

# 1. Example

## 1. (a) Example 1

This template is useful if every single homework question requires you to upload a separate pdf file or image. You may even upload 20 separate files/images just to hand in your homework!

Bonus 1: When you're uploading a pdf file, does the server split the pdf file into individual pages? Can you find a way to generate an image for every page in a pdf?

Bonus 2: This template has very little whitespace. Less scrolling is needed!

## 1. (b) Example 2

**Lemma 1.** *If  $x < y$  then  $-y < -x$ .*

*Proof.* Here is the proof:

$$\begin{aligned}x &< y \\x + (-x - y) &< y + (-x - y) \\-y &< -x\end{aligned}$$

It was trivial.



## 2. Longer example

Let  $F(1, b) = 1$  and  $F(a, 1) = 1$ . Also, for  $a > 1$  and  $b > 1$ , let  $F(a, b) = F(a - 1, b) + F(a, b - 1)$ .

We show that for all  $a \geq 1$  and  $b \geq 1$ , any set of  $F(a, b)$  people would satisfy at least one of these conditions:

- There exists a crew of size  $a$  that know each other
- There exists a crew of size  $b$  that don't know each other

The case where  $a = 1$  or  $b = 1$  is easy. The set is size 1. Pick this person from the set, and make a crew.

Let's solve the case where  $a > 1$  and  $b > 1$ . Assume that  $F(a - 1, b)$  and  $F(a, b - 1)$  are already proven.

Ask the first person. Put all of the known people into a set  $S$ , and all of the unknown people into set  $T$ . Note that  $F(a, b) = 1 + |S| + |T|$  and  $F(a, b) = F(a - 1, b) + F(a, b - 1)$ . So we have  $|S| + |T| = F(a - 1, b) + F(a, b - 1) - 1$ . By the pigeonhole principle, we have  $|S| \geq F(a - 1, b)$  or  $|T| \geq F(a, b - 1)$ . So we have two cases.

- Case 1:  $|S| \geq F(a - 1, b)$ . We can use  $F(a - 1, b)$  to create two cases.
  - Case 1a: We can find  $b$  people in  $S$  that don't know each other. In this case, we are already done.
  - Case 1b: There are  $a - 1$  people in  $S$  that know each other. The first person knows everyone in  $S$ , so we can form a crew with the first person and the  $a - 1$  people in  $S$ . Then there is a crew of  $a$  people that know each other, and we are done.
- Case 2:  $|T| \geq F(a, b - 1)$ . This case is similar to case 1.

The last step is to use induction to finish off the proof. This step is easy and is left to the reader as a short exercise.

Note: You may notice that  $F(a, b) = \binom{a + b - 2}{a - 1}$ .

