

Fizika 2 - izpeljave

insightfulbriyan

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1 Valovna enačba

1.1

Faradejev 97 in Amperov 99 zakon lahko združimo v valovno enačbo.

1.2

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-

$$\vec{J} = 0$$

$$\vec{D} = \epsilon_0 \vec{E}$$

$$\vec{B} = \mu_0 \vec{H}$$

$$\vec{E} = (0, E_y(x, t), E_z(x, t))$$

$$\vec{H} = (0, H_y(x, t), H_z(x, t))$$

1.3

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}, \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) = \left(0, -\frac{\partial E_z}{\partial x}, \frac{\partial E_y}{\partial x} \right) =$$
$$= -\mu_0 \left(\frac{\partial H_x}{\partial t}, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) = -\mu_0 \left(0, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) \quad (1)$$

$$\frac{\partial E_z}{\partial x} = \mu_0 \frac{\partial H_y}{\partial t} \quad (2)$$

$$\frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \quad (3)$$

1.4

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}, \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}, \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) = \left(0, -\frac{\partial H_z}{\partial x}, \frac{\partial H_y}{\partial x} \right) =$$
$$= \epsilon_0 \left(\frac{\partial H_x}{\partial t}, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) = \epsilon_0 \left(0, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) \quad (4)$$

$$\frac{\partial H_z}{\partial x} = -\epsilon_0 \frac{\partial E_y}{\partial t} \quad (5)$$

$$\frac{\partial H_y}{\partial x} = \epsilon_0 \frac{\partial E_z}{\partial t} \quad (6)$$

1.5

$$\begin{aligned}\frac{\partial E_y}{\partial x} &= -\mu_0 \frac{\partial H_z}{\partial t} & \frac{\partial H_z}{\partial x} &= -\epsilon_0 \frac{\partial E_y}{\partial t} \\ \frac{\partial^2 E_y}{\partial x \partial t} &= -\mu_0 \frac{\partial^2 H_z}{\partial t^2} & \frac{\partial^2 H_z}{\partial x^2} \frac{1}{-\epsilon_0} &= \frac{\partial^2 E_y}{\partial t \partial x}\end{aligned}$$

1.6

$$\frac{\partial^2 E_y}{\partial x^2} \frac{1}{-\mu_0} = -\epsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad (7)$$

1.7

$$E_y = E_0 \cos(\omega t - kx); \omega = 2\pi f, k = \frac{\omega}{c_0} \quad (8)$$

$$\frac{\partial^2 E_y}{\partial x^2} = -k^2 E_y; \frac{\partial^2 E_y}{\partial t^2} = -\omega^2 E_y \quad (9)$$

$$k^2 E_y \frac{1}{-\epsilon_0 \mu_0} = \omega^2 E_y = \frac{\omega^2}{c_0^2} \frac{1}{-\epsilon_0 \mu_0} E_y \quad (10)$$

1.8

$$\frac{1}{\epsilon_0 \mu_0} = c_0^2 \quad (11)$$

2 Lomni količník

2.1

$$c = \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_r \mu_0}}; c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \quad (12)$$

$$n = \frac{c}{c_0} = \sqrt{\frac{\epsilon_0 \mu_0}{\epsilon_r \epsilon_0 \mu_r \mu_0}} = \frac{1}{\sqrt{\epsilon_r \mu_r}} \quad (13)$$

3 Lomni zakon

3.1

Minimiziramo čas preleta žarkov od točke A do točke B, ki sta v različnih medijih z različnimi n .

3.2

$t(\alpha, \beta)$ ima ekstrem, če je $dt = 0$

$$t = t_1 + t_2 \quad (14)$$

$$t = \frac{s_1}{c_1} + \frac{s_2}{c_2} \quad (15)$$

$$t = \frac{h_1}{\cos(\alpha) * c_1} + \frac{h_2}{\cos(\beta) * c_2} \quad (16)$$

$$dt = \frac{\partial t}{\partial \alpha} d\alpha + \frac{\partial t}{\partial \beta} d\beta = 0 \quad (17)$$

$$\frac{h_1 \sin(\alpha)}{\cos^2(\alpha) * c_1} d\alpha = - \frac{h_2 \sin(\beta)}{\cos^2(\beta) * c_2} d\beta \quad (18)$$

$$\frac{d\alpha}{d\beta} = - \frac{h_2 c_1 \sin(\beta) \cos^2(\alpha)}{h_1 c_2 \sin(\alpha) \cos^2(\beta)} \quad (19)$$

3.3

L je razdalja med točkama po y osi, ker je konstantna, je $dL = 0$

$$L = l_1 + l_2 = \frac{h_1}{\tan(\alpha)} + \frac{h_2}{\tan(\beta)} \quad (20)$$

$$dL = \frac{h_1}{\cos^2(\alpha)} d\alpha + \frac{h_2}{\cos^2(\beta)} d\beta = 0 \quad (21)$$

$$\frac{h_1}{\cos^2(\alpha)} d\alpha = - \frac{h_2}{\cos^2(\beta)} d\beta \quad (22)$$

$$\frac{d\alpha}{d\beta} = - \frac{h_2 \cos^2(\beta)}{h_1 \cos^2(\alpha)} \quad (23)$$

3.4

$$- \frac{h_2 c_1 \sin(\beta) \cos^2(\alpha)}{h_1 c_2 \sin(\alpha) \cos^2(\beta)} = - \frac{h_2 \cos^2(\beta)}{h_1 \cos^2(\alpha)} \quad (24)$$

$$\Rightarrow \frac{\sin(\beta)}{\sin(\alpha)} = \frac{c_1}{c_2} = \frac{\frac{c_0}{n_1}}{\frac{c_0}{n_2}} = \frac{n_2}{n_1} \quad (25)$$

$$\Rightarrow n_1 \sin(\alpha) = n_2 \sin(\beta) \quad (26)$$

4 Odbojni zakon

$$t = t_1 + t_2 \quad (27)$$

$$t = \frac{s_1}{c} + \frac{s_2}{c} \quad (28)$$

$$t = \frac{h}{\cos(\alpha) * c} + \frac{h}{\cos(\beta) * c} \quad (29)$$

$$dt = \frac{\partial t}{\partial \alpha} d\alpha + \frac{\partial t}{\partial \beta} d\beta = 0 \quad (30)$$

$$\frac{h \sin(\alpha)}{\cos^2(\alpha) * c} d\alpha = -\frac{h \sin(\beta)}{\cos^2(\beta) * c} d\beta \quad (31)$$

$$\frac{d\alpha}{d\beta} = -\frac{\sin(\beta) \cos^2(\alpha)}{\sin(\alpha) \cos^2(\beta)} \quad (32)$$

4.1

L je razdalja med točkama po y osi, ker je konstantna, je $dL = 0$

$$L = l_1 + l_2 = \frac{h}{\tan(\alpha)} + \frac{h}{\tan(\beta)} \quad (33)$$

$$dL = \frac{h}{\cos^2(\alpha)} d\alpha + \frac{h}{\cos^2(\beta)} d\beta = 0 \quad (34)$$

$$\frac{h}{\cos^2(\alpha)} d\alpha = -\frac{h}{\cos^2(\beta)} d\beta \quad (35)$$

$$\frac{d\alpha}{d\beta} = -\frac{\cos^2(\beta)}{\cos^2(\alpha)} \quad (36)$$

4.2

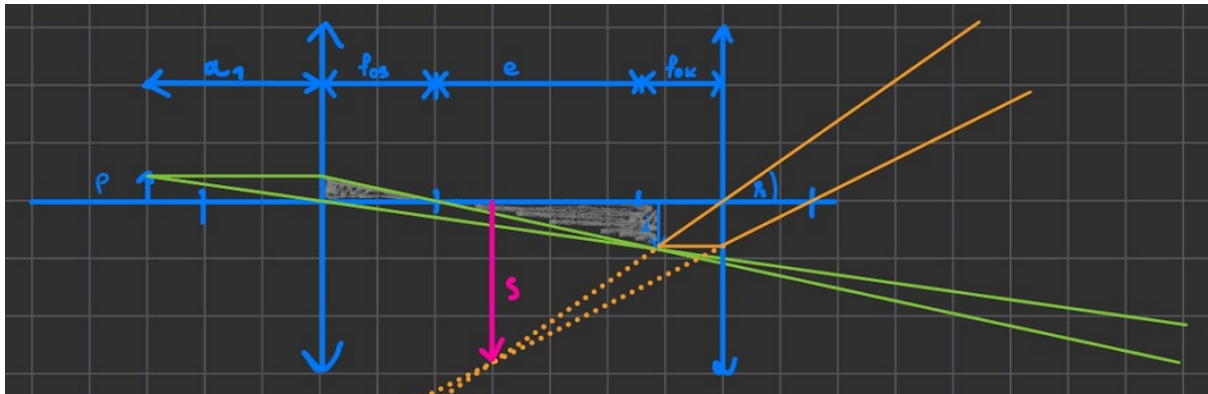
$$-\frac{\sin(\beta) \cos^2(\alpha)}{\sin(\alpha) \cos^2(\beta)} = -\frac{\cos^2(\beta)}{\cos^2(\alpha)} \quad (37)$$

$$\Rightarrow \frac{\sin(\beta)}{\sin(\alpha)} = 1 \quad (38)$$

$$\Rightarrow \sin(\alpha) = \sin(\beta) \quad (39)$$

$$\Rightarrow \alpha = \beta \quad (40)$$

5 Mikroskop



$$\tan(\varphi_1) = \frac{p}{x_0} \quad (41)$$

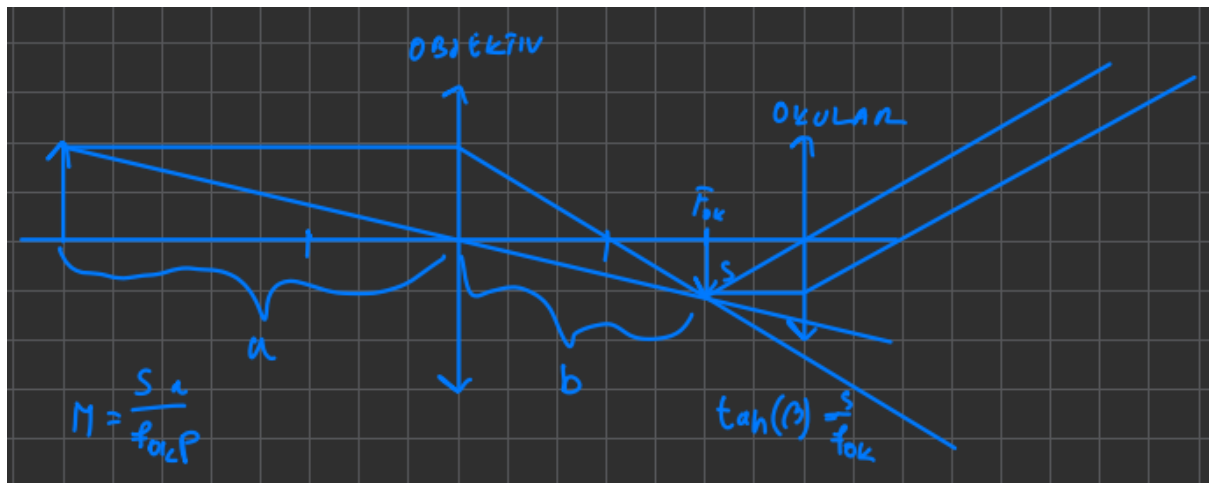
$$\tan(\varphi_2) = \frac{i}{f_{ok}} \quad (42)$$

$$\frac{i}{e} = \frac{p}{f_{ob}} \quad (43)$$

$$M = \frac{\tan(\varphi_2)}{\tan(\varphi_1)} \quad (44)$$

$$M = \frac{ex_0}{f_{ok}f_{ob}} \quad (45)$$

6 Daljnogled



$$\tan(\varphi_1) = \frac{p}{a} = \frac{s}{b} \quad (46)$$

$$\tan(\varphi_2) = \frac{s}{f_{ok}} \quad (47)$$

$$\frac{1}{f_{ob}} = \frac{1}{a} + \frac{1}{b} \Rightarrow b = \frac{a f_{ob}}{a - f_{ob}} \quad (48)$$

$$M = \frac{\tan(\varphi_2)}{\tan(\varphi_1)} = \frac{b}{f_{ok}} \quad (49)$$

$$M = \frac{a f_{ob}}{f_{ok}(a - f_{ob})} \quad (50)$$

7 Absorpcija EM valovanja

Telo debeline x in koeficientom absorpcije μ bo absorbira svetlobni tok j . Tanko telo debeline dx absorbira svetlobni tok dj

$$dj = -\mu j dx \quad (51)$$

$$\frac{1}{j} dj = -\mu dx \quad (52)$$

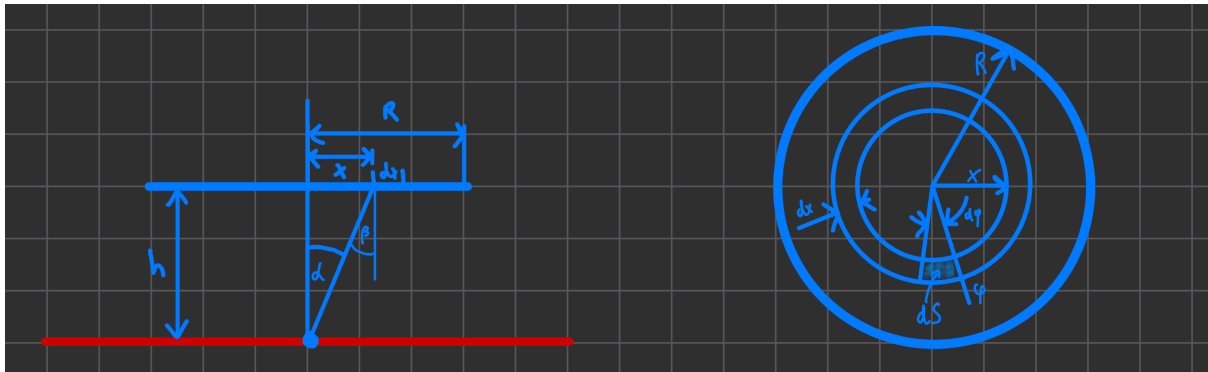
$$\int_{j_0}^{j(x)} \frac{1}{j} dj = -\mu \int_0^x dx \quad (53)$$

$$\ln(j(x)) - \ln(j_0) = -\mu x \quad (54)$$

$$\ln\left(\frac{j(x)}{j_0}\right) = -\mu x \quad (55)$$

$$j(x) = j_0 e^{-\mu x} \quad (56)$$

8 Okrogla luč po Lambertovem zakonu



8.1

$$B(\theta) = B_0 \quad (57)$$

$$dI = B(\theta) \cos(\beta) dS \quad (58)$$

$$dS = x d\varphi dx \quad (59)$$

$$dS = \int_0^{2\pi} x dx d\varphi \quad (60)$$

$$dS = 2\pi x dx \quad (61)$$

$$dI = B_0 \cos(\beta) 2\pi x dx \quad (62)$$

$$dE = \frac{dI}{r^2} \cos(\beta) \quad (63)$$

$$dE = \frac{B_0}{r^2} \cos^2(\beta) 2\pi x dx \quad (64)$$

8.2

$$r = \sqrt{x^2 + h^2} \quad (65)$$

$$\cos(\beta) = \frac{h}{r} \quad (66)$$

$$r = \frac{h}{\cos(\beta)} \quad (67)$$

$$\tan(\beta) = \frac{x}{h} \Rightarrow x = h \tan \beta \quad (68)$$

$$\frac{1}{\cos^2(\beta)} d\beta = \frac{1}{h} dx \quad (69)$$

$$dx = h \frac{1}{\cos^2(\beta)} d\beta \quad (70)$$

8.3

$$\beta(x) = \arctan\left(\frac{x}{h}\right) \quad (71)$$

$$\beta(0) = 0 \quad (72)$$

$$\beta(R) = \beta_0 = \arctan\left(\frac{R}{h}\right) \quad (73)$$

8.4

$$E = \int dE = \int_0^{\beta} \frac{B_0 \cos^2(\beta) \cos^2(\beta) 2\pi h \sin(\beta) h}{h^2 \cos(\beta) \cos^2(\beta)} d\beta \quad (74)$$

$$E = \int_0^{\beta} B_0 \sin(\beta) \cos(\beta) 2\pi d\beta \quad (75)$$

$$E = B_0 \pi \int_0^{\beta} \sin(2\beta) d\beta \quad (76)$$

$$E = B_0 \pi \int_0^{2\beta} \sin(u) du; u = 2\beta \Rightarrow du = 2d\beta \quad (77)$$

$$E = \frac{B_0 \pi}{2} (-\cos(u)) \Big|_{u=0}^{2\beta} \quad (78)$$

$$E = \frac{B_0 \pi}{2} (1 - \cos(2\beta)) \quad (79)$$

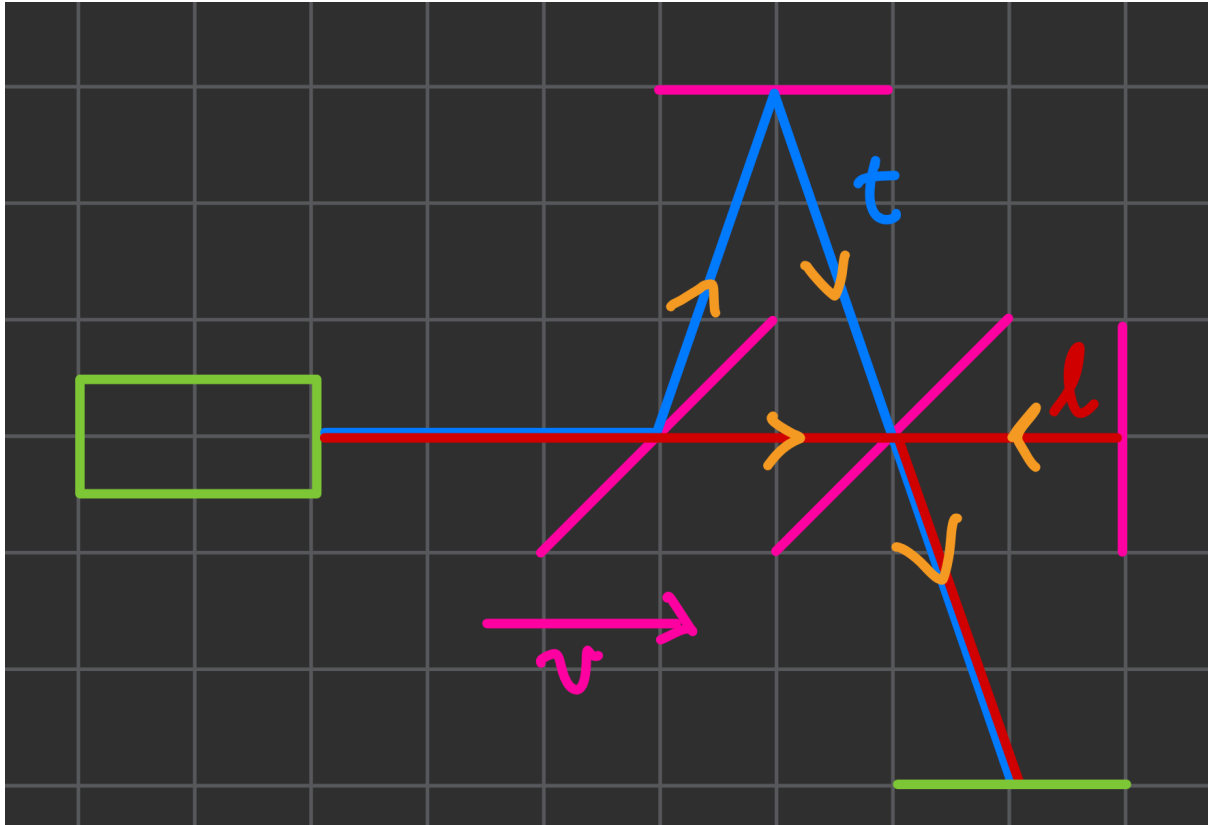
$$(80)$$

8.5

Če je R velik

$$\lim_{R \rightarrow \infty} E = \lim_{\beta \rightarrow \frac{\pi}{2}} \frac{B_0 \pi}{2} (1 - \cos(2\beta_0)) = \frac{B_0 \pi}{2} (1 - \cos(\pi)) = \frac{B_0 \pi}{2} (1 + 1) = B_0 \pi \quad (81)$$

9 Michelson-Morley interferometer



9.1

$$T_t = \frac{2L}{\sqrt{c^2 - v^2}}$$

$$T_l = \frac{L}{c-v} + \frac{L}{c+v} = \frac{L(c+v) + L(c-v)}{(c-v)(c+v)}$$

$$T_t = \frac{2L}{\sqrt{c^2(1 - \frac{v^2}{c^2})}}$$

$$T_l = \frac{2Lc}{c^2 - v^2}$$

$$T_t = \frac{2L}{c} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$T_l = \frac{2L}{c} \frac{1}{1 - \frac{v^2}{c^2}}$$

9.2

$$T_l - T_t = \frac{2L}{c} \left(\frac{1}{1 - \frac{v^2}{c^2}} - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right) \quad (82)$$

$$(T_l - T_t)c = k\lambda = 2L \left(\left(1 + \frac{v^2}{c^2}\right) - \left(1 + \frac{1}{2} \frac{v^2}{c^2}\right) \right) \quad (83)$$

$$k\lambda = 2L \frac{1}{2} \frac{v^2}{c^2} = L \frac{v^2}{c^2} \quad (84)$$

9.3

Če interferometer zavrtimo za 90° lahko zapišemo $k\lambda = -L \frac{v^2}{c^2}$

$$n = \frac{k_1\lambda - k_2\lambda}{\lambda} = \frac{L \frac{v^2}{c^2} + L \frac{v^2}{c^2}}{\lambda} = \frac{2L v^2}{\lambda c^2} \quad (85)$$

(86)

Žal se teoretični izračun ne ujema z eksperimentom

9.4

Če upoštevamo Lorentzovo transformacijo 11.4 lahko zapišemo

$$T_l = \frac{2L \sqrt{1 - \frac{v^2}{c^2}}}{c} \frac{1}{1 - \frac{v^2}{c^2}} = \frac{2L}{c} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = T_t \quad (87)$$

10 Stefanov zakon

Lahko ga izpeljemo iz Planckovega zakona 11.4

$$j = \int_0^\infty \frac{dj}{d\lambda} d\lambda = \int_0^\infty \frac{2\pi h c_0^2}{\lambda^5 \left(e^{\frac{hc_0}{\lambda k_B T}} - 1 \right)} d\lambda \quad (88)$$

10.1

$$x = \frac{hc_0}{\lambda k_B T} \Rightarrow d\lambda = -\frac{hc_0}{k_B T x^2} dx \text{ in } \lambda \rightarrow 0 \Rightarrow x \rightarrow \infty, \lambda \rightarrow \infty \Rightarrow x \rightarrow 0$$

$$j = \int_\infty^0 \frac{2\pi h c_0^2}{\left(\frac{hc_0}{k_B T x} \right)^5 (e^x - 1)} \left(-\frac{hc_0}{k_B T x^2} \right) dx \quad (89)$$

$$j = \frac{2\pi (k_B T)^4}{h^3 c_0^3} \int_0^\infty \frac{x^3}{e^x - 1} dx \quad (90)$$

$$j = \frac{2\pi (k_B T)^4}{h^3 c_0^3} \cdot \frac{\pi^4}{15} = \frac{2\pi^5 k_B^4 T^4}{15 h^3 c_0^3} \quad (91)$$

$$j = \sigma T^4; \sigma = \frac{2\pi^5 k_B^4}{15 h^3 c_0^3} = 5,67 \cdot 10^{-8} \frac{W}{m^2 K^4} \quad (92)$$

11 Uporabljene enačbe

11.1 Matematika

Gradient

$$\vec{\nabla} f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right) \quad (93)$$

Divergenca

$$\vec{\nabla} \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \quad (94)$$

Rotacija

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} \quad (95)$$

11.2 Maxwellove enačbe

Faradejev zakon v integralni obliki

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{S} \quad (96)$$

Amperov zakon v integralni obliki

$$\int \vec{H} \cdot d\vec{s} = \int J d\vec{S} + \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S} \quad (98)$$

Faradejev zakon v diferencialni obliki

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (97)$$

Amperov zakon v diferencialni obliki

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (99)$$

11.3 Optika

Enačba preslikave

$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b} \quad (100)$$

Lomni zakon (3)

$$n_1 \sin(\alpha) = n_2 \sin(\beta) \quad (103)$$

Enačba leče

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1 \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right); \quad (101)$$

$R > 0$, če je središče krivulje na nasprotni strani leče, kot površina, ki jo opisuje

Mikroskop (5)

$$M = \frac{ex_0}{f_{ok} f_{ob}} \quad (104)$$

Skupek leč

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad (102)$$

Daljnogled (6)

$$M = \frac{a f_{ob}}{f_{ok} (a - f_{ob})} \quad (105)$$

11.4 Moderna fizika

Lorentzova transformacija

$$x' = \gamma(x - vt) \quad x = \gamma(x' + vt') \quad (106)$$

$$y' = y \quad y = y' \quad (107)$$

$$z' = z \quad z = z' \quad (108)$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \quad t = \gamma\left(t' + \frac{v}{c^2}x'\right) \quad (109)$$

Galilejeva transformacija

$$x' = x - vt \quad x = x' + vt' \quad (111)$$

$$y' = y \quad y = y' \quad (112)$$

$$z' = z \quad z = z' \quad (113)$$

$$t' = t \quad t = t' \quad (114)$$

Lorentzov faktor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (110)$$

Planckov zakon

$$\frac{dj}{d\lambda} = \frac{2\pi hc^2}{\lambda^5 \left(e^{\frac{hc}{\lambda k_B T}} - 1 \right)} \quad (115)$$