Framework Overview 1.2
A Finite-Hilbert-Space Approach to Emergent Structure

Ben Bray

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Abstract

This document presents a clarified and conservative overview of the Hilbert Substrate research program. The core aim is to investigate how structured, classical-like behavior can arise inside finite-dimensional quantum systems with local Hamiltonians and constraint operators. The discussion is organized into three strictly separated tiers: (1) mathematical definitions, (2) numerical or structural observations, and (3) interpretive or speculative hypotheses.

No claims are made about the true structure of the Standard Model, spacetime, or quantum gravity. All statements about "emergence" refer solely to patterns observed within finite toy models. Interpretive ideas are presented as hypotheses to be tested. Throughout this work, in keeping with a disciplined scientific posture, *if the data do not show it, we do not say it.*

1 Axioms (Non-Speculative)

The program begins with three minimal assumptions:

A1. Hilbert-Space Realism. Physical states are taken to be vectors in a Hilbert space, without additional ontological primitives added at the outset.

A2. Unitary Evolution. Dynamics arise from a time-independent Hermitian Hamiltonian H acting locally on the Hilbert space.

A3. Emergent Structure. Classical-like variables, fields, or objects—when they appear at all—are interpreted as coarse-grained, dynamically stable patterns in the underlying quantum substrate.

These axioms are deliberately conservative and do not exceed standard quantum mechanics. The research question is how far one can go with *only* these ingredients.

2 Finite-Hilbert Substrates

In this program, the starting point is a finite-dimensional Hilbert space

$$\mathcal{H} = \mathcal{H}_{\text{sites}} \otimes \mathcal{H}_{\text{links}} \otimes \cdots$$

constructed from:

- local matter sites (e.g. 4-level "spinor" spaces),
- gauge or auxiliary qubits on links (e.g. Z_2 gauge variables),

- optional environmental or coherence-control registers.

The total dimensionality is finite, making exact simulation feasible for small lattices. All structure is encoded in:

$$H = \sum_{\text{local}} H_{\text{term}},$$

with each term acting on a small region of the lattice.

3 Constraints and Locality

Two forms of structural constraint have been especially important in the numerical explorations to date.

3.1 Local Gauge-Like Constraints

At each site s, a local constraint operator of the form

$$G_s = (-1)^{n_s} \prod_{\ell \in \text{star}(s)} \sigma_x(\ell)$$

can be defined, coupling matter occupancy to surrounding link qubits. The operator is analogous to a Z_2 Gauss law in lattice gauge theory.

Violation of this constraint is measured by

$$V_s = \left\langle (I - G_s)^2 \right\rangle .$$

When included in the Hamiltonian as

$$H_{\text{Gauss}} = \lambda_G \sum_s (I - G_s)^2,$$

the system energetically suppresses configurations that deviate from local consistency.

3.2 Locality and Information Routing

Because G_s links occupancy to nearby links, it is natural to interpret it as a rule governing how "information" about local configuration is routed through neighboring degrees of freedom. This interpretation is not asserted as a physical law of nature, but as a useful lens through which to read the structure of the toy model.

4 Numerical Toy Models

Simulations were performed on 2×2 lattices with matter sites and Z_2 gauge links. Three classes of initial states were examined:

- one localized skyrmion-like pattern,
- two overlapping patterns,
- two spatially separated patterns.

For each, the quantities measured included:

- the energy expectation $\langle H \rangle$,
- the total occupation N,
- local and global Gauss-violation measures.

4.1 Empirical Observations

Across parameter sweeps, the following empirical features were seen:

O1. Gauss-Law Differentiation. Overlapping patterns generally exhibited larger total Gauss-violation than separated patterns.

O2. Energy Amplification under Constraint. Increasing λ_G amplified the energy difference between overlapping and separated states approximately linearly.

O3. Pattern Selectivity. For sufficiently large λ_G, overlapping states became heavily suppressed relative to separated states.

These are strictly numerical outputs of the defined finite models. No physical claims extend beyond the simulations.

5 Interpretive Hypotheses (Speculative)

The following statements represent hypotheses motivated by the observed structure. They are not asserted as physical truths and do not extend beyond the toy models.

- Local Gauss constraints can be viewed as enforcing a form of *local conservation of informational flux*: matter occupancy and gauge links must form a consistent local pattern.
- Overlapping skyrmion-like patterns attempt to compress too much structured information into a single local region, causing large violations of G_s and thus large energy penalties when λ_G is significant.
- Separated patterns distribute informational structure spatially, allowing the constraints to be satisfied more easily.

This behavior is structurally reminiscent of exclusion-like behavior in constrained systems. However, no claim is made that these observations reproduce real fermionic statistics or the Pauli exclusion principle.

6 Long-Term Research Aims (Speculative)

The broader research direction includes:

- studying larger lattices and richer constraint structures;
- projecting exactly into the constraint-satisfying subspace;

- exploring connections to stabilizer codes and error correction;
- characterizing the space of Hamiltonians that support stable classical-like emergent patterns;
- investigating how gauge-like redundancy shapes emergent dynamics.

These aims remain conceptual until further numerical or analytic evidence is produced. In all future work, the principle remains:

If the data do not show it, we do not say it.