
MATH 233 - Linear Algebra I

Lecture Notes

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$$n = \text{rank}(A) + \text{nullity}(A)$$

$$U^T U = I$$

$$A = P^{-1}DP$$

$$\|v\| = \sqrt{\langle v, v \rangle} \quad A^{-1} = \frac{1}{\det A} \text{Cof}(A)^T$$

$$\mathbb{R}^n = \text{span}\{v_1, v_2, \dots, v_n\} \quad \det(\lambda I - A) = 0$$

$$Ax = \lambda x$$

$$A^T = A$$

$$\text{tr} A = \lambda_1 + \lambda_2 + \dots + \lambda_n$$

$$R = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

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