MSPR 10: Gaussian Mixture Models and Kernel Smoothing Exercises (Due: 8.11.2015 12h)

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1. (Feedback) Please give us feedback on the last lecture and homework: `http://goo.gl/forms/N6KnaX6deF` Thanks!

2. During a great adventure trip by some mysterious divine intervention you are trapped in a Greek monastery `http://www.prayerfoundation.org/lent_41.jpg`. You listen to the chant of a Byzantine monk. Also you find an old manuscript by *Chrysanthos of Madytos* which explains how the `First Plagal` scale divides the octave into the following parts: $0, \frac{10}{72}, \frac{18}{72}, \frac{30}{72}, \frac{42}{72}, \frac{52}{72}, \frac{60}{72}, 1$. The only way to escape from the monastery is to analyse the tones of the scale of the song and to find out how much the most prominent tone deviates in the singing from its pitch given in Chrysanthos' manuscript. [1] Load the `firstPlagal.mat` file into matlab. This file contains the following variables: the variables `t_sec` (the time in s), `f_log2` (the pitch trajectory in fractions of an octave, relative to the tonic, corresponding to the time in `t_sec`), `tick` (the pitch positions of the first Plagal scale, as fractions of an octave, relative to the tonic, according to Chrysanthos' manuscript) and `labs` (a cell array with numbers i, ii, iii, iv and so on, indicating the scale tone).

   (a) Plot pitch versus time. Indicate the pitches of the scale notes according to Chrysanthos' manuscript as ticks on the y-axis (the pitch) as well as horizontal lines. (bonus points 10 P)

   (b) (**Histogram**) Plot vertical lines from 0 to 4 at the positions of the scale notes according to Chrysanthos (the `ticks` contain the pitch values in fractions of an octave). Fix the range of the image from -1 to 1 on the x-axis and from 0 to 4 on the y-axis. Plot the histogram of the pitches in `f_log2`. Experiment with numbers of bins between 10 and 1000. Choose the bin number that gives a histogram, for which the local maxima of the histogram (the pitches actually sung) match as well as possible with the vertical lines (the scale pitches according to Chrysanthos' manuscript). In order so that the area under the histogram is one (to be compared with a probability density function, e.g. a Gaussian), normalize the histogram bar heights `h` for two adjacent

[1] More details (but not necessary to understand this assignment): Maria Panteli & Hendrik Purwins (2013) A Quantitative Comparison of Chrysanthine Theory and Performance Practice of Scale Tuning, Steps, and Prominence of the Octoechos in Byzantine Chant, Journal of New Music Research, 42:3, 205-221, DOI: 10.1080/09298215.2013.827215

center bins `b(1)` and `b(2)` according to: `h_norm=h/(length(f_log2)*(b(2)-b(1)))` (bonus points 20 P)

(c) (**Univariate Parametric Density Estimation**) Fit a single Gaussian to the pitch histogram. Plot the obtained probability density function in the same plot as the histogram. How does it fit the histogram? (bonus points 20 P)

(d) (**Multivariate Parametric Density Estimation**) Use a Gaussian mixture model to fit the histogram of the pitch trajectory. Try different numbers of Gaussians between 2 and 10. To make sure that the GMM converges, find an appropriate maximal iteration number itNo via `opt=statset('MaxIter',itNo);`. Choose the number of Gaussians so that the Gaussian mixture model fits the histogram best. Plot the density estimated in this way in the same plot as the histogram and the density estimate obtained by 1 fitted Gaussian. Comment on the fit of the GMM. (bonus points 20 P)

(e) (**(non-parametric) Kernel Smoothing**) Use (non-parametric) kernel smoothing to approximate the pitch histogram. Try bandwidths between 0.005 and 0.5. Choose the bandwidth, so that the kernel-smoothed density estimate fits the pitch histogram as well as possible. Especially, the local maxima of the smoothed histogram should be close to scale tone pitches according to Chrysanthos, and as few as possible local maxima should occur between the local maxima related to Chrysanthos. Plot the estimated density in the same plot as the histogram and the other density estimates. Comment on the fit of the kernel smoother. (bonus points 20 P)

(f) According to the kernel smoother, what is is the most prominent scale tone (i, ii, iii, iv, v, vi, or vii) and how far is it away (in fractions of an octave) from the pitch given by Chrysanthos?(bonus points 10 P)

3. Self Assessment: Check the exercises that you have seriously worked on.

2 a	2 b	2 c	2 d	2 e	2 f