Fizika 2 - izpeljave

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1 Valovna enačba

1.1

Faradejev 44 in Amperov 46 zakon lahko združimo v valovno enačbo.

1.2

- $$\vec{J} = 0$$
- $$\vec{D} = \epsilon_0 \vec{E}$$
- $$\vec{B} = \mu_0 \vec{H}$$
- $$\vec{E} = (0, E_y(x,t), E_z(x,t))$$
- $$\vec{H} = (0, H_y(x,t), H_z(x,t))$$

1.3

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}, \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) = \left(0, -\frac{\partial E_z}{\partial x}, \frac{\partial E_y}{\partial x} \right) =$$

$$= -\mu_0 \left(\frac{\partial H_x}{\partial t}, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) = -\mu_0 \left(0, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) \quad (1)$$

$$\frac{\partial E_z}{\partial x} = \mu_0 \frac{\partial H_y}{\partial t} \quad (2) \qquad \frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \quad (3)$$

1.4

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}, \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}, \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) = \left(0, -\frac{\partial H_z}{\partial x}, \frac{\partial H_y}{\partial x} \right) =$$

$$= \epsilon_0 \left(\frac{\partial H_x}{\partial t}, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) = \epsilon_0 \left(0, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) \quad (4)$$

$$\frac{\partial H_z}{\partial x} = -\epsilon_0 \frac{\partial E_y}{\partial t} \quad (5) \qquad \frac{\partial H_y}{\partial x} = \epsilon_0 \frac{\partial E_z}{\partial t} \quad (6)$$

1.5

$$\frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \qquad \frac{\partial H_z}{\partial x} = -\epsilon_0 \frac{\partial E_y}{\partial t}$$

$$\frac{\partial^2 E_y}{\partial x \partial t} = -\mu_0 \frac{\partial^2 H_z}{\partial t^2} \qquad \frac{\partial^2 H_z}{\partial x^2} \frac{1}{-\epsilon_0} = \frac{\partial^2 E_y}{\partial t \partial x}$$

1.6

$$\frac{\partial^2 H_z}{\partial x^2} \frac{1}{-\epsilon_0} = -\mu_0 \frac{\partial^2 H_z}{\partial t^2}$$
$$\equiv$$
$$\frac{\partial^2 E_z}{\partial x^2} \frac{1}{-\epsilon_0} = -\mu_0 \frac{\partial^2 E_z}{\partial t^2}$$

1.7

$$E_y = E_0 \cos(\omega t - kx); \omega = 2\pi f, k = \frac{\omega}{c_0} \tag{7}$$

$$\frac{\partial^2 E_y}{\partial x^2} = -k^2 E_y; \frac{\partial^2 E_y}{\partial t^2} = -\omega^2 E_y \tag{8}$$

$$k^2 E_y \frac{1}{-\epsilon_0 \mu_0} = \omega^2 E_y = \frac{\omega^2}{c_0^2} \frac{1}{-\epsilon_0 \mu_0} E_y \tag{9}$$

1.8

$$\frac{1}{\epsilon_0 \mu_0} = c_0^2 \tag{10}$$

2 Lomni kotnik

2.1

$$c = \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_r \mu_0}}; c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \tag{11}$$

$$n = \frac{c}{c_0} = \sqrt{\frac{\epsilon_0 \mu_0}{\epsilon_r \epsilon_0 \mu_r \mu_0}} = \frac{1}{\sqrt{\epsilon_r \mu_r}} \tag{12}$$

3 Lomni zakon

3.1

Minimiziramo čas preleta žarkov od točke A do točke B, ki sta v različnih medijih z različnimi n.

3.2

$t(\alpha, \beta)$ ima ekstrem, če je $dt = 0$

$$t = t_1 + t_2 \tag{13}$$

$$t = \frac{s_1}{c_1} + \frac{s_2}{c_2} \tag{14}$$

$$t = \frac{h_1}{\cos(\alpha) * c_1} + \frac{h_2}{\cos(\beta) * c_2} \tag{15}$$

$$dt = \frac{\partial t}{\partial \alpha} d\alpha + \frac{\partial t}{\partial \beta} d\beta = 0 \tag{16}$$

$$\frac{h_1 \sin(\alpha)}{\cos^2(\alpha) * c_1} d\alpha = -\frac{h_2 \sin(\beta)}{\cos^2(\beta) * c_2} d\beta \tag{17}$$

$$\frac{d\alpha}{d\beta} = -\frac{h_2}{h_1} \frac{c_1}{c_2} \frac{\sin(\beta)}{\sin(\alpha)} \frac{\cos^2(\alpha)}{\cos^2(\beta)} \tag{18}$$

3.3

L je razdalja med točkama po y osi, ker je konstantna, je $dL = 0$

$$L = l_1 + l_2 = \frac{h_1}{\tan(\alpha)} + \frac{h_2}{\tan(\beta)} \tag{19}$$

$$dL = \frac{h_1}{\cos^2(\alpha)} d\alpha + \frac{h_2}{\cos^2(\beta)} d\beta = 0 \tag{20}$$

$$\frac{h_1}{\cos^2(\alpha)} d\alpha = -\frac{h_2}{\cos^2(\beta)} d\beta \tag{21}$$

$$\frac{d\alpha}{d\beta} = -\frac{h_2}{h_1} \frac{\cos^2(\beta)}{\cos^2(\alpha)} \tag{22}$$

3.4

$$-\frac{h_2}{h_1} \frac{c_1}{c_2} \frac{\sin(\beta)}{\sin(\alpha)} \frac{\cos^2(\alpha)}{\cos^2(\beta)} = -\frac{h_2}{h_1} \frac{\cos^2(\beta)}{\cos^2(\alpha)} \tag{23}$$

$$\Rightarrow \frac{\sin(\beta)}{\sin(\alpha)} = \frac{c_1}{c_2} = \frac{\frac{c_0}{n_1}}{\frac{c_0}{n_2}} = \frac{n_2}{n_1} \tag{24}$$

$$\Rightarrow n_1 \sin(\alpha) = n_2 \sin(\beta) \tag{25}$$

4 Odbojni zakon

$$t = t_1 + t_2 \tag{26}$$
$$t = \frac{s_1}{c} + \frac{s_2}{c} \tag{27}$$
$$t = \frac{h}{\cos(\alpha) * c} + \frac{h}{\cos(\beta) * c} \tag{28}$$
$$dt = \frac{\partial t}{\partial \alpha} d\alpha + \frac{\partial t}{\partial \beta} d\beta = 0 \tag{29}$$
$$\frac{h \sin(\alpha)}{\cos^2(\alpha) * c} d\alpha = -\frac{h \sin(\beta)}{\cos^2(\beta) * c} d\beta \tag{30}$$
$$\frac{d\alpha}{d\beta} = -\frac{\sin(\beta)}{\sin(\alpha)} \frac{\cos^2(\alpha)}{\cos^2(\beta)} \tag{31}$$

4.1

L je razdalja med točkama po y osi, ker je konstantna, je $dL = 0$

$$L = l_1 + l_2 = \frac{h}{\tan(\alpha)} + \frac{h}{\tan(\beta)} \tag{32}$$
$$dL = \frac{h}{\cos^2(\alpha)} d\alpha + \frac{h}{\cos^2(\beta)} d\beta = 0 \tag{33}$$
$$\frac{h}{\cos^2(\alpha)} d\alpha = -\frac{h}{\cos^2(\beta)} d\beta \tag{34}$$
$$\frac{d\alpha}{d\beta} = -\frac{\cos^2(\beta)}{\cos^2(\alpha)} \tag{35}$$

4.2

$$-\frac{\sin(\beta)}{\sin(\alpha)} \frac{\cos^2(\alpha)}{\cos^2(\beta)} = -\frac{\cos^2(\beta)}{\cos^2(\alpha)} \tag{36}$$
$$\Rightarrow \frac{\sin(\beta)}{\sin(\alpha)} = 1 \tag{37}$$
$$\Rightarrow \sin(\alpha) = \sin(\beta) \tag{38}$$
$$\Rightarrow \alpha = \beta \tag{39}$$

5 Uporabljene enačbe

5.0.1 Matematika

Gradient

$$\vec{\nabla} f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right) \tag{40}$$

Divergenca

$$\vec{\nabla} \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \tag{41}$$

Rotacija

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} \tag{42}$$

5.0.2 Maxwellove enačbe

Faradejev zakon v integralni obliki

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{S} \tag{43}$$

Faradejev zakon v diferencialni obliki

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{44}$$

Amperov zakon v integralni obliki

$$\int \vec{H} \cdot d\vec{s} = \int J d\vec{S} + \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S} \tag{45}$$

Amperov zakon v diferencialni obliki

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{46}$$