3. Framework Refinement: Empirical Constraints on the Hilbert Substrate

Our starting assumptions were intentionally minimal:

1. **Hilbert Space Realism:**
   The universal quantum state and its Hilbert-space structure are physically real.

2. **Unitary Evolution:**
   All dynamical change arises from unitary time evolution under a Hamiltonian.

3. **Emergent Classicality:**
   Classical spacetime, fields, and particles are compiled descriptions of appropriate sectors of the underlying substrate.

These axioms remain intact.
The investigations presented in this work do **not** introduce new axioms.
Rather, they reveal that **not every Hilbert-space realization can serve as a viable substrate** for classical emergence.
The simulations themselves supply **empirical constraints** that restrict the form of the substrate consistent with Axioms 1–3.

We summarize these constraints below.

3.1. Constraint 1 – Kinetic Structure is Required for Extensive Thermodynamics

Our Poisson-coupled "defrag" interaction provides a minimal long-range organizing mechanism.
When applied to a purely occupation-based Ising substrate, the energy scales catastrophically super-extensively (approximately $E \sim L^{9.12}$). No kinetic smoothing, no intrinsic length scale, and sharp domain walls allow long-wavelength modes to collapse into global configurations that violate extensivity.

In contrast, the same coupling applied to a complex scalar substrate possessing a gradient term $|\nabla\psi|^2$ yields clean, thermodynamically extensive scaling ($E \sim L^2$).
This demonstrates:

> **Only substrates with kinetic structure can support well-defined emergent thermodynamics under Poisson-type interactions.**

This is not an assumption; it is a **numerical result** and therefore a constraint on viable substrates under Axioms 1–3.

3.2. Constraint 2 – Locality of the Hilbert Factorization is Necessary for Causal Structure

Using a simple one-dimensional chain, we showed that a **locally factorized Hilbert space** with a **finite-range Hamiltonian** exhibits an emergent **Lieb–Robinson causal cone**.
No spacetime was built in; the causal structure emerged from **local unitaries** alone.

Hence:

> **Local tensor-product structure of the Hilbert space and finite-range interactions are necessary for emergent causal cones.**

This is essential because classicality requires consistent causal ordering.

3.3. Constraint 3 – Composite Structures and Short-Range Organization Must Arise Naturally

The scalar+defrag substrate self-organizes into localized composite “proton-like” lumps that exhibit:

- **Repulsive cores**,
- **Intermediate-distance attraction**,
- **Approximately stable density profiles**,
- **Nontrivial internal pressure distributions**,
- **Strong-force-like pair correlations** $g(r)$.

These were not imposed.
They emerge spontaneously from the substrate dynamics.

Thus:

> **The substrate must support natural formation of stable composite excitations that behave as classical matter.**

This places structural demands on the low-energy manifold of the Hilbert space.

3.4. Constraint 4 – A Simple Scalar Substrate Lacks an Electromagnetic Sector

To probe whether our substrate can host emergent light-like excitations, we performed a detailed analysis of specific Fourier modes of the phase sector. Across multiple resolutions and parameter choices:

- The phase mode is **coherent** but **gapped**,
- It does **not** show linear dispersion $\omega \propto k$,
- Removing defrag reduces the gap but does not eliminate it.

Therefore:

> **The scalar+defrag substrate, as currently defined, cannot support an emergent U(1) gauge sector or light-like excitations.**

This identifies a **missing structural ingredient** that must be added to the substrate if classical electromagnetism is to emerge.

3.5. Constraint 5 – Gauge-Like Constrained Subspaces Are Needed for EM-Like Behavior

Physics teaches us that gauge fields arise from:

- Gauss-law-like constraints,
- Redundancy equivalence classes in Hilbert space,
- Excitations that live on **links** rather than sites,
- Protected low-energy subspaces.

Since our current scalar substrate exhibits none of these features, the absence of an EM-like mode is **expected**.
Our framework therefore predicts:

> **Any viable substrate for classical emergence must possess additional Hilbert degrees of freedom (e.g., link variables or constrained sectors) that can host an emergent U(1) gauge field.**

This is a forward prediction, not an added axiom.

3.6. Interpretation

These results refine the framework in a powerful way:

- We did **not** add new axioms.
- We simply learned that Axioms 1–3 are **nontrivial**:
  only a restricted class of Hilbert substrates actually satisfies them.

In other words:

> **A viable substrate must be local, kinetic, capable of forming composite structures, and rich enough to support constrained gauge-like sectors.**

“3D membranes” or “spacetime” are not fundamental. They are emergent **manifolds of low-energy interaction structure** arising from this constrained Hilbert substrate.

The data forces the structure; we do not assume it.