. If the data set is small than a hand-design system is a way to give manual knowledge into the algorithm.

What is neural network?

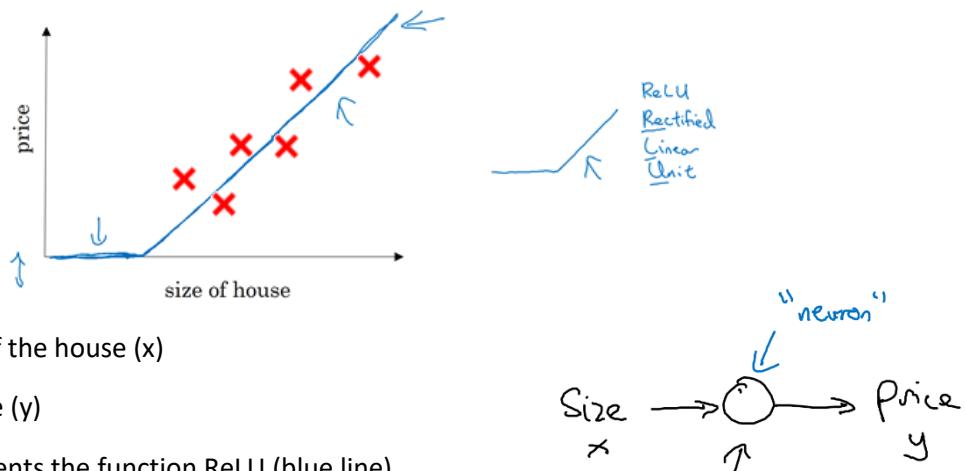
It is a powerful learning algorithm inspired by how the brain works.

Example 1 – single neural network

Given data about the size of houses on the real estate market and you want to fit a function that will predict their price. It is a linear regression problem because the price as a function of size is a continuous output.

We know the prices can never be negative so we are creating a function called Rectified Linear Unit (ReLU) which starts at zero.

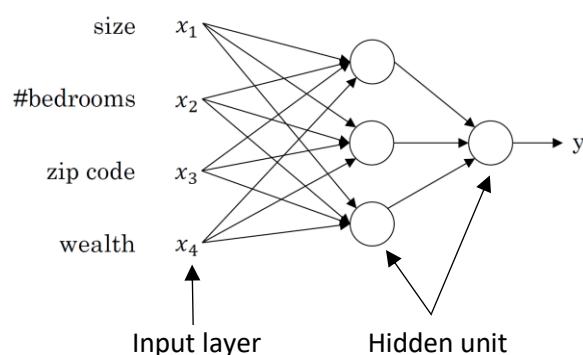
Housing Price Prediction



Example 2 – Multiple neural network

The price of a house can be affected by other features such as size, number of bedrooms, zip code and wealth. The role of the neural network is to predict the price and it will automatically generate the hidden units. We only need to give the inputs x and the output y.

Housing Price Prediction



Supervised learning for Neural Network

In supervised learning, we are given a data set and already know what our correct output should look like, having the idea that there is a relationship between the input and the output.

Supervised learning problems are categorized into "regression" and "classification" problems. In a regression problem, we are trying to predict results within a continuous output, meaning that we are trying to map input variables to some continuous function. In a classification problem, we are instead trying to predict results in a discrete output. In other words, we are trying to map input variables into discrete categories.

Here are some examples of supervised learning

Input(x)	Output (y)	Application
Home features	Price	Real Estate
Ad, user info	Click on ad? (0/1)	Online Advertising
Image	Object (1,...,1000)	Photo tagging
Audio	Text transcript	Speech recognition
English	Chinese	Machine translation
Image, Radar info	Position of other cars	Autonomous driving

There are different types of neural network, for example Convolution Neural Network (CNN) used often for image application and Recurrent Neural Network (RNN) used for one-dimensional sequence data such as translating English to Chinese or a temporal component such as text transcript. As for the autonomous driving, it is a hybrid neural network architecture.

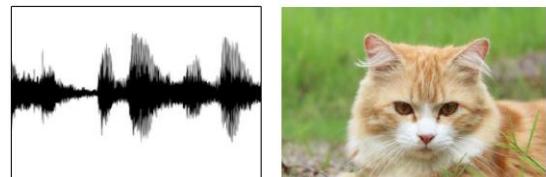
Structured vs unstructured data

Structured data refers to things that has a defined meaning such as price, age whereas unstructured data refers to things like pixel, raw audio, text.

Structured Data

Size	#bedrooms	...	Price (1000\$)
2104	3		400
1600	3		330
2400	3		369
:	:		:
3000	4		540

Unstructured Data



Audio

Image

User Age	Ad Id	...	Click
41	93242		1
80	93287		0
18	87312		1
:	:		:
27	71244		1

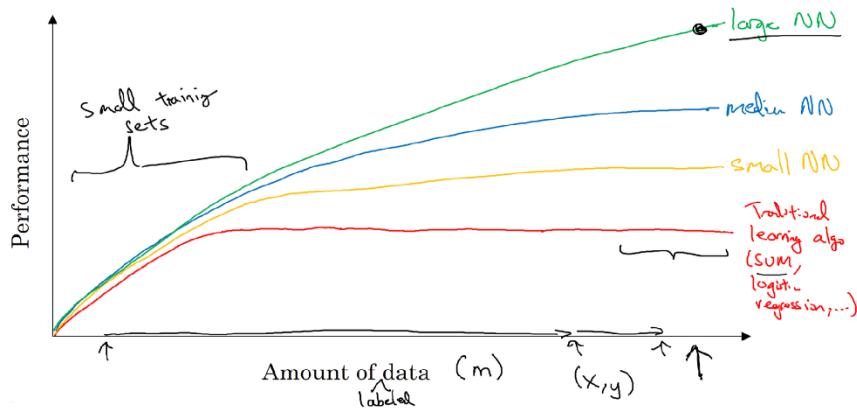
Four scores and seven years ago...

Text

Why is deep learning taking off?

Deep learning is taking off due to a large amount of data available through the digitization of the society, faster computation and innovation in the development of neural network algorithm.

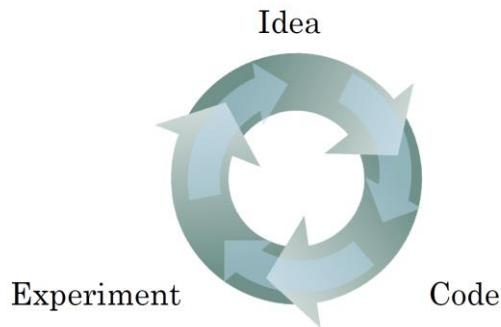
Scale drives deep learning progress



Two things have to be considered to get to the high level of performance:

1. Being able to train a big enough neural network
2. Huge amount of labeled data

The process of training a neural network is iterative.



It could take a good amount of time to train a neural network, which affects your productivity. Faster computation helps to iterate and improve new algorithm.

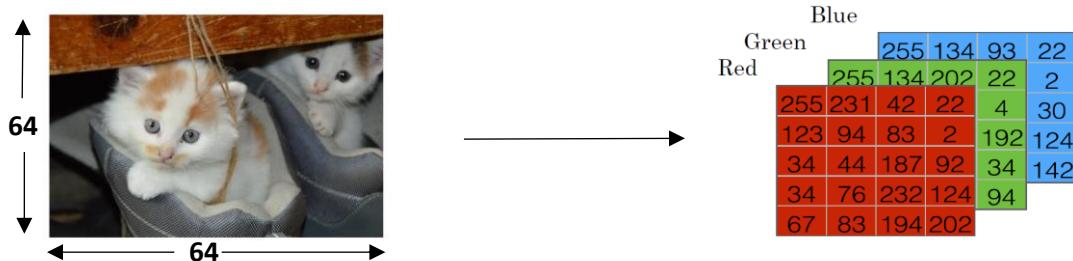
Binary Classification

In a binary classification problem, the result is a discrete value output.

- For example
- account hacked (1) or compromised (0)
 - a tumor malign (1) or benign (0)

Example: Cat vs Non-Cat

The goal is to train a classifier that the input is an image represented by a feature vector, x , and predicts whether the corresponding label y is 1 or 0. In this case, whether this is a cat image (1) or a non-cat image (0).



An image is stored in the computer in three separate matrices corresponding to the Red, Green, and Blue color channels of the image. The three matrices have the same size as the image, for example, the resolution of the cat image is 64 pixels X 64 pixels, the three matrices (RGB) are 64 X 64 each.

The value in a cell represents the pixel intensity which will be used to create a feature vector of n-dimension. In pattern recognition and machine learning, a feature vector represents an object, in this case, a cat or no cat.

To create a feature vector, x , the pixel intensity values will be “unroll” or “reshape” for each color. The dimension of the input feature vector x is $n_x = 64 \times 64 \times 3 = 12,288$.

$$x = \begin{bmatrix} 255 \\ 231 \\ 42 \\ \vdots \\ 255 \\ 134 \\ 202 \\ \vdots \\ 255 \\ 134 \\ 93 \\ \vdots \end{bmatrix} \left[\begin{array}{l} \text{red} \\ \text{green} \\ \text{blue} \end{array} \right]$$

Standard notations for Deep Learning

This document has the purpose of discussing a new standard for deep learning mathematical notations.

1 Neural Networks Notations.

General comments:

- superscript (i) will denote the i^{th} training example while superscript [l] will denote the l^{th} layer

Sizes:

· m : number of examples in the dataset

· n_x : input size

· n_y : output size (or number of classes)

· $n_h^{[l]}$: number of hidden units of the l^{th} layer

In a for loop, it is possible to denote $n_x = n_h^{[0]}$ and $n_y = n_h^{[\text{number of layers} + 1]}$.

· L : number of layers in the network.

Objects:

· $X \in \mathbb{R}^{n_x \times m}$ is the input matrix

· $x^{(i)} \in \mathbb{R}^{n_x}$ is the i^{th} example represented as a column vector

· $Y \in \mathbb{R}^{n_y \times m}$ is the label matrix

· $y^{(i)} \in \mathbb{R}^{n_y}$ is the output label for the i^{th} example

· $W^{[l]} \in \mathbb{R}^{\text{number of units in next layer} \times \text{number of units in the previous layer}}$ is the weight matrix, superscript [l] indicates the layer

· $b^{[l]} \in \mathbb{R}^{\text{number of units in next layer}}$ is the bias vector in the l^{th} layer

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Common forward propagation equation examples:

$a = g^{[l]}(W_x x^{(i)} + b_1) = g^{[l]}(z_1)$ where $g^{[l]}$ denotes the l^{th} layer activation function

$\hat{y}^{(i)} = \text{softmax}(W_h h + b_2)$

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Examples of cost function:

· $J_{CE}(\hat{y}, y) = -\sum_{i=0}^m y^{(i)} \log \hat{y}^{(i)}$

· $J_1(\hat{y}, y) = \sum_{i=0}^m |y^{(i)} - \hat{y}^{(i)}|$

2 Deep Learning representations

For representations:

- nodes represent inputs, activations or outputs
- edges represent weights or biases

Here are several examples of Standard deep learning representations

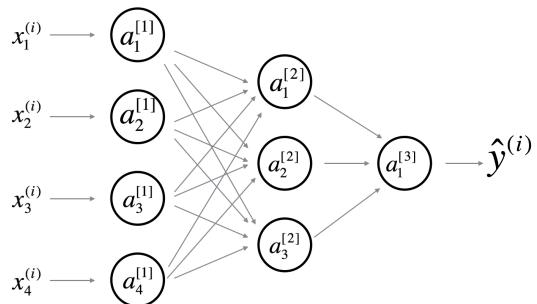


Figure 1: Comprehensive Network: representation commonly used for Neural Networks. For better aesthetic, we omitted the details on the parameters ($w_{ij}^{[l]}$ and $b_i^{[l]}$ etc...) that should appear on the edges

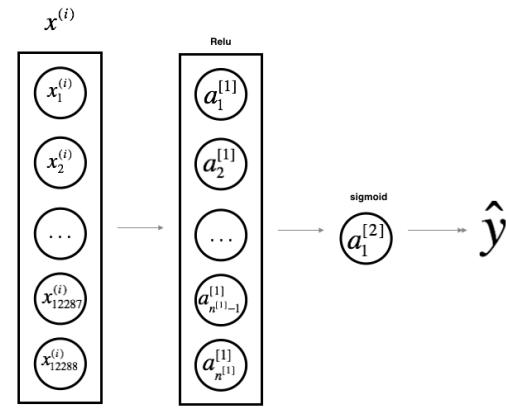


Figure 2: Simplified Network: a simpler representation of a two layer neural network, both are equivalent.

Logistic Regression

Logistic regression is a learning algorithm used in a supervised learning problem when the output y are all either zero or one. The goal of logistic regression is to minimize the error between its predictions and training data.

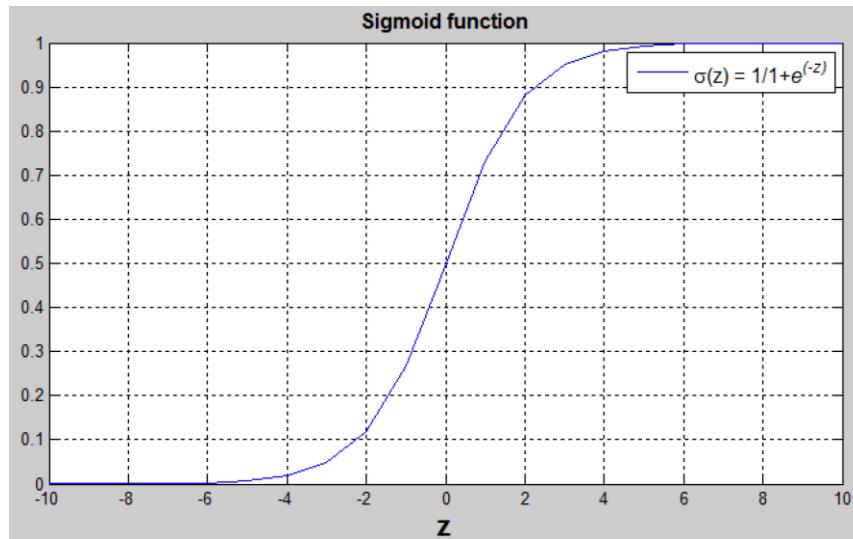
Example: Cat vs No - cat

Given an image represented by a feature vector x , the algorithm will evaluate the probability of a cat being in that image.

$$\text{Given } x, \hat{y} = P(y = 1|x), \text{ where } 0 \leq \hat{y} \leq 1$$

The parameters used in Logistic regression are:

- The input features vector: $x \in \mathbb{R}^{n_x}$, where n_x is the number of features
- The training label: $y \in \{0,1\}$
- The weights: $w \in \mathbb{R}^{n_x}$, where n_x is the number of features
- The threshold: $b \in \mathbb{R}$
- The output: $\hat{y} = \sigma(w^T x + b)$
- Sigmoid function: $s = \sigma(w^T x + b) = \sigma(z) = \frac{1}{1+e^{-z}}$



$(w^T x + b)$ is a linear function ($ax + b$), but since we are looking for a probability constraint between $[0,1]$, the sigmoid function is used. The function is bounded between $[0,1]$ as shown in the graph above.

Some observations from the graph:

- If z is a large positive number, then $\sigma(z) = 1$
- If z is small or large negative number, then $\sigma(z) = 0$
- If $z = 0$, then $\sigma(z) = 0.5$

Logistic Regression: Cost Function

To train the parameters w and b , we need to define a cost function.

Recap:

$$\hat{y}^{(i)} = \sigma(w^T x^{(i)} + b), \text{ where } \sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$$

$x^{(i)}$ the i-th training example

Given $\{(x^{(1)}, y^{(1)}), \dots, (x^{(m)}, y^{(m)})\}$, we want $\hat{y}^{(i)} \approx y^{(i)}$

Loss (error) function:

The loss function measures the discrepancy between the prediction ($\hat{y}^{(i)}$) and the desired output ($y^{(i)}$). In other words, the loss function computes the error for a single training example.

$$L(\hat{y}^{(i)}, y^{(i)}) = \frac{1}{2}(\hat{y}^{(i)} - y^{(i)})^2$$

$$L(\hat{y}^{(i)}, y^{(i)}) = -(y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)}))$$

- If $y^{(i)} = 1$: $L(\hat{y}^{(i)}, y^{(i)}) = -\log(\hat{y}^{(i)})$ where $\log(\hat{y}^{(i)})$ and $\hat{y}^{(i)}$ should be close to 1
- If $y^{(i)} = 0$: $L(\hat{y}^{(i)}, y^{(i)}) = -\log(1 - \hat{y}^{(i)})$ where $\log(1 - \hat{y}^{(i)})$ and $\hat{y}^{(i)}$ should be close to 0

Cost function

The cost function is the average of the loss function of the entire training set. We are going to find the parameters w and b that minimize the overall cost function.

$$J(w, b) = \frac{1}{m} \sum_{i=1}^m L(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m} \sum_{i=1}^m [-(y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)}))]$$

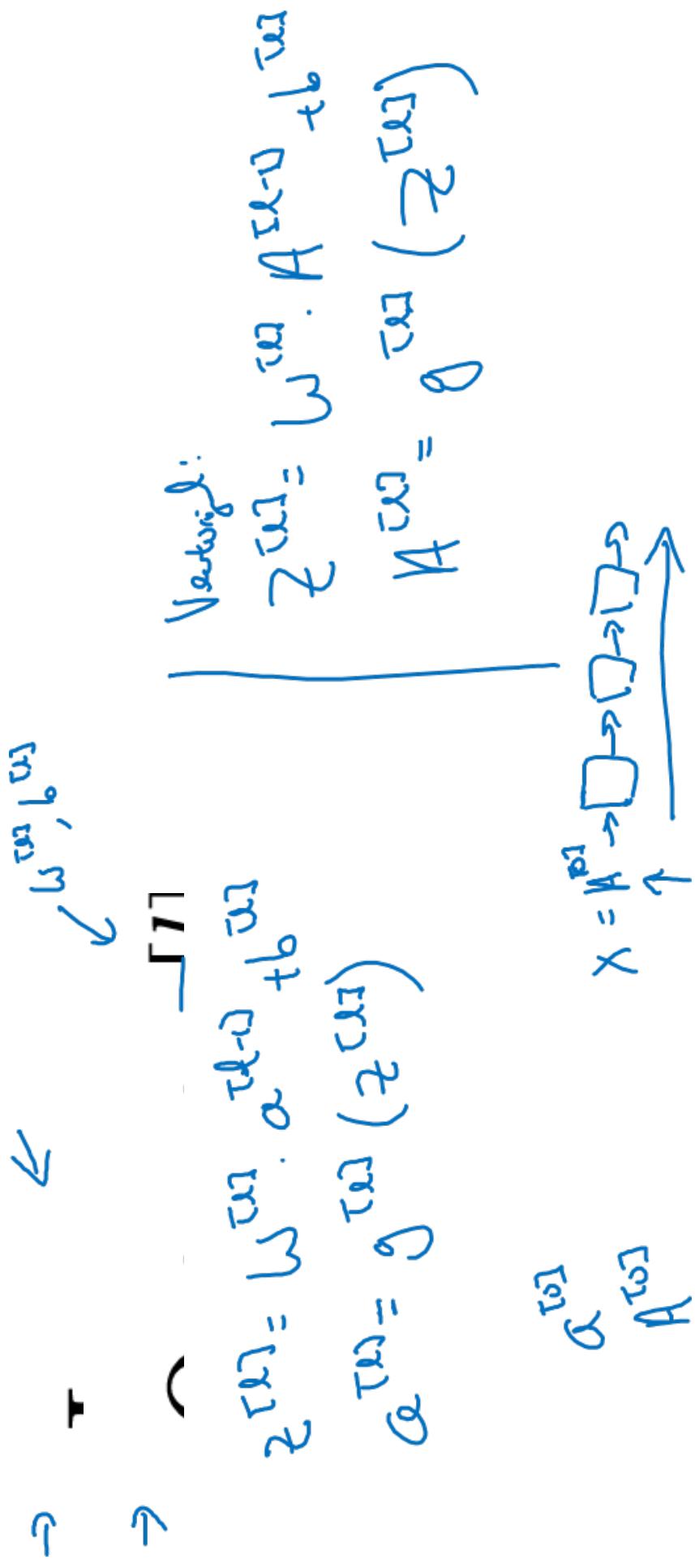
Deep Neural Networks

Forward and backward
propagation

deeplearning.ai



Forward propagation for layer l



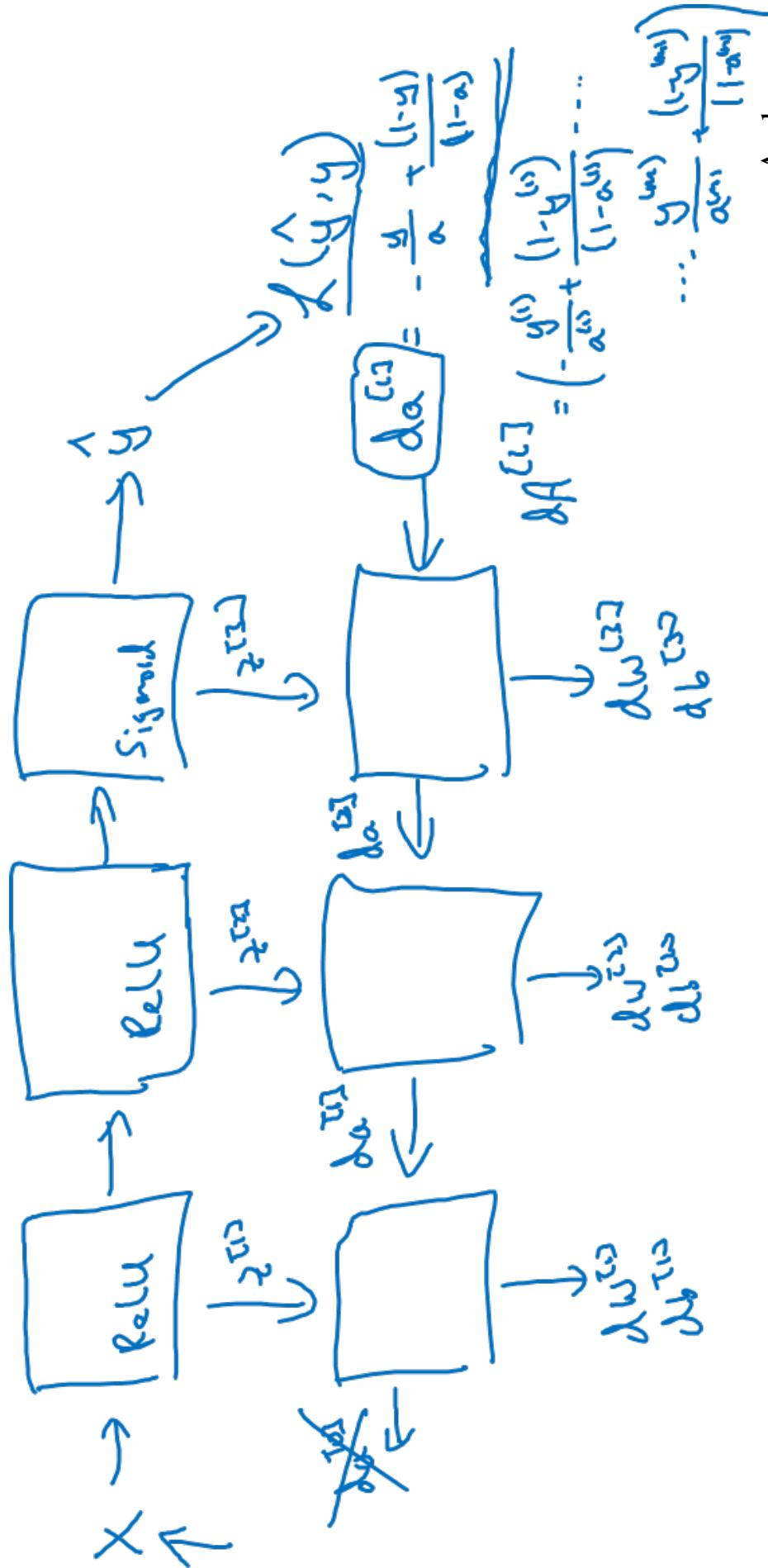
Backward propagation for layer l

$$\begin{aligned}
 & \theta \times \varphi^2 \cdot \varphi^2 = m^2 \varphi \\
 & \varphi^2 \cdot \varphi^2 = m^2 \varphi \\
 & \varphi^2 = m^2 \varphi \\
 & \varphi = m \varphi
 \end{aligned}$$

$$\begin{aligned}
 & \delta z^{\text{new}} = \delta A^{\text{new}} \times g^{\text{new}}(z^{\text{old}}) \\
 & \delta A^{\text{new}} = \frac{1}{m} \sum_{i=1}^m \delta z^{\text{new}} \cdot A^{\text{old}, i} \\
 & \underbrace{\delta b^{\text{new}}}_{\delta A^{\text{old}, i}} = \frac{1}{m} \sum_{i=1}^m \delta z^{\text{new}} \cdot (\delta z^{\text{old}}, \text{axis}=1, \text{keepdim}=\text{true}) \\
 & \boxed{\delta A^{\text{old}, i} = \text{sum}(\delta z^{\text{old}}, \text{axis}=1, \text{keepdim}=\text{true})}
 \end{aligned}$$

Andrew

Summary



Andrew

Standard notations for Deep Learning

This document has the purpose of discussing a new standard for deep learning mathematical notations.

1 Neural Networks Notations.

General comments:

- superscript (i) will denote the i^{th} training example while superscript [l] will denote the l^{th} layer

Sizes:

· m : number of examples in the dataset

· n_x : input size

· n_y : output size (or number of classes)

· $n_h^{[l]}$: number of hidden units of the l^{th} layer

In a for loop, it is possible to denote $n_x = n_h^{[0]}$ and $n_y = n_h^{[\text{number of layers} + 1]}$.

· L : number of layers in the network.

Objects:

· $X \in \mathbb{R}^{n_x \times m}$ is the input matrix

· $x^{(i)} \in \mathbb{R}^{n_x}$ is the i^{th} example represented as a column vector

· $Y \in \mathbb{R}^{n_y \times m}$ is the label matrix

· $y^{(i)} \in \mathbb{R}^{n_y}$ is the output label for the i^{th} example

· $W^{[l]} \in \mathbb{R}^{\text{number of units in next layer} \times \text{number of units in the previous layer}}$ is the weight matrix, superscript [l] indicates the layer

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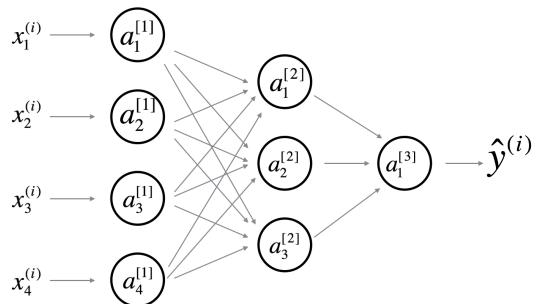


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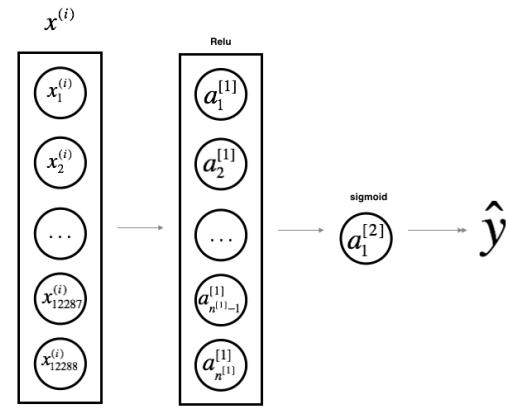


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