Introduction to Computer Programing C++ Real Problem

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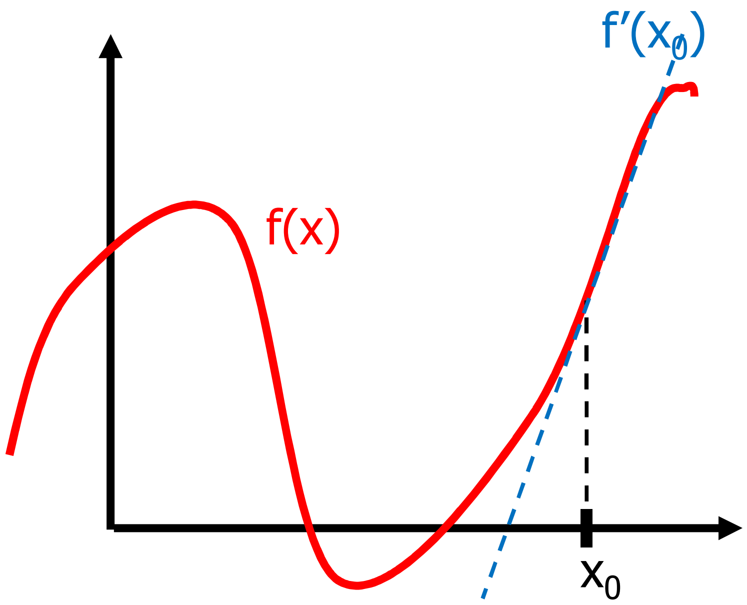
Performing Differentiation



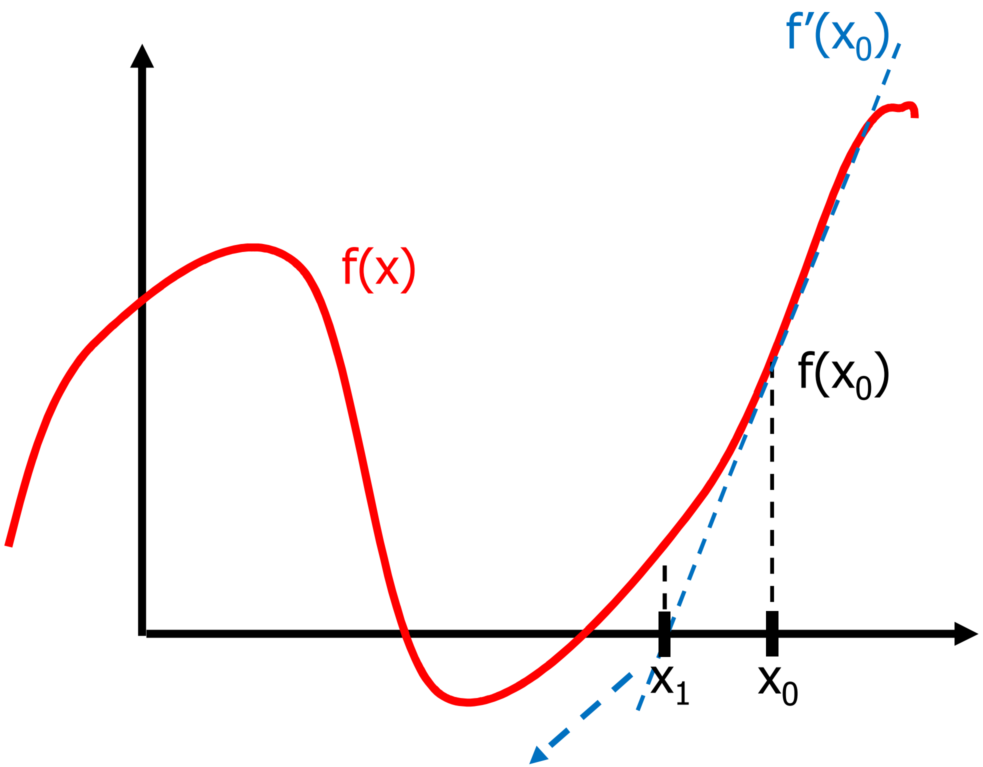
1. How to perform differentiation?

f'(x0)=???

2. How small dx should be?

Find:
y'(1), y'(-4), and y'(4)
for
$y=x^3 - 8x^2 - 6x + 50$

Finding the real root(s) of a function



$y-f(x_0)=f'(x_0)(x-x_0)$, it intersects with x axis at $x_1=x_0-f(x_0)/f'(x_0)$

….

$x_{i+1}=x_i-f(x_i)/f'(x_i)$

For any given function f(x):

- Start from x0 (initial guess)
- Find the tangent line at (x_0, $f(x_0)$)
- Find new x1, and the tangent line at (x_1, $f(x_1)$)
- Repeat the above steps until x_i~~x_{i+1}

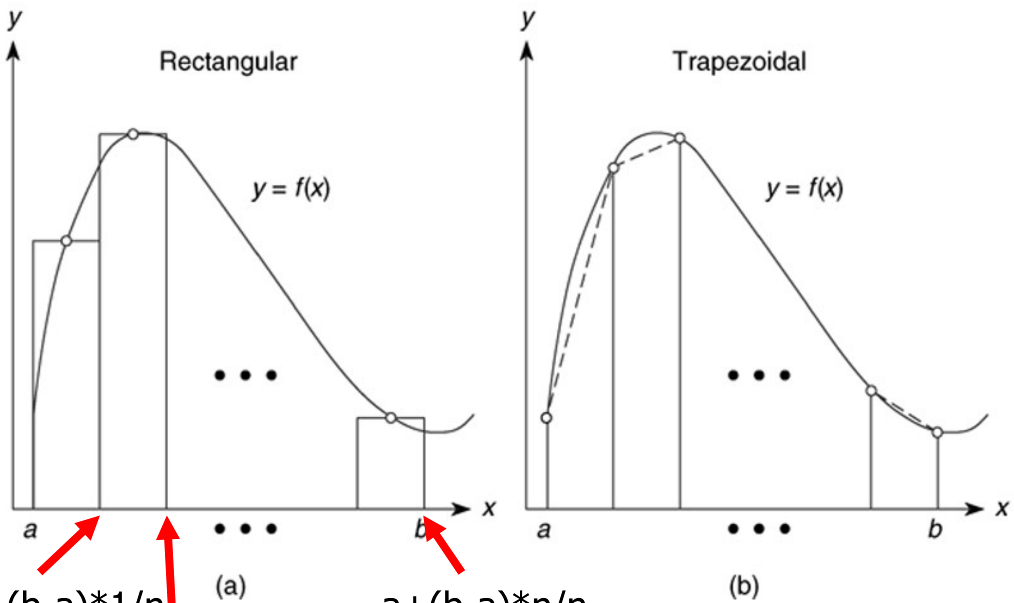
Notes:

- Requires a "reasonably close guess"
- Does not guarantee finding the (closest) root.

Test Your Understanding

- Find root(s) for
  - $y=x^3 - 8x^2 - 6x + 50$
  - Question: How to guarantee that all real roots are found?
- Using a "for" loop, find the roots for the following functions. Please plot using Excel first, then test using multiple initial guesses. Please use 10 iterations (i=1~10), and display the new x after each iteration.
  - $y=x^3-5x^2+3x+5$
  - $y=x^3-5x^2+3x+5-\exp(x)/3$.

Performing Numerical Integration



a+(b-a)*1/n

a+(b-a)*2/n

a+(b-a)*n/n

$$\int_a^b f(x)\,\mathrm{d}x = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\frac{(b-a)}{n}$$

$$\cong \sum_{i=1}^{n} f(x_i)\frac{(b-a)}{n}$$

Test Your Understanding

- Find the approximated value of $\int_{-2}^{2} \sin(x)\,\mathrm{d}x$ with 100 increments and compare with the actual number.
- Find the approximated value of $\int_{0}^{2} x^2\,\mathrm{d}x$ with 100 increments and compare with the actual number.
- Find the approximated value of $\int_{2}^{4} \exp(x) * x^2\,\mathrm{d}x$ with 100 increments.

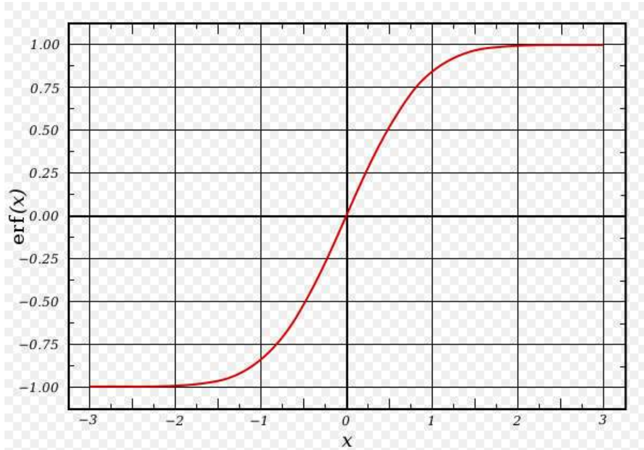
- Ask user to input a positive number t, and find approximated value of $\int_{0}^{\mathrm{t}} x^2\,\mathrm{d}x$ with 100 increments and compare with the actual number.

Challenging Problem

- Calculate erf(0.1) with 10 increments between 0 and 0.1
- Find n needed such as the error for erf(0.1) is within 0.001 of the actual number
- Ask the user to input x, then output erf(x)

Error function is defined as:

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)\, dt$$



x	erf(x)
0	0
0.02	0.022 564 575
0.04	0.045 111 106
0.06	0.067 621 594
0.08	0.090 078 126
0.1	0.112 462 916
0.2	0.222 702 589
0.3	0.328 626 759
0.4	0.428 392 355
0.5	0.520 499 878
0.6	0.603 856 091
0.7	0.677 801 194
0.8	0.742 100 965
0.9	0.796 908 212
1	0.842 700 793

x	erf(x)
1	0.842 700 793
1.1	0.880 205 07
1.2	0.910 313 978
1.3	0.934 007 945
1.4	0.952 285 12
1.5	0.966 105 146
1.6	0.976 348 383
1.7	0.983 790 459
1.8	0.989 090 502
1.9	0.992 790 429
2	0.995 322 265
2.1	0.997 020 533
2.2	0.998 137 154
2.3	0.998 856 823

Notes for error function:

- Complementary error function erfc(x)=1-erf(x)
- Error function is the solution of a 1D transient heat transfer problem.

Terminal Velocity of Falling Particles

Based on the force balance on a spherical particle, the terminal velocity in a fluid is given:

$$v_t = \sqrt{\frac{4g(\rho_p - \rho)D_p}{3C_D\rho}}$$

v_t is the terminal velocity
g=9.8m/s^2
ρ_p: particle density in kg/m^3
ρ: fluid density in kg/m^3
D_p: particle diameter in m
C_D: drag coefficient
$Re=D_p v_t \rho/\mu$

$$C_D = \frac{24}{Re} \qquad \text{for} \qquad Re < 0.1$$

$$C_D = \frac{24}{Re}(1 + 0.14\,Re^{0.7}) \qquad \text{for} \qquad 0.1 \le Re \le 1000$$

$$C_D = 0.44 \qquad \text{for} \qquad 1000 < Re \le 350000$$

$$C_D = 0.19 - 8\times10^4/Re \qquad \text{for} \qquad 350000 < Re$$

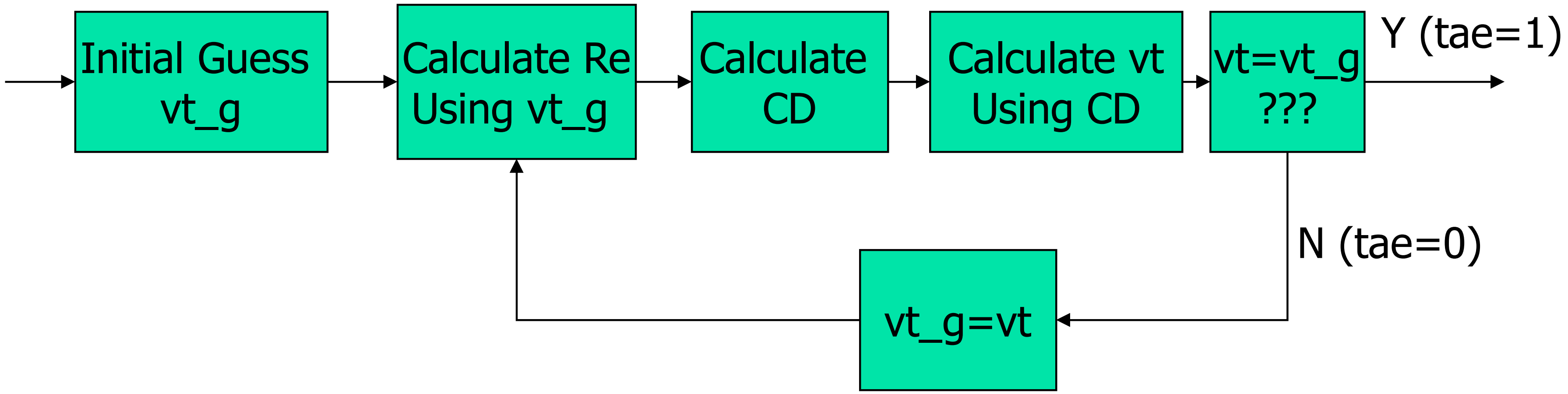
Find the v_t for a coal particle (ρ_p=1800 kg/m^3, D_p=0.208*10^{-3} m) falling in water (ρ=994.6 kg/m^3, μ=8.931*10^{-4} kg/m.s)

How to Solve – Strategy

$v_t=v_t(C_D)$
$C_D=C_D(Re)=C_D(v_t)$

For their given forms, substitute v_t into C_D or C_D into v_t are not practical.

$\longrightarrow$ Solving this problem "iterately"



How to find vt?
Note: You'll need the ReCD(x) you created earlier.

Solving for vt

1. Machine precision – monitor v_t at each iteration?
2. Count number of iterations done – How to do this?
3. Is it necessary to do so many iterations?
4. How to set the new criteria such if the variation is within 0.01% then give the result?