Gaussian elimination pivoting method

- find the row with the largest value in each element

$\longrightarrow k = 0, \ldots, N-1$

$\downarrow$ m rows

$$\begin{matrix} A_{00} & A_{01} & A_{02} & A_{0N-1} \\ A_{10} & & & \\ A_{20} & & & \\ A_{N-10} & & & \end{matrix}$$

if current row $\neq$ largest row, then replace m_{max}

→ swap rows so m_{max}^{th} row $= k^{th}$ row

e.g. for $k=0$ (1st step)

$$\begin{pmatrix} A_{00} \\ A_{10} \\ A_{20} \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \Rightarrow M_{max} = 2$$

$$\therefore \begin{pmatrix} A_{20} & A_{21} & A_{22} & A_{2N-1} \\ A_{10} & A_{11} & A_{12} & A_{1N-1} \\ A_{00} & A_{01} & A_{02} & A_{0N-1} \end{pmatrix}$$

so have to swap entire rows

$A_{M_{max}c} = A_{kc}$ for $c = 0, \ldots, N-1$

for rows below (1st row) $\therefore > k$

→ need to perform row operation after swapping

after swapping & taking away a multiple of the row from each row below it, you leave the 1st row alone when computing subsequent ones

$\therefore$ for $r = k+1, \ldots, N-1$

$$\lambda_r = \frac{A_{rk}}{A_{kk}}$$

& for that we need to take away a λ_r multiple for $c = 0, \ldots, N-1$ (each column)

$$A_{rc} = A_{rc} - \lambda_r A_{rc}$$

& for $k = 1$

$\Rightarrow r = 2$ $\therefore$ just the last row

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} u_0 \\ u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix}$$

if a_{10}, a_{20} & $a_{21} = 0$ then

$$u_2 = \frac{b_2}{a_{22}}$$

Then $a_{11} u_1 + a_{12} u_2 = b_1$

$$\Rightarrow u_1 = \frac{b_1 - a_{12} u_2}{a_{11}}$$

$$u_0 = \frac{b_0 - (a_{02} u_2 + a_{01} u_1)}{a_{00}}$$

for $i = N-1, \ldots, 0$

$$u_i = \left(b_i - \sum_{j=i+1}^{N-1} a_{ij} u_j\right) / a_{ii}$$