Fizika 2 - izpeljave

insightfulbriyan

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1 Valovna enačba

1.1

Faradejev 108 in Amperov 110 zakon lahko združimo v valovno enačbo.

1.2

-
-
-
-
-

$$
\begin{gathered}
\vec{J} = 0 \\
\vec{D} = \epsilon_0 \vec{E} \\
\vec{B} = \mu_0 \vec{H} \\
\vec{E} = (0, E_y(x,t), E_z(x,t)) \\
\vec{H} = (0, H_y(x,t), H_z(x,t))
\end{gathered}
$$

1.3

$$
\begin{aligned}
\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} &= \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}, \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) = \left(0, -\frac{\partial E_z}{\partial x}, \frac{\partial E_y}{\partial x} \right) = \\
&= -\mu_0 \left(\frac{\partial H_x}{\partial t}, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) = -\mu_0 \left(0, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) \qquad (1)
\end{aligned}
$$

$$
\frac{\partial E_z}{\partial x} = \mu_0 \frac{\partial H_y}{\partial t} \qquad (2) \qquad\qquad \frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \qquad (3)
$$

1.4

$$
\begin{aligned}
\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} &= \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}, \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}, \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) = \left(0, -\frac{\partial H_z}{\partial x}, \frac{\partial H_y}{\partial x} \right) = \\
&= \epsilon_0 \left(\frac{\partial H_x}{\partial t}, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) = \epsilon_0 \left(0, \frac{\partial H_y}{\partial t}, \frac{\partial H_z}{\partial t} \right) \qquad (4)
\end{aligned}
$$

$$
\frac{\partial H_z}{\partial x} = -\epsilon_0 \frac{\partial E_y}{\partial t} \qquad (5) \qquad\qquad \frac{\partial H_y}{\partial x} = \epsilon_0 \frac{\partial E_z}{\partial t} \qquad (6)
$$

1.5

$$\frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \qquad \frac{\partial H_z}{\partial x} = -\epsilon_0 \frac{\partial E_y}{\partial t}$$

$$\frac{\partial^2 E_y}{\partial x \partial t} = -\mu_0 \frac{\partial^2 H_z}{\partial t^2} \qquad \frac{\partial^2 H_z}{\partial x^2} \frac{1}{-\epsilon_0} = \frac{\partial^2 E_y}{\partial t \partial x}$$

1.6

$$\frac{\partial^2 E_y}{\partial x^2} \frac{1}{-\mu_0} = -\epsilon_0 \frac{\partial^2 E_y}{\partial t^2} \tag{7}$$

1.7

$$E_y = E_0 \cos(\omega t - kx); \omega = 2\pi f, k = \frac{\omega}{c_0} \tag{8}$$

$$\frac{\partial^2 E_y}{\partial x^2} = -k^2 E_y; \frac{\partial^2 E_y}{\partial t^2} = -\omega^2 E_y \tag{9}$$

$$k^2 E_y \frac{1}{-\epsilon_0 \mu_0} = \omega^2 E_y = \frac{\omega^2}{c_0^2} \frac{1}{-\epsilon_0 \mu_0} E_y \tag{10}$$

1.8

$$\frac{1}{\epsilon_0 \mu_0} = c_0^2 \tag{11}$$

2 Lomni količnik

2.1

$$c = \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_r \mu_0}}; c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \tag{12}$$

$$n = \frac{c}{c_0} = \sqrt{\frac{\epsilon_0 \mu_0}{\epsilon_r \epsilon_0 \mu_r \mu_0}} = \frac{1}{\sqrt{\epsilon_r \mu_r}} \tag{13}$$

3 Lomni zakon

3.1

Minimiziramo čas preleta žarkov od točke A do točke B, ki sta v različnih medijih z različnimi n.

3.2

$t(\alpha, \beta)$ ima ekstrem, če je $dt = 0$

$$t = t_1 + t_2 \tag{14}$$
$$t = \frac{s_1}{c_1} + \frac{s_2}{c_2} \tag{15}$$
$$t = \frac{h_1}{\cos(\alpha) * c_1} + \frac{h_2}{\cos(\beta) * c_2} \tag{16}$$
$$dt = \frac{\partial t}{\partial \alpha} d\alpha + \frac{\partial t}{\partial \beta} d\beta = 0 \tag{17}$$
$$\frac{h_1 \sin(\alpha)}{\cos^2(\alpha) * c_1} d\alpha = -\frac{h_2 \sin(\beta)}{\cos^2(\beta) * c_2} d\beta \tag{18}$$
$$\frac{d\alpha}{d\beta} = -\frac{h_2}{h_1} \frac{c_1}{c_2} \frac{\sin(\beta)}{\sin(\alpha)} \frac{\cos^2(\alpha)}{\cos^2(\beta)} \tag{19}$$

3.3

L je razdalja med točkama po y osi, ker je konstantna, je $dL = 0$

$$L = l_1 + l_2 = \frac{h_1}{\tan(\alpha)} + \frac{h_2}{\tan(\beta)} \tag{20}$$
$$dL = \frac{h_1}{\cos^2(\alpha)} d\alpha + \frac{h_2}{\cos^2(\beta)} d\beta = 0 \tag{21}$$
$$\frac{h_1}{\cos^2(\alpha)} d\alpha = -\frac{h_2}{\cos^2(\beta)} d\beta \tag{22}$$
$$\frac{d\alpha}{d\beta} = -\frac{h_2}{h_1} \frac{\cos^2(\beta)}{\cos^2(\alpha)} \tag{23}$$

3.4

$$-\frac{h_2}{h_1} \frac{c_1}{c_2} \frac{\sin(\beta)}{\sin(\alpha)} \frac{\cos^2(\alpha)}{\cos^2(\beta)} = -\frac{h_2}{h_1} \frac{\cos^2(\beta)}{\cos^2(\alpha)} \tag{24}$$
$$\Rightarrow \frac{\sin(\beta)}{\sin(\alpha)} = \frac{c_1}{c_2} = \frac{\frac{c_0}{n_1}}{\frac{c_0}{n_2}} = \frac{n_2}{n_1} \tag{25}$$
$$\Rightarrow n_1 \sin(\alpha) = n_2 \sin(\beta) \tag{26}$$

4 Odbojni zakon

$$t = t_1 + t_2 \tag{27}$$
$$t = \frac{s_1}{c} + \frac{s_2}{c} \tag{28}$$
$$t = \frac{h}{\cos(\alpha) * c} + \frac{h}{\cos(\beta) * c} \tag{29}$$
$$dt = \frac{\partial t}{\partial \alpha} d\alpha + \frac{\partial t}{\partial \beta} d\beta = 0 \tag{30}$$
$$\frac{h \sin(\alpha)}{\cos^2(\alpha) * c} d\alpha = -\frac{h \sin(\beta)}{\cos^2(\beta) * c} d\beta \tag{31}$$
$$\frac{d\alpha}{d\beta} = -\frac{\sin(\beta)}{\sin(\alpha)} \frac{\cos^2(\alpha)}{\cos^2(\beta)} \tag{32}$$

4.1

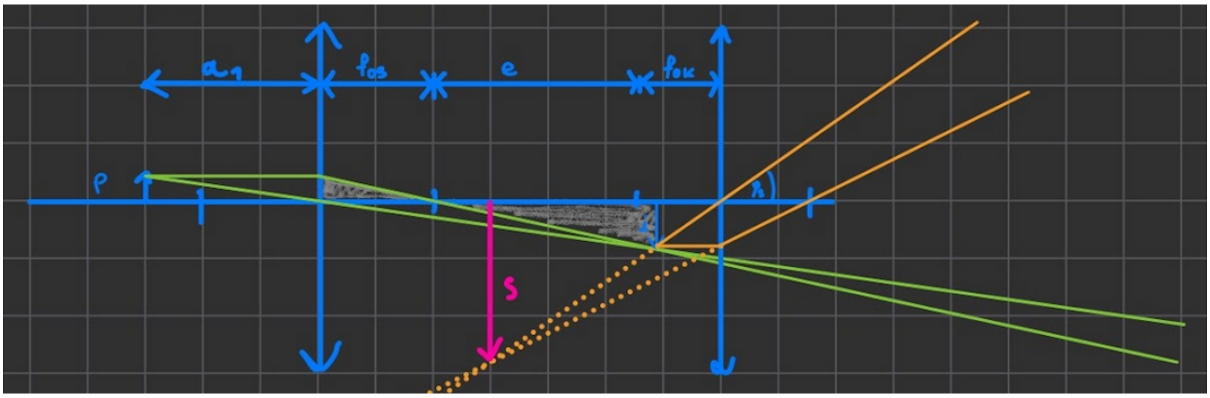
L je razdalja med točkama po y osi, ker je konstantna, je $dL = 0$

$$L = l_1 + l_2 = \frac{h}{\tan(\alpha)} + \frac{h}{\tan(\beta)} \tag{33}$$
$$dL = \frac{h}{\cos^2(\alpha)} d\alpha + \frac{h}{\cos^2(\beta)} d\beta = 0 \tag{34}$$
$$\frac{h}{\cos^2(\alpha)} d\alpha = -\frac{h}{\cos^2(\beta)} d\beta \tag{35}$$
$$\frac{d\alpha}{d\beta} = -\frac{\cos^2(\beta)}{\cos^2(\alpha)} \tag{36}$$

4.2

$$-\frac{\sin(\beta)}{\sin(\alpha)} \frac{\cos^2(\alpha)}{\cos^2(\beta)} = -\frac{\cos^2(\beta)}{\cos^2(\alpha)} \tag{37}$$
$$\Rightarrow \frac{\sin(\beta)}{\sin(\alpha)} = 1 \tag{38}$$
$$\Rightarrow \sin(\alpha) = \sin(\beta) \tag{39}$$
$$\Rightarrow \alpha = \beta \tag{40}$$

5 Mikroskop



$$\tan(\varphi_1) = \frac{p}{x_0} \tag{41}$$

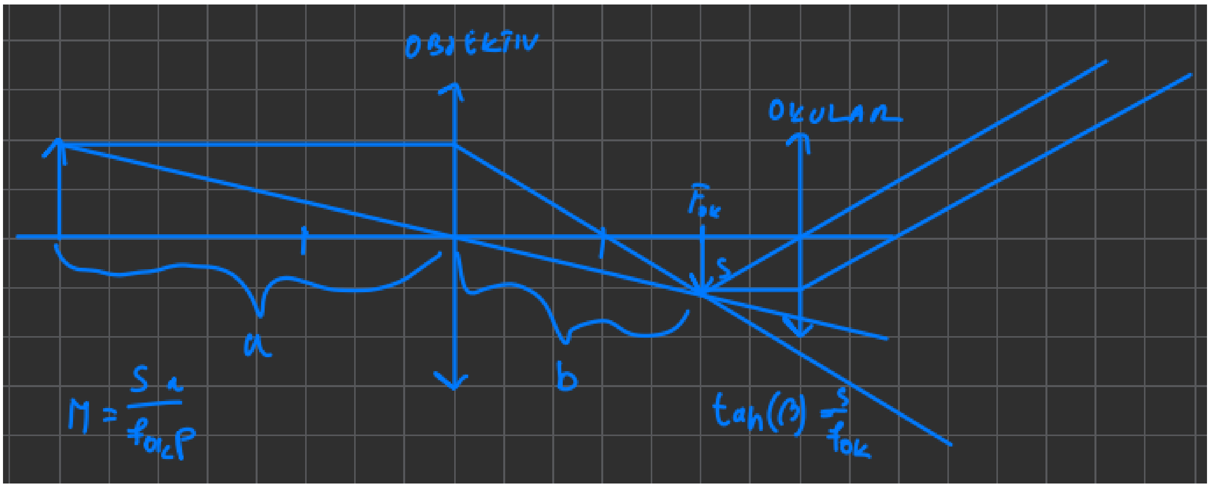
$$\tan(\varphi_2) = \frac{i}{f_{ok}} \tag{42}$$

$$\frac{i}{e} = \frac{p}{f_{ob}} \tag{43}$$

$$M = \frac{\tan(\varphi_2)}{\tan(\varphi_1)} \tag{44}$$

$$M = \frac{e x_0}{f_{ok} f_{ob}} \tag{45}$$

6 Daljnogled



$$\tan(\varphi_1) = \frac{p}{a} = \frac{s}{b} \tag{46}$$

$$\tan(\varphi_2) = \frac{s}{f_{ok}} \tag{47}$$

$$\frac{1}{f_{ob}} = \frac{1}{a} + \frac{1}{b} \Rightarrow b = \frac{a f_{ob}}{a - f_{ob}} \tag{48}$$

$$M = \frac{\tan(\varphi_2)}{\tan(\varphi_1)} = \frac{b}{f_{ok}} \tag{49}$$

$$M = \frac{a f_{ob}}{f_{ok}(a - f_{ob})} \tag{50}$$

7 Absorpcija EM valovanja

Telo debeline x in koeficientom absorpcije μ bo absorbira svetlobni tok j. Tanko telo debeline dx absorbira svetlobni tok dj

$$dj = -\mu j dx \tag{51}$$

$$\frac{1}{j} dj = -\mu dx \tag{52}$$

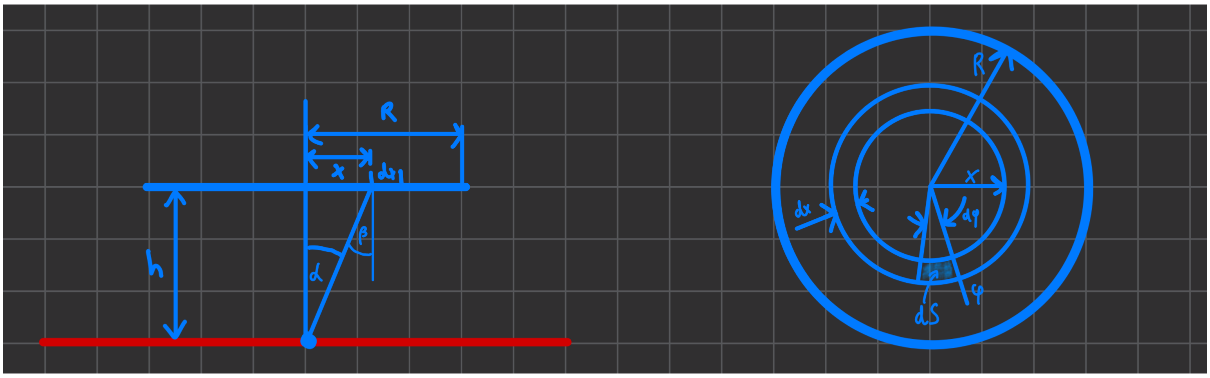
$$\int_{j_0}^{j(x)} \frac{1}{j} dj = -\mu \int_0^x dx \tag{53}$$

$$\ln(j(x)) - \ln(j_0) = -\mu x \tag{54}$$

$$\ln\left(\frac{j(x)}{j_0}\right) = -\mu x \tag{55}$$

$$j(x) = j_0 e^{-\mu x} \tag{56}$$

8 Okrogla luč po Lambertovem zakonu



8.1

$$B(\theta) = B_0 \tag{57}$$

$$dI = B(\theta) \cos(\beta) dS \tag{58}$$

$$dS = x d\varphi dx \tag{59}$$

$$dS = \int_0^{2\pi} x dx d\varphi \tag{60}$$

$$dS = 2\pi x dx \tag{61}$$

$$dI = B_0 \cos(\beta) 2\pi x dx \tag{62}$$

$$dE = \frac{dI}{r^2} \cos(\beta) \tag{63}$$

$$dE = \frac{B_0}{r^2} \cos^2(\beta) 2\pi x dx \tag{64}$$

8.2

$$r = \sqrt{x^2 + h^2} \tag{65}$$

$$\cos(\beta) = \frac{h}{r} \tag{66}$$

$$r = \frac{h}{\cos(\beta)} \tag{67}$$

$$\tan(\beta) = \frac{x}{h} \Rightarrow x = h \tan\beta \tag{68}$$

$$\frac{1}{\cos^2(\beta)} d\beta = \frac{1}{h} dx \tag{69}$$

$$dx = h \frac{1}{\cos^2(\beta)} d\beta \tag{70}$$

8.3

$$\beta(x) = \arctan\left(\frac{x}{h}\right) \tag{71}$$

$$\beta(0) = 0 \tag{72}$$

$$\beta(R) = \beta_0 = \arctan\left(\frac{R}{h}\right) \tag{73}$$

8.4

$$E = \int dE = \int_0^\beta \frac{B_0 \cos^2(\beta)\cos^2(\beta) 2\pi h \sin(\beta) h}{h^2 \cos(\beta)\cos^2(\beta)} d\beta \tag{74}$$

$$E = \int_0^\beta B_0 \sin(\beta)\cos(\beta) 2\pi d\beta \tag{75}$$

$$E = B_0 \pi \int_0^\beta \sin(2\beta) d\beta \tag{76}$$

$$E = B_0 \pi \int_0^{2\beta} \sin(u) du; u = 2\beta \Rightarrow du = 2d\beta \tag{77}$$

$$E = \frac{B_0 \pi}{2} (-\cos(u))|_{u=0}^{2\beta} \tag{78}$$

$$E = \frac{B_0 \pi}{2} (1 - \cos(2\beta)) \tag{79}$$

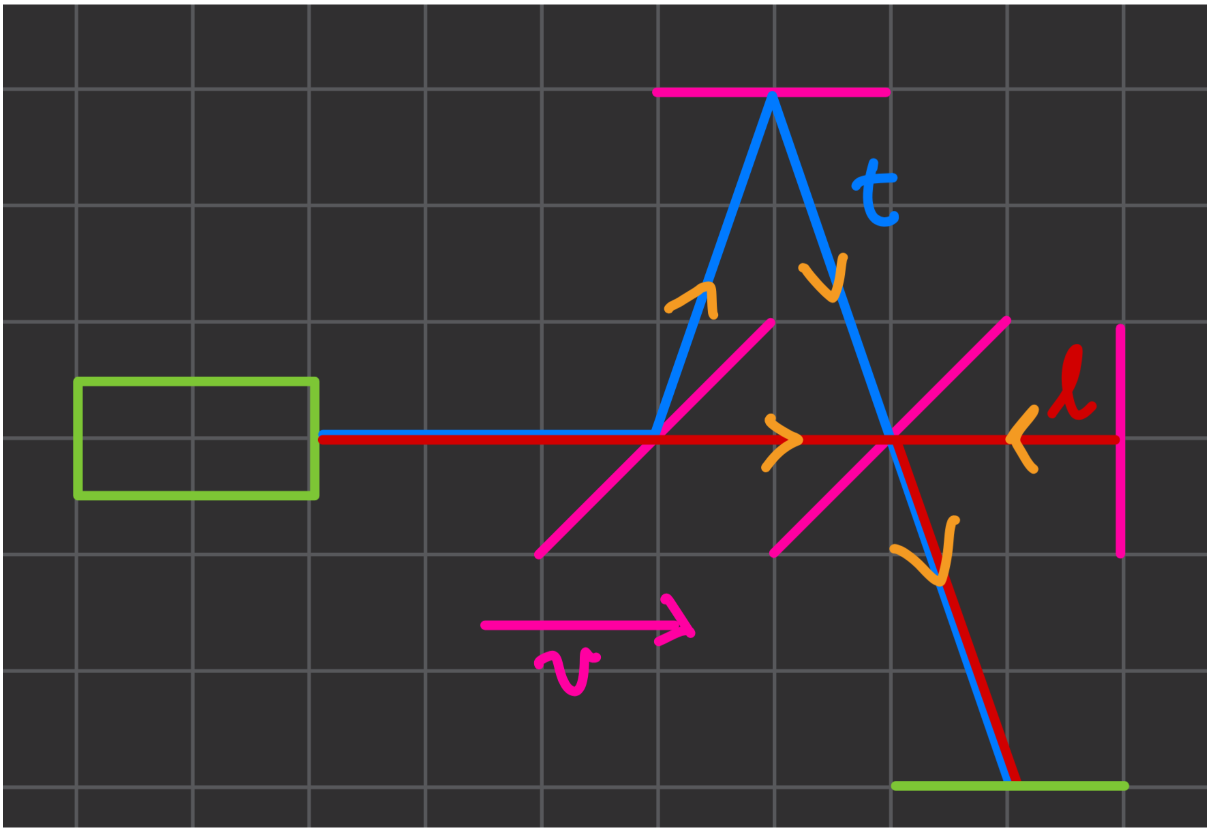
$$\tag{80}$$

8.5

Če je R velik

$$\lim_{R\to\infty} E = \lim_{\beta \to \frac{\pi}{2}} \frac{B_0\pi}{2}(1 - \cos(2\beta_0)) = \frac{B_0\pi}{2}(1 - \cos(\pi)) = \frac{B_0\pi}{2}(1+1) = B_0\pi \tag{81}$$

9 Michelson-Morleyev interferometer



9.1

$$T_t = \frac{2L}{\sqrt{c^2-v^2}}$$

$$T_t = \frac{2L}{\sqrt{c^2(1-\frac{v^2}{c^2})}}$$

$$T_t = \frac{2L}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$T_l = \frac{L}{c-v} + \frac{L}{c+v} = \frac{L(c+v)+L(c-v)}{(c-v)(c+v)}$$

$$T_l = \frac{2Lc}{c^2-v^2}$$

$$T_l = \frac{2L}{c}\frac{1}{1-\frac{v^2}{c^2}}$$

9.2

$$T_l - T_t = \frac{2L}{c}\left(\frac{1}{1-\frac{v^2}{c^2}} - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right) \tag{82}$$

$$(T_l - T_t)c = k\lambda = 2L\left((1+\frac{v^2}{c^2}) - (1+\frac{1}{2}\frac{v^2}{c^2})\right) \tag{83}$$

$$k\lambda = 2L\frac{1}{2}\frac{v^2}{c^2} = L\frac{v^2}{c^2} \tag{84}$$

9.3

Če interferometer zavrtimo za 90° lahko zapišemo $k\lambda = -L\frac{v^2}{c^2}$

$$n = \frac{k_1\lambda - k_2\lambda}{\lambda} = \frac{L\frac{v^2}{c^2} + L\frac{v^2}{c^2}}{\lambda} = \frac{2L}{\lambda}\frac{v^2}{c^2} \tag{85}$$

$$\tag{86}$$

Žal se teoretični izračun ne ujema z eksperimentom

9.4

Če upoštevamo Lorentzovo transformacijo 12.4 lahko zapišemo

$$T_l = \frac{2L\sqrt{1-\frac{v^2}{c^2}}}{c}\frac{1}{1-\frac{v^2}{c^2}} = \frac{2L}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = T_t \tag{87}$$

10 Stefanov zakon

Lahko ga izpeljemo z integriranjem Planckovega zakona 12.4

$$j = \int_0^\infty \frac{dj}{d\lambda}d\lambda = \int_0^\infty \frac{2\pi h c_0^2}{\lambda^5\left(e^{\frac{hc_0}{\lambda k_B T}} - 1\right)}d\lambda \tag{88}$$

10.1

$x = \frac{hc_0}{\lambda k_B T} \Rightarrow d\lambda = -\frac{hc_0}{k_B T x^2}dx$ in $\lambda \to 0 \Rightarrow x \to \infty, \lambda \to \infty \Rightarrow x \to 0$

$$j = \int_\infty^0 \frac{2\pi h c_0^2}{\left(\frac{hc_0}{k_B T x}\right)^5 (e^x - 1)}\left(-\frac{hc_0}{k_B T x^2}\right)dx \tag{89}$$

$$j = \frac{2\pi (k_B T)^4}{h^3 c_0^3}\int_0^\infty \frac{x^3}{e^x - 1}dx \tag{90}$$

$$j = \frac{2\pi (k_B T)^4}{h^3 c_0^3} \cdot \frac{\pi^4}{15} = \frac{2\pi^5 k_B^4 T^4}{15 h^3 c_0^3} \tag{91}$$

$$j = \sigma T^4; \sigma = \frac{2\pi^5 k_B^4}{15 h^3 c_0^3} = 5,67 \cdot 10^{-8}\frac{\mathrm{W}}{\mathrm{m^2K^4}} \tag{92}$$

11 Wienov zakon

Lahko ga izpeljemo z odvajanjem Planckovega zakona 12.4

$$\frac{dj}{d\lambda} = \frac{2\pi h c_0^2}{\lambda^5\left(e^{\frac{hc_0}{\lambda k_B T}} - 1\right)} \tag{93}$$

$$\frac{dj}{dx} = \frac{2\pi k_B^5 T^5}{h^4 c_0^3}\frac{x^5}{e^x - 1}; x = \frac{hc_0}{\lambda k_B T} \tag{94}$$

11.1

$$\frac{dj}{dx} = A\frac{x^5}{e^x - 1} = 0 \tag{95}$$
$$\frac{5x^4(e^x - 1) - x^5 e^x}{(e^x - 1)^2} = 0 \tag{96}$$
$$5x^4(e^x - 1) - x^5 e^x = 0 \tag{97}$$
$$5(e^x - 1) = xe^x \tag{98}$$
$$5 = x\frac{e^x}{e^x - 1} \tag{99}$$
$$x \approx 4.9651 \tag{100}$$
$$\tag{101}$$

11.2

$$\lambda_{max} = \frac{hc_0}{k_B T x} = \frac{hc_0}{4.9651 k_B T} \tag{102}$$
$$\lambda_{max} T = k_W = 2.898 \cdot 10^{-3}\text{m K} \tag{103}$$

12 Uporabljene enačbe

12.1 Matematika

Gradient

$$\vec{\nabla} f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) \tag{104}$$

Divergenca

$$\vec{\nabla} \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \tag{105}$$

Rotacija

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} \tag{106}$$

12.2 Maxwellove enačbe

Faradejev zakon v integralni obliki

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{S} \tag{107}$$

Faradejev zakon v diferencialni obliki

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{108}$$

Amperov zakon v integralni obliki

$$\int \vec{H} \cdot d\vec{s} = \int J d\vec{S} + \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S} \tag{109}$$

Amperov zakon v diferencialni obliki

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{110}$$

12.3 Optika

Enačba preslikave

$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b} \tag{111}$$

Enačba leče

$$\frac{1}{f} = (\frac{n_2}{n_1} - 1)(\frac{1}{R_1} + \frac{1}{R_2}); \tag{112}$$

$R > 0$, če je središče krivulje na nasprotni strani leče, kot površina, ki jo opisuje

Skupek leč

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \tag{113}$$

Lomni zakon (3)

$$n_1 \sin(\alpha) = n_2 \sin(\beta) \tag{114}$$

Mikroskop (5)

$$M = \frac{e x_0}{f_{ok} f_{ob}} \tag{115}$$

Daljnogled (6)

$$M = \frac{a f_{ob}}{f_{ok}(a - f_{ob})} \tag{116}$$

12.4 Moderna fizika

Lorentzova transformacija

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt') \tag{117}$$
$$y' = y \qquad y = y' \tag{118}$$
$$z' = z \qquad z = z' \tag{119}$$
$$t' = \gamma(t - \frac{v}{c^2}x) \qquad t = \gamma(t' + \frac{v}{c^2}x') \tag{120}$$

Lorentzov faktor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{121}$$

Galilejeva transformacija

$$x' = x - vt \qquad x = x' + vt' \tag{122}$$
$$y' = y \qquad y = y' \tag{123}$$
$$z' = z \qquad z = z' \tag{124}$$
$$t' = t \qquad t = t' \tag{125}$$

Planckov zakon

$$\frac{dj}{d\lambda} = \frac{2\pi hc^2}{\lambda^5 \left(e^{\frac{hc}{\lambda k_B T}} - 1\right)} \tag{126}$$