

# Fizika 2 - izpeljave

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# 1 Valovna enačba

## 1.1

Faradejev 86 in Amperov 88 zakon lahko združimo v valovno enačbo.

## 1.2

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- 
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- 
- 

$$\vec{J} = 0$$

$$\vec{D} = \epsilon_0 \vec{E}$$

$$\vec{B} = \mu_0 \vec{H}$$

$$\vec{E} = (0, E_y(x, t), E_z(x, t))$$

$$\vec{H} = (0, H_y(x, t), H_z(x, t))$$

## 1.3

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \left( \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}, \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) = \left( 0, -\frac{\partial E_z}{\partial x}, \frac{\partial E_y}{\partial x} \right) =$$
$$= -\mu_0 \left( \frac{\partial H_x}{\partial t}, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) = -\mu_0 \left( 0, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) \quad (1)$$

$$\frac{\partial E_z}{\partial x} = \mu_0 \frac{\partial H_y}{\partial t} \quad (2)$$

$$\frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \quad (3)$$

## 1.4

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}, \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}, \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) = \left( 0, -\frac{\partial H_z}{\partial x}, \frac{\partial H_y}{\partial x} \right) =$$
$$= \epsilon_0 \left( \frac{\partial H_x}{\partial t}, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) = \epsilon_0 \left( 0, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) \quad (4)$$

$$\frac{\partial H_z}{\partial x} = -\epsilon_0 \frac{\partial E_y}{\partial t} \quad (5)$$

$$\frac{\partial H_y}{\partial x} = \epsilon_0 \frac{\partial E_z}{\partial t} \quad (6)$$

## 1.5

$$\begin{aligned}\frac{\partial E_y}{\partial x} &= -\mu_0 \frac{\partial H_z}{\partial t} & \frac{\partial H_z}{\partial x} &= -\epsilon_0 \frac{\partial E_y}{\partial t} \\ \frac{\partial^2 E_y}{\partial x \partial t} &= -\mu_0 \frac{\partial^2 H_z}{\partial t^2} & \frac{\partial^2 H_z}{\partial x^2} \frac{1}{-\epsilon_0} &= \frac{\partial^2 E_y}{\partial t \partial x}\end{aligned}$$

## 1.6

$$\frac{\partial^2 E_y}{\partial x^2} \frac{1}{-\mu_0} = -\epsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad (7)$$

## 1.7

$$E_y = E_0 \cos(\omega t - kx); \omega = 2\pi f, k = \frac{\omega}{c_0} \quad (8)$$

$$\frac{\partial^2 E_y}{\partial x^2} = -k^2 E_y; \frac{\partial^2 E_y}{\partial t^2} = -\omega^2 E_y \quad (9)$$

$$k^2 E_y \frac{1}{-\epsilon_0 \mu_0} = \omega^2 E_y = \frac{\omega^2}{c_0^2} \frac{1}{-\epsilon_0 \mu_0} E_y \quad (10)$$

## 1.8

$$\frac{1}{\epsilon_0 \mu_0} = c_0^2 \quad (11)$$

## 2 Lomni količník

### 2.1

$$c = \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_r \mu_0}}; c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \quad (12)$$

$$n = \frac{c}{c_0} = \sqrt{\frac{\epsilon_0 \mu_0}{\epsilon_r \epsilon_0 \mu_r \mu_0}} = \frac{1}{\sqrt{\epsilon_r \mu_r}} \quad (13)$$

### 3 Lomni zakon

#### 3.1

Minimiziramo čas preleta žarkov od točke A do točke B, ki sta v različnih medijih z različnimi  $n$ .

#### 3.2

$t(\alpha, \beta)$  ima ekstrem, če je  $dt = 0$

$$t = t_1 + t_2 \quad (14)$$

$$t = \frac{s_1}{c_1} + \frac{s_2}{c_2} \quad (15)$$

$$t = \frac{h_1}{\cos(\alpha) * c_1} + \frac{h_2}{\cos(\beta) * c_2} \quad (16)$$

$$dt = \frac{\partial t}{\partial \alpha} d\alpha + \frac{\partial t}{\partial \beta} d\beta = 0 \quad (17)$$

$$\frac{h_1 \sin(\alpha)}{\cos^2(\alpha) * c_1} d\alpha = -\frac{h_2 \sin(\beta)}{\cos^2(\beta) * c_2} d\beta \quad (18)$$

$$\frac{d\alpha}{d\beta} = -\frac{h_2 c_1 \sin(\beta) \cos^2(\alpha)}{h_1 c_2 \sin(\alpha) \cos^2(\beta)} \quad (19)$$

#### 3.3

$L$  je razdalja med točkama po  $y$  osi, ker je konstantna, je  $dL = 0$

$$L = l_1 + l_2 = \frac{h_1}{\tan(\alpha)} + \frac{h_2}{\tan(\beta)} \quad (20)$$

$$dL = \frac{h_1}{\cos^2(\alpha)} d\alpha + \frac{h_2}{\cos^2(\beta)} d\beta = 0 \quad (21)$$

$$\frac{h_1}{\cos^2(\alpha)} d\alpha = -\frac{h_2}{\cos^2(\beta)} d\beta \quad (22)$$

$$\frac{d\alpha}{d\beta} = -\frac{h_2 \cos^2(\beta)}{h_1 \cos^2(\alpha)} \quad (23)$$

#### 3.4

$$-\frac{h_2 c_1 \sin(\beta) \cos^2(\alpha)}{h_1 c_2 \sin(\alpha) \cos^2(\beta)} = -\frac{h_2 \cos^2(\beta)}{h_1 \cos^2(\alpha)} \quad (24)$$

$$\Rightarrow \frac{\sin(\beta)}{\sin(\alpha)} = \frac{c_1}{c_2} = \frac{\frac{c_0}{n_1}}{\frac{c_0}{n_2}} = \frac{n_2}{n_1} \quad (25)$$

$$\Rightarrow n_1 \sin(\alpha) = n_2 \sin(\beta) \quad (26)$$

## 4 Odbojni zakon

$$t = t_1 + t_2 \quad (27)$$

$$t = \frac{s_1}{c} + \frac{s_2}{c} \quad (28)$$

$$t = \frac{h}{\cos(\alpha) * c} + \frac{h}{\cos(\beta) * c} \quad (29)$$

$$dt = \frac{\partial t}{\partial \alpha} d\alpha + \frac{\partial t}{\partial \beta} d\beta = 0 \quad (30)$$

$$\frac{h \sin(\alpha)}{\cos^2(\alpha) * c} d\alpha = -\frac{h \sin(\beta)}{\cos^2(\beta) * c} d\beta \quad (31)$$

$$\frac{d\alpha}{d\beta} = -\frac{\sin(\beta) \cos^2(\alpha)}{\sin(\alpha) \cos^2(\beta)} \quad (32)$$

### 4.1

$L$  je razdalja med točkama po  $y$  osi, ker je konstantna, je  $dL = 0$

$$L = l_1 + l_2 = \frac{h}{\tan(\alpha)} + \frac{h}{\tan(\beta)} \quad (33)$$

$$dL = \frac{h}{\cos^2(\alpha)} d\alpha + \frac{h}{\cos^2(\beta)} d\beta = 0 \quad (34)$$

$$\frac{h}{\cos^2(\alpha)} d\alpha = -\frac{h}{\cos^2(\beta)} d\beta \quad (35)$$

$$\frac{d\alpha}{d\beta} = -\frac{\cos^2(\beta)}{\cos^2(\alpha)} \quad (36)$$

### 4.2

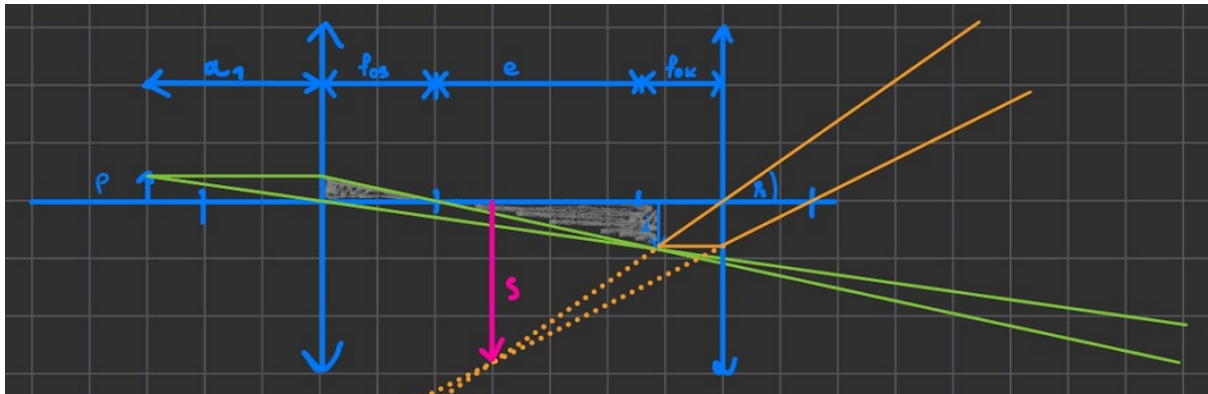
$$-\frac{\sin(\beta) \cos^2(\alpha)}{\sin(\alpha) \cos^2(\beta)} = -\frac{\cos^2(\beta)}{\cos^2(\alpha)} \quad (37)$$

$$\Rightarrow \frac{\sin(\beta)}{\sin(\alpha)} = 1 \quad (38)$$

$$\Rightarrow \sin(\alpha) = \sin(\beta) \quad (39)$$

$$\Rightarrow \alpha = \beta \quad (40)$$

## 5 Mikroskop



$$\tan(\varphi_1) = \frac{p}{x_0} \quad (41)$$

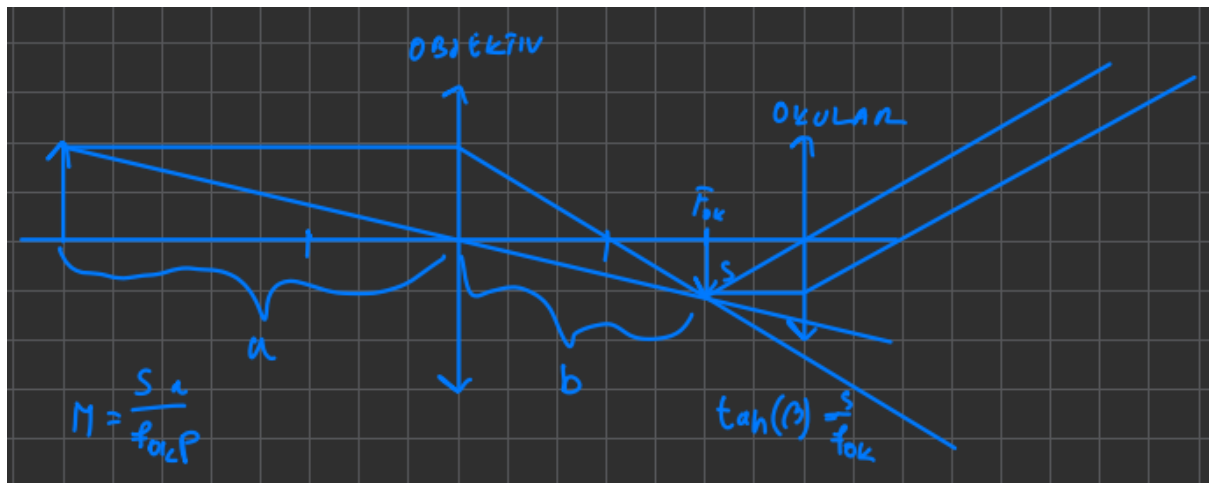
$$\tan(\varphi_2) = \frac{i}{f_{ok}} \quad (42)$$

$$\frac{i}{e} = \frac{p}{f_{ob}} \quad (43)$$

$$M = \frac{\tan(\varphi_2)}{\tan(\varphi_1)} \quad (44)$$

$$M = \frac{ex_0}{f_{ok}f_{ob}} \quad (45)$$

## 6 Daljnogled



$$\tan(\varphi_1) = \frac{p}{a} = \frac{s}{b} \quad (46)$$

$$\tan(\varphi_2) = \frac{s}{f_{ok}} \quad (47)$$

$$\frac{1}{f_{ob}} = \frac{1}{a} + \frac{1}{b} \Rightarrow b = \frac{a f_{ob}}{a - f_{ob}} \quad (48)$$

$$M = \frac{\tan(\varphi_2)}{\tan(\varphi_1)} = \frac{b}{f_{ok}} \quad (49)$$

$$M = \frac{a f_{ob}}{f_{ok}(a - f_{ob})} \quad (50)$$



## 7 Absorpcija EM valovanja

Telo debeline  $x$  in koeficientom absorpcije  $\mu$  bo absorbira svetlobni tok  $j$ . Tanko telo debeline  $dx$  absorbira svetlobni tok  $dj$

$$dj = -\mu j dx \quad (51)$$

$$\frac{1}{j} dj = -\mu dx \quad (52)$$

$$\int_{j_0}^{j(x)} \frac{1}{j} dj = -\mu \int_0^x dx \quad (53)$$

$$\ln(j(x)) - \ln(j_0) = -\mu x \quad (54)$$

$$\ln\left(\frac{j(x)}{j_0}\right) = -\mu x \quad (55)$$

$$j(x) = j_0 e^{-\mu x} \quad (56)$$

## 8 Okrogla luč po Lambertovem zakonu



### 8.1

$$B(\theta) = B_0 \quad (57)$$

$$dI = B(\theta) \cos(\beta) dS \quad (58)$$

$$dS = x d\varphi dx \quad (59)$$

$$dS = \int_0^{2\pi} x dx d\varphi \quad (60)$$

$$dS = 2\pi x dx \quad (61)$$

$$dI = B_0 \cos(\beta) 2\pi x dx \quad (62)$$

$$dE = \frac{dI}{r^2} \cos(\beta) \quad (63)$$

$$dE = \frac{B_0}{r^2} \cos^2(\beta) 2\pi x dx \quad (64)$$

## 8.2

$$r = \sqrt{x^2 + h^2} \quad (65)$$

$$\cos(\beta) = \frac{h}{r} \quad (66)$$

$$r = \frac{h}{\cos(\beta)} \quad (67)$$

$$\tan(\beta) = \frac{x}{h} \Rightarrow x = h \tan \beta \quad (68)$$

$$\frac{1}{\cos^2(\beta)} d\beta = \frac{1}{h} dx \quad (69)$$

$$dx = h \frac{1}{\cos^2(\beta)} d\beta \quad (70)$$

## 8.3

$$\beta(x) = \arctan\left(\frac{x}{h}\right) \quad (71)$$

$$\beta(0) = 0 \quad (72)$$

$$\beta(R) = \beta_0 = \arctan\left(\frac{R}{h}\right) \quad (73)$$

## 8.4

$$E = \int dE = \int_0^{\beta} \frac{B_0 \cos^2(\beta) \cos^2(\beta) 2\pi h \sin(\beta) h}{h^2 \cos(\beta) \cos^2(\beta)} d\beta \quad (74)$$

$$E = \int_0^{\beta} B_0 \sin(\beta) \cos(\beta) 2\pi d\beta \quad (75)$$

$$E = B_0 \pi \int_0^{\beta} \sin(2\beta) d\beta \quad (76)$$

$$E = B_0 \pi \int_0^{2\beta} \sin(u) du; u = 2\beta \Rightarrow du = 2d\beta \quad (77)$$

$$E = \frac{B_0 \pi}{2} (-\cos(u)) \Big|_{u=0}^{2\beta} \quad (78)$$

$$E = \frac{B_0 \pi}{2} (1 - \cos(2\beta)) \quad (79)$$

$$(80)$$

## 8.5

Če je  $R$  velik

$$\lim_{R \rightarrow \infty} E = \lim_{\beta \rightarrow \frac{\pi}{2}} \frac{B_0 \pi}{2} (1 - \cos(2\beta_0)) = \frac{B_0 \pi}{2} (1 - \cos(\pi)) = \frac{B_0 \pi}{2} (1 + 1) = B_0 \pi \quad (81)$$

## 9 Uporabljene enačbe

### 9.1 Matematika

#### Gradient

$$\vec{\nabla} f = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right) \quad (82)$$

#### Divergenca

$$\vec{\nabla} \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \quad (83)$$

#### Rotacija

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} \quad (84)$$

### 9.2 Maxwellove enačbe

#### Faradejev zakon v integralni obliki

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{S} \quad (85)$$

#### Amperov zakon v integralni obliki

$$\int \vec{H} \cdot d\vec{s} = \int J d\vec{S} + \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S} \quad (87)$$

#### Faradejev zakon v diferencialni obliki

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (86)$$

#### Amperov zakon v diferencialni obliki

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (88)$$

### 9.3 Optika

#### Enačba preslikave

$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b} \quad (89)$$

#### Mikroskop (5)

$$M = \frac{ex_0}{f_{ok}f_{ob}} \quad (92)$$

#### Enačba leče

$$\frac{1}{f} = \left( \frac{n_2}{n_1} - 1 \right) \left( \frac{1}{R_1} - \frac{1}{R_2} \right); \quad (90)$$

$R > 0$ , če je središče krivulje na levi strani

#### Daljnogled (6)

#### Lomni zakon (3)

$$n_1 \sin(\alpha) = n_2 \sin(\beta) \quad (91)$$

$$M = \frac{af_{ob}}{f_{ok}(a - f_{ob})} \quad (93)$$

### 9.4 Moderna fizika

#### Lorentzova transformacija

$$x' = \gamma(x - vt) \quad x = \gamma(x' + vt') \quad (94)$$

$$y' = y \quad y = y' \quad (95)$$

$$z' = z \quad z = z' \quad (96)$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \quad t = \gamma\left(t' + \frac{v}{c^2}x'\right) \quad (97)$$

**Lorentzov faktor**

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (98)$$