



Matrices

▼ Class

Algebra 2

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Matrix: arrangement of numbers

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Dimensions: rows x columns (ex. 3×3)

Adding and Subtracting Matrices

You can add or subtract matrices only if **their dimensions are the same**.

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$$

Diagram illustrating matrix addition. A yellow arrow points from the top-left element of the first matrix (3) to the top-left element of the result matrix (7), with the calculation $3+4=7$ written above it. Another yellow arrow points from the top-left element of the second matrix (4) to the same result element (7).

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 3 & 15 \end{bmatrix}$$

Diagram illustrating matrix subtraction. A yellow arrow points from the top-left element of the first matrix (3) to the top-left element of the result matrix (-1), with the calculation $3-4=-1$ written above it. Another yellow arrow points from the top-left element of the second matrix (4) to the same result element (-1).

Multiplication with Matrices

Scalar Multiplication

- Multiplying 1 number and a matrix

$$2 \cdot \begin{bmatrix} 10 & 6 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 2 \cdot 10 & 2 \cdot 6 \\ 2 \cdot 4 & 2 \cdot 3 \end{bmatrix}$$

You can solve for variables in matrices.

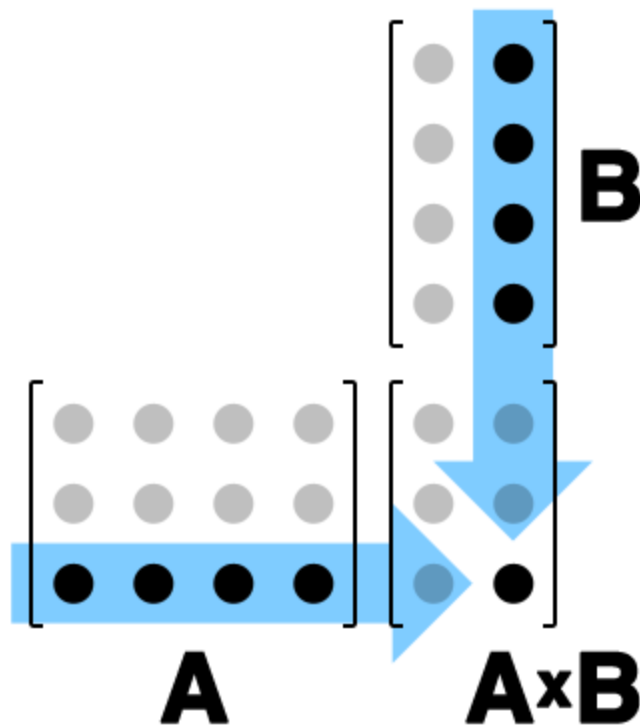
Multiplying Matrices

You can only multiply two matrices if...

- The number of columns in the first matrix = The number of rows in the second matrix

Stacking → one matrix on top of the other

- First in row (A) * First in column (B) + Second in row (A) * Second in column (B) + ...



Determinants

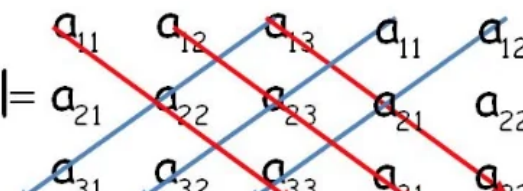
- Only for square matrices

Determinant of a 2x2 Matrix

Matrix:	Determinant:
$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	$\det(A) = A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Determinant of a 3x3 Matrix

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$|A| =$


Sum of products along **red arrow**
minus sum of products along **blue arrow**

$$\det(A) = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

Area of a Triangle From Vertices

- Uses a 3x3 Matrix Determinant Equation

Area of Triangle in Determinant Form

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle

then its area is :

$$\text{Area of } \triangle ABC = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

OR

$$\text{Area of } \triangle ABC = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

Matrix Inverses

- Determinant cannot be 0

Inverse of 2x2 Matrix

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Diagram illustrating the formula for the inverse of a 2x2 matrix A . The formula is $A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. The components are labeled:

- A^{-1} : Inverse of A
- $ad - bc$: Determinant of A
- $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$: Adjoint of A

Note: A^{-1} exists only when $ad - bc \neq 0$

Row Operations

- Switch rows
- Multiply rows by nonzero constants
- Add or subtract rows

Swap

$$\begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \begin{bmatrix} 3 & 6 & 9 \\ 0 & 3 & 6 \\ -2 & 4 & 8 \end{bmatrix} \quad R_2 \leftrightarrow R_3 \quad \begin{bmatrix} 3 & 6 & 9 \\ -2 & 4 & 8 \\ 0 & 3 & 6 \end{bmatrix}$$

Scale

$$\begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \begin{bmatrix} 3 & 6 & 9 \\ -2 & 4 & 8 \\ 0 & 3 & 6 \end{bmatrix} \quad \frac{1}{3}R_1 \rightarrow R_1 \quad \begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & 8 \\ 0 & 3 & 6 \end{bmatrix}$$

Add

$$\begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & 8 \\ 0 & 3 & 6 \end{bmatrix} \quad R_2 + 2R_1 \rightarrow R_2 \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 8 & 14 \\ 0 & 3 & 6 \end{bmatrix}$$

Solving Systems of Equations with Matrices

- Put coefficients and values into matrix
- Solve to *reduced row-echelon form*