The Hilbert Substrate Framework: Persistent Heterogeneity in Information Propagation

Ben Bray

December 25, 2025

Abstract

Classical structure persists in physical systems despite underlying unitary dynamics that, in principle, permit widespread delocalization of information. Understanding how such persistence arises without introducing additional degrees of freedom remains an open problem. In this work, we study a minimal Hilbert-space substrate constrained only by unitarity, locality of interactions, and standard information-theoretic restrictions such as no-signaling and no-forgetting.

We introduce a diagnostic quantity, termed *heterogeneity in information propagation* (HIP), which measures spatial variation in the ability of localized perturbations to influence neighboring subsystems under unitary time evolution. HIP is defined operationally through reduced-state distinguishability and does not assume any underlying geometric or classical structure.

Using numerical simulations on finite interaction graphs, we find that HIP generically develops nonuniform structure over time. Moreover, this heterogeneity is not purely transient: certain regions of the substrate exhibit persistent dominance in information transport, as quantified by node-level intensity and rank-stability diagnostics. While absolute magnitudes fluctuate, the identity and ordering of high-intensity regions remain stable across extended temporal intervals.

These results indicate that persistent heterogeneity in information propagation can emerge dynamically from minimal constraints, without postulating additional fields or mechanisms. We suggest that HIP persistence serves as an order parameter distinguishing structured regimes from homogeneous or purely diffusive dynamics, providing a concrete diagnostic for emergent structure in Hilbert-space–based models.

1 Introduction

Unitary quantum dynamics generically disperses information across available degrees of freedom. Absent additional structure, local perturbations are expected to spread, entangle, and lose distinguishability over time. From this perspective, the persistence of localized or structured features—such as stable subsystems, records, or effective classical behavior—presents a nontrivial problem. Understanding how such persistence can arise without assuming classical spacetime, particles, or additional fields remains an open question in quantum foundations.

Many existing approaches address this issue by introducing supplementary structure at the outset, including preferred bases, external environments, geometric backgrounds, or explicit classical degrees of freedom. While often effective, these assumptions obscure which features of observed structure are fundamental and which are contingent. An alternative strategy is to begin with a minimal description—Hilbert space, unitary evolution, and basic information-theoretic constraints— and to ask what forms of structure are unavoidable under those conditions alone [1, 2].

The Hilbert Substrate Framework adopts this latter perspective. Rather than postulating classical entities, it treats the quantum state space itself as the primary arena and seeks operational diagnostics capable of identifying emergent structure directly within unitary dynamics. The guiding principle is conservative: new physical entities are not introduced unless required by observed behavior. Instead, attention is focused on measurable properties of information flow that may distinguish homogeneous dynamics from regimes exhibiting persistent organization.

In this work, we introduce and study one such diagnostic, termed *heterogeneity in information propagation* (HIP). HIP quantifies spatial variation in the ability of localized perturbations to influence neighboring subsystems over time. It is defined entirely in terms of reduced-state distinguishability under unitary evolution, using standard operational notions from quantum information theory [3, 2], and does not presuppose any underlying geometric or classical organization.

While heterogeneity in information propagation can arise transiently in finite systems, it is not *a priori* clear whether such effects persist or merely reflect short-lived fluctuations. The central question addressed here is therefore not whether HIP appears, but whether it exhibits temporal stability indicative of meaningful structure. By combining edge-level permeability measures with node-level persistence and rank-stability diagnostics, we show that HIP generically develops long-lived, nonuniform patterns under minimal assumptions.

The results presented here do not establish the emergence of spacetime, particles, or classical objects. Rather, they demonstrate that persistent heterogeneity in information transport can arise dynamically within a purely unitary substrate constrained by locality and no-signaling [4, 5]. This persistence provides a concrete, operationally defined distinction between homogeneous information flow and structured regimes, and serves as a candidate order parameter for emergent organization in Hilbert-space–based models.

2 Constraints

The Hilbert Substrate Framework is defined not by a specific microscopic Hamiltonian, but by a set of minimal operational constraints that any admissible substrate dynamics must satisfy. These constraints are not introduced to reproduce known physics by assumption, but to exclude classes of models that cannot support stable, observer-accessible structure.

2.1 Unitarity

Global evolution of the substrate is assumed to be unitary, in the standard quantum-mechanical sense [1]. This ensures conservation of total information and excludes fundamental loss or creation of degrees of freedom. Unitarity is treated here as a minimal consistency requirement rather than a dynamical principle.

2.2 No-Signaling

The substrate dynamics must respect no-signaling: operations performed on one subsystem cannot be used to transmit information instantaneously to spacelike-separated subsystems. This constraint is imposed operationally and is independent of any assumed spacetime background.

2.3 No-Forgetting

While local subsystems may decohere through interaction with their environment, information is not destroyed but redistributed. The framework therefore excludes dynamics that erase correlations without trace. Records may delocalize, but they must persist within the global state.

2.4 Locality of Information Propagation

Information propagation through the substrate is assumed to be constrained by finite effective speeds, in the sense that correlations cannot be established arbitrarily fast across the interaction graph. This requirement does not presuppose geometric locality, but rather bounds on the rate at which influence can spread.

2.5 Structural Stability

Finally, the framework requires that certain patterns of correlation persist over time under the allowed dynamics. Without such stability, no notion of effective subsystems, particles, or observers could be meaningfully defined.

3 Substrate Model

We consider a finite-dimensional quantum system composed of a collection of subsystems, each associated with a local Hilbert space. The total Hilbert space is given by the tensor product of these local spaces. No assumption is made regarding an underlying spatial geometry or classical background; all structure arises from the pattern of interactions specified below, consistent with standard formulations of composite quantum systems [1].

3.1 Interaction Graph

Locality of interactions is encoded by an undirected interaction graph. Each vertex corresponds to a subsystem, and an edge between two vertices indicates the presence of a direct coupling term in the Hamiltonian. The graph serves purely as an interaction topology and is not assumed to carry metric, dimensional, or geometric significance. This use of graphs as bookkeeping devices for interaction structure is standard in studies of locally interacting quantum systems [4, 5].

3.2 Hamiltonian Structure

The system evolves under a time-independent Hamiltonian consisting of local terms and pairwise interaction terms associated with the edges of the interaction graph. Schematically, the Hamiltonian may be written as

$$H = \sum_i H_i + \sum_{(i,j)\in E} H_{ij}, \tag{1}$$

where H_i acts on a single subsystem and H_{ij} couples subsystems connected by an edge. The specific operator content of these terms is chosen for simplicity and numerical tractability, and does not encode preferred bases or classical structure. This decomposition follows standard practice in lattice and network models of quantum dynamics [1].

3.3 System and Environment Partition

In some simulations, the collection of subsystems is partitioned into a distinguished set of "system" degrees of freedom and an auxiliary set of "environment" degrees of freedom. This partition is introduced solely to probe information redistribution and does not represent an external classical environment. All degrees of freedom are treated quantum mechanically, and the combined evolution remains unitary, in line with standard treatments of open-system diagnostics within a closed global system [2].

3.4 Initial States

Initial states are chosen from simple, low-complexity ensembles, including product states and single-excitation configurations. These choices are made to facilitate interpretation of information propagation and to avoid embedding structure in the initial conditions. Such ensembles are commonly used in studies of operator spreading and information transport in unitary quantum systems [4]. The qualitative behavior reported here is not tied to a specific choice of initial ensemble.

This substrate model defines the minimal setting required to study information propagation under unitary dynamics. Importantly, it introduces no preferred geometry, classical observables, or additional dynamical principles. Any persistent organization identified in subsequent sections must therefore arise from the interaction topology and constrained unitary evolution alone.

4 Heterogeneity in Information Propagation (HIP)

We introduce a diagnostic quantity that characterizes how unevenly information propagates across an interacting quantum system. We refer to this diagnostic as *heterogeneity in information propagation* (HIP). HIP is defined operationally, entirely within standard quantum mechanics and quantum information theory, and does not presuppose geometric, classical, or field-theoretic structure.

4.1 Directed Information Permeability

Consider a localized perturbation applied to subsystem i at an initial time. After unitary evolution for a time t, the influence of this perturbation on a neighboring subsystem j may be quantified by the distinguishability it induces in the reduced state of j. We define the directed information permeability from i to j at time t as

$$P_{i\to j}(t) = \sup_{\rho,\, O_i} \left\| \mathrm{Tr}_{\bar{j}}\Big(U(t)\, O_i \rho O_i^\dagger\, U^\dagger(t)\Big) - \mathrm{Tr}_{\bar{j}}\Big(U(t)\, \rho\, U^\dagger(t)\Big) \right\|_1 , \qquad (2)$$

where ρ ranges over a chosen ensemble of initial states, O_i denotes a local operator acting on subsystem i, $U(t) = e^{-iHt}$ is the unitary time-evolution operator, and $\| \cdot \|_1$ denotes the trace norm. The trace norm provides an operational measure of state distinguishability and is standard in quantum detection and information theory [3, 2].

This quantity measures the maximal detectable effect of a local perturbation at i on subsystem j after time t, subject to the constraints imposed by the interaction structure and unitary dynamics.

4.2 Edge-Level Permeability

For subsystems connected by an edge in the interaction graph, we define a symmetric edge permeability as

$$P_{\{i,j\}}(t) = \max(P_{i\to j}(t),\, P_{j\to i}(t))\,. \qquad (3)$$

This symmetrization removes directional bias and yields a scalar quantity associated with each edge, characterizing its effectiveness as a channel for information propagation. Similar edge-based characterizations of influence and correlation transport appear in studies of locality-bounded quantum dynamics [4, 5].

4.3 HIP as a Variance Diagnostic

Heterogeneity in information propagation is quantified by the statistical dispersion of edge permeabilities across the interaction graph. Specifically, we define

$$\mathrm{HIP}(t) = \mathrm{Var}_{\{i,j\}\in E}\big[P_{\{i,j\}}(t)\big]\,, \qquad (4)$$

where the variance is taken over all edges in the graph. By construction, HIP vanishes when information propagates uniformly across all interaction pathways. Nonzero values indicate uneven propagation, with some edges acting as more effective channels than others.

Importantly, HIP does not measure the absolute magnitude of information flow, but rather the degree to which that flow is spatially nonuniform. This distinction separates HIP from global measures of entanglement or entropy, which are insensitive to transport structure.

4.4 Interpretation

HIP is a diagnostic quantity rather than a dynamical variable. It introduces no new degrees of freedom and does not modify the underlying unitary evolution. Instead, it provides an operational measure of how constrained information transport becomes distributed across the substrate.

While nonzero HIP can arise transiently in finite systems due to interference effects, its significance depends on temporal behavior. The following section therefore examines whether HIP-induced structure persists over time, or whether it reflects only short-lived fluctuations.

5 Persistence as an Order Parameter

The presence of nonzero heterogeneity in information propagation does not, by itself, imply meaningful structure. In finite systems, spatial variation in information transport may arise transiently due to interference effects or short-time fluctuations. To distinguish such behavior from persistent organization, it is therefore necessary to examine the temporal stability of HIP-induced structure.

5.1 Node-Level Information Intensity

To probe persistence, we introduce a node-level diagnostic derived from the directed permeabilities defined in Sec. 4. For each subsystem i, we define the outgoing information intensity at time t as

$$I_i(t) = \sum_{j \in \mathcal{N}(i)} P_{i \to j}(t), \tag{5}$$

where the sum runs over subsystems directly coupled to i in the interaction graph. $I_i(t)$ quantifies the total influence that perturbations localized at i can exert on its neighbors after time t.

This quantity does not represent information stored at a node, but rather the effectiveness of that node as a conduit for information propagation within the substrate. Similar node-level summaries of influence and transport appear in analyses of operator spreading and correlation growth in locally interacting quantum systems [4, 5].

5.2 Temporal Persistence

We evaluate $I_i(t)$ across a sequence of times under fixed unitary evolution. While the absolute magnitudes of $I_i(t)$ generally fluctuate, we observe that a subset of nodes consistently exhibits elevated intensity relative to the rest of the system. In particular, nodes identified as high-intensity at early non-trivial times tend to remain dominant over extended temporal intervals.

This behavior is assessed without requiring intensities to remain constant. Instead, persistence is identified through the stability of relative ordering among nodes. Such rank-based criteria are commonly used to distinguish structural stability from transient fluctuations in dynamical systems and information-theoretic analyses [2].

5.3 Rank Stability

To quantify ordering stability, we compare the rank ordering of node intensities at different times. Once HIP becomes appreciable, the relative ordering of high-intensity nodes is found to remain largely unchanged, even as individual values fluctuate. This indicates that persistent pathways for information propagation emerge dynamically, rather than being washed out by continued unitary evolution.

Importantly, this persistence does not correspond to equilibration to a static configuration. Information continues to propagate throughout the system, but does so preferentially along stable channels determined by the interaction structure and dynamics. Persistence therefore appears at the level of identity and ordering, not frozen magnitude.

5.4 Interpretation

The emergence of persistent node-level structure suggests that HIP can serve as an order parameter distinguishing structured regimes from homogeneous or purely diffusive dynamics. Unlike global entanglement measures or coarse-grained entropy, HIP-based diagnostics are sensitive to how information moves through a system rather than how much information it contains.

These results do not imply the formation of classical objects or spacetime geometry. Rather, they demonstrate that minimal quantum and informational constraints are sufficient to support long-lived heterogeneity in information transport. Related ideas concerning robust emergent structure appear in broader studies of quantum matter and topological order [6], though the present work remains agnostic regarding specific physical realizations.

6 Visualization of HIP Structure

The diagnostics introduced above yield spatially and temporally resolved quantities defined on the interaction graph. To aid interpretation, we visualize these quantities directly on the graph structure, producing representations that make patterns of information propagation explicit while remaining agnostic about geometric interpretation. Visualization is employed here as an interpretive tool rather than as an independent source of evidence.

6.1 Edge-Level Permeability Fields

Figure 1 displays the distribution of edge-level permeability $P_{\{i,j\}}(t)$ at a representative intermediate time. Edges are colored according to their effective permeability, providing a visual map of how readily information propagates across different interaction pathways. Such visualizations are commonly used to illustrate transport and correlation structure in locally interacting quantum systems, where scalar summaries alone may obscure spatial variation [4, 5].

The appearance of nonuniform edge permeability indicates that information transport becomes heterogeneous under unitary evolution, even when microscopic interaction rules are homogeneous. Importantly, these plots do not assume or impose spatial geometry. The layout of the interaction graph is chosen solely for readability, and distances or angles in the figure carry no physical meaning.

6.2 Node-Level Intensity Maps

Complementary insight is provided by node-level visualizations based on the outgoing information intensity $I_i(t)$ defined in Sec. 5. In these representations, nodes are colored according to their total outgoing permeability, while edges are shown only to indicate interaction connectivity. Node-based summaries of influence are widely used in networked and many-body systems to highlight dominant pathways of interaction and transport [2].

High-intensity nodes in these maps should not be interpreted as localized information storage or classical objects. Rather, they identify subsystems whose perturbations exert disproportionately large downstream effects on neighboring degrees of freedom within the unitary dynamics.

6.3 Temporal Interpretation

Viewed across time, these visualizations illustrate the distinction between transient fluctuations and persistent structure. While local intensities and permeabilities vary continuously under unitary evolution, the identity of prominent nodes and pathways remains stable once HIP becomes appreciable. This persistence is not readily apparent from scalar diagnostics alone, but becomes clear when visualized in conjunction with the interaction topology.

The role of visualization in this context is therefore interpretive rather than demonstrative. By rendering information propagation directly on the substrate, these figures clarify how persistent heterogeneity manifests within ongoing unitary dynamics without introducing additional assumptions or geometric structure.

Taken together, these visualizations provide an intuitive representation of HIP and its temporal stability, complementing the quantitative diagnostics presented earlier and preparing the ground for the broader discussion that follows.

7 Discussion

The results presented in this work demonstrate that heterogeneous information propagation can arise and persist under minimal and widely accepted constraints. Starting from unitary dynamics governed by local interactions, we observe the emergence of long-lived, nonuniform patterns in information transport. These patterns are not imposed by initial conditions, geometric assumptions, or external structure, but develop dynamically within the Hilbert-space substrate itself.

A central observation is that persistence manifests at the level of identity and ordering rather than static magnitude. While local permeabilities and node intensities fluctuate continuously under unitary evolution, the subsystems that dominate information propagation remain stable across time. This behavior distinguishes persistent heterogeneity from transient interference effects commonly encountered in finite quantum systems. The result aligns with broader insights from studies of locality-bounded dynamics, in which information spreads subject to structural constraints rather than homogenizing instantaneously [4, 5].

Importantly, the diagnostics employed here do not rely on geometric interpretation. Although the interaction graph admits visual representations, these serve solely as tools for organizing and displaying information-theoretic quantities. The observed persistence therefore does not presuppose spatial locality in a classical sense, nor does it require an underlying metric structure. Any geometric interpretation must arise as a secondary inference rather than an input to the framework.

The present results also clarify what is *not* implied. Persistent heterogeneity in information propagation does not, by itself, constitute the emergence of particles, spacetime, or classical objects. Nor does it imply equilibration to a static configuration. Instead, the system remains fully dynamical, with structure appearing as stable pathways embedded within ongoing unitary evolution. This distinction parallels observations in other contexts where robust organization emerges without freezing of microscopic dynamics [6].

From a broader perspective, HIP persistence provides a concrete candidate for an order parameter distinguishing structured regimes from homogeneous or purely diffusive dynamics. Unlike global entanglement measures or coarse-grained entropy, HIP-based diagnostics are sensitive to how information moves through a system rather than how much information it contains. This focus on transport rather than storage complements existing information-theoretic approaches to emergent structure [7].

Several limitations of the present study should be noted. All simulations reported here involve finite systems and specific choices of interaction graphs and Hamiltonians. While the qualitative behavior appears robust across simple initial state ensembles, systematic exploration of scaling behavior and model dependence remains an open direction. In particular, whether HIP persistence sharpens, weakens, or undergoes qualitative transitions with increasing system size is an important question for future work.

Taken together, these considerations position the Hilbert Substrate Framework not as a complete theory of emergent classicality, but as a diagnostic approach for identifying when and how structure becomes unavoidable in unitary quantum systems. The results presented here establish that persistent heterogeneity in information propagation is one such unavoidable feature under minimal constraints, providing a firm foundation for further investigation.

8 Conclusion

In this work, we have examined how persistent structure can arise within a purely unitary quantum system subject only to minimal and widely accepted constraints. By focusing on information dynamics rather than assumed ontology, we introduced a diagnostic quantity—heterogeneity in information propagation (HIP)—that characterizes uneven transport of information across an interacting quantum substrate.

Through numerical investigation, we found that HIP not only develops dynamically but can also exhibit temporal persistence. This persistence manifests as stable identity and ordering of dominant information pathways, even as local magnitudes fluctuate under ongoing unitary evolution. Such behavior distinguishes structured regimes from homogeneous or purely diffusive dynamics without requiring imposed geometry, classical degrees of freedom, or additional dynamical principles.

The results presented here do not establish the emergence of spacetime, particles, or classical objects. Rather, they demonstrate that persistent heterogeneity in information transport is an unavoidable feature of constrained unitary dynamics. As such, HIP persistence provides a concrete, operationally defined order parameter for emergent organization within Hilbert-space–based models.

More broadly, the Hilbert Substrate Framework offers a diagnostic approach to foundational questions concerning the origin of structure in quantum systems. By identifying what must emerge under minimal assumptions, it complements more constructive or model-specific theories and provides a disciplined basis for future investigations into scaling behavior, robustness, and possible secondary interpretations.

References

[1] Michael A. Nielsen and Isaac L. Chuang. *Quantum Computation and Quantum Information*. Cambridge University Press, 10th anniversary edition edition, 2010.

[2] John Watrous. *The Theory of Quantum Information*. Cambridge University Press, 2018.

[3] Carl W. Helstrom. *Quantum Detection and Estimation Theory*. Academic Press, 1976.

[4] Elliott H. Lieb and Derek W. Robinson. The finite group velocity of quantum spin systems. *Communications in Mathematical Physics*, 28:251–257, 1972.

[5] Sergey Bravyi, Matthew B. Hastings, and Frank Verstraete. Lieb-robinson bounds and the generation of correlations and topological quantum order. *Physical Review Letters*, 97:050401, 2006.

[6] Xiao-Gang Wen. Choreographed entanglement dances: Topological states of quantum matter. *Science*, 363(6429):eaal3099, 2019.

[7] Wojciech H. Zurek. Quantum darwinism. *Nature Physics*, 5:181–188, 2009.

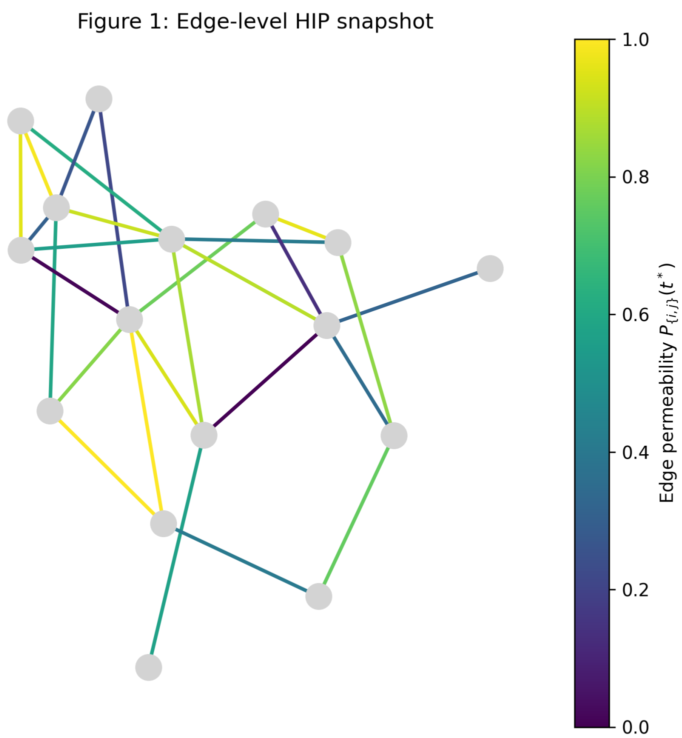

Figure 1: Edge-level heterogeneity in information propagation at an intermediate time under unitary evolution. Each edge is colored by its effective permeability, defined as the maximum distinguishability induced on a neighboring subsystem by a localized perturbation. The resulting pattern illustrates that information transport becomes spatially nonuniform, despite homogeneous microscopic rules and the absence of any imposed geometric or classical structure.

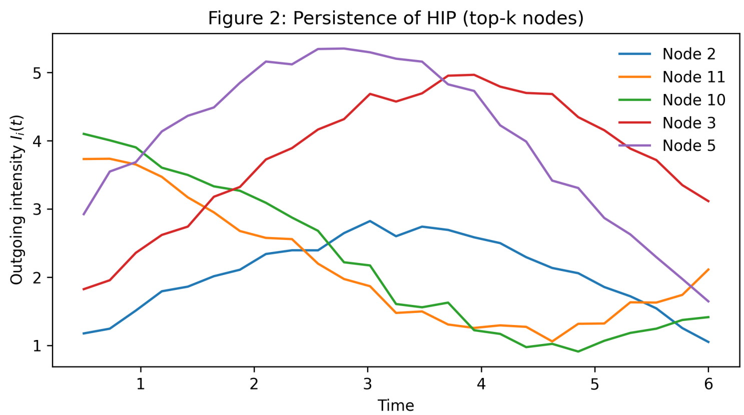

Figure 2: Temporal persistence of heterogeneous information propagation at the node level. Shown is the time evolution of outgoing information intensity for a subset of nodes selected by cumulative permeability. While absolute intensities fluctuate under unitary dynamics, the identity and relative ordering of high-intensity nodes remain stable across extended time intervals, indicating persistent structure in information transport rather than transient fluctuations.