Exercise 3.2, 3.3(a-b), 3.6 and 3.7

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3.2

Verify that the generator polynomial g(X) = 1 + X + X2 + X3 generates a binary cyclic code Ccyc(8, 5) and determine the code polynomial for the message vector m = (10101) in systematic form.

```
clear; clc;
g = [1 1 1 1];
```

f(n) = X^n+1 = X^8+1

```
f = [1 0 0 0 0 0 0 0 1];
```

```
[qu re] = gfdeconv(f, g)
```

```
qu =

     1     1     0     0     1     1


re =

     0
```

Because the remainder of f/g is 0, the polynomial g generates a Ccyc(8,k) . r can be obtained by inspecting g. k can now be verified:

```
r = 3;
n = 8;
k = n - r
```

```
k =

     5
```

Since k = 5 g generates Ccyc(8,5).

The following is based on ex. 3.2 from the book.

```
m = [1 0 1 0 1];

Xr = [0 0 0 1];

XrMX = gfconv(Xr, m)

[qu p] = gfdeconv(XrMX, g)

cX = mod([ p zeros(1, n - length(p))] + XrMX, 2)
```

```
XrMX =

     0     0     0     1     0     1     0     1


qu =

     0     1     1     1     1


p =

     0     1


cX =

     0     1     0     1     0     1     0     1
```

Given cX the code polynomial is: c(X) = X + X^3 + X^5 + X^7

```
clear;
```

3.3

A binary linear cyclic code Ccyc(n, k) has code length n = 7 and generator polynomial g(X) = 1 + X2 + X3 + X4.

a) Find the code rate, the generator and parity check matrices of the code in systematic form, and its Hamming distance.

```
g = [1 0 1 1 1];
n = 7;
r = 4; % degree as seen from g
```

CodeRate

```
k = n - r;

codeRate = k/n
```

```
codeRate =
```

```
    0.4286
```

Generator matrix

```
g1 = gfconv([0 1],g);
g2 = gfconv([0 0 1],g);

G = [g 0 0 ; g1 0; g2]

GRowReduced = mod(rref(G),2)

GRowReducedSystematic = [GRowReduced(:,4 : 1 : end) GRowReduced(:,1 : 1 : 3)]
```

```
G =

     1     0     1     1     1     0     0
     0     1     0     1     1     1     0
     0     0     1     0     1     1     1


GRowReduced =

     1     0     0     1     0     1     1
     0     1     0     1     1     1     0
     0     0     1     0     1     1     1


GRowReducedSystematic =

     1     0     1     1     1     0     0
     1     1     1     0     0     1     0
     0     1     1     1     0     0     1
```

Parity matrix

```
P = GRowReducedSystematic(:,1 : 1 : 4)

H = [eye(n-k) P']
```

```
P =

     1     0     1     1
     1     1     1     0
     0     1     1     1


H =

     1     0     0     0     1     1     0
     0     1     0     0     0     1     1
```

```
     0     0     1     0     1     1     1
     0     0     0     1     1     0     1
```

Hamming distance

```
hamDist = gfweight(GRowReducedSystematic)
```

```
hamDist =

     4
```

b) If all the information symbols are ‘1’s, what is the corresponding code vector? Based on example 3.2, so the resulting code vector is systematic.

```
m = [1 1 1];

Xr = [0 0 0 0 1];

XrMX = gfconv(Xr, m)

[qu p] = gfdeconv(XrMX, g)

cX = mod([ p zeros(1,n - length(p))] + XrMX, 2)

clear;
```

```
XrMX =

     0     0     0     0     1     1     1

qu =

     0     0     1

p =

     0     0     1

cX =

     0     0     1     0     1     1     1
```

3.6

a) Determine the table of code vectors of the binary linear cyclic block code Ccyc(6, 2) generated by the polynomial g(X) = 1 + X + X3 + X4.

```
g = [1 1 0 1 1];
n = 6;
k = 2;

M = [0 0; 1 0; 0 1; 1 1]


        M =

             0     0
             1     0
             0     1
             1     1
```

These code vectors are not in systematic form.

```
C1 = gfconv(M(1,:), g);
C2 = gfconv(M(2,:), g);
C3 = gfconv(M(3,:), g);
C4 = gfconv(M(4,:), g);

C = [C1 0; C2 0; C3; C4]


        C =

             0     0     0     0     0     0
             1     1     0     1     1     0
             0     1     1     0     1     1
             1     0     1     1     0     1
```

b) Calculate the minimum Hamming distance of the code, and its error- correction capability.

The hamming distance is the minimum weight of all non-zero code vectors.

```
Dmin = 4;
```

Error correction

```
Edetect = Dmin - 1

Ecorrect = floor((Dmin-1)/2)


clear;


        Edetect =

             3


        Ecorrect =
```

```
            1
```

3.7

A binary linear cyclic block code with a code length of n = 14 has the generator polynomial g(X) = 1 + X2 + X6.

```
g = [1 0 1 0 0 0 1];
n = 14;
r = 6; % degree as seen from g
```

a) Determine the number of information and parity check bits in each code vector.

information bits

```
k = n - r
```

```
k =

     8
```

parity bits

```
parBits = n - k
```

```
parBits =

     6
```

b) Determine the number of code vectors in the code.

```
numCodeVectors = 2^k
```

```
numCodeVectors =

   256
```

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