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Biologically-Inspired Iterative Learning Control Design: A Modular-Based Approach

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***Abstract*— Iterative learning control is a feedforward control scheme designed for systems operating in a repetitive setting to achieve high performance tracking for a single fixed reference, with fast learning of a control signal often only achieved when an accurate model of the system is known. On the other hand, biological control systems achieve fast learning without accurate *a priori* modelling, by learning dynamics and control signals simultaneously. Sensorimotor control studies the motion of humans and animals, and a key observation from this field is that a modular structure facilitates the generalisation of learnt skill, which inspires a new modular approach to iterative learning control design that accurately tracks trial-varying references.**

I. Introduction

Iterative learning control (ILC) is a learning-based control scheme designed to achieve high-accuracy tracking of a fixed finite-duration reference repeatedly, improving the feedforward control signal after each trial using information from the previous trials [1]. Existing work applies ILC to learn parameters of a polynomial or rational transfer function [2], but these representations are restrictive. The literature in the field of sensorimotor control offers a more general candidate structure: biological motion in vertebrates is controlled by linear combinations of pattern-generating modules in the central nervous system [3], and this modularity facilitates efficient learning of complex system dynamics. The linear combination of modules is enticing for control design because the parameters can be found analytically, and applies to systems tracking *different* references on each trial – which is a focus of recent work in ILC [2], [4], [5].

This paper extends existing work to explicitly mimic the modular structure seen in sensorimotor control to achieve accurate tracking of trial-varying references, by choosing a performance index that promotes *both* minimal tracking error *and* accurate parametrisation of the system dynamics.

II. Problem Formulation

In this section, stable minimum phase discrete time single-input-single-output (SISO) linear time invariant (LTI) systems are considered, with dynamics described by the state space model below

$$x_k(t+1) = Ax_k(t) + Bu_k(t), \qquad x_k(0) = x_0, \tag{1}$$

$$y_k(t) = Cx_k(t), \qquad t \in [0, N], \tag{2}$$

where t is the time index, k indicates the trial, N is the finite trial length, and $u_k(t)$, $x_k(t) \in \mathbb{R}^n$ (n is the system order), and $y_k(t) \in \mathbb{R}$ are respectively the input, state and output on trial k at time t of the system defined by matrices $\{A, B, C\}$ of appropriate dimensions. The reference to track on each trial is denoted $r(t)$ and the system resets to the initial state x_0 after each trial. Assume the system has relative degree one, i.e., $CB \neq 0$, and then the system dynamics can be described in lifted form as $y_k = Pu_k + d$, where P is the system matrix, i.e.,

$$P = \begin{bmatrix} CB & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N-1}B & CA^{N-2}B & \cdots & CB \end{bmatrix}, \tag{3}$$

and the initial conditions are contained in d as defined below

$$d = \begin{bmatrix} CAx_0 & CA^2x_0 & \cdots & CA^Nx_0 \end{bmatrix}^\top. \tag{4}$$

The vectors u_k and $y_k \in \mathbb{R}^N$ are given by

$$u_k = \begin{bmatrix} u_k(0) & u_k(1) & \cdots & u_k(N-1) \end{bmatrix}^\top, \tag{5}$$

$$y_k = \begin{bmatrix} y_k(1) & y_k(2) & \cdots & y_k(N) \end{bmatrix}^\top. \tag{6}$$

Further, assume, without loss of generality, the initial state $x_0 = 0$, then we have $d = 0$.

The problem is to design u_{k+1} using previous trial information $\{u_k, y_k\}$ and the reference r (defined analogously to (6)), such that the tracking error $e_{k+1} = r - y_{k+1}$ is as small as possible – ideally as the learning progresses the tracking error converges to zero.

There are several ILC designs in the literature; a well established scheme is norm-optimal iterative learning control (NO-ILC) which is a model-based ILC design which solves the optimisation problem below [1]

$$u_{k+1} = \arg\min_u \{\lambda||e_{k+1}||_2^2 + ||u - u_k||_2^2\}, \tag{7}$$

where $\lambda > 0$ is a weighting parameter used to balance fast learning and robustness. The analytic solution to (7) is given by

$$u_{k+1} = u_k + \lambda(I + \lambda P^\top P)^{-1}P^\top e_k. \tag{8}$$

With access to the model P, this design achieves monotonic convergence of the tracking error to zero in the case of *fixed* reference r [1], but not a *trial-varying* reference r_k.

III. A Modular Based ILC Design

To introduce the modular based design, enforce upon u_k the modular structure shown in Figure 1, i.e., let it be described by a linear combination of M modules as

$$u_k = \sum_{i=1}^{M} \theta_{i,k}\phi_i r_k = C_{ff_k} r_k, \tag{9}$$

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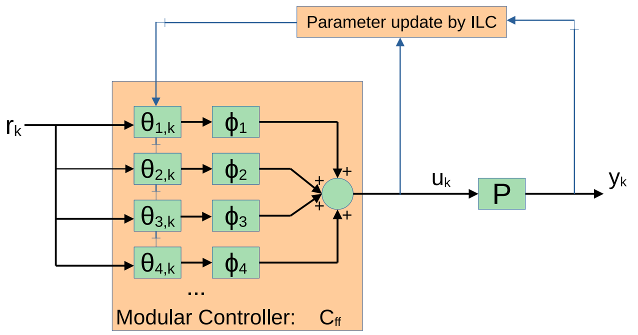

Fig. 1. Modular feedforward controller comprising M feedforward blocks.

where $\{\theta_{i,k}\}_{i=1}^{M}$ are parameters being estimated, and are collected into the vector $\theta_k = \begin{bmatrix}\theta_{1,k} & \theta_{2,k} & \cdots & \theta_{M,k}\end{bmatrix}^\top$. The modules ϕ_i are $N \times N$ matrices and the optimal controller is $C_{ff}^{*} = P^{-1}$ since this yields $y_k = r_k, \forall r_k \in \mathbb{R}^N$.

Our design minimises the following performance index

$$\theta_{k+1} = \arg\min_{\theta} \{\lambda_1 ||r_{k+1} - P\phi_{r_{k+1}}\theta||_2^2 + \lambda_2 ||u_k - \phi_{y_k}\theta||_2^2 + ||\theta - \theta_k||_2^2\}, \quad (10)$$

where $\lambda_1, \lambda_2 > 0$ are the weighting parameters and $\{\phi_{y_k}, \phi_{r_{k+1}}\}$ are the $N \times M$ matrices defined by

$$\phi_{y_k} = \begin{bmatrix}\phi_1 y_k & \phi_2 y_k & \cdots & \phi_M y_k\end{bmatrix}, \quad (11)$$

$$\phi_{r_{k+1}} = \begin{bmatrix}\phi_1 r_{k+1} & \phi_2 r_{k+1} & \cdots & \phi_M r_{k+1}\end{bmatrix}, \quad (12)$$

such that the parameters θ_{k+1} are estimated iteratively in order for *i)* the tracking error for the next trial to be small and *ii)* $C_{ff_{k+1}}$ approaches C_{ff}^{*}. The optimal updated estimate is given by the following proposition:

Proposition 1: The control updating law for the proposed modular-based ILC design is given by (9) with θ_{k+1} given by

$$\theta_{k+1} = \theta_k + (I + \lambda_1 \phi_{r_{k+1}}^\top P^\top P \phi_{r_{k+1}} + \lambda_2 \phi_{y_k}^\top \phi_{y_k})^{-1} \{\lambda_1 \phi_{r_{k+1}}^\top P^\top (r_{k+1} - P\phi_{r_{k+1}}\theta_k) + \lambda_2 \phi_{y_k}^\top \epsilon_k\}, \quad (13)$$

in which $\epsilon_k = u_k - \phi_{y_k}\theta_k$ is a measure of the model error. The proof of Proposition 1 is omitted here for brevity.

Remark 1: The introduction of the term $\lambda_2 ||u_k - \phi_{y_k}\theta||_2^2$, when compared to existing work [5], has the benefit of promoting parameters θ_{k+1} that match the measured data ϕ_{y_k}. As such, $C_{ff_{k+1}}$ is an improved estimate of the true dynamics, enabling the algorithm to achieve accurate tracking on varying references even when the *a priori* estimate of P used in (13) is inaccurate.

Remark 2: By parametrising the input as in (9), the reference r_{k+1} is also permitted to vary in length unlike existing schemes (e.g. [4]) which require fixed trial lengths.

IV. Numerical Example

The algorithm (13) is applied to the test system with

$$A = \begin{bmatrix}0.5 & 0.3\\ 0.4 & 0.2\end{bmatrix}, \quad B = \begin{bmatrix}1\\ 0\end{bmatrix}, \quad C = \begin{bmatrix}1 & 1\end{bmatrix}, \quad (14)$$

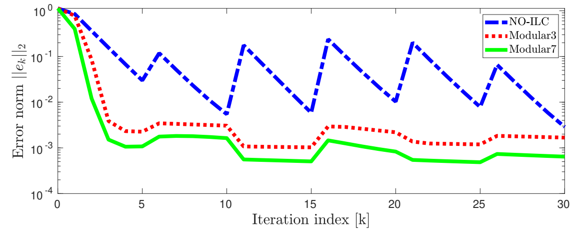

Fig. 2. Algorithms convergence history.

and with modules $\{\phi_i\}$ corresponding with the first M Laguerre polynomials. The cases $M = 3$ and $M = 7$ are labelled as "Modular3" and "Modular7" respectively. The error norms for NO-ILC and the proposed algorithm are shown in Figure 2. The proposed algorithm (with $\{\lambda_1, \lambda_2\} = \{0.1, 10\}$) is initialised with $\theta_0 = 0$ and outperforms NO-ILC (with $\lambda = 0.1$) when sequentially tracking four sinusoidal references. The learning is accelerated because (10) utilises *two* sources of information; the term with λ_1 uses an *a priori* estimate of the model P, whereas the term with λ_2 uses measured data. The optimal choice of $\{\lambda_1, \lambda_2\}$ is expected to be related to the uncertainty in the estimate of P and the uncertainty in the measurements of u_k and y_k.

Additionally, the performance of NO-ILC is severely degraded on trials $k = 5, 10, 15, \ldots$ i.e. when the reference switches. The error norm returns to approximately pre-learning levels and the new optimal control sequence is slowly approached. By comparison, the modular approaches *continue* to learn when the reference changes and therefore maintain high accuracy.

V. Conclusions and Future Work

This paper presents an ILC controller design composed of a linear combination of modules, which matches the structure of high-performance biological controllers and significantly outperforms NO-ILC when the reference changes.

Of future interest is the rigorous proof of the convergence properties of the proposed algorithm, along with the design – or online learning – of appropriate sets of modules $\{\phi_1, \phi_2, \cdots\}$ to efficiently represent the desired tasks. An additional direction for future work is to choose $\{\lambda_1, \lambda_2\}$ optimally to balance model uncertainty and measurement uncertainty.

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