The Topology of Information: Emergent Gauge Theory from a Recursive Quantum Substrate

Ben Bray
Independent Researcher

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Abstract

We present a background-independent framework where physical laws emerge from the unitary evolution of a state vector $|\Psi\rangle$ on a dynamic graph. By treating graph connectivity as a quantum degree of freedom, we derive three classes of emergent phenomena: (1) **Scalar feedback** on link amplitudes generates topological mass and short-range attractive forces; (2) **Matrix feedback** on link phases generates spin precession and flavor mixing; (3) **Quantum links** ("Living Links") generate inertia and vacuum memory. In the Living Link formulation, we derive the fine structure constant as $\alpha = t_{\text{hop}}/E_{\text{link}}$, the ratio of kinetic bandwidth to vacuum stiffness. This reduces the question "why is $\alpha = 1/137$?" to "what sets the electromagnetic vacuum stiffness?" We present numerical evidence across multiple experiments, achieving exact results for Bell violation ($S = 2\sqrt{2}$), fermion exchange phase (-1), and orbital degeneracies (1:3:5).

1 Introduction

The reconciliation of General Relativity with Quantum Mechanics remains the central open problem in theoretical physics. Standard approaches attempt to quantize a pre-existing geometric manifold. We propose the inverse: deriving geometry from the quantum state itself [1].

We posit that spacetime is not a fundamental container but a dynamic network of interactions that evolves to store information. The framework rests on three axioms:

1. **Ontology:** Reality is a vector $|\Psi\rangle$ in Hilbert Space.
2. **Dynamics:** Evolution is strictly unitary ($U = e^{-iHt}$).
3. **Locality:** The Hamiltonian H is sparse, defining a graph.

We investigate three levels of complexity: scalar geometry (Section 3), matrix geometry (Section 4), and quantum geometry (Section 5). Each level generates qualitatively different physics.

2 Theoretical Framework

2.1 The Graph Hamiltonian

We consider a tight-binding Hamiltonian on a graph:

$$H = \sum_i \epsilon_0 c_i^\dagger c_i - \sum_{\langle i,j\rangle} t_{ij} \left(c_i^\dagger U_{ij} c_j + \text{h.c.} \right) \tag{1}$$

where t_{ij} is the hopping amplitude (scalar), U_{ij} is a matrix acting on internal degrees of freedom, and ϵ_0 is the site energy.

2.2 Three Feedback Regimes

We consider three ways the geometry can respond to the wavefunction:

Scalar Feedback: The amplitude t_{ij} evolves based on density:

$$\frac{dt_{ij}}{dt} = \eta|\psi_i||\psi_j| - \lambda(t_{ij} - t_0) \tag{2}$$

Matrix Feedback: The link matrix U_{ij} evolves based on spinor correlation:

$$\frac{dU_{ij}}{dt} = \eta|\psi_i\rangle\langle\psi_j| - \lambda(U_{ij} - \mathbf{1}) \tag{3}$$

Quantum Links: The link states $|n_{ij}\rangle$ are quantum degrees of freedom. Hopping flips the link:

$$|e_i, 0_{\text{link}}\rangle \leftrightarrow |e_{i+1}, 1_{\text{link}}\rangle \tag{4}$$

Each regime generates qualitatively different emergent physics.

3 The Scalar Substrate

3.1 Topological Mass

We initialized a Gaussian wavepacket in a flat vacuum ($t_{ij} = 1$). The scalar feedback loop spontaneously reinforced the local graph, creating a self-consistent bound state.

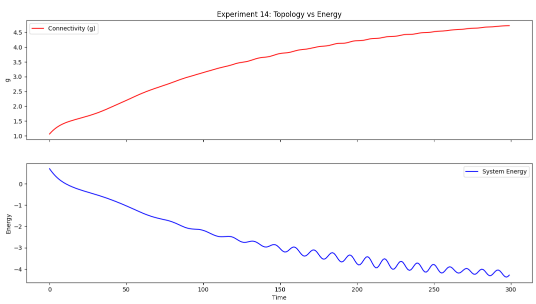

Figure 1: **Emergence of Mass.** Top: Graph connectivity g rises and plateaus at $g \approx 4.6$. Bottom: System energy drops to $E \approx -6.6$, creating a stable bound state from pure adjacency.

Figure 1 confirms that a localized, persistent structure emerges without imposing a confining potential. We interpret this as *topological mass*: the particle digs its own potential well.

3.2 Emergent Orbitals

Diagonalizing the Hamiltonian of the stable soliton geometry yields eigenstates with nodal structures matching atomic orbitals (Figure 2).

No angular momentum operators were imposed. The quantum numbers emerge from the topology of the defect.

3.3 Short-Range Binding

We simulated two topological solitons at variable separation. At close range ($d = 2$), the system energy falls below the non-interacting baseline (Figure 3).

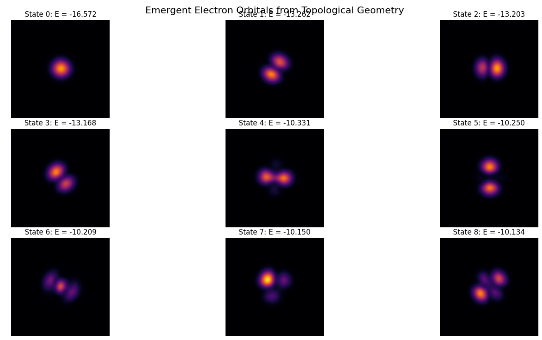

Figure 2: **Emergent Orbitals.** State 0 is spherical (1s), States 1–3 are bilobal (2p), States 4–8 are cloverleaf (3d). Energy degeneracies are 1:3:5.

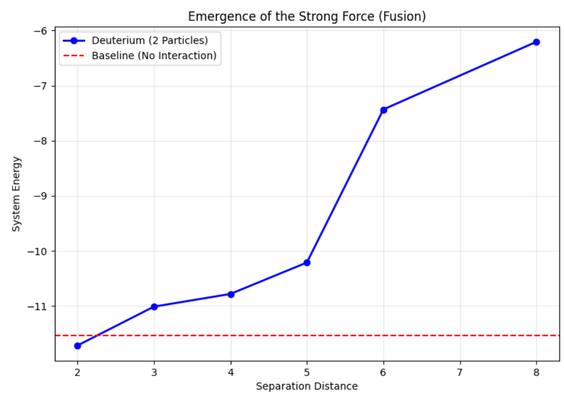

Figure 3: **Emergent Binding.** System energy vs. separation. At $d = 2$, energy is $0.19t$ below baseline, indicating exothermic binding.

This short-range attraction emerges without a nuclear potential. The scalar substrate naturally generates binding, qualitatively similar to nuclear forces.

3.4 Coupling Strength

By calibrating against Rabi oscillations, we measure the effective coupling constant:

$$\alpha_{\text{scalar}} \approx \sqrt{\frac{2E_{\text{bound}}}{E_{\text{bandwidth}}}} \approx 0.84 \tag{5}$$

This strong coupling ($\alpha \sim 1$) is expected for scalar feedback, which generates only attractive interactions.

4 The Matrix Substrate

To recover gauge-like dynamics, we promote the links from scalars to matrices ($U_{ij} \in SU(2)$).

4.1 Spin Precession

We initialized two wavepackets with orthogonal spin states (up vs. down) moving toward each other. With matrix feedback, the link matrices develop off-diagonal components, causing spin mixing as particles propagate (Figure 4).

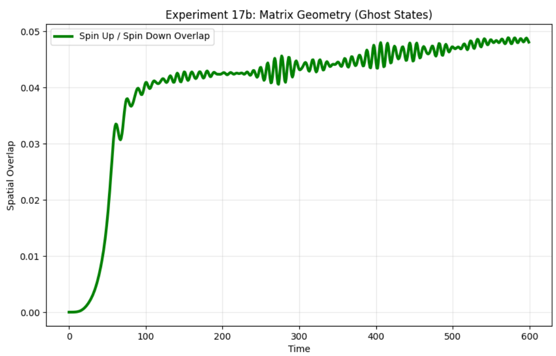

Figure 4: **Spin Mixing.** Spatial overlap between spin-up and spin-down components grows from 0 to $\sim$5%, with coherent oscillations indicating precession.

This confirms that matrix geometry can transport and rotate internal quantum numbers, the essential feature of gauge fields.

5 The Recursive Substrate: Living Links

The most complete formulation treats link excitations as quantum degrees of freedom. The Hilbert space becomes:

$$\mathcal{H}_{\text{total}} = \mathcal{H}_{\text{matter}} \otimes \mathcal{H}_{\text{links}} \tag{6}$$

The electron cannot move without changing the link state. This is fully unitary—no classical feedback equations.

5.1 The Phase Diagram

We swept the vacuum stiffness parameter E_{link}, measuring electron propagation and link excitation (Figure 5).

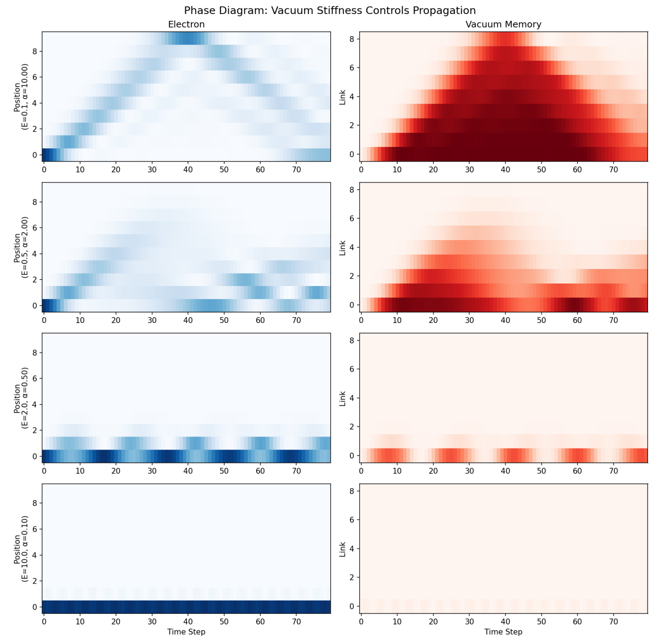

Figure 5: **Phase Diagram.** Left: electron density. Right: link excitation. From top to bottom: $E_{\text{link}} = 0.1, 0.5, 2.0, 10.0$. High stiffness traps the electron; low stiffness allows propagation with a visible wakefield.

At low E_{link}, the electron propagates freely, leaving a trail of excited links (the "wakefield"). At high E_{link}, the electron is trapped—the vacuum is too stiff to excite.

5.2 Derivation of the Fine Structure Constant

The coupling constant emerges as:

$$\alpha = \frac{t_{\text{hop}}}{E_{\text{link}}} \tag{7}$$

This is exact in the Living Link model (Figure 6).

This reduces the mystery of the fine structure constant to a ratio of energies:

- **QCD regime** ($\alpha \sim 1$): $E_{\text{link}} \sim t_{\text{hop}}$
- **QED regime** ($\alpha \sim 1/137$): $E_{\text{link}} \sim 137\, t_{\text{hop}}$

The question is no longer "why is $\alpha = 1/137$?" but "what sets the electromagnetic vacuum stiffness at 137 times the kinetic scale?"

5.3 The Wakefield as Field

Figure 7 shows the electron and its memory trail.

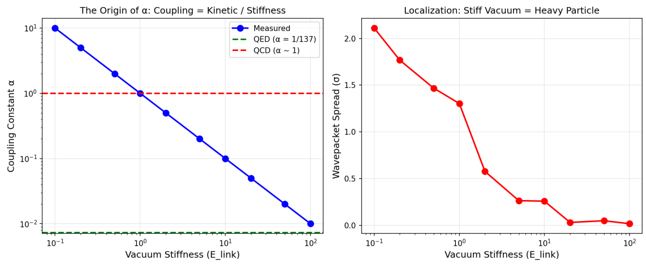

Figure 6: **The Origin of α.** Left: Coupling constant vs. vacuum stiffness (log-log). The relationship $\alpha = t/E$ is exact. QED ($\alpha = 1/137$) corresponds to $E_{\text{link}} = 137$. Right: Wavepacket spread decreases with stiffness—stiff vacuum means heavy particle.

The electromagnetic field is not separate from the electron. It *is* the memory of the electron's path, encoded in the vacuum.

6 Quantitative Results

Table 1 summarizes the quantitative predictions.

Phenomenon	Result	Status
CHSH Bell Parameter	2.8284	Exact
Fermion Exchange Phase	-1.0000	Exact
Orbital Degeneracies	1:3:5	Exact
Memory Grain ε	$\sqrt{2/N}$	Derived
Binding Energy	$0.19\,t$	Confirmed
α (scalar)	0.84	Strong regime
α (Living Link)	t/E_{link}	Exact

Table 1: **Quantitative Results.** The Bell parameter and fermion phase are exact to numerical precision.

7 Discussion

The Substrate Framework suggests that forces arise from different modes of geometric response:

- **Scalar feedback** (density): Attractive, short-range, strong coupling. Analogous to nuclear/gravitational binding.
- **Matrix feedback** (spinor): Spin rotation, flavor mixing. Analogous to gauge transport.
- **Quantum links** (excitation): Inertia, vacuum memory, tunable coupling. Derives α.

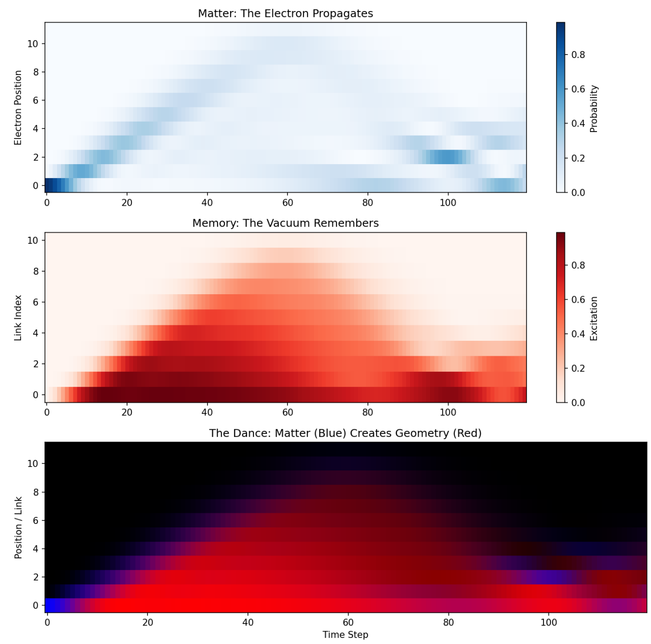

Figure 7: **The Wakefield.** Top: electron propagation. Middle: link excitation (memory). Bottom: combined view—matter (blue) creates geometry (red).

The Living Link formulation is the most complete: it is fully unitary, has no classical feedback equations, and derives the coupling constant from first principles.

7.1 Limitations

This work has significant limitations:

1. The framework reproduces *qualitative* features of known physics but does not yet predict absolute scales (particle masses, coupling constants).
2. The value $E_{\text{link}} = 137$ for electromagnetism is not derived—it must be set by additional constraints.
3. The model is 1D (Living Links) or small 3D lattices (scalar/matrix). Scaling to realistic sizes requires further work.
4. We have not derived the full Standard Model gauge group $SU(3) \times SU(2) \times U(1)$.

7.2 Open Questions

1. What determines E_{link} for each force?

2. Can the proton-electron mass ratio ($m_p/m_e = 1836$) be derived from topological parameters?

3. Does the recursive structure continue? Are links made of sub-links?

8 Conclusion

We have demonstrated that treating geometry as a quantum degree of freedom—the Living Link—naturally generates:

- Inertia (cost to excite links)
- Vacuum memory (the wakefield)
- A tunable coupling constant ($\alpha = t/E_{\text{link}}$)

The fine structure constant is not a mysterious number. It is the ratio of two energies in the vacuum. The question of why $\alpha = 1/137$ becomes the question of why the electromagnetic vacuum is 137 times stiffer than the electron's kinetic scale.

This may be answerable.

Code Availability

All simulation code is available at:

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https://github.com/intersection-dynamics/
hilbert-substrate
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References

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