Université de Paris-Saclay
Institut Polytechnique de Paris
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Lab 1
2024 – 2025
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Refresher in Source Coding

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Lab 1 – Entropy – Quantization

Aim of the Lab

The aim of this lab is to build entropy evaluation functions, to evaluate the entropy of text, audio signals, and to get familiar with quantization, rate-distortion, and quantizer optimization.

1 First part - entropy

Consider a vector `x` containing a sequence of integers. The aim is to build a function to estimate the entropy of the source which has generated `x`, assuming that this source is memoryless.

1. Evaluate the frequency of occurence of each distinct symbol in `x`. The `unique` matlab function may be useful for that purpose.
2. Build a function `entropy` which takes as input `x` and provides its entropy.
3. Load the file `Declaration1789.txt` using

```
>>  fid = fopen('Declaration1789.txt','r');
>>  F = fread(fid);
>>  x = char(F')
```

4. What is its entropy ?

The source which has generated `x` is no more considered as memoryless.

5. Evaluate the frequency of occurence of each distinct pair of symbols in `x`.
6. Build a function `entropy1` which takes as input `x` and provides its first-order entropy.
7. Evaluate the first order entropy of `Declaration1789.txt`.

2 Second part - quantization

The aim of this part is to build and compare the performance of two types of uniform quantizers, the mid-rise and mid-tread quantizers. The performance of these quantizers will be compared considering realizations of a unit-variance Gaussian source.

1. Generate a vector `x` containing $N = 10000$ realizations of a iid Gaussian random variables with zero-mean and unit variance.

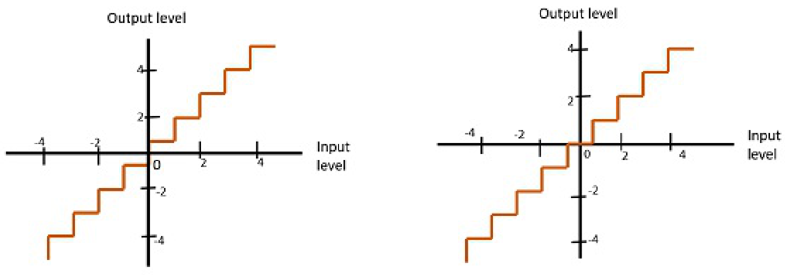

Fig 1 : Mid-Rise type Uniform Quantization   Fig 2 : Mid-Tread type Uniform Quantization

FIGURE 1 – Mid-rise and mid-tread quantizers

2. Build a function

   `[qx,idx] = quant_midrise(x,Delta)`

   performing a mid-rise quantization of `x` with a step size `Delta`. The input-output characteristic of this quantizer is in Figure 1 (a). The output of this function is the vector `qx` of the quantized components of `x` and `idx` of the corresponding indexes. The quantizer should not be bounded, *i.e.*, `idx` may take any integer value.
3. For different values of `Delta`, evaluate the distortion introduced by the quantizer, considering a quadratic distorsion measure. Evaluate also the entropy of `idx`.
4. Plot the distortion as a function of the entropy obtained with the mid-rise quantizer.
5. Build a function

   `[qx,idx] = quant_midtread(x,Delta)`

   performing a mid-tread quantization of `x` with a step size `Delta`. The output of this function is the vector `qx` of the quantized components of `x` and `idx` of the corresponding indexes. The quantizer should not be bounded, *i.e.*, `idx` may take any integer value. The input-output characteristic of this quantizer is in Figure 1 (b).
6. Plot the distortion as a function of the entropy obtained with the mid-tread quantizer.

In what follows, one will consider a constraint on the number of output bits of each quantizer. One will consider that quantization indexes have to be represented on R bits, in which case, only $M = 2^R$ different indexes may be represented.

7. Build two functions

   `[qx,idx] = quant_midrise(x,Delta,M)`

   and

   `[qx,idx] = quant_midtread(x,Delta,M)`

   performing a mid-rise quantization and a mid-tread quantization of `x` with a step size `Delta`, where only M different output indexes may be represented.

8. For different values of $M = 2$, $M = 4$, $M = 8$, and $M = 16$, plot the distortion obtained when quantizing `x` with different values of `Delta`. Show that for each value of M, there is a value of `Delta` minimizing the distortion.
9. For each value of M, plot the distortion as a function of the entropy parametrized in `Delta`.
10. What is the value of the optimal value of `Delta` when the variance of the sources which have generated `x` is 2 or 4 ?
11. Repeat the previous experiments with a uniformly distributed source in the interval $[-1, 1]$.
12. Repeat the previous experiments with a Laplacian source with variance 1 and 2.

One will now quantize audio signals.

13. Read the sound file `Ding.wav` using

    ```
    >> [x,Fs]=audioread('Ding.wav')
    ```

    and play the sound file using

    ```
    >> sound(x,Fs)
    ```

14. Perform the quantization of the audio signal `x` with different values of $M = 2$, $M = 4$, $M = 8$, and $M = 16$. Plot the distortion obtained when quantizing `x` with different values of `Delta`. Show that for each value of M, there is a value of `Delta` minimizing the distortion. Listen the quantized audio signal for that value of `Delta`.

Sometimes, in the mid-tread quantizer, the quantization interval containing zero is larger than the other intervals. In that case, one obtains a mid-tread quantizer with dead-zone.

15. Build a function

    ```
    [qx,idx] = quant_midtread(x,Delta,M,Deltadz)
    ```

    performing a mid-tread quantization of `x` with a step size `Delta` and a dead-zone `Deltadz`, where only M different output indexes may be represented.
16. What is the usefulness of this dead-zone quantizer ?