MSPR 2 Linear Algebra Exercises (Due: Sunday 20.9.2015 12h pm (noon))

Dr. Hendrik Purwins, Assistant Professor
Jan Stian Banas, TA
Dept. Architecture, Design and Media Technology, Aalborg University Copenhagen
A.C. Meyers Vænge 15, DK-2450 Copenhagen SV, Denmark

1. (Feedback) Please give us feedback on the last lecture and homework: `http://goo.gl/forms/sbIm0Vq57K` Thanks!

2. Consider the matrix

$$\mathbf{A} = \begin{bmatrix} -1 & 5 & -1 \\ 5 & 0 & 5 \\ -1 & 5 & -1 \end{bmatrix}$$

   Is it diagonal? Is it symmetric? Is it invertible? (12P)

3. Describe geometrically (line, plane, or all of $\mathbb{R}^3$) all linear combinations of

   a) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$ b) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$ c) $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$ (12 P)

4. Calculate $\mathbf{u} \cdot \mathbf{v}^T$ and $\mathbf{u} \cdot \mathbf{w}^T$ and $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})^T$ and $\mathbf{w} \cdot \mathbf{v}^T$:

$$\mathbf{u} = \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

   Compute the lengths $||\mathbf{u}||$ and $||\mathbf{v}||$ and $||\mathbf{w}||$ of those vectors.

   Find unit vectors in the directions of $\mathbf{v}$ and $\mathbf{w}$. (20 P)

5. From Moodle, download the 5 remixes *snare_kick.wav, gtr_vox.waw, gtr_bass.wav, bass_kick.wav, bass_vox.wav* of the 5 (unknown) original sound tracks: *snare.wav, gtr.wav, bass.wav, kick.wav, vox.wav.* You also know the mixing coefficients for each of the remixes:

$$\mathbf{a}_1 = \begin{bmatrix} 0.4 \\ 0.0 \\ 0.0 \\ 0.6 \\ 0.0 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0.0 \\ 0.5 \\ 0.0 \\ 0.0 \\ 0.5 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 0.0 \\ 0.5 \\ 0.5 \\ 0.0 \\ 0.0 \end{bmatrix}, \mathbf{a}_4 = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.2 \\ 0.8 \\ 0.0 \end{bmatrix}, \mathbf{a}_5 = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.5 \\ 0.0 \\ 0.5 \end{bmatrix}.$$

Here, applying the mixing coefficient $\mathbf{a}_1$ means, that a *snare* track multiplied with 0.4 is added to the *kick* track which is multiplied by 0.6. None of the other original tracks (*gtr, bass, vox*) are used (i.e. values 0.0 in the components of $\mathbf{a}_1$). Put the mixing coefficients into a matrix $\mathbf{A} = \left[\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5\right]$. Use the Matlab function `rank` to determine if $\mathbf{A}$ is invertible. (If `rank` gives a value of 5, $\mathbf{A}$ is invertible, otherwise it is not). If it is invertible, invert it, using the Matlab function `inv` and retrieve the original sound tracks *snare.wav, gtr.wav, bass.wav, kick.wav, vox.wav*. Listen to them. What would happen if you use less than 5 mixtures or more than 5 mixtures? Could you then uniquely determine the original sources? (16 P)

6. Self Assessment: Check the exercises that you have seriously worked on.

2	3	4	5