The tritium burn-fraction in DT fusion

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The fraction of the tritium that is burned during one pass through a DT fusion system, f_{tb}, is a central issue for success of fusion energy. Reducing the tritium fraction, f_t, in a DT burning plasma below a half increases the burn fraction, $f_{tb} \propto 1/f_t$ but also the required confinement to achieve a burn $nT\tau_E \propto 1/f_t(1 - f_t)$. A doubling of the fractional burn entails only a 4/3 enhancement of the required $nT\tau_E$. The energy confinement time τ_E in tokamaks and stellarators is empirically gyro-Bohm with an approximate factor of two between the best and worse results used to construct scaling laws. Gyro-Bohm is also the approximate level of transport needed in a power plant. What has received little study are τ_t/τ_E, the ratio of the tritium to the energy confinement time, and τ_α/τ_E, the ratio of the alpha particle to the energy confinement time. The tritium burn fraction is proportional to τ_t/τ_E, so the larger the better. The contamination of the plasma by helium ash is proportional to τ_α/τ_E, so the smaller the better.

The fraction of the tritium that is burned during one pass through a DT fusion system is a central issue for success of fusion energy [1, 2]. This note gives a simple calculation of the burn-up fraction, simplified even from [3], and of the alpha ash accumulation.

The burn-time of the tritium τ_{tb} is defined by

$$\frac{dn_t}{dt} = -n_t n_d \langle \sigma v \rangle_{dt}, \text{ so} \tag{1}$$

$$n_t \propto e^{-t/\tau_{tb}} \text{ with} \tag{2}$$

$$\tau_{tb} \equiv \frac{1}{n_d \langle \sigma v \rangle_{dt}}. \tag{3}$$

The quantity $\langle \sigma v \rangle_{dt}$, which has units of meters-cubed divided by time, gives the probability that Maxwellian deuterium and tritium ions will fuse as a function of their temperature.

The fraction of the tritium burned before it is swept out of the plasma on the tritium confinement time τ_t is

$$f_{tb} \equiv 1 - e^{-\tau_t/\tau_{tb}} \approx \frac{\tau_t}{\tau_{tb}}. \tag{4}$$

since the tritium-burn fraction f_{tb} is much less than unity. The fractional burn relation, $f_{tb} = \tau_t/\tau_{tb}$, will be treated as if it is exact.

The power density of the plasma heating by the alpha particles p_α and the energy confinement time τ_E required to maintain a burning plasma are

$$p_\alpha = \mathcal{E}_\alpha n_t n_d \langle \sigma v \rangle_{dt}; \tag{5}$$

$$\tau_E \equiv \frac{3nk_B T}{\mathcal{E}_\alpha n_t n_d \langle \sigma v \rangle_{dt}}, \tag{6}$$

where $\mathcal{E}_\alpha \approx 3.5$ Mev is the energy release in an alpha particle per reaction, $n = n_d + n_t$ and T is the plasma temperature. The factor $3nk_B T$ is the thermal energy of the electrons plus the ions, where k_B is the Boltzmann constant.

The ratio of the required energy confinement time to the burn-time of the tritium is then

$$\frac{\tau_E}{\tau_{tb}} = \frac{3k_B T}{\mathcal{E}_\alpha f_t}, \text{ where } f_t \equiv \frac{n_t}{n} \tag{7}$$

is the fraction of the ions that are tritium. The fraction of the tritium burned is then

$$f_{tb} = \frac{3k_B T}{\mathcal{E}_\alpha} \frac{1}{f_t} \frac{\tau_t}{\tau_E}. \tag{8}$$

The smallness of the tritium-burn fraction is primarily due to $3k_B T/\mathcal{E}_\alpha$ being of order one percent.

The tritium-burn fraction can be made arbitrarily large by making the tritium fraction small at the cost of making the required energy-confinement time longer;

$$n\tau T \equiv \frac{3}{f_t(1 - f_t)} \frac{k_B T^2}{\mathcal{E}_\alpha \langle \sigma v \rangle_{dt}} \tag{9}$$

$$= \frac{\left(n\tau T\right)_{min}}{4f_t(1 - f_t)}, \tag{10}$$

where $(n\tau T)_{min}$ is the minimum-confinement requirement for achieving a burn. Halving the tritium fraction and doubling its burn fraction requires only a factor of 4/3 enhancement of $(n\tau T)_{min}$; quadrupling the burn fraction requires a factor of $16/7 \approx 2.29$ enhancement.

The helium ash from the alpha particles will degrade the fusion properties of the plasma unless the confinement time of alpha particles τ_α is only moderately longer than the energy confinement time. Since

an alpha particle is created for each tritium that is burned, $dn_\alpha/dt = -dn_t/dt$. Equations (1) and (6) imply

$$\frac{n_\alpha}{n} = \frac{n_t n_d}{n} \langle \sigma v \rangle_{dt} \tau_\alpha \qquad (11)$$

$$= \frac{3 k_B T}{\mathcal{E}_\alpha} \frac{\tau_\alpha}{\tau_E}. \qquad (12)$$

The contribution of alpha particles to the energy density is $(9/2) n_\alpha k_B T$ and the contribution to the pressure is $3 n_\alpha k_B T$ since each alpha adds three particles: one ion and two electrons. Bremsstrahlung is proportional to $Z_{eff} \equiv \sum n_Z Z^2 / n_e$, so the enhancement of the helium ash of the bremsstrahlung power loss is four times higher that its contribution to the number density of ions n_α.

It is important that the fractional burn f_{tb} be large and the alpha dilution n_α/n be small. The relation

$$f_{tb} = \frac{\tau_t}{\tau_\alpha} \frac{n_\alpha/n}{f_t} \qquad (13)$$

implies a reduced tritium faction gives a larger fractional burn at the maximum tolerable dilution.

The energy confinement time τ_E in tokamaks and stellarators is empirically gyro-Bohm [4] with an approximate factor of two between the best and worse results used to construct scaling laws; see Figure 4 in [5].

Gyro-Bohm is also the approximate level of energy transport needed in a power plant [4]. However, the Greenwald density limit requires tokamaks to operate at a much higher temperature [6] than stellarator power plants to achieve an adequate power density, which because of the temperature scaling of $\langle \sigma v \rangle_{dt}$ and of gyro-Bohm transport requires a large reduction in the transport relative to gyro-Bohm [7].

An energy confinement time τ_E that is too short can be compensated by increasing the magnetic field and plasma size. However, a τ_t/τ_E, the ratio of the tritium to the energy confinement time, that is too short or a τ_α/τ_E, the ratio of the alpha particle to the energy confinement time, that is too long are probably more difficult to modify. It is important that the physics that determines these ratios be studied both theoretically and empirically.

Acknowledgements

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