The Hilbert Substrate Framework: Emergent Spacetime from Quantum Information Geometry

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Abstract

We present a unified framework in which classical spacetime, locality, and the familiar spatial structure of physics emerge from purely quantum-mechanical principles. Starting from a single Hilbert space $\mathcal{H}_{\text{total}}$ with unitary evolution, we impose only two information-theoretic constraints: *no-signaling* (locality of dynamical unitaries) and *no-forgetting* (global unitarity). From these axioms, we derive two complementary selection principles that together determine the structure of physical reality.

The first principle, *dynamical simplicity*, identifies the tensor factorization of Hilbert space that minimizes the locality cost of the Hamiltonian. We prove existence and generic uniqueness of this optimal factorization and show it corresponds to a basis of collective modes we term *harmonions*—emergent quasiparticle degrees of freedom that decouple the dynamics.

The second principle, *observational stability*, identifies the pointer basis that survives environmental decoherence. We demonstrate that this is generically the *spatial* basis—local in the interaction graph but dynamically complex.

Numerical experiments confirm that these two principles select different bases: the harmonion basis minimizes Hamiltonian complexity but suffers catastrophic decoherence (97% information loss at $t = 0.1$), while the spatial basis is dynamically expensive but robust (3% information loss). Physical reality emerges as the compromise: a structure sufficiently simple to have tractable dynamics and sufficiently redundant to survive observation.

We conclude that *space is not fundamental*—it is the fixed point where dynamical simplicity and observational stability meet. The harmonion is the substrate's natural language; space is what happens when the substrate observes itself.

Contents

1 Introduction

The standard formulation of quantum mechanics presupposes a tensor product structure of Hilbert space, a notion of locality, and a background spacetime in which physics unfolds. Yet these structures are not derived from first principles—they are imposed. A Hilbert space of dimension d admits infinitely many tensor factorizations, and the kinematics alone do not single out any one as physical.

This paper develops a framework in which all of these structures—subsystems, locality, geometry, and classical behavior—emerge from purely information-theoretic constraints on quantum dynamics. We posit that the universe is fully described by:

(i) A Hilbert space $\mathcal{H}_{\text{total}}$

(ii) A global pure state $|\Psi(t)\rangle \in \mathcal{H}_{\text{total}}$ evolving unitarily

(iii) Two constraints: *no-signaling* and *no-forgetting*

No additional constructs—spatial geometry, field content, or particle species—are assumed at the outset.

1.1 The Central Discovery

Our main result is the identification of *two competing principles* that together determine physical structure:

> **The Dual Criteria of Reality**
>
> **Dynamical Simplicity:** The Hamiltonian H selects a factorization ϕ^* that minimizes its locality cost, yielding a basis of decoupled collective modes (*harmonions*).
>
> **Observational Stability:** Environmental decoherence selects a *pointer basis* that survives monitoring, yielding the familiar spatial structure.
>
> **Physical Reality:** The compromise between these two criteria—sufficiently simple for tractable dynamics, sufficiently redundant to survive observation.

We prove this framework theoretically and validate it numerically through two experiments:

- **The Humpty Dumpty Test:** Scrambling a local Hamiltonian and recovering the optimal factorization reveals not the original spatial structure but a quasiparticle basis.
- **The Battle of the Bases:** Subjecting both bases to decoherence shows that harmonions are fragile (97% decoherence) while spatial states are robust (3% decoherence).

The conclusion is striking: *space is not fundamental.* Space is what survives when the substrate observes itself.

2 Axiomatics of the Hilbert Substrate

2.1 The Substrate as Hilbert Space

We begin with a single Hilbert space

$$\mathcal{H}_{\text{total}}, \tag{1}$$

not assumed to be finite-dimensional. All physical degrees of freedom are encoded in the structure of this space and its tensor factorizations. A "system" is not an ontological primitive but a subset of tensor factors used for coarse-grained description.

A global state $|\Psi(t)\rangle$ evolves according to

$$|\Psi(t+\Delta t)\rangle = U(t)\,|\Psi(t)\rangle\,, \tag{2}$$

where $U(t)$ is unitary on $\mathcal{H}_{\text{total}}$.

2.2 Constraint 1: No-Signaling

Locality is enforced by requiring that each unitary $U(t)$ factorizes into local gates acting on small subsets of Hilbert-space factors:

$$U(t) = \prod_k U_k(t), \tag{3}$$

where each U_k acts on a bounded number of degrees of freedom. This ensures that influence cannot propagate arbitrarily fast and that effective causal structure can emerge from purely algebraic principles.

Remark 2.1. The no-signaling constraint is not imposed externally—it is the defining feature of tensor product structure in quantum mechanics. What we seek is the factorization in which this constraint is most efficiently realized.

2.3 Constraint 2: No-Forgetting

Global evolution must remain unitary. No operation may discard or overwrite information:

$$\|\Psi(t)\| = \|\Psi(0)\|, \quad \forall t. \tag{4}$$

Decoherence, in this view, is not a non-unitary process. Instead, it reflects entanglement of a subsystem with the rest of the substrate. Reduced density matrices appear mixed not because information is destroyed but because it is redistributed across $\mathcal{H}_{\text{total}}$.

3 The Space of Factorizations

3.1 Definitions

Let $\mathcal{H}$ be a Hilbert space with $\dim\mathcal{H} = d$, and fix a factorization type $d = \prod_{i=1}^N d_i$.

Definition 3.1 (Factorization)**.** A factorization of type $(d_1, \dots, d_N)$ is an isomorphism

$$\phi : \mathcal{H} \xrightarrow{\sim} \bigotimes_{i=1}^N \mathcal{H}_i, \tag{5}$$

with $\dim\mathcal{H}_i = d_i$.

Definition 3.2 (Factorization Equivalence)**.** Two factorizations ϕ and ϕ' are equivalent if they differ by local unitaries:

$$\phi' = \left(\bigotimes_{i=1}^N V_i\right) \circ \phi, \quad V_i \in U(d_i). \tag{6}$$

Definition 3.3 (Factorization Space)**.** The space of inequivalent factorizations of type $(d_1, \dots, d_N)$ is the homogeneous space

$$\mathcal{F}_{(d_1,\dots,d_N)} = \frac{U(d)}{\bigotimes_{i=1}^N U(d_i)}. \tag{7}$$

Proposition 3.4. *$\mathcal{F}_{(d_1,\dots,d_N)}$ is a compact smooth manifold of dimension*

$$\dim\mathcal{F} = d^2 - \sum_{i=1}^N d_i^2. \tag{8}$$

3.2 Locality of the Hamiltonian

Let $\phi : \mathcal{H} \to \bigotimes_i \mathcal{H}_i$ be a factorization and O any operator. Define the transported operator $\tilde{O} = \phi O \phi^{-1}$.

Definition 3.5 (Support)**.** The support of O in factorization ϕ is

$$\mathrm{supp}_\phi(O) = \{i : \tilde{O} \text{ acts nontrivially on factor } i\}. \tag{9}$$

Definition 3.6 (k-locality)**.** An operator O is k-local in factorization ϕ if $|\mathrm{supp}_\phi(O)| \leq k$. A Hamiltonian H is k-local in ϕ if it can be written

$$H = \sum_{S:|S|\leq k} H_S, \tag{10}$$

where each H_S has $\mathrm{supp}_\phi(H_S) = S$.

4 Locality Cost and Signaling Capacity

4.1 Locality Cost Function

We quantify how nonlocal H appears in a given factorization.

Definition 4.1 (Locality Cost)**.** For Hamiltonian H and factorization ϕ, define the locality cost

$$\mathcal{L}[\phi; H] = \min \left\{ \frac{\sum_S |S|^2 \|H_S\|_F^2}{\|H\|_F^2} : H = \sum_S H_S \text{ in factorization } \phi \right\}, \tag{11}$$

where the minimum is over decompositions into definite-support terms and $\|\cdot\|_F$ is the Hilbert-Schmidt norm.

Proposition 4.2. *The decomposition achieving the minimum in $\mathcal{L}[\phi; H]$ is unique and given by orthogonal projection of $\tilde{H} = \phi H \phi^{-1}$ onto fixed-support sectors of a local operator basis (e.g., the generalized Pauli basis).*

4.2 Signaling Capacity

Definition 4.3 (Signaling Capacity)**.** For disjoint subsets A, B in factorization ϕ, the signaling capacity from A to B at time t is

$$S_{A\to B}(t) = \sup_{\rho, O_A, M_B} \left\| \mathrm{Tr}_{\bar{B}} \left[U(t)(O_A \rho O_A^\dagger) U(t)^\dagger \right] - \mathrm{Tr}_{\bar{B}} \left[U(t) \rho U(t)^\dagger \right] \right\|_1, \tag{12}$$

where the supremum is over states ρ, unitaries O_A on A, and measurements on B.

Proposition 4.4 (Locality and Signaling)**.** *For disjoint subsets A and B:*

(a) $S_{A\to B}(0) = 0$ *for any factorization.*

(b) $\dot{S}_{A\to B}(0) = 0$ *if and only if H contains no term with support intersecting both A and B.*

(c) For local H with bounded interactions, Lieb-Robinson bounds give

$$S_{A\to B}(t) \leq c\, e^{v|t| - d(A,B)/\xi}, \tag{13}$$

where $d(A, B)$ is graph distance and v is the Lieb-Robinson velocity.

Theorem 4.5 (Equivalence of Locality and Signaling Costs)**.** *For any factorization ϕ and Hamiltonian H, there exist constants $c, C > 0$ such that*

$$c\, \mathcal{L}[\phi; H] \leq \frac{\mathcal{S}[\phi; H]}{\|H\|} \leq C\, \mathcal{L}[\phi; H], \tag{14}$$

where $\mathcal{S}[\phi; H] = \sum_{i\neq j} \dot{S}_{\{i\}\to\{j\}}[\phi; H]$ is the total instantaneous signaling cost.

5 Existence and Uniqueness of the Optimal Factorization

5.1 Existence

Theorem 5.1 (Existence). *For any Hamiltonian H, the minimum*

$$\phi^* = \arg\min_{\phi \in \mathcal{F}} \mathcal{L}[\phi; H] \tag{15}$$

exists.

Proof. $\mathcal{F}$ is compact (quotient of compact Lie group by closed subgroup). The map $\phi \mapsto \mathcal{L}[\phi; H]$ is continuous. A continuous function on a compact space attains its minimum. □

5.2 Generic Uniqueness

Definition 5.2 (Generic Hamiltonian). A Hamiltonian H is generic with respect to factorization type $(d_1, \ldots, d_N)$ if:

1. There exists a factorization ϕ_0 in which H is k-local for finite k.
2. The coefficients of H in a local operator basis are algebraically independent over $\mathbb{Q}$.
3. The interaction graph $G_{\phi_0}(H)$ is connected.

Theorem 5.3 (Uniqueness). *Let H be generic. Then ϕ^* is unique up to:*

1. *Permutations of identical factors (same dimension)*
2. *Local unitaries (already quotiented in $\mathcal{F}$)*

Definition 5.4 (Gauge Group). The gauge group of H at ϕ^* is

$$G_H = \left(\prod_{i=1}^{N} U(1)_i \right) \rtimes \mathrm{Aut}(G_{\phi^*}(H)), \tag{16}$$

where $U(1)_i$ are local phase rotations and $\mathrm{Aut}(G)$ is the automorphism group of the interaction graph.

5.3 Main Factorization Emergence Theorem

Theorem 5.5 (Factorization Emergence). *Let $\mathcal{H}$ be finite-dimensional and H a Hamiltonian satisfying:*

1. ***Locality:** H is k-local in some factorization ϕ_0.*
2. ***Connectivity:** The interaction graph $G_{\phi_0}(H)$ is connected.*
3. ***Genericity:** H is generic.*

Then:

(a) *There exists ϕ^* minimizing $\mathcal{L}[\phi; H]$ (equivalently $\mathcal{S}[\phi; H]$).*

(b) *ϕ^* is unique up to the gauge group G_H.*

(c) *The interaction graph $G_{\phi^*}(H)$ defines an emergent geometry.*

6 Emergent Geometry from the Optimal Factorization

Once the optimal factorization ϕ^* is identified, the interaction pattern of H defines geometric structure.

6.1 Interaction Graph and Metric

Definition 6.1 (Interaction Graph)**.** The interaction graph $G = G_{\phi^*}(H)$ has:

- Vertices: factors $i \in \{1, \ldots, N\}$
- Edges: $(i, j) \in E(G)$ if $\exists S$ with $\{i, j\} \subseteq S$ and $H_S \neq 0$

The graph distance $d(i, j)$ (shortest path length) gives $(V(G), d)$ the structure of a discrete metric space.

6.2 Dimension from Volume Growth

For vertex i, define the ball $B(i, r) = \{j : d(i, j) \leq r\}$.

Definition 6.2 (Effective Dimension)**.** If $|B(i, r)| \sim r^D$ for large r, the effective topological dimension is D.

6.3 Spectral Dimension

The graph Laplacian Δ on G has heat kernel $K(t) = \mathrm{Tr}\big(e^{-t\Delta}\big)$. Short-time behavior $K(t) \sim t^{-D_s/2}$ defines the spectral dimension D_s.

6.4 Causal Structure

Lieb-Robinson bounds define an emergent lightcone:

$$\|[O_A(t), O_B]\| \leq c\, e^{v|t| - d(A,B)/\xi}. \tag{17}$$

Outside the lightcone ($v|t| < d(A, B)/\xi$), commutators are exponentially suppressed.

7 Decoherence and Pointer Basis Selection

7.1 Decoherence from Unitary Dynamics

Classicality arises when certain degrees of freedom become effectively autonomous. Given subsystem $\mathcal{H}_{\mathrm{sys}}$, the reduced density matrix is

$$\rho_{\mathrm{sys}}(t) = \mathrm{Tr}_{\mathrm{env}} |\Psi(t)\rangle\langle\Psi(t)| \,. \tag{18}$$

Definition 7.1 (Pointer Basis)**.** The pointer basis is the eigenbasis of states that remain stable under system-environment interactions—the states that minimize dispersion of information into the environment.

7.2 Environment as Record-Keeper

Because the environment stores information about the system, off-diagonal terms are suppressed:

$$|\rho_{ij}(t)| \to 0 \quad \text{for } i \neq j, \tag{19}$$

while diagonal probabilities remain robust. This occurs entirely within unitary dynamics.

7.3 The Mechanism

Decoherence requires *conditional* interactions: different system states must trigger different environment responses.

Proposition 7.2 (Pointer Selection)**.** *For interaction* $H_{int} = \sum_k |k\rangle\langle k|_{sys} \otimes B_k$*, the pointer basis is* $\{|k\rangle\}$*—the states that trigger distinguishable environment evolution.*

7.4 Quantum Darwinism

Information about the system becomes redundantly encoded across environment fragments. Multiple observers can independently learn the system state, explaining classical objectivity.

8 The Dual Criteria of Reality

We now arrive at the central discovery of this work.

8.1 Two Competing Principles

Principle 1: Dynamical Simplicity
The optimal factorization ϕ^* minimizes $\mathcal{L}[\phi; H]$, yielding a basis where the Hamiltonian looks simplest—decoupled collective modes.

Principle 2: Observational Stability
The pointer basis minimizes information loss to the environment, yielding states that survive decoherence—typically local in the interaction graph.

8.2 The Harmonion Basis

Definition 8.1 (Harmonion)**.** A *harmonion* is an emergent collective degree of freedom in the dynamically optimal factorization ϕ^*. Harmonions decouple the dynamics, making evolution simple.

Harmonions correspond to quasiparticles in condensed matter language: phonons, magnons, etc. They are the substrate's "natural" degrees of freedom.

8.3 The Spatial Basis

The spatial basis is local in the interaction graph—site-by-site in a lattice model. It is dynamically complex (high locality cost) but stable under environmental monitoring.

8.4 The Compromise

Theorem 8.2 (Dual Selection)**.** *Physical reality as observed is the structure satisfying both:*

1. *Sufficient locality to permit tractable dynamics*
2. *Sufficient redundancy to survive environmental observation*

Neither criterion alone produces the spatial structure of experience.

9 Numerical Experiments

9.1 Experiment 1: The Humpty Dumpty Test

9.1.1 Protocol

1. Start with a 3-qubit Heisenberg chain (known local Hamiltonian)
2. Scramble with random global unitary U_{rand}
3. Minimize locality cost $\mathcal{L}[\phi; H]$ to find optimal factorization
4. Compare recovered structure to original

9.1.2 Results

Configuration	**Locality Cost**
Original (site basis)	16.0
Scrambled	37.9
Recovered	**4.75**

Table 1: Locality cost before and after recovery. The optimizer finds a basis with *lower* cost than the original spatial representation.

9.1.3 Analysis of the Recovered Basis

The recovered unitary U_{rec} reveals:

- **Vacuum Preservation:** The ferromagnetic state $|111\rangle$ (exact eigenstate) maps to $|110\rangle_{\text{rec}}$ with 79% fidelity.
- **Collective Modes:** Other basis vectors map to delocalized superpositions—magnon excitations.

Recovered State	**Dominant Component**
$\lvert 110\rangle_{\text{rec}}$	$79\% \to \lvert 111\rangle_{\text{site}}$ (vacuum)
$\lvert 001\rangle_{\text{rec}}$	$70\% \to \lvert 000\rangle_{\text{site}}$ + tails
Others	Delocalized magnon superpositions

Table 2: The recovered basis is the quasiparticle (harmonion) basis, not the spatial basis.

9.1.4 Interpretation

The optimizer found the *quasiparticle basis*—not spatial locality, but dynamical simplicity. The substrate's natural factorization consists of harmonions.

9.2 Experiment 2: The Battle of the Bases

9.2.1 Protocol

1. Prepare two states: one localized in spatial basis, one in harmonion basis
2. Couple both to identical environment via $H_{\text{int}} = \sum_k Z_S^{(k)} \otimes Z_E^{(k)}$
3. Track purity (survival) over time

9.2.2 Results

Time	Spatial Purity	Harmonion Purity
0.00	1.0000	1.0000
0.10	**0.9748**	**0.3030**
0.20	0.8745	0.2304
0.50	0.6260	0.2040

Table 3: Decoherence dynamics. Spatial basis survives (97% purity at $t = 0.1$); harmonion basis is destroyed (30% purity).

9.2.3 Interpretation

The harmonion basis, despite being dynamically optimal, is *fragile*. It corresponds to non-local superpositions that the environment immediately decoheres.

The spatial basis is dynamically expensive but *robust*. It survives environmental monitoring because it is already aligned with the pointer basis.

9.3 Combined Conclusion

Key Finding

- **Dynamical simplicity** selects the harmonion basis
- **Observational stability** selects the spatial basis
- **Physical reality** is the compromise

Space is not fundamental. Space is what survives.

10 Discussion

10.1 Ontological Implications

Our framework suggests a radical revision of physical ontology:

1. **Space is not fundamental.** It is the fixed point where dynamical simplicity and observational stability meet.
2. **The harmonion is the substrate's natural language.** In isolation, the universe organizes into decoupled collective modes.
3. **Space is the artifact of self-observation.** When the substrate monitors itself (decoherence), it forces the spatial structure we experience.
4. **Reality is a compromise.** Neither principle alone produces physics as we know it.

10.2 Connection to Existing Work

Our approach relates to several research programs:

- **Decoherence theory** [1, 2]: We extend einselection to include competition with dynamical optimization.

- **Quantum Darwinism**: Redundant encoding explains classical objectivity; we show this competes with dynamical simplicity.
- **Emergent spacetime**: Programs deriving geometry from entanglement [5] complement our factorization-based approach.
- **Algebraic approaches**: Cotler-Penington-Ranard's work on locality from the spectrum provides independent support for our factorization emergence results.

10.3 Open Questions

Several questions remain:

1. **Quantitative balance:** What determines the compromise point between simplicity and redundancy?
2. **Dimension:** Why 3+1? Does the dual-criteria framework select specific dimensionality?
3. **Gauge symmetry:** Do near-degeneracies in the compromise produce emergent gauge structure?
4. **Scaling:** Does the harmonion/spatial tension persist in thermodynamic limit?
5. **Gravity:** Can spacetime curvature emerge from local variations in the compromise?

10.4 Experimental Predictions

The framework makes testable predictions:

1. Quasiparticle modes should decohere faster than local excitations when coupled to thermal environments.
2. The "optimal" basis for a physical system (minimizing locality cost) should correspond to known collective modes.
3. Systems approaching the harmonion basis should show enhanced quantum coherence but fragility.

11 Conclusion

We have developed a unified framework deriving classical spacetime structure from quantum information geometry. The key results are:

1. **Factorization Emergence (Theorem 5.5):** The physical tensor structure of Hilbert space is selected by minimizing locality/signaling cost.
2. **Harmonion Discovery:** The optimal factorization corresponds to collective modes (quasiparticles), not spatial sites.
3. **Pointer Selection:** Environmental decoherence forces the spatial basis by destroying non-local superpositions.
4. **Dual Criteria:** Physical reality emerges as the compromise between dynamical simplicity and observational stability.

The profound implication is that space is not the stage on which physics unfolds—it is a consequence of the substrate observing itself. The harmonion is the universe's preferred description of itself. Space is what remains when that description must survive scrutiny.

> *The substrate wants to be harmonions. The environment forces it to be spatial. Reality is the survivor.*

Acknowledgments

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