MSPR 5: Probabilities

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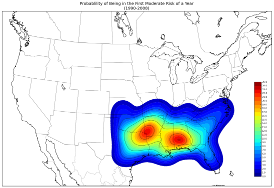
This lecture is based on presentations made in my course 'Advanced Topics in in Music Technology' at Music Technology Group, Universitat Pompeu Fabra between 2007-2011, Barcelona, especially the ones by Stefan Kersten and Srikanth Cherla, closely following

- *Andrew Moore's machine learning tutorial lectures: Gaussians, Gaussian Mixture Models,* `http://www.autonlab.org/tutorials/`
- *Christopher Bishop: Pattern Recognition and Machine Learning: Chapter 1 (Introduction) 1.2.3 (The Gaussian Distribution) p. 24 - 27, 2.3 (The Gaussian Distribution) p. 78, 84 bottom - 85 top.*

Outline

Applications of Probability Density Estimation

- Of few customers we know the income. Do we have more customers with income around 300 000 DKK or 1 000 000 DKK?
- Estimation of crime rates in different areas before buying a house there
- Updating information in computer vision (on-line object tracking) as new data becomes available
- Stormrisks in the US (Image)



http://www.pmarshwx.com/blog/2011/02/03/aotw-storm-prediction-center-moderate-high-risks/
http://www.cs.utah.edu/~lifeifei/papers/kernelsigmod13.pdf

Random Variables

- *Random variable Part* of the world whose *exact value* is uncertain
- *Boolean variables* $\{\mathrm{true}, \mathrm{false}\}$, e.g. *Crown Prince Frederik will win the next KMD Ironman Copenhagen*
- *Discrete variables* take a value from a countable domain, e.g. *this piece's genre is one of* $\{$*jazz, hiphop, country*$\}$
- *Continuous variables* take a value from a continuous domain, e.g. *this piece's tempo is close to 94.5 bpm*

- From the *Kolmogorov's axioms* it follows (a simple foundation of probability theory)

$$
\begin{aligned}
&0 \le P(A) \le 1 \\
&P(\mathrm{true}) = 1 \\
&P(\mathrm{false}) = 0 \\
&P(A \vee B) = P(A) + P(B) - P(A \wedge B)
\end{aligned}
$$

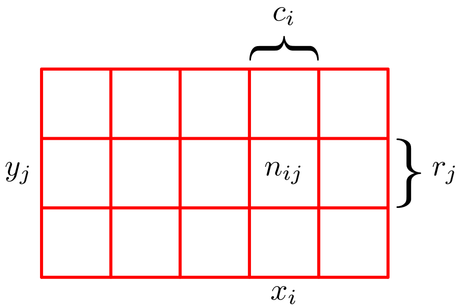
- The *joint distribution* denotes the probabilities of all combinations of two or more random variables
- *Conditional probability*

$$P(Y|X) = \frac{P(Y, X)}{P(X)}$$

- *Sum rule*

$$P(X) = \sum_Y P(X, Y)$$

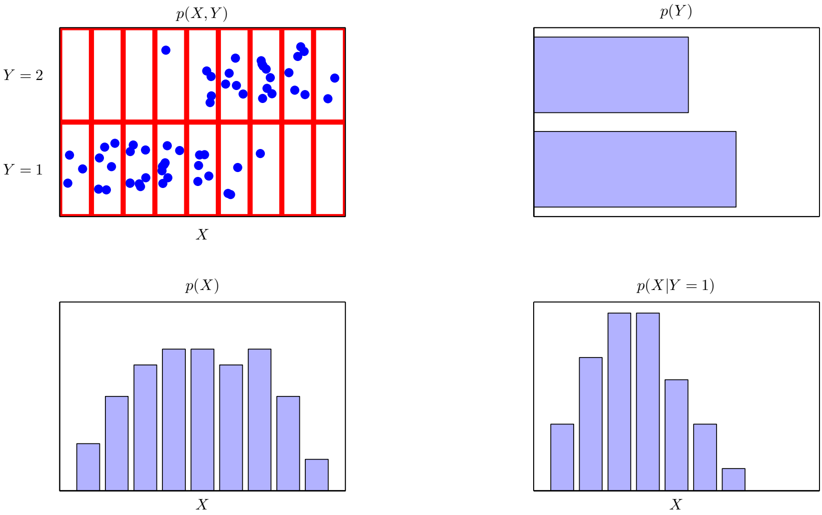
- *Marginalization* over the variable Y given the joint probability $P(X, Y)$



- *Product rule*

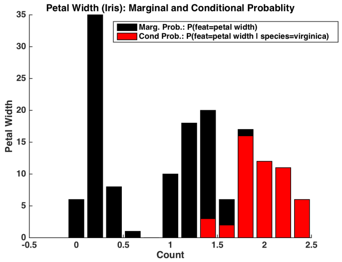
$$P(X, Y) = P(Y|X)P(X)$$

Q: Calculate joint, marginal, and conditional probabilites from a co-occurrence table.



$p(X,Y)$
$Y = 2$
$Y = 1$
X
$p(Y)$
$p(X)$
X
$p(X|Y = 1)$
X

For the Iris data set, the histogram as an estimate for the conditional Probability of petal width under the condition that we have iris species virginica:
P(feat='petal width' — species='virginica'):



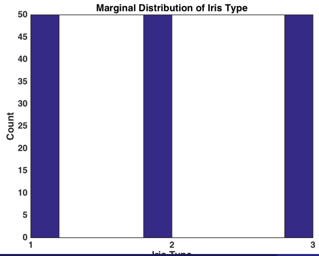

```
adata = textscan(fid,'%f%f%f%f%s','delimiter',',');
X=[adata{1} adata{2} adata{3} adata{4}];
idx_virginica=find(strcmp(species,'Iris-virginica')==1);
idx_versicolor=find(strcmp(species,'Iris-versicolor')==1);
[hvir bvir]=hist(X(idx_virginica,4),(0:0.2: 2.5));
 [hmarg bmarg]=hist(X(:,4),(0:0.2: 2.5));
bar(bmarg,hmarg,'k'); hold on; bar(bvir,hvir,'r');
```

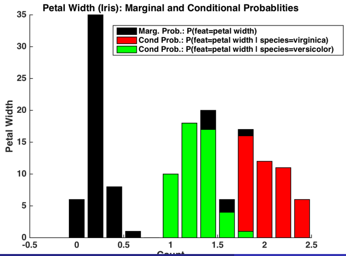
Class Assignment

- *For the Iris data set, the histogram as an estimate for the conditional Probability of petal width under the condition that we have iris species versicolor:*
  P(petal width |species='versicolor').
- *By a histogram, give the marginal probability of the iris type.*

```
idx_virginica=find(strcmp(species,'Iris-virginica')==1);
idx_versicolor=find(strcmp(species,'Iris-versicolor')==1);
[hvir bvir]=hist(X(idx_virginica,4),(0:0.2: 2.5));
[hmarg bmarg]=hist(X(:,4),(0:0.2: 2.5));
[hver bver]=hist(X(idx_versicolor,4),(0:0.2: 2.5)); hold on;
bar(bmarg,hmarg,'k'); bar(bvir,hvir,'r');bar(bver,hver,'g');

iris_type=strcmp(species,'Iris-virginica')+2*strcmp(species,'
    Iris-versicolor')+3*strcmp(species,'Iris-setosa');
hist(iris_type)
```





Q: Derive Bayes rule from the sum and the product rule.

- Bayes' rule is derived from the *product rule*:

$$\begin{aligned} P(X,Y) &= P(Y,X) \\ P(X|Y)P(Y) &= P(Y|X)P(X) \end{aligned}$$

- *Bayes' rule* then becomes

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)}$$

- Bayes rule can be stated as

$$posterior \propto likelihood \ x \ prior$$

Independence

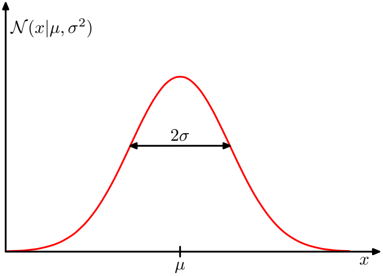
- We say that X is independent of Y, if knowing Y doesn't influence our degree of belief in X
- $P(X|Y) = P(X)$

- *Normal distribution* of a single variable x (*univariate*)

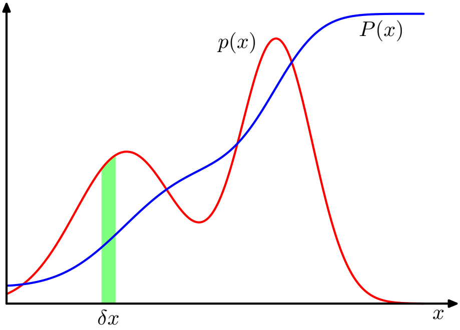
$$\mathcal{N}(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{(x-\mu)^2}{2\sigma^2}\}$$

- *Mean* μ and *Variance* σ^2
- Desirable properties for a PDF

  Positive $\mathcal{N}(x|\mu,\sigma^2) > 0$



  Normalized $\int_{-\infty}^{\infty} \mathcal{N}(x|\mu,\sigma^2)dx = 1$



The probability of x is given by probability distribution $p(x)$.

```
mu = 0; sigma = 2;
pd = makedist('Normal',mu,
    sigma);
```

The probability of $x \in (-\infty, z)$ is given by the *cumulative distribution function* (CDF) $P(x)$ Matlab `y = cdf(pd,x)` Consider a probability in the range δx E.g. that a normally distributed random variable falls within $\pm\sigma$:

```
y = cdf(pd,[a b])}
x = [-sigma sigma]; y =
    cdf(pd,x)y(2)-y(1)
```

Result: Probability: 0.6827

Class Assignment

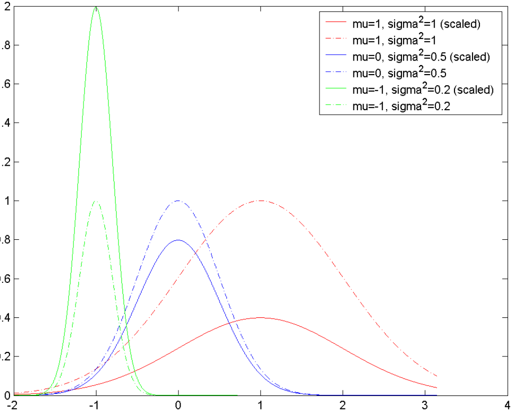
Use the cumulative distribution function in Matlab to calculate what the probability is that a normally distributed random variable falls farther away than 2 σ from the mean μ. How does this value relate to the significance level α of a statistical test?

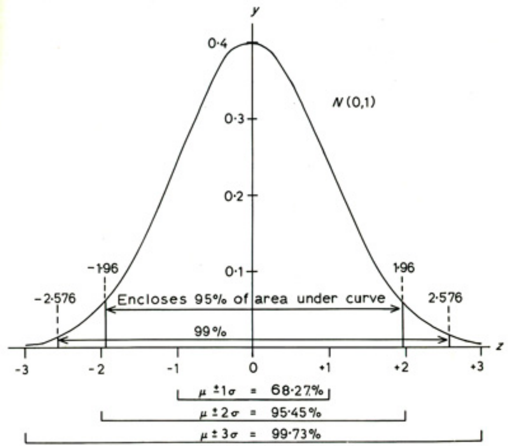
Use the cumulative distribution function in Matlab to calculate what the probability is that a normally distributed random variable falls farther away than 2 σ from the mean μ. How does this value relate to the significance level α of a statistical test?

```
mu = 0; sigma = 2; pd = makedist('Normal',mu,sigma);
x = [-2*sigma 2*sigma]; y = cdf(pd,x)
y(1) + 1-y(2) % or
2*y(1)
```

Result: 0.0455
Significance level $\alpha = 0.05$ gives an upper limit of the probability that, when rejecting the Nullhypothesis it may still be true.

Examples of Gaussians with Different Means / Variances




y
0·4
0·3
0·2
0·1
N (0,1)
−1·96
1·96
−2·576
2·576
Encloses 95% of area under curve
99%
z
-3
-2
-1
0
+1
+2
+3
μ ±1σ = 68·27%
μ ±2σ = 95·45%
μ ±3σ = 99·73%

Fitting a Gaussian to a Histogram of 1-D Data

Data: I observations of just one feature (1-D, just a number=scalar):

$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_I \end{bmatrix}$. Fit a 1-dimensional Gaussian to the data histogram by finding the right estimates $\bar{x}$ for mean μ and s^2 for σ^2:

$$\mathcal{N}(x|\bar{x}, s^2) = \frac{1}{\sqrt{2\pi s^2}} e^{\frac{(x-\bar{x})^2}{2s^2}} \quad (\mathrm{Matlab}: \mathtt{n} = \mathtt{pdf}('\mathtt{norm}', \mathtt{x}, \mathtt{xm}, \mathtt{sqrt}(\mathtt{s2}));)$$

$$\bar{x} = \frac{1}{I} \sum_{i=1}^{I} x_i \quad (\text{sample mean},\ \mathrm{Matlab}: \mathtt{mean}(\mathtt{x}))$$

$$s^2 = \frac{1}{I-1} \sum_{i=1}^{I} (x_i - \bar{x})^2 \quad (\text{sample variance},\ \mathrm{Matlab}: \mathtt{var}(\mathtt{x}))$$

Class Assignment

Load the data `gauss.mat` *and fit a Gaussian to the data A0_tr How good is the fit? Use Matlab functions* `[h p]=hist(x), bar(p,h,'b')` *. Plot the pdf in the same plot. Normalize the height of the histogram bars* `h` *by dividing it with the distance* `b(2) - b(1)` *between two bar positions and the number of components in the data vector of that class.*

Solution

```
M0=mean(A0_tr); S0=var(A0_tr);
tr_no= size(A0_tr,1);
x=-10:0.1:10;
h=figure; n0=pdf('norm',x,M0,sqrt(S0));
[h0 p0]=hist(A0_tr);
bar(p0,h0/size(A0_tr,1)/(p0(2)-p0(1)),'b'); hold on; plot(x,
    n0,'b')
% Plot gaussians
```

Covariance Matrix I

- Covariance between feature j (j-th data matrix column) and k (k-th data matrix column):

$$\mathrm{cov}(\mathbf{X}_{[:,j]}, \mathbf{X}_{[:,k]}) = \frac{1}{I-1} \sum_{i=1}^{I} (x_{ij} - \bar{\mathbf{X}}_{[:,j]})(x_{ik} - \bar{\mathbf{X}}_{[:,k]})$$

- $J \times J$ covariance matrix:

$$\mathbf{\Sigma} = \begin{bmatrix} \mathrm{var}(\mathbf{X}_{[:,1]}) & \mathrm{cov}(\mathbf{X}_{[:,1]}, \mathbf{X}_{[:,2]}) & \dots & \mathrm{cov}(\mathbf{X}_{[:,1]}, \mathbf{X}_{[:,J]}) \\ \mathrm{cov}(\mathbf{X}_{[:,2]}, \mathbf{X}_{[:,1]}) & \mathrm{var}(\mathbf{X}_{[:,2]}) & \dots & \mathrm{cov}(\mathbf{X}_{[:,2]}, \mathbf{X}_{[:,J]}) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}(\mathbf{X}_{[:,J]}, \mathbf{X}_{[:,1]}) & \mathrm{cov}(\mathbf{X}_{[:,J]}, \mathbf{X}_{[:,2]}) & \dots & \mathrm{var}(\mathbf{X}_{[:,J]}) \end{bmatrix}$$

The diagonal elements are the variances of each features $1, \dots, J$ Matrix is symmetric

Covariance

1. Sample covariance Matrix $\mathbf{C}$ of all features (columns of centirized data matrix $\mathbf{X}_c$):

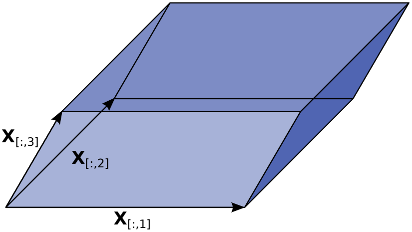
$$\mathbf{C} = \frac{1}{I-1}\mathbf{X}_c^T\mathbf{X}_c$$

2. Matlab: `C=Xc'*Xc/(I-1)` or `C=cov(X)`

Determinant

$$\mathbf{X} := \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$$

The determinant det assigns a number to square matrix $\mathbf{X}$: $c = \det(\mathbf{X})$, Matlab: `c=det(X)`.



- The volumn V of the object (parallepiped) that is sided by the parallelograms defined by the column vectors $\mathbf{X}_{[:,1]}, \mathbf{X}_{[:,2]}, \mathbf{X}_{[:,3]}$ of the matrix $\mathbf{X}$ is the absolut value of the determinant of $\mathbf{X}$: $V = |det(\mathbf{X})|$.
- If $\det(\mathbf{X}) \neq 0$, $\mathbf{X}$ is invertible, for $\det(\mathbf{X}) = 0$, $\mathbf{X}$ is *not* invertible.

Determinant

Class Assignment

Calculate the determinant of the following matrices and invert them if possible and show that you found the inverse. What was the definition of invertibility for a matrix?

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}$$

```
A1=[1 0; 0 1]; A2=[1 2; -1 -2]; A3=[1 -0.5; -0.5 1];
det(A1); A1I=inv(A1); A1*A1I % det(A1)=1
det(A2); inv(A2) % det(A1)=0 (A2 NOT INVERTIBLE!)
det(A3);A3I=inv(A3);A3*A3I % det(A1)=0.75
```

Multi-variate Gaussians

- The multivariate Gaussian distribution for a J-dimensional variable $\mathbf{x}$ is given by

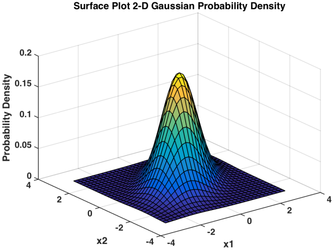
$$\mathcal{N}(\mathbf{x}|\mu, \mathbf{\Sigma}) = \frac{1}{2\pi^{J/2} \det \mathbf{\Sigma}^{1/2}} e^{-\frac{1}{2}(\mathbf{x}-\mu)^T \mathbf{\Sigma}^{-1}(\mathbf{x}-\mu)}$$

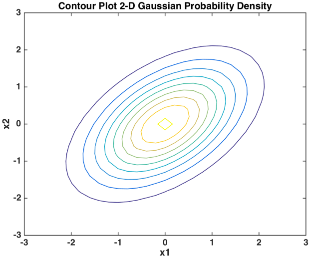
- Defining parameters: μ and $\mathbf{\Sigma}$
- Mean and Covariance
  - Mean (vector) $= \mu$ (J dimensional)
  - Covariance (matrix) $= \mathbf{\Sigma}$ ($J \times J$)
  - $\det \mathbf{\Sigma}$ denotes the determinant of $\mathbf{\Sigma}$

Example Multivariate Gaussian

$$\mathbf{\Sigma}_1 = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

x_1/x_2 axis correlated






```
mu = [0 0]; Sigma1 = [1 .5; .5 1];
x1 = -3:.2:3; x2 = -3:.2:3; [X1,X2] = meshgrid(x1,x2);
F = mvnpdf([X1(:) X2(:)],mu,Sigma1);
F = reshape(F,length(x2),length(x1));
surf(x1,x2,F);
figure; contour(X1,X2,F)
```

Class Assignment

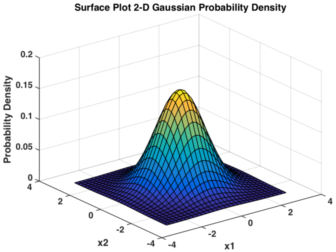
For the following covariance matrices predict the shape of the normal distribution using that covariance matrix and then plot the distribution as a verification:

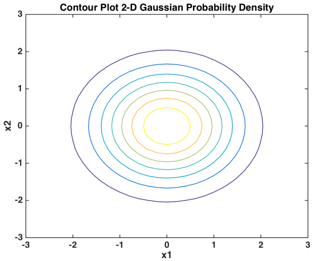
$$\mathbf{\Sigma}_2 = \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{\Sigma}_5 = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 2 \end{bmatrix} \qquad \mathbf{\Sigma}_3 = \begin{bmatrix} 1 & -0.8 \\ -0.8 & 1 \end{bmatrix} \qquad \mathbf{\Sigma}_4 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\mathbf{\Sigma}_2 = \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

x_1 / x_2-axis uncorrelated

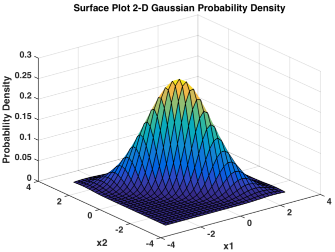


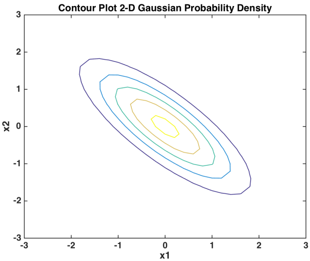



```
mu = [0 0]; Sigma2 = [1 0; 0 1];
x1 = -3:.2:3; x2 = -3:.2:3; [X1,X2] = meshgrid(x1,x2);
F = mvnpdf([X1(:) X2(:)],mu,Sigma1);
F = reshape(F,length(x2),length(x1));
surf(x1,x2,F);
figure; contour(X1,X2,F)
```

$$\mathbf{\Sigma}_3 = \begin{bmatrix} 1 & -0.8 \\ -0.8 & 1 \end{bmatrix}$$

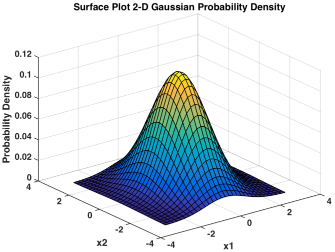
x_1/x_2-axis negatively correlated

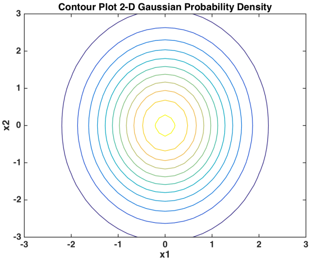




$$\mathbf{\Sigma}_4 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

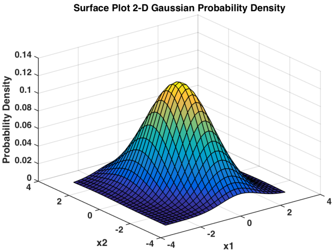
x_1/x_2-axis uncorrelated, stretched in x_2 direction

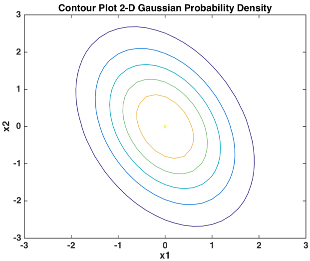




$$\boldsymbol{\Sigma}_5 = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 2 \end{bmatrix}$$

x_1/x_2-axis negatively correlated stretched in x_2 direction





Fitting a Gaussian to Multdimensional Data

Data: I observations of J features:

$$\mathbf{X} := \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1J} \\ x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2J} \\ x_{31} & x_{32} & \cdots & x_{3j} & \cdots & x_{3J} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{i1} & x_{i2} & \cdots & x_{ij} & \cdots & x_{iJ} \\ & & \cdots & & \cdots & \\ x_{I1} & x_{I2} & \cdots & x_{Ij} & \cdots & x_{IJ} \end{pmatrix}$$

Sample mean vector $\bar{\mathbf{x}}$ estimates true mean vector μ (Matlab: `mean(X)`):

$$\bar{\mathbf{x}} = \frac{1}{I} \underbrace{(1, 1 \ldots, 1)}_{I\times} \mathbf{X}$$

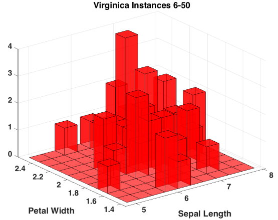
Fit a J-dimensional Gaussian to the data by finding the right estimates for the mean vector μ for covariance matrix σ: $\mathcal{N}(\mathbf{x}|\mu, \mathbf{\Sigma})$

$$= \frac{1}{2\pi^{J/2}|\mathbf{\Sigma}|^{1/2}} e^{-\frac{1}{2}(\mathbf{x}-\mu)^T \mathbf{\Sigma}^{-1}(\mathbf{x}-\mu)}$$

Sample covariance matrix $\mathbf{C}$ estimates true covariance matrix $\mathbf{\Sigma}$ (Matlab: `cov(X)`)

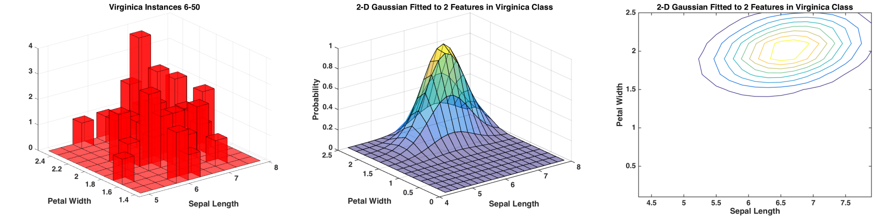
$$C = \frac{1}{I-1}\mathbf{X}'\mathbf{X}$$

3-D Histogram of components 'Sepal Length' and 'Petal Width' only for the Virginica Iris type




```
fid = fopen([data_path 'iris.data']);
adata = textscan(fid,'%f%f%f%f%s','
    delimiter',',');
fclose(fid);
X=[adata{1} adata{4}];
species=adata{5};
idx_virginica=find(...
  strcmp(species,'Iris-virginica')==1);
idx_virginica_tr=idx_virginica(6:50);
mean_virginica=...
  mean(X(idx_virginica_tr,:));
sig_virginica=cov(X(idx_virginica_tr,:))
hist3(X(idx_virginica_tr,:),...
  'FaceAlpha',.65,'FaceColor','red');
```

Fitting Gaussian with Full Cov. Matrix to 3-D Histogram

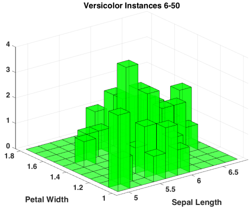


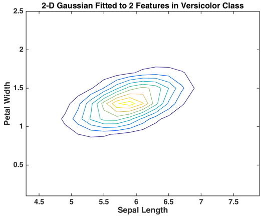

```
x1 = min(X(:,1)):.2:max(X(:,1));
x2=min(X(:,2)):.2:max(X(:,2));
[X1,X2] = meshgrid(x1,x2);
F_virginica = ...
    mvnpdf([X1(:) X2(:)],mean_virginica,sig_virginica);
F_virginica = reshape(F_virginica,length(x2),length(x1));
surf(x1,x2,F_virginica,'FaceAlpha',.5);
contour(X1,X2,F_virginica);
```

Class Assignment

Plot 3-D histogram for instances 5-50 of features 'Sepal Length' and 'Petal Width' the versicolor iris species. Fit a 2-D Gaussian to the histogram and do a surface and contour plot it.

Plot 3-D histogram for instances 5-50 of features 'Sepal Length' and 'Petal Width' the versicolor iris species. Fit a 2-D Gaussian to the histogram and do a surface and contour plot it.





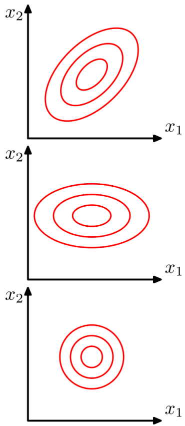

```
...
hist3(X(idx_versicolor_tr,:),'FaceAlpha',.65,'FaceColor','green');
surf(x1,x2,F_versicolor,'FaceAlpha',.5,'FaceColor','green');
```

Parameters to Estimate

- In its most general form, the multi-variate Gaussian of J dimensions has
  - $J(J+1)/2$ parameters for the covariance matrix
  - J parameters for the mean vector.
- The number of parameters increases quadratically with J and hence poses a problem both in parameter estimation and matrix inversion.

Reduction of Parmeters to be Estimated

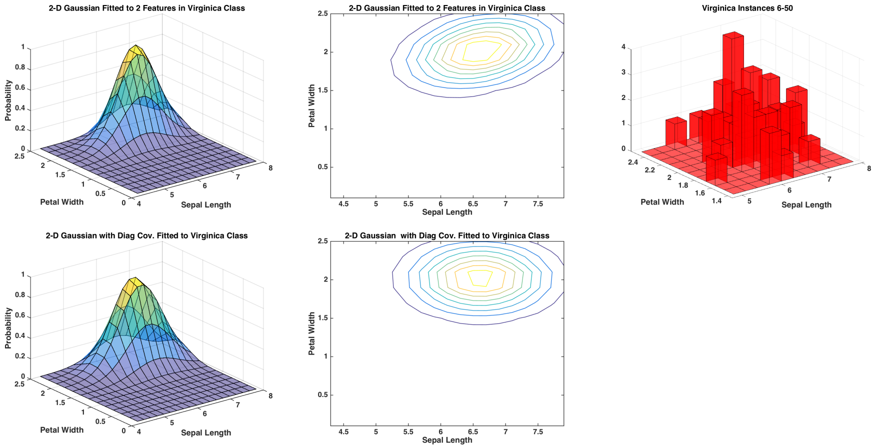
There are two ways of simplifying this problem



- Assume diagonal covariance matrix ($2 \times J$ parameters).
- Assume equal covariance $\mathbf{\Sigma} = \sigma^2 \mathbf{I}$ ($(J + 1)$ parameters).

However, these simplifications also limit the modeling capability of the Gaussian

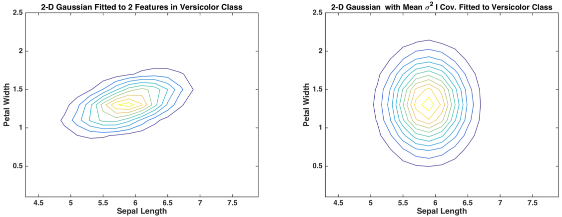
Fit of the Virginica Iris Class with a Gaussian with Diagonal Covariance Matrix



Class Assignment

Fit a Gaussian with covariance matrix of type $\sigma = \sigma^2 \mathbf{I}$ to 3-D histogram of features 'Sepal Length' and 'Petal Width' of the versicolor class considering instance 6-50. For σ^2 take the average of the variances for 'Sepal Length' and 'Petal Width' .

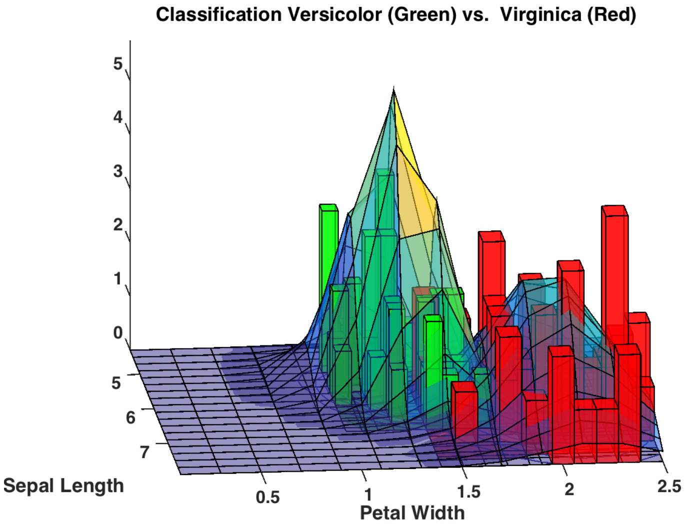
Fit a Gaussian with covariance matrix of type $\sigma = \sigma^2 \mathbf{I}$ to 3-D histogram of features 'Sepal Length' and 'Petal Width' of the versicolor class considering instance 6-50.




```
sig_versicolor=mean(diag(sig_versicolor))*diag(ones(2,1));
```

Gaussian Fit for Predicting the Iris Type Based on Sepal Length and Petal Width only

- We had fitted two Gaussians $\mathcal{N}(\mathbf{x}|\mu_{virg}, \mathbf{\Sigma}_{virg})\mathcal{N}(\mathbf{x}|\mu_{vers}, \mathbf{\Sigma}_{vers})$ to the points 6-50 of Virginica and Versicolor Iris data ('Sepal Length' and 'Petal Width').
- The first 5 points of Virginica and Versicolor iris data ('Sepal Length' and 'Petal Width') have not been used for the fitting.
- Let us use the fitted Gaussians, to determine whether these points belong to Virginica or Versicolor according to the rule: If for a point $\mathbf{x}$ $\mathcal{N}(\mathbf{x}|\mu_{virg}, \mathbf{\Sigma}_{virg}) > \mathcal{N}(\mathbf{x}|\mu_{vers}, \mathbf{\Sigma}_{vers})$ we predict its class to be Virginica, otherwise Versicolor.
- let us count the wrong predictions.


Classification Versicolor (Green) vs. Virginica (Red)
5
4
3
2
1
0
5
6
7
Sepal Length
0.5
1
1.5
2
2.5
Petal Width

```
sig_virginica=cov(X(idx_virginica_tr,:));
sig_versicolor=cov(X(idx_versicolor_tr,:));
idx_virginica_tst=idx_virginica(1:5);
idx_versicolor_tst=idx_versicolor(1:5);
F_versicolor_tst = mvnpdf(X([idx_virginica_tst;
    idx_versicolor_tst],:),...
     mean_versicolor,sig_versicolor);
F_virginica_tst = mvnpdf(X([idx_virginica_tst;
    idx_versicolor_tst],:),...
     mean_virginica,sig_virginica);
[F_virginica_tst'; F_versicolor_tst']
%ans =
%   0.18  0.43  0.69   0.66   0.69  0.03  0.16  0.09  0.02  0.15
%   0.00  0.00  0.00   0.10   0.00  0.08  0.98  0.19  1.30  0.80
```

Perfect Prediction!

Class Assignment

- *Fit two Gaussians* $\mathcal{N}(\mathbf{x}|\mu_{virg}, \mathbf{\Sigma}_{virg})\mathcal{N}(\mathbf{x}|\mu_{vers}, \mathbf{\Sigma}_{vers})$ *using a covariance matrix of type* $\sigma^2\mathbf{I}$ *with* σ^2 *being the mean variance for 'Sepal Length' and 'Petal Width'. Use to the features 'Sepal Length' and 'Petal Width' of instances 6-50 of the Virginica and Versicolor Iris data.*
- *Use the fitted Gaussians, to determine whether these points belong to Virginica or Versicolor according to the rule: If for a point* $\mathbf{x}$ $\mathcal{N}(\mathbf{x}|\mu_{virg}, \mathbf{\Sigma}_{virg}) > \mathcal{N}(\mathbf{x}|\mu_{vers}, \mathbf{\Sigma}_{vers})$ *we predict its class to be Virginica, otherwise Versicolor.*
- *Count the wrong predictions for the first 5 points of Virginica and Versicolor iris data.*

Solution

```
sig_virginica=mean(diag(sig_virginica))*diag(ones(2,1));
sig_versicolor=mean(diag(sig_versicolor))*diag(ones(2,1));
F_versicolor_tst = mvnpdf(X([idx_virginica_tst;
    idx_versicolor_tst],:),mean_versicolor,sig_versicolor);
F_virginica_tst = mvnpdf(X([idx_virginica_tst;
    idx_versicolor_tst],:),mean_virginica,sig_virginica);
[F_virginica_tst'; F_versicolor_tst']
```

```
ans =
    0.33  0.17  0.39  0.48  0.58  0.22  0.34  0.31  0.02  0.36
    0.00  0.32  0.00  0.25  0.02  0.01  0.38  0.02  0.69  0.25
```

4 Errors!