Matrices

Class	Algebra 2

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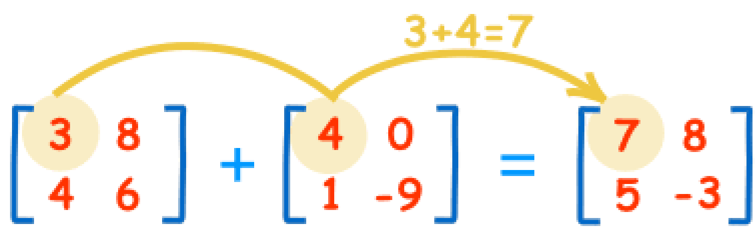
Matrix: arrangement of numbers

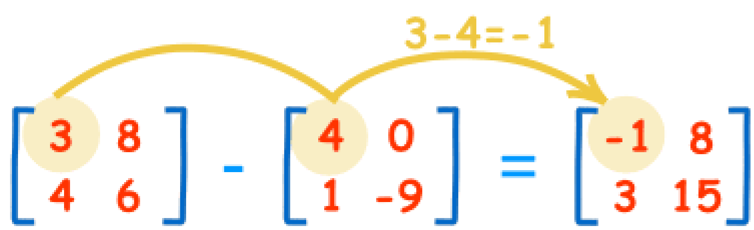
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Dimensions: rows x columns (ex. 3×3)

Adding and Subtracting Matrices

> You can add or subtract matrices only if **their dimensions are the same**.





Multiplication with Matrices

Scalar Multiplication

- Multiplying 1 number and a matrix

$$2 \cdot \begin{bmatrix} 10 & 6 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 2 \cdot 10 & 2 \cdot 6 \\ 2 \cdot 4 & 2 \cdot 3 \end{bmatrix}$$

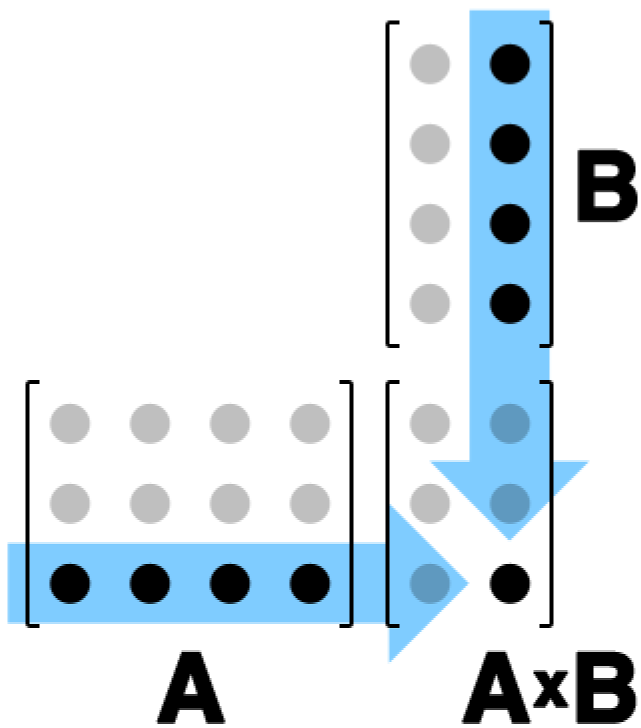
You can solve for variables in matrices.

Multiplying Matrices

> You can only multiply two matrices if...
>
> - The number of columns in the first matrix = The number of rows in the second matrix

Stacking → one matrix on top of the other

- First in row (A) * First in column (B) + Second in row (A) * Second in column (B) + ...



Determinants

- Only for square matrices

Determinant of a 2x2 Matrix

Matrix:

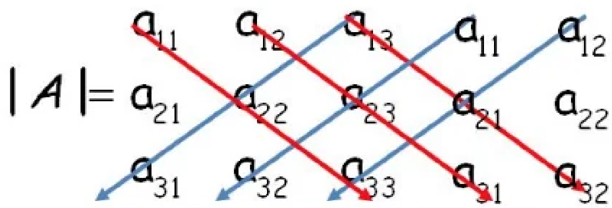
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Determinant:

$$\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Determinant of a 3x3 Matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$



Sum of products along red arrow
minus sum of products along blue arrow

$$\det(A) = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

Area of a Triangle From Vertices

- Uses a 3×3 Matrix Determinant Equation

Area of Triangle in Determinant Form

If $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are the vertices of a triangle

then its area is :

$$\text{Area of } \Delta\, ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

OR

$$\text{Area of } \Delta\, ABC = \frac{1}{2}\left| x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) \right|$$

Matrix Inverses

- Determinant cannot be 0

Inverse of 2x2 Matrix

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then

$$\boxed{\underbrace{A^{-1}}_{\text{Inverse of A}} = \frac{1}{\underbrace{ad - bc}_{\text{Determinant of A}}} \underbrace{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}_{\text{Adjoint of A}}}$$

Note: A^{-1} exists only when $ad - bc \neq 0$

Row Operations

- Switch rows
- Multiply rows by nonzero constants
- Add or subtract rows

Swap

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 3 & 6 & 9 \\ 0 & 3 & 6 \\ -2 & 4 & 8 \end{bmatrix} \quad R_2 \leftrightarrow R_3 \quad \begin{bmatrix} 3 & 6 & 9 \\ -2 & 4 & 8 \\ 0 & 3 & 6 \end{bmatrix}$$

Scale

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 3 & 6 & 9 \\ -2 & 4 & 8 \\ 0 & 3 & 6 \end{bmatrix} \quad \frac{1}{3}R_1 \to R_1 \quad \begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & 8 \\ 0 & 3 & 6 \end{bmatrix}$$

Add

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & 8 \\ 0 & 3 & 6 \end{bmatrix} R_2 + 2R_1 \to R_2 \begin{bmatrix} 1 & 2 & 3 \\ 0 & 8 & 14 \\ 0 & 3 & 6 \end{bmatrix}$$

Solving Systems of Equations with Matrices

- Put coefficients and values into matrix
- Solve to *reduced row-echelon form*