Hilbert Substrate Framework 1.2

A Clarified, Non-Speculative Summary

Ben Bray

November 20, 2025

Abstract

This document presents a clarified and professional restatement of the Hilbert Substrate Framework. Version 1.2 restructures the material to clearly distinguish between (i) standard mathematical constructs, (ii) toy-model numerical experiments, and (iii) speculative interpretive hypotheses. No claims are made about the true structure of the Standard Model, spacetime, or gravity. All results are described strictly as features of finite Hilbert-space simulations.

1 Introduction

The Hilbert Substrate Framework explores how structured behavior can arise from finite-dimensional quantum systems. The goal is to probe how constraint structures—such as local Gauss operators, gauge-covariant hopping, and coherence or penalty terms—shape pattern formation inside a small quantum model.

In this version, the focus is on a clean separation between:

1. **Mathematical definitions:** the finite Hilbert spaces, operators, and Hamiltonians used;
2. **Numerical observations:** what small simulations actually compute and return;
3. **Interpretive hypotheses:** how one might view these structures in terms of "information flux" or exclusion-like behavior.

No claims are made that these toy models reproduce real particle physics or quantum field theory.

2 Mathematical Structure (Non-Speculative)

The core formalism uses standard tools from lattice gauge theory and quantum information. For concreteness, consider a 2×2 lattice of matter sites, coupled to Z_2 gauge degrees of freedom defined on links.

2.1 Hilbert-space factors

- **Matter (sites):** Each lattice site carries a four-dimensional local Hilbert space $\mathcal{H}_{\text{site}} \cong \mathbb{C}^4$, which can be interpreted as a coarse "spinor" structure (e.g. vacuum/spin and excited/spin states). The total matter space on N_{sites} sites is

$$\mathcal{H}_{\text{matter}} = \bigotimes_{s=1}^{N_{\text{sites}}} \mathcal{H}_{\text{site}}.$$

- **Gauge (links):** Each nearest-neighbor link carries a two-dimensional Hilbert space $\mathcal{H}_{\text{link}} \cong \mathbb{C}^2$, representing a Z_2 gauge qubit. The gauge-space factor is

$$\mathcal{H}_{\text{gauge}} = \bigotimes_{\ell=1}^{N_{\text{links}}} \mathcal{H}_{\text{link}}.$$

- **Total space:** The full Hilbert space is

$$\mathcal{H} = \mathcal{H}_{\text{matter}} \otimes \mathcal{H}_{\text{gauge}}.$$

2.2 Local operators

On each matter site, one introduces:

- A number operator n_{field} (e.g. acting as 0 on two "vacuum-spin" states and 1 on two "excited-spin" states).
- Spin-like operators S_x, S_y, S_z (implemented as 4×4 matrices) that act within site space.
- Ladder operators $a, a^\dagger$ that connect the vacuum and excited subspaces.

On each gauge link, one introduces the usual Pauli operators σ_x and σ_z acting on $\mathbb{C}^2$. The operator σ_z encodes the Z_2 "link variable," while σ_x generates gauge flips and enters the gauge kinetic term.

2.3 Gauge-covariant hopping

A basic matter-gauge coupling term is built using a gauge-covariant hopping operator of the form

$$H_{\text{hop}} = -J_{\text{hop}} \sum_{\langle i,j \rangle} \big(a_i^\dagger U_{ij} a_j + a_j^\dagger U_{ji} a_i \big), \tag{1}$$

where $\langle i, j \rangle$ denotes nearest-neighbor sites and U_{ij} is the gauge field on the link connecting i to j. In the Z_2 toy model, one takes

$$U_{ij} = \sigma_z(\ell_{ij}), \tag{2}$$

with ℓ_{ij} the link between i and j. This is a standard discrete analog of gauge-covariant hopping.

2.4 Gauge kinetic term

The gauge degrees of freedom have their own dynamics via a simple kinetic term

$$H_{\text{gauge}} = -g_{\text{gauge}} \sum_{\ell} \sigma_x(\ell), \tag{3}$$

which flips gauge qubits and provides gauge-field fluctuations. This is again a standard Z_2-like choice in lattice gauge models.

2.5 Local Gauss operators

At each site s, one can define a local Gauss operator

$$G_s = (-1)^{n_s} \prod_{\ell \in \mathrm{star}(s)} \sigma_x(\ell), \tag{4}$$

where n_s is some local number operator (e.g. n_{field} at site s), and the product ranges over all links in the "star" of s (those touching s). In a Z_2 gauge theory, a physical state would satisfy

$$G_s |\psi\rangle = +|\psi\rangle \quad \text{for all sites } s, \tag{5}$$

expressing a local gauge constraint (analogous to a Gauss law).

2.6 Gauss-law penalty

Instead of restricting to the $G_s = +1$ subspace exactly, one can add a penalty term

$$H_{\text{Gauss}} = \lambda_G \sum_s \big(I - G_s\big)^2, \tag{6}$$

with $\lambda_G \geq 0$ a tunable parameter. When λ_G is large compared to other couplings, states that strongly violate the $G_s = +1$ conditions acquire large energies. This is a standard way of energetically enforcing a constraint subspace in finite toy models.

2.7 Total Hamiltonian

The toy total Hamiltonian used in the numerical experiments combines these elements:

$$H = H_{\text{hop}} + H_{\text{spin}} + H_{\text{mass}} + H_{\text{onsite}} + H_{\text{gauge}} + H_{\text{Gauss}}, \tag{7}$$

where:

- H_{hop} is the gauge-covariant hopping term,
- H_{spin} includes nearest-neighbor spin interactions (e.g. $S_z S_z$),
- H_{mass} is a mass-like term proportional to n_{field},
- H_{onsite} is an onsite "stiffness" proportional to n_{field}^2,
- H_{gauge} is the gauge kinetic term,
- H_{Gauss} is the Gauss-law penalty described above.

All of these are standard, finite-dimensional operators defined on $\mathcal{H}$.

3 Numerical Experiments (Empirical Only)

3.1 Lattice and implementation

Numerical experiments were performed on 2×2 lattices using `QuTiP` for operator construction and expectation values. The Hilbert-space dimension in this case is $\dim(\mathcal{H}) = 4^{N_{\text{sites}}} 2^{N_{\text{links}}}$, which remains tractable for small N_{sites} and N_{links}.

The emphasis is solely on the internal behavior of these finite models: no continuum limit is taken, and no claim is made that the spectrum or correlation structure reproduces any real-world field theory.

3.2 States considered

Three classes of initial states were studied:

1. **One localized skyrmion-like pattern:** a spatially localized, spin-textured excitation centered in the lattice.

2. **Two overlapping patterns:** a higher-amplitude configuration designed to resemble two such patterns occupying the same region.

3. **Two separated patterns:** a superposition of two skyrmion-like patterns localized near different sites, combined to approximate a "two-object, separated" configuration.

For each state, the gauge degrees of freedom were initialized in a uniform $|+\rangle$ state on each link (an eigenstate of σ_x with eigenvalue $+1$), so that the Gauss operators meaningfully probe the matter configuration.

3.3 Quantities measured

For each configuration, the following quantities were computed:

- Total energy $E = \langle\psi|H|\psi\rangle$,
- Total matter occupation $N = \sum_s \langle n_{\text{field}}(s)\rangle$,
- Local Gauss-violation measures
$$V_s = \left\langle \left(I - G_s\right)^2 \right\rangle,$$
as well as their sums over sites.

These are strictly numerical outputs of the defined toy Hamiltonian on the given finite Hilbert space.

4 Observations (Not Interpretations)

Across multiple parameter choices (e.g. varying λ_G, onsite penalties, and hopping strengths), the following empirical features were repeatedly observed in this toy model:

1. **Gauss-law violation:** Configurations designed to represent "two overlapping" skyrmion-like patterns consistently exhibit larger values of $\sum_s V_s$ than configurations representing "two separated" patterns.

2. **Energy under Gauss penalty:** As the Gauss-penalty coefficient λ_G is increased, the energy of overlapping states rises *more rapidly* than that of separated states, reflecting the larger Gauss-law violation in the overlapping case.

3. **Energy differences:** The separation-dependent energy difference
$$\Delta E = E_{\text{overlap}} - E_{\text{separated}} \tag{8}$$
tends to grow approximately linearly with λ_G once λ_G is large compared to other scales in the Hamiltonian.

These statements are purely empirical descriptions of the finite-model simulations. No interpretation in terms of real fermionic statistics or physical exclusion principles is implied by these observations alone.

5 Interpretive Hypotheses (Speculative)

The structure of the toy model suggests a possible interpretive angle in terms of *local conservation of informational flux*:

- The Gauss operators G_s tie together matter parity at site s and the surrounding gauge links, enforcing a compatibility condition between "what is present" at the site and "how flux is routed" around it.
- Under this view, a state close to the $G_s = +1$ sector at each site can be interpreted as one in which information about local occupancy and gauge configuration is internally consistent, or where "information flux" is locally conserved.

Within this interpretive framework, the numerical observations can be summarized as:

- Configurations resembling overlapping skyrmion-like patterns attempt to place more structured pattern information into the same local region. In this toy model, these configurations typically exhibit larger violations of the Gauss constraints (larger V_s), and thus incur significant energy penalties when λ_G is large.
- Separated skyrmion-like patterns distribute this pattern information over multiple sites, so that the local Gauss constraints can be more easily satisfied. As a result, these configurations tend to have lower Gauss violations and lower energies for large λ_G.

This behavior bears a structural resemblance to excitation patterns in constrained systems where certain configurations are effectively forbidden or highly suppressed. However, this resemblance is strictly analogical: the toy model is *not* claimed to reproduce real fermionic statistics, the Pauli exclusion principle, or any feature of the Standard Model. The "informational flux" language is presented as a hypothesis for organizing intuition, not as an established physical doctrine.

6 Limitations and Scope

To avoid misunderstanding, the limitations of this work are made explicit:

- The framework does *not* claim to derive the Standard Model, its particle content, or its gauge structure.
- The framework does *not* claim to model real spacetime, general relativity, or quantum gravity.
- The framework does *not* claim to provide a new physical law or a replacement for quantum field theory.

All results described here are features of finite-dimensional Hilbert-space models with specific, user-defined Hamiltonians. No continuum limit, renormalization procedure, or direct link to experimental data is provided in this version.

Interpretive statements about "information flux" or "exclusion-like behavior" should be read as speculative hypotheses about how constraint structures might organize patterns in toy models, not as statements about nature.

7 Future Work

Several directions for further exploration are possible, all within the same carefully bounded scope:

- **Larger lattices:** Extending simulations to larger lattices could reveal richer constraint structure, including more complex patterns of Gauss-law satisfaction and violation.
- **Explicit projection:** Instead of enforcing Gauss constraints with penalties, one could project states directly into the $G_s = +1$ subspace and study dynamics and spectra restricted to that subspace.
- **Connections to stabilizer codes:** The Gauss operators act as local stabilizers in a Z_2 gauge theory. Exploring precise connections to stabilizer codes and quantum error-correction may sharpen the "information-flux" picture in a more formal way.
- **Refined toy models:** Additional toy Hamiltonians that incorporate different forms of locality, symmetry, or constraint could be examined to see which features of the observed behavior are robust.

In all future work, the guiding principle will remain the same:

1. Clearly state the mathematical structure,
2. Clearly present numerical observations,
3. Clearly label any interpretive or speculative steps.

Acknowledgements

The author thanks the broader physics and quantum information literature on lattice gauge theories, stabilizer codes, and constrained dynamics for conceptual background and inspiration.