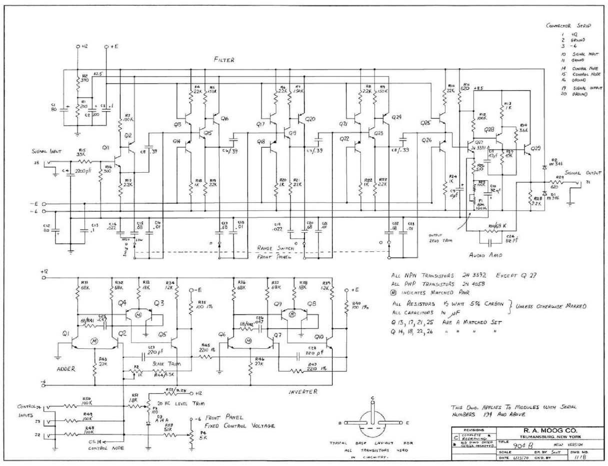
A STUDY ON ROBERT MOOG'S 904B HIGH PASS FILTER

DIEGO DI CARLO, MATTIA PATERNA



Emulation, analysis and discrete-time implementation of the analog circuit

Msc in Sound and Music Computing

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I happen to think that computers are the most important thing to happen to musicians since the invention of cat-gut which was a long time ago.

My training as an engineer has enabled me to design the stuff, but the reason I do it is not to make music but for the opportunity to work with musicians.

Robert Arthur Moog

1934 – 2005

We wish to thank Stefano d'Angelo for his help and useful suggestion, and for have let us do this challenging project.

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ACRONYMS

DSP	Digital Signal Processing
RC	Resistor-Capacitor (circuit, or filter)
FCV	Fixed Control Voltage
TPT	Topology-Preserving Transform
DC	Direct Current
AC	Alternate Current
HP	High Pass (filter)
LP	Low Pass (filter)

Part I

INTRODUCTION

In this section, a short description of the *904B* Moog high pass filter is given, both from a theoretical and a musical point of view

1 INTRODUCTION

With this project, we've aimed to analyse the original analog circuit of the Moog 904B highpass filter, extract some useful and interesting information from it and try to replicate it in the discrete-time using the *virtual analog* emulation technique.

This project has started from the suggestion by Stefano D'Angelo, DSP Research Engineer at Arturia, a French software company specialized in recreating legendary analogue synthesizers in digital format[1]. This company has been developing for 2003 Moog modular V, a virtual analog emulation of Moog Modular, formerly one of the most famous analog synthesizer developed and built by Robert Moog in 1964.

1.1 MOOG MODULAR AND THE 904B HIGH PASS FILTER MODULE

The Moog Modular system consists of a number of various modules mounted in a cabinet. Each module performs a specific signal-generating or modifying function, such as sound waveforms, envelopes generators, filters and spectral modulators.

The 904B high pass filter module is a voltage controlled filter (VCF) with a high pass behaviour. It comes as a single module, but it is usually coupled with the other filter of the Moog 904 Series: the 904A Low Pass Filter (LPF) and 904C Coupler Filter, which allows for band pass filtering due to the conjunction of the frequency response of both the two single filters.

The 904B module features a 4-pole voltage controlled high pass filter, giving an attenuation of 24 dB/oct (80 db/dec) for the part of a signal below a specific cutoff frequency, that is commonly referred as ω. That action of the attenuation given by a specific filter is also named *rolloff*. In this kind of circuit, the cutoff frequency is modulated by the control signal E, which could be any complex tone generated by other Moog's modules. The FCV cutoff point is raised or lowered in octave per volt control inputs.

The Moog VCF 904B has also a frequency range switch, that simply sets the frequency band over which the FCV operates. The *Low* frequency range acts between 4 Hz and 20 KHz, while the *High* range encompasses 10 Hz to 50 KHz. That is, the High shifts 1 and 1/2 octaves up the previous range. In the figure, they are referred as *range switch*.

CIRCUIT SPECIFICS As explained in the US Patent $3,475,623$ (fig. 1, possibly the most famous music patent ever, the high pass filter contains these main sections, electronically speaking:

- an *adder*, that is *per se* an operational amplifier whose output is proportional to the sum of the input voltages plus the voltage across a diode which defines the operating point for the filter;
- an *inverter* which comprises a unity gain operational amplifier and basically does the opposite of the adder;

1 https://www.arturia.com/company/history

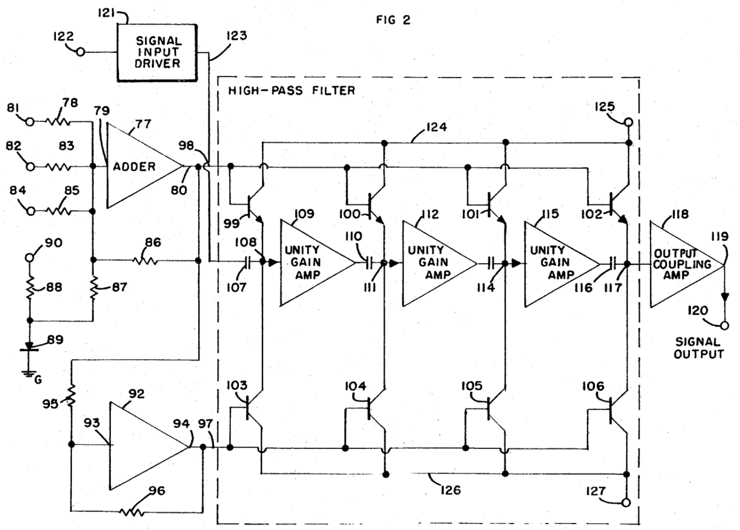

Figure 1: Moog 904B block diagram, US patent by Robert A. Moog, 1969.

- four identical stages each comprising a pair of transistors, a capacitor and a unity gain amplifier. Transistors of a given pair are complementary, one PNP and the other NPN. The base of all the NPN transistors are connected together and to the output of the adder, while the base of all the PNP transistors are connected together and to the output of the inverter.

As show in fig. 2, the original circuit consists in two different parts: the lower half of the circuit, that is the control voltage mixer and a following inverter, and the upper half, that is formerly the 4-pole voltage controlled high pass filter, so that the latter can be referred as the *audio* part of the circuit while the former is named as the *control* part.

The audio part consists of the following:

- An attenuation stage of almost -40 dB as a voltage divider (R16 and R15);

- 4 filter stages that are passed in series. Each stage consists of a complementary *Darlington* cell (or Sziklay pair, Q11 and Q12) and a complementary amplifier (e.g. Q13-Q14), both of which work as a voltage controlled resistor and yield to a high pass filter behaviour in series with a capacitors set. Two of the capacitors can be decoupled from the circuit by the frequency range switch;

- an output amplifier (named in the circuit *Audio Amp*) that recreates the amplitude of the signal to the original value. It has therefore an amplification factor of 42 dB. The gain of the amplifier is set so that, for frequencies above the cutoff frequency of the filter, the net gain from signal input to signal output is unity. The complete module has an overall amplification of factor 2.

The breakthrough of Moog's work has been to overcome some significant problems arising when dealing with a musical application of filters. First, voltage-tuning a filter over a wide range is a difficult electrical engineering problem. Second, it is required for the cutoff frequency to be fixed in the

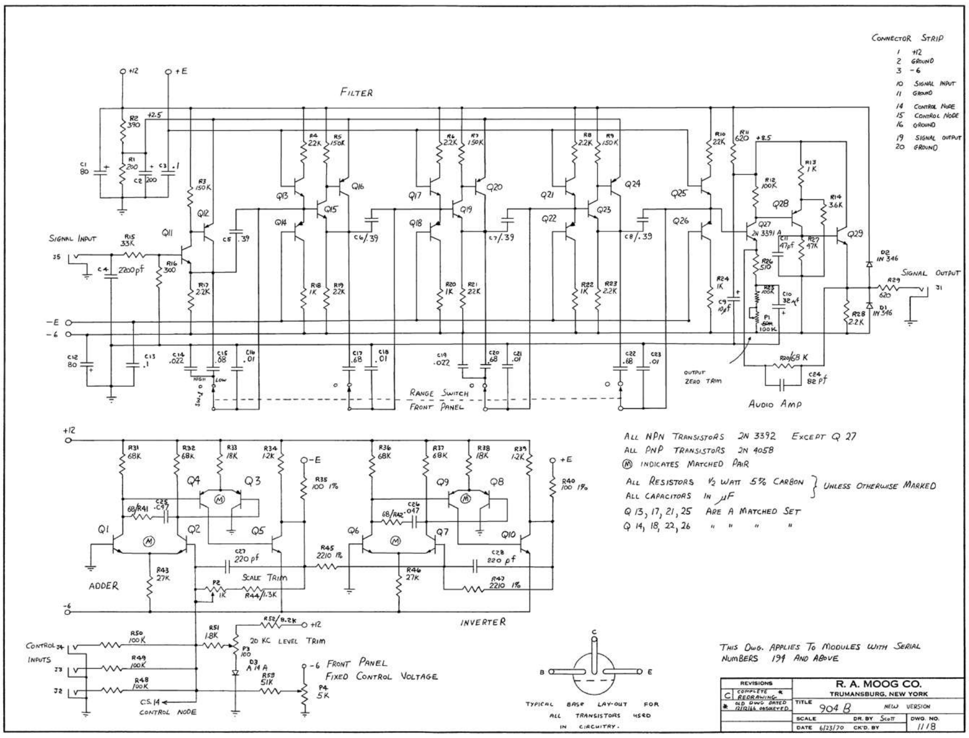

Figure 2: Moog 904B original schematics by Robert A. Moog.

presence of control which causes the filter to peak and even oscillate. In other words, the resonance should be adjustable without affecting the tuned frequency, whereas the resonant frequency were proportional to the square root of the amplifier gain in other popular VCF. The 4-stage design address perfectly that kind of problem. Third, The cutoff frequency is proportional to the exponential of the sum of the input control voltages applied to the adder. That allows for a more musical *meaning*, since the human hearing works on a logarithmic scale.

MUSICAL APPLICATION As the well-known Moog *ladder* low pass filter, which contributed to create the *mood* and the legendary sound of the Moog Modular, much appreciated among all musicians ranging from pop (e.g. The Beatles) and classical music (e.g. Wendy Carlos and her seminal work *Switched-on Bach*), the voltage controlled high pass filter can be used in altering any timbre of an input signal by deleting the predominance of the fundamental partial in a complex tone[2].

The voltage control of this module creates a spectral sweep that is dramatically different from those associated to acoustic instruments. For instance, high pass filter with slow control voltage and white noise as input signal provides the basis for some kind of *crispy* cymbals and snare drum sound which is constantly changing.

What makes the Moog filter sound special has been the subject of many academic studies and much speculation. The filter can certainly be overdriven in a musically pleasing way, as all the transistor stages clip gradually.

2 http://moogarchives.com/m904b.htm

Moreover, that 24 dB/oct filter slope resonant sound contributed to define the peculiarities and the essence of the Moog sound[3].

3 http://www.uaudio.com/blog/moog-ladder-filter/

Part II

ANALYSIS AND IMPLEMENTATION

This part deals with the derivation of the function in Laplace domain, the choice of integration of the RC circuit and its digital implementation

2 FILTER ANALYSIS

2.1 PRELIMINARIES

Once we get the more truthful schematics, we started to spice the circuit (Figure at the bottom of the next page) Lot of time was necessary to search for the right model for the vintage components, such as old transistors (2n3392 and 2n4058) and diodes (1n346). We performed some basic analysis such as check the independence of every amplification stages, that is each pole of the filter and the audio amplification part.

2.2 CIRCUIT ANALYSIS

As explained in the 1, the high-pass filter consists of an adder and inverter portion for control voltage processing, and a filter portion for signal voltage processing. Figure 3 shows the SPICE model version of only the audio part of the original circuit. It consist in a attenuation stage, the four 1-pole HPF unit, each one following the Sziklai pair and an output amplifier as last stage.

Figure 4 depicts a detail of the the Higi Pass Filter module. Each filter stage consists of capacitor set (C_5, C_{14}, C_{15} and C_{16}) in series with the two dynamic resistances of the two junction diodes of the two transistors (Q_{13} and Q_{14}), that is the voltage controlled resistors. Moreover Figure 5 shows the output amplification stage which has not been analysed and implemented yet.

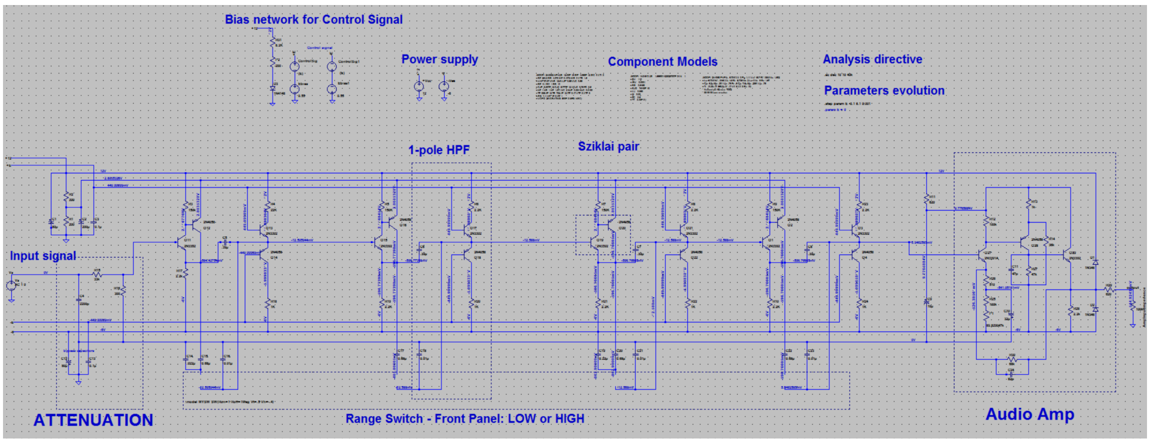

Figure 3: SPICE model version of only the audio part of the circuit subtitled with "DATE 6/23/70 DWG. NO 1118".

DC ANALYSIS Performing an *DC operating point* analysis in SPICE, we have been able to figure out the voltage and current variable involved, especially around the Sziklai pairs. With some basic analysis tool provided by the simulation application, the independence of each filter stage have been shown: thinking at the input of each stage as a voltage divider, at the worst

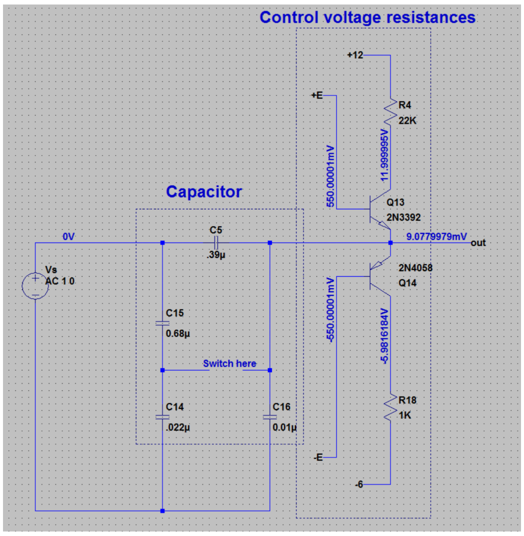

Figure 4: SPICE model version of only one high pass module.

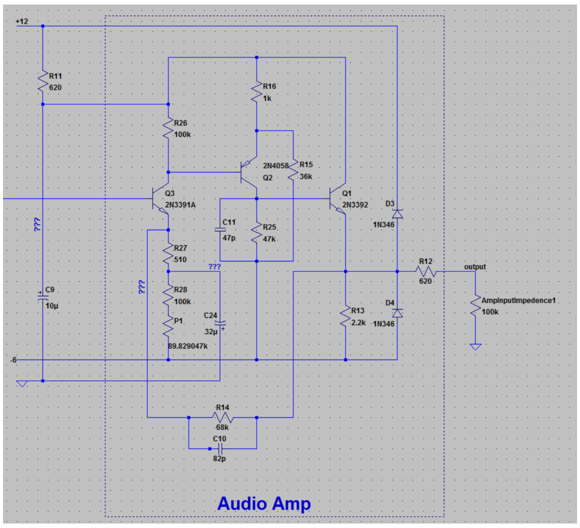

Figure 5: SPICE model version of only the audio part.

case the output voltage V_o is 0.991 of the input voltage V_i, that is a amplitude attenuation of $0.03\,\text{dB}$, a good approximation for Real-time virtual analog. This is true for only the first three stage of the filter, in fact the last one is affected by a low input frequency-variant impedance of the output stage. Figure 6 shows the transfer functions of each HPF stage varying the impedance of a load resistance. It can be seen that only the last HPF is affected. In this first step, the voltage and the current values have been stored and studied in order to understand the operating point of each transistor, that is their function mode and their internal small-signal parameters.

AC ANALYSIS Performing some trivial *ac analysis*, we began to plot some *raw transfer* functions of every blocks of the circuit (Figure 7, 8, 9)

Thanks to these plots, we have been able to debug the circuit, highlight some non linear behaviours and understand its evolution with respect to different value of the control signal E. Again from these plots, we extracted

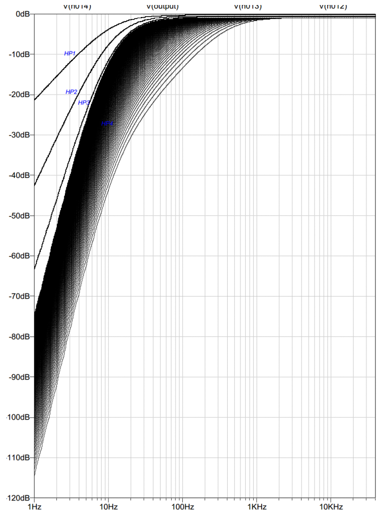

Figure 6: Magnitude of the The transfer function for each of the HPF stage varing the input impedance of the final ampification stage

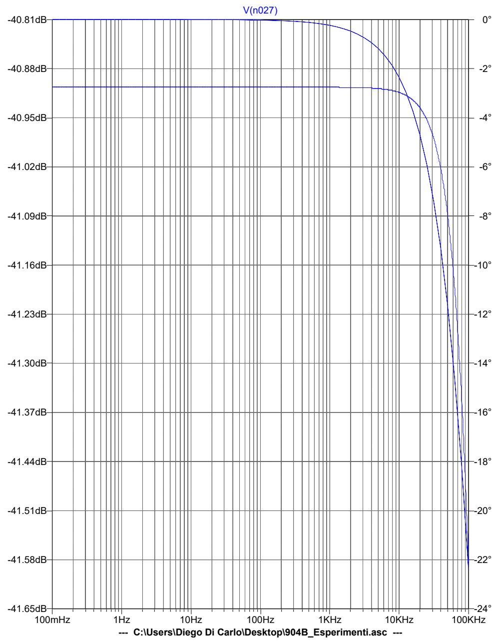

Figure 7: The transfer function (magnitude and phase) of the *attenuation stage*

the frequency response of every blocks and implemented a first black box model of our circuits in MATLAB.

2.3 LAPLACE ANALYSIS

SINGLE STAGE TRANSFER FUNCTION In order to simplify calculations in the frequency and Laplace domains, variable (sinusoidal) voltage and current waves are commonly represented as complex-valued functions of Laplace's variable s denoted as $V(s)$ and $I(s)$. More complete and deep

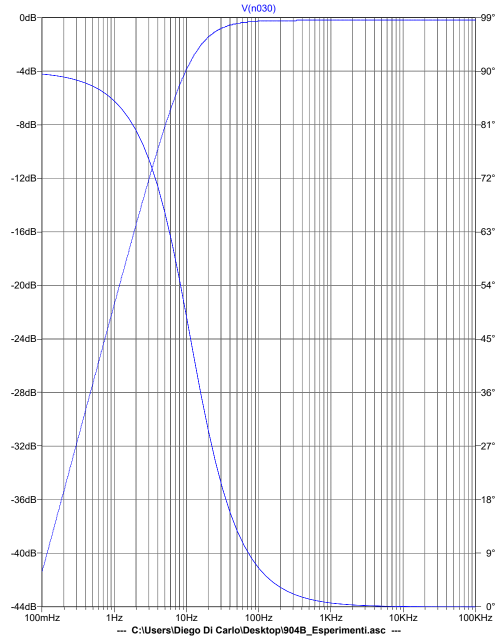

Figure 8: The transfer function (magnitude and phase) of a one *single-pole HPF stage*

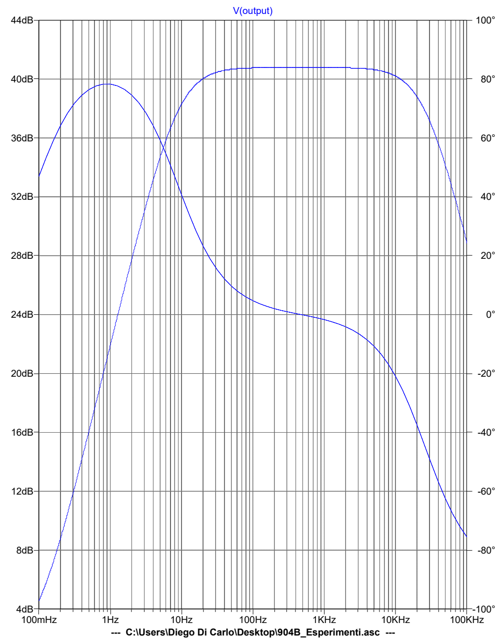

Figure 9: The transfer function (magnitude and phase) of the *output amplifier stage*

theory related to this can be found in [1]. This notation led to a generalization of the *Ohm's law*:

$$V(s) = \dot{Z}(s)I(s) \tag{1}$$

Thus, the quantity $\dot{Z}(s) = \frac{V(s)}{I(s)}$ represents the *impedance* of the circuit element. It assumes values in $\mathbb{C}$ and in *Cartisian form* is defined as

$$\dot{Z}(s) = R(s) + jX(s) \tag{2}$$

where the real part of impedance is the *resistance* $R(s)$ and the imaginary part is the *reactance* $X(s)$. The convenience of this notation is that now the dynamic behaviour of an electrical network with non linear components, such as capacitors and inductors, can be studied as a network of series and parallel of complex impedences using only the Ohm's law, neglecting integration and derivative relationships. The resistor is a linear time-invariant and frequency-constant, that is $Z_R(s) = R \forall s \in \mathbb{C}$. Therefore for the capacitors follows that:

$$i(t) = C\frac{dv(t)}{dt} \tag{3}$$

where C is the capacitance of the capacitor. Now, solving for voltage:

$$v(t) = \frac{1}{C}\int_0^\infty i(t)dt \tag{4}$$

Then, transforming this equation into the Laplace domain, we get the following:

$$V(s) = \frac{1}{C}\frac{1}{s}I(s) \tag{5}$$

Again, it can be expressed in the form of Ohm's law, that is the following ratio:

$$\dot{Z}_C(s) = \frac{V(s)}{I(s)} = \frac{1}{sC}I(s) \tag{6}$$

As it show in figure 10 The RC circuit can be seen as a voltage divider of complex impedences:

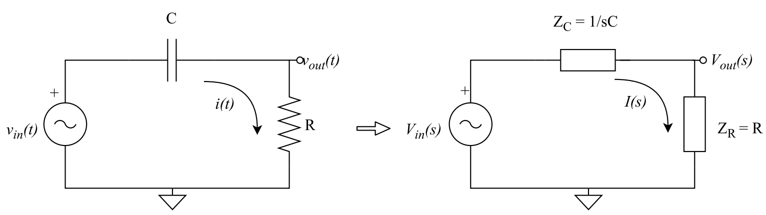

Figure 10: RC circuit as a voltage divider with complex impedences in Laplace domain

$$V_{out}(s) = \frac{\dot{Z_C}}{\dot{Z_R} + \dot{Z_C}}V_{in}(s) = \frac{R}{R + \frac{1}{Cs}}V_{in}(s) \tag{7}$$

Rearranging the equation we obtain:

$$V_{out}(s) = \frac{s}{s + \frac{1}{RC}} V_{in}(s) \tag{8}$$

which is the well-know relation for an first order high pass filter (HPF) with cut-off frequency $\omega_c = 1/RC$

TOTAL TRANSFER FUNCTIONS Since each stage is a single-pole HPF and their are independent thanks to the Sziklay-pair unit buffers, it naturally follows that the Laplace transfer function of the i-th stage is:

$$H_i(s) = \frac{V_{i,out}(s)}{V_{i,in}(s)} = \frac{s}{s + \omega_c} \tag{9}$$

where $\omega_c = 1/RC$ is the cut-off frequency, R is the dynamic resistance of the two emitter diodes of the transistors in parallel and C the equivalent capacitor of the capacitor set. So the overall transfer function $H_{HPF}(s)$ is:

$$\begin{aligned} H_{HPF}(s) &= H_1(s)H_2(s)H_3(s)_4H(s) \\ &= \frac{s^4}{s^4+4\omega_c s^3+6\omega_c^2 s^2+4\omega_c^3 s+\omega_c^4} \end{aligned} \tag{10}$$

R AND C PARAMETERS DERIVATION As the final part of this analysis and to complete the analog description of each HPF, the cut-off frequency of the filters must be derived and expressed with respect to the control signals $+E$ and $-E$. As explained in the previous sections, the bipolar junction transistors (BJTs) can act as a voltage controlled resistors, that is they can be seen as a resistors that varies its own resistance according magnitude of a certain input signals. This propriety is know as *dynamic resistance* of the transistors, or *emitter resistance*. However this behaviour is not guaranteed for every operating point, i.e. the current and voltage values forced by the rest of the electrical network. After some SPICE simulation, we were able to demonstrate that the transistors works always in the so-called *forward active* zone, that is as a controlled resistors. Because it is non-linear and dynamic, we can not say that it has a particular resistance, but it depends on the voltage-current values of the component.

Thus, the dynamic resistance of each transistor is computed thanks to the *Ebers-Moll* model for BJT transistor and its physics equations [1], both for the PNP and for the NPN type. According to this model, the NPN BJT transistor can be represented as in figure 11.

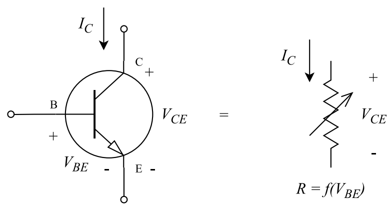

Figure 11: BJT NPN transistor as voltage-controlled resistor. The equivalent resistance R is a non-linear function of the voltage V_{BE}

In order to represent it a voltage controlled potentiometer, we need to evaluate the value of the equivalent resistance as $R = V/I$, which in this context are the values of the collector-emitter voltage $v_{BE}(t)$ and the collector current $i_C(t)$ across the transistor (see figure 12. Thus, basically the resistance value is the value of the slope of the curve $v - i$. That is:

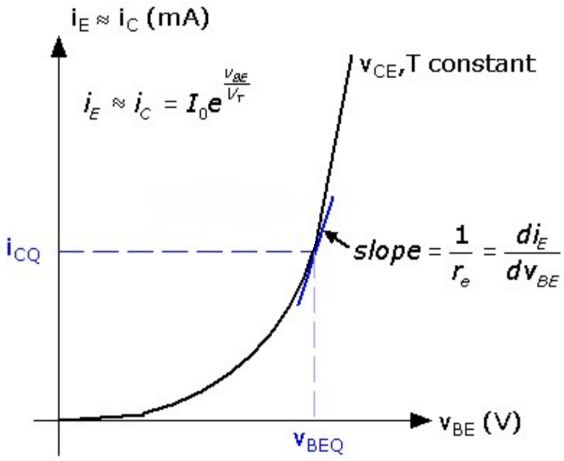

Figure 12: Voltage-current curve and dynamic (emitter) resistance of the transistor in *forward active* region. Picture curtesy of [1]

$$R = \frac{V}{I} \Rightarrow \frac{1}{R} = \frac{I}{V} \tag{11}$$

Thus for non time-constant variables:

$$\frac{1}{R} = \frac{di(t)}{dv(t)} \tag{12}$$

From now on for sake of simplicity we omit the trivial time dependency in the equations. The collector current value in given by the following equation for the transistor:

$$i_C = I_S \cdot e^{\frac{v_{BE}}{V_T}} \left(1 + \frac{v_{CB}}{V_A}\right) \tag{13}$$

where V_A is the constant Early voltage, V_T is the constant thermal voltage, v_{BE} the voltage difference between base and emitter and v_{CB} the voltage difference between collector and base. Since in practical application $v_{CE} \cong v_{CB}$, this formula is often written as

$$i_C = I_S \cdot e^{\frac{v_{BE}}{V_T}} \left(1 + \frac{v_{CE}}{V_A}\right) \tag{14}$$

which is a non-linear function of v_{BE}. In our case, we have that $v_{BE} = +E$. To evaluate the dynamic equivalent resistance of this component, we must compute the derivative.

$$\frac{1}{R} = \frac{\partial i_C}{\partial v_{CE}} = \frac{I_S \cdot e^{\frac{V_{BE}}{V_T}}}{V_A} \tag{15}$$

The previous equation describes the behaviour of the resistance for the NPN BJT, however for the PNP their the same with the only difference that it works with negative voltage values.

As can be seen in the resulting plots, R is inversely proportional to the average diode current, which in turn is proportional to the exponential of the voltage control signal $v_{be} = +E$ for the NPN transistor [*risp.* $-E$ for the dual PNP transistor] (Figure 13). The capacitor values have been derived using SPICE simulation and its analysis tools. These formulas have been implemented in MATLAB for the real-time simulation and the code can be found as appendix.

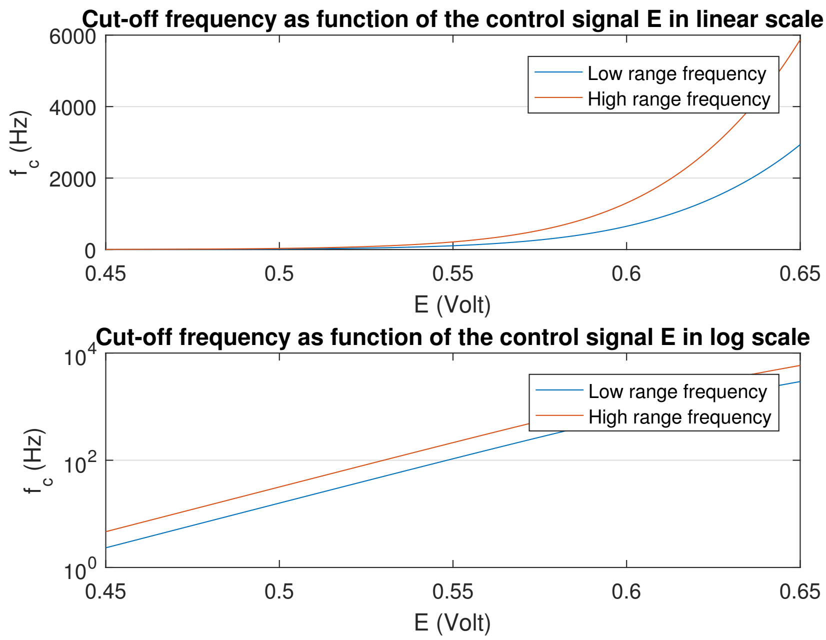

Figure 13: Dynamic resistance of the transistors

3 VA FILTER IMPLEMENTATION

Several implementation methods for the Moog $904B$ filter have been evaluated. In the following chapter, a brief explanation of the 1-pole highpass analog filter, the approach we chose to convert the analog filter model to discrete time and further details on *time-discretization* are proposed.

3.1 ANALOG 1-POLE FILTER

A 1-pole RC highpass filter could be represented using a block diagram form like this:

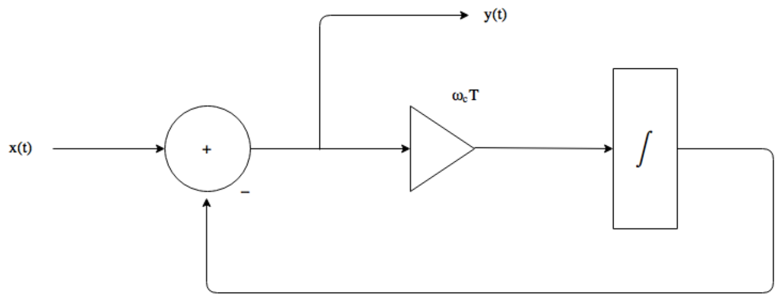

Figure 14: 1-pole RC highpass filter.

where the gain coefficient ω_c expresses $1/RC$ being R and C the resistor and the capacitor of the analog circuit. That is, the *gain element* represented by the triangle multiplies the input signal. Moreover, T represents the sampling rate and, for this explanation's sake, it could be set to 1. It could be also noticed how the summator has inverting input.

Compared to the similar block diagram form of the analog 1-pole RC lowpass filter, the main difference is the position where the output signal $y(n)$ is taken. In the latter, the output of the integrator *is* the output signal, while in the former the output signal is obtained before the integrator.

A STRAIGHTFORWARD NOTATION It could be demonstrated that a complex sinusoid $e^{j\omega t}$ comes out from an integrator as the same signal having a different amplitude, being $1/S$ the scaling factor. In the analysis of this filter for its discretization, the extra DC term coming from the integration could be neglected since we are assuming the filter is *stable* and the initial moment $t_0 \ll 0$. The integrator could therefore assume this form:

$$\int e^{s\tau}\delta\tau = \frac{1}{s}e^{st} \tag{16}$$

where $s = j\omega$ and τ is the analog time variable.

The block diagram form which the integrator is replaced by a gain element with a factor of $1/S$ is shown in fig. 15.

Let us suppose a complex input signal $x(t) = X(s)e^{st}$ and a known otput signal $y(t)$. The input signal for the integrator in fig. 15 is $\omega_c y(t)$

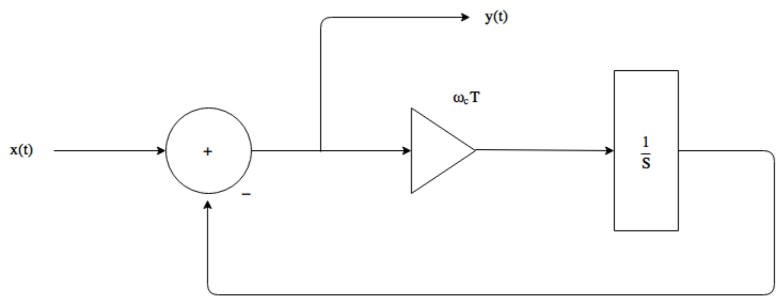

Figure 15: 1-pole RC highpass filter with $1/S$ notation for the integrator.

(assumed $T = 1$). The output signal of the integrator is $\omega_c \frac{y(t)}{s}$ since the integrator simply multiplies the amplitude of the input signal by its gain factor. From this, we can state:

$$y(t) = x(t) - \omega_c \frac{y(t)}{s}$$

or, substituting with the complex signal

$$sY(s) = sX(s) - \omega_c Y(s)$$

or

$$Y(s) = \frac{sX(s)}{s + \omega_c}$$

from where, we can get the *transfer function* for the analog filter:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s}{s + \omega_c} \tag{17}$$

HIGHPASS FILTER SPECIFICS Given the transfer function in eq. 17 and fig. 15, it is possible to define the highpass filter behaviour. As said before, it basically differs in which voltage is picked up as output signal, that is the *resistor* voltage. The amplitude response of such a filter is shown in fig. 16.

This response is effectively a mirrored version of the lowpass filter one. For analog frequencies $\omega \ll \omega_c$ the transfer function is $H(s) = s/\omega_c$. Therefore, when the frequency is dropped by an octave (that is, halved), the amplitude gains drops by approximately $6dB$. That is, the 1-pole highpass filter has a $6dB/oct$, or $20db/dec$ roll-off.

It is also possible to derive the transfer function of the highpass filter from the one of the lowpass filter using their amplitude response symmetry property. [1] Such a transfer function can be obtained by the *LP to HP substitution*:

$$s \leftarrow 1/S$$

1 we are herein referring to the case of 1-pole analog filter

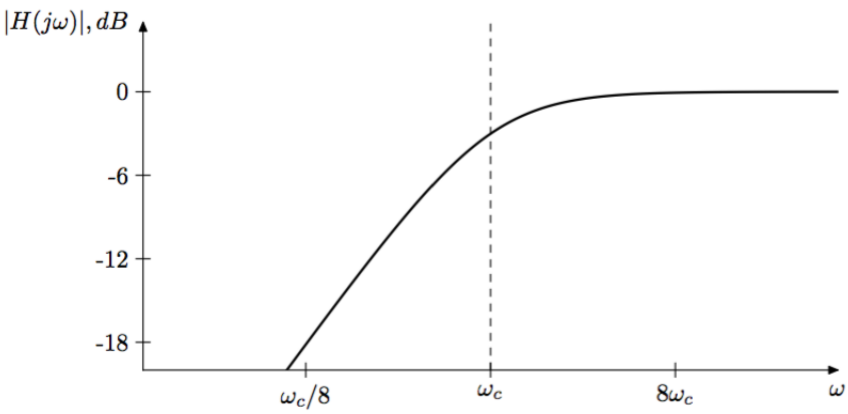

Figure 16: Amplitude response of a 1-pole highpass filter.

The LP to HP substitution could be performed not only algebraically, but also directly on a block diagram. Recalling that the integrator can be expressed as a gain factor of $1/s$, we can directly substitute that gain factor allowing for a circuit where integrator has been replace by *differentiators*, whose transfer function is $H(s) = s$.

3.2 DISCRETE-TIME INTEGRATORS

In order to convert analog filter models to the discrete time, we have to develop a strategy that let us switch from Laplace to Fourier domain, so to yield to an implementation routine.

Doing so, we need to build a digital model of the integrator, that is the filtering stage recalling 3.1. Doing so, several methods have been proposed in the literature and we have decided to come up with two different transfer function, one for the *naive* and the other one for the *trapezoidal* integration, so to demostrate how a digital model could vary depending on the chosen discretization, and consequently also its code implementation.

THE UNIT DELAY The counterpart of a continuous-time integrator is given by *unit delays*. A unit delay basically delays the signal by just one sample. Since we have to keep in memory the last sample of the incoming signal, it is said for a unit delay element to have a *state*. That is, it requires a variable in the implementation. This quality makes the unit delay similar to the analog integrator somewhat. It is convention to denote the unity delay with a box containing a z^{-1} notation.

3.2.1 *Naive integration*

A naive highpass filter approach yields to such a block diagram as shown in fig. 17:

Using this method, we have the first summation input be $x - \omega_c(y + yz^{-1})$, as well as the integrator input is $\omega_c y$ and its output just the second

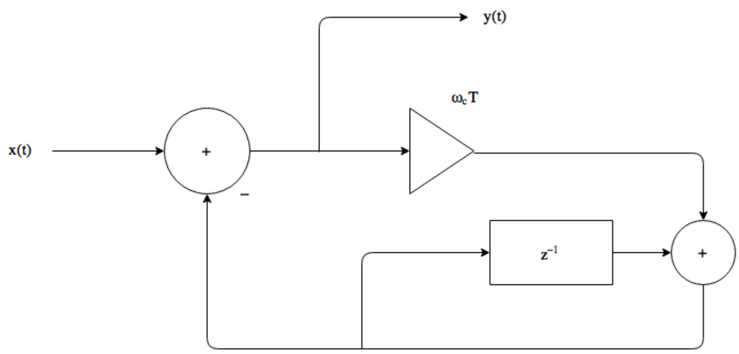

Figure 17: Naive 1-pole highpass filter.

term of the previous addition. From there, we can get the transfer function in z and assuming $T = 1$:

$$H(s) = \frac{1}{(\omega_c + 1) + \omega_c z^{-1}} \tag{18}$$

3.2.2 *Trapezoidal integration, or TPT*

A trapezoidal integrator, particularly its *transposed canonical* form is shown in fig. 18:

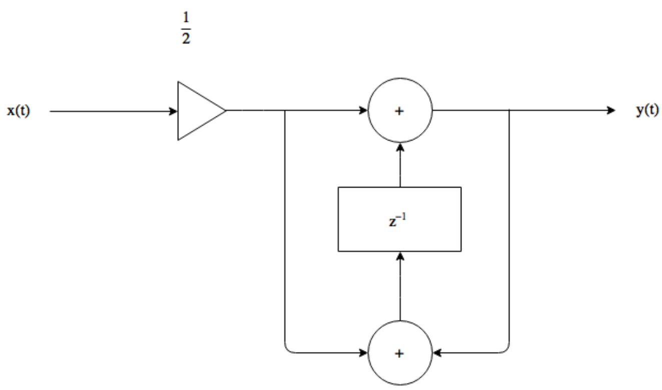

Figure 18: Trapezoidal integration (direct form II, or *transposed canonical*).

As explained by Zavalishin in his *The art of VA filter design*, such an implementation has several advantages compared to the naive one. In particular, it has been demostrated that its transfer function is exactly the *bilinear transform*

$$H(s) = \frac{\omega_c}{2} \frac{1 + z^{-1}}{1 - z^{-1}}$$

so that it could also be named *bilinear integrator*, or BLT integrator. The bilinear trasform allows for a precise mapping of the analog frequency response ω from the range $[0, +\infty)$ into the digital frequency range $[0, \pi fs)$, that is from zero to Nyquist, whithout changing the frequency response at all. In more specific terms, the bilinear transform maps the left complex semiplane in the s-domain into the inner region of the unit circle in the z-domain.

Since the goal of a virtual analog design is to maintain the original structure of the circuit, the trapezoidal integration does a good job. It tries to emulate explicitly the analog integration behaviour, that is preserving its *topology*. So, this replacement technique is also defined as *topology-preserving transform*, or TPT. This is also superior compared to a standard application of the bilinear transform to the 1-pole transfer function.

Finally, whereas a classical bilinear transform approach doesn't preserve the original structure and a naive implementation has some issue on the transfer function replacement, the trapezoidal integration achieves both goals simultaneously and it has been therefore chosen for the code implementation.

3.3 IMPLEMENTATION

Recalling 1, a digital filter for musical purposes should have some qualities, such as the decoupling of the filter cutoff frequency from the resonance parameter and a good strategy to handle with coefficient update at a very update, e.g. on a sample basis. Furthermore, it should be stable enough for parameters inside the allowed range. Finally, it should be capable of self-resonance, if talking of resonance filter. All these characteristics could be found in several analog filter and particularly in the Moog 904 family, of which the ladder filter could be considered the seminal.

PROPOSED MODEL This model consists of four identical 1-pole filter sections and it closely follows the analog structure. The basis for the implementation arises from the work of Zavalishin. He presented in his book *The art of VA filter design* an algorithm for the trapezoidal integration, which contains also all the necessary improvements regarding some possible issues when dealing with such an integration, that is *frequency prewarping* and *zero-dalay feedback* implementation[2]. Moreover, the implementation provided by Will Pirkle, which offers a filter implementation of the ladder filter, has also been examined.

In particular, it has been found out that Pirkle's implementation refers both to Zavalishin and to Välimäki. The latter, together with Huovilanen, proposed an improved version of the Moog ladder filter, starting from the seminal work of Smith and Stilson.

To sum up, in our implementation we have decided to keep the integral structure of the RC circuit as proposed by Zavalishin. It has to be point out how the frequency prewarping is useless in our proposed implementation. In fact, since we aimed to provide a filter cutoff not in terms of frequency, but in terms of *voltage tension* control, it is not necessary deriving the analog frequency from the chosen digital one. Though, once the desired value of E is provided, the analog cutoff frequency is computed using the Ebers-Moll BJT model. Moreover, the frequency range shift has also been implemented, so that the *virtual-analog* cutoff frequency value depends also on the position of that shifter. Finally, the application of the hyperbolic tangent as a *gentle* saturation stage immediately before the sample goes into the first filter stage has been applied, as shown in the previous implementation by Välimäki.

2 for further details concerning the algorithm, please refer to chapter 3 *Time-discretization* in the book mentioned herein

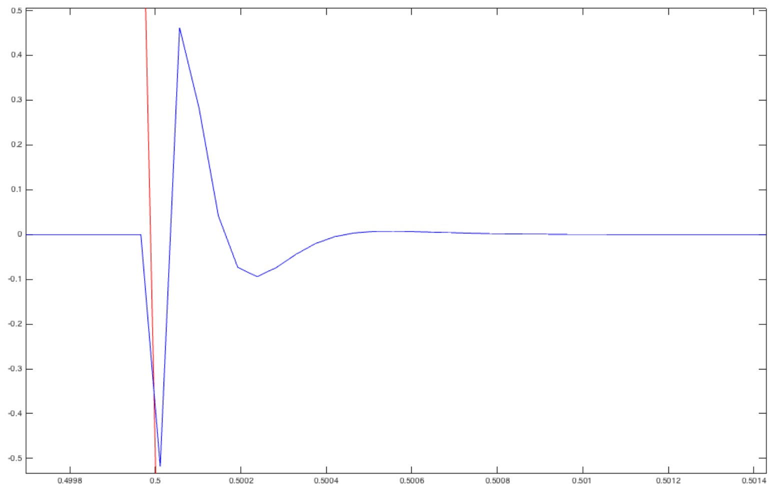

Figure 19: Effect of the discretized 904B on a pulse wave with duty cycle of 0.5. It can be noticed how the filter has a *smoothly* action on the square wave.

The filter evaluation has been conducted with a pulse wave with a duty of 0.5. Some results could be seen in fig. 19 and 20. An evaluation of the frequency range shift has been done, so that we demonstrated that, for a voltage tension value of 0.05, we obtained a frequency value of 1954.2 Hz for the *High* setting and a frequency value of 651.39 Hz for the *Low* value, that is exactly a shifting of $1 + 1/2$ octaves lower, as stated directly by Moog.

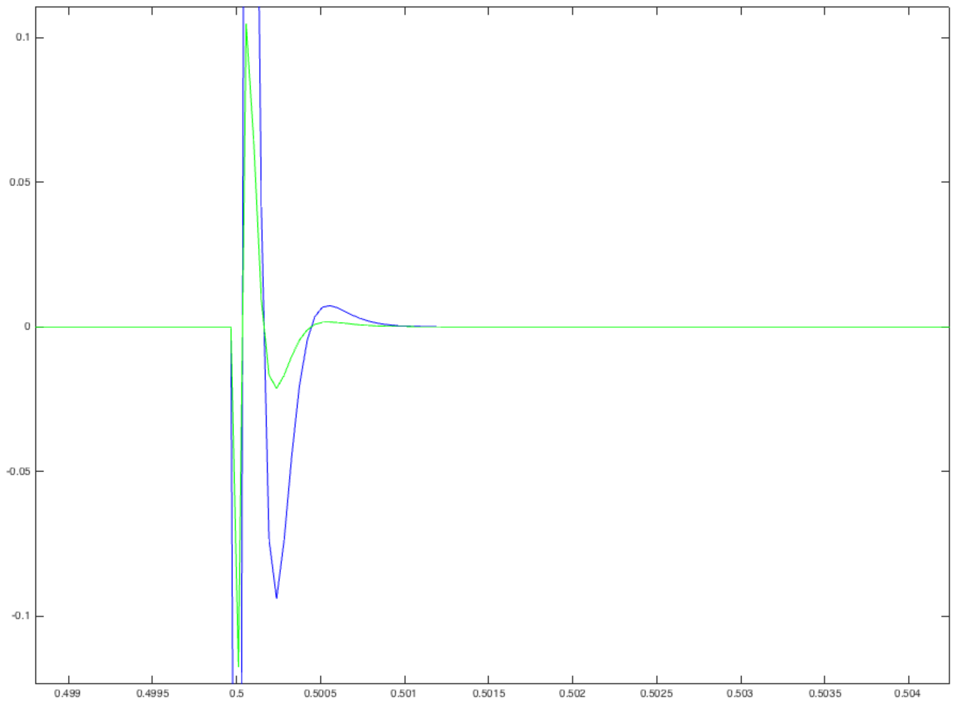

Figure 20: Comparison between our proposed model and the model from Välimäki. For a value of $C_{RES} = 0$ and specific weighting coefficient values, it can be demonstrated how both of them affect in similar way the pulse wave. That is, the $904B$ highpass filter can be considered as a ladder filter without recursion. Nevertheless, those implementation differ somewhat, so that we couldn't obtain the same behaviour.

BIBLIOGRAPHY

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