

# Project Definitions

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September 2015

## 1 Introduction

Introduction coming soon to a pdf near you.

## 2 Definitions

### 2.1 Graphs

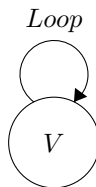
A graph, is an ordered pair  $G = (V, E)$  which represents a set of objects where some of these objects are linked. In the denotation  $G = (V, E)$ ,  $V$  stands for the vertexes or objects, and  $E$  stands for the edges or links. Edges in a graph can be directed or undirected, however, we will focus on undirected edges in our application.

#### 2.1.1 Componentets

A component or connected component of a graph is a subgraph in which any two vertices are connected to each other by paths, and which is connected to no additional vertices in the supergraph.

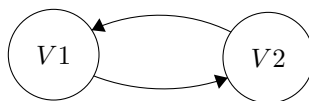
#### 2.1.2 Loops

A loop is when a vertex has an edge connecting it to itself.



#### 2.1.3 Multiple Edges

A graph with multiple edges is one which has two or more edges connect the same two vertices.



## 2.2 Types of Graphs

### 2.2.1 Connected Graphs

A graph is connected when there is a path between every pair of vertices. In a connected graph every vertex is reachable. A graph with just one vertex is connected.

### 2.2.2 Simple Graphs

Is an unweighted, undirected graph, containing no loops or multiple edges. A simple graph may either be connected or disconnected.

### 2.2.3 Planer Graphs

A planer graph is one that can be embedded in the plane. This means that it can be drawn on the plane in such a way that its edges intersect only at their endpoints (no edges cross each other).

- A triangulation, also referred to as a maximal planar graph, is a planar graph in which there is no way to add another edge and have the graph continue to be planar. In practice these means that each face is bounded by three edges.

## 2.3 Embedding

An embedding of a graph  $G$  on a surface  $\Sigma$  is a representation of  $G$  on  $\Sigma$  in which points of  $\Sigma$  are associated to vertices and simple arcs are associated to edges in such a way that:

- the endpoints of the arc associated to the edge  $e$  are the points associated to the end vertices of  $e$ .
- no arcs include points associated with other vertices.
- two arcs never intersect at a point which is interior to either of the arcs.

### 2.3.1 Straight-Line Embedding

A straight-line embedding is a embedding of a planar graph in which all arcs are straight.

## 2.4 Euler's Formula

Euler's formula states that if a finite, connected, planar graph is drawn in the plane without any edge intersections, and  $v$  is the number of vertices,  $e$  is the number of edges and  $f$  is the number of faces (including the outer region), then:

$$v - e + f = 2 \quad (1)$$

If  $c$  is the number of components in a graph, then a more general form of Euler's formula is:

$$v - e + f = 1 + c \quad (2)$$

## 2.5 Circle Packing

We say that two circles drawn in a plane kiss (or osculate) when they intersect in exactly one point. A circle packing is a graph formed by a set of circles which have no overlapping interiors where each circle kisses its surrounding circles. More formally, a circle packing is a connected collection of circles whose interiors are disjoint. The intersection graph of a circle packing is the graph having a vertex for each circle and an edge for every pair of circles that are tangent. If the circle packing is on the plane or the sphere, then its intersection graph is called a coin graph. Coin graphs are always connected, simple and planar.

### 2.5.1 Liftings

### 2.5.2 Stereographic Projections

The stereographic projection is a mapping that projects a sphere onto a plane. The projection is defined on the entire sphere except for the projection point. The mapping is bijective which means that it can be reversed.

### 2.5.3 Mobius Transformations

A Mobius Transformation of a plane can be obtained by performing the stereographic projection of the plane onto a sphere, then rotating or moving the sphere and then performing the stereographic projection back onto the plane. A Mobius Transformation is a function of the form:

$$f(z) = \frac{az + b}{cz + d} \quad (3)$$

where  $z$  is a complex variable and  $a, b, c, d$  are complex numbers satisfying the equation  $ad - bc \neq 0$ .

## 2.6 Dual Graph

The dual graph of a plane graph  $G$  is a graph that has a vertex for each face of  $G$ . The dual graph has an edge whenever two faces of  $G$  are separated from

each other by an edge. Thus each edge  $e$  of  $G$  has a corresponding dual edge, the edge that connects the two faces on either side of  $e$ .

### 2.6.1 Dual Packing

The dual packing is the circle packing of the dual graph. In this packing, each circle passes through the points where the original circles kiss. The dual packing does not form a triangulation whereas the original circle packing does.

## 3 Data Structures

### 3.1 Half Edge

Also called a doubly-connected edge list, a half edge is a data structure where one edge on the graph corresponds to two half edges. Each half edge stores a vertex, a face, the next half edge, the previous half edge and the twin, which is the half edge going in the other direction. Half edges are typically directional, going counter clockwise with respect to the face which they define.

## 4 Conclusion

This is how you cite a source. [1]

## References

- [1] D. Adams. *The Hitchhiker's Guide to the Galaxy*. San Val, 1995.