Hilbert Substrate Framework 1.3: A Finite-Hilbert Quantum Substrate with Gauge-Like Constraints, Defragmentation Potentials, and Emergent Exchange-Antisymmetric Ground States

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We present an updated formulation of the Hilbert Substrate Framework, a finite-dimensional quantum substrate designed to investigate how exchange antisymmetry, spatial exclusion, and nonclassical correlations may arise from local constraint structures and information-balance terms. The framework combines: (1) a two-excitation quantum substrate defined on a small periodic lattice; (2) a Gauss-like information-flux penalty; (3) a defragmentation potential that encourages spatial coherence; (4) a contact spin interaction analogous to the singlet–triplet structure of two-body quantum mechanics; and (5) a CHSH/Bell-sector subsystem used as a calibration of the quantum machinery.

In the two-excitation substrate, we find that the ground state is exactly exchange antisymmetric under swap, and that all spatial-overlap weight is concentrated in the spin-singlet channel. This provides a concrete demonstration that a constrained finite Hilbert system can prefer fermion-like symmetry sectors without explicitly postulating fermionic operators. The results are not claimed to reproduce Pauli exclusion or the Standard Model, but they illustrate that constraint-driven information balance can shape symmetry sectors in a controlled finite model. Throughout, we adopt the guiding principle: *if the data do not show it, we do not claim it.*

CONTENTS

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I. INTRODUCTION

Hilbert Substrate 1.3 refines and formalizes the conceptual and computational investigations reported in previous versions of this framework. The central philosophical motivation is to study how *local constraints on information distribution* can shape the symmetry structure of low-energy states within a finite Hilbert space. The question is not whether this reproduces real fermions, gauge theories, or gravity, but whether constraint and routing principles can bias a Hilbert system toward familiar quantum-sector behavior.

To remain controlled and falsifiable, we restrict ourselves entirely to finite-dimensional Hilbert spaces, local Hamiltonians acting on them, and direct numerical diagonalization of the two-excitation sector. We then test the quantum engine with a CHSH subsystem whose results can be compared to the Tsirelson bound.

A pair of guiding principles shape the framework:

1. **Finite Hilbert substrate:** All degrees of freedom exist inside a explicitly-constructed finite Hilbert space. There are no continuum fields, no functional integrals, and no infinite sums.

2. **Empirical modesty:** We do not claim more structure than what emerges from the model. *If the data do not show it, we do not say it.*

In this version, the framework achieves a significant milestone: the ground state of the two-excitation substrate becomes *exactly exchange antisymmetric*, and the spin structure of same-site configurations becomes *purely singlet*. This is a clean, interpretable, finite-Hilbert analogue of "fermion-like" behavior arising from constraint rules.

II. CONFIGURATION SPACE AND THE SUBSTRATE HILBERT SPACE

A. Configuration space in finite models

For two distinguishable excitations on a lattice with N_s sites, each carrying a two-level internal degree of freedom (spin), the configuration space is

$$\mathcal{C} = \{(r_1, s_1; r_2, s_2) \mid r_i \in \{0, \ldots, N_s - 1\},\ s_i \in \{\uparrow, \downarrow\}\}. \tag{1}$$

The Hilbert space is

$$\mathcal{H} = \mathbb{C}^{(N_s\times 2)\ (N_s\times 2)} \cong \mathbb{C}^{4N_s^2}. \tag{2}$$

In the present work, $N_s = 4$ (a 2×2 lattice), so $\dim \mathcal{H} = 64$.

We emphasize: nothing in the construction presupposes bosonic or fermionic symmetry sectors. The swap operator

$$P_{\mathrm{ex}}\,|r_1, s_1; r_2, s_2\rangle = |r_2, s_2; r_1, s_1\rangle \tag{3}$$

is available as an observable. The model is free to place weight in symmetric, antisymmetric, or mixed symmetry subspaces.

B. Local geometric structure: neighbors and periodicity

The lattice is periodic: each site has four neighbors. Hopping terms act only along these edges, so there is a clear notion of locality.

III. HAMILTONIAN TERMS

The substrate Hamiltonian is

$$H = H_{\mathrm{hop}} + H_{\mathrm{mass}} + H_{\mathrm{defrag}} + H_{\mathrm{Gauss}} + H_{\mathrm{contact}}. \tag{4}$$

A. Hopping

Each excitation hops to its nearest neighbors with amplitude $-J_{\mathrm{hop}}$. This produces delocalization and kinetic spread.

B. Mass term

Each excitation contributes $+m$ to the energy. This plays no structural role but keeps the model interpretable.

C. Defrag potential

A Gaussian potential centered on the lattice midpoint,

$$V_{\text{defrag}}(r) = -\exp\left(-\frac{\text{dist}(r, \text{center})^2}{2\sigma^2}\right), \tag{5}$$

encourages clumping and coherence. It introduces a "direction" toward which the substrate tends to compress or self-organize.

D. Gauss-like information-flux penalty

Let $\rho(r)$ be the occupancy of site r and $\rho_0 = 2/N_s$ the uniform occupancy. Define

$$H_{\text{Gauss}} = \frac{\lambda_G}{2} \sum_r (\rho(r) - \rho_0)^2 . \tag{6}$$

This term enforces local "information balance": states with uneven occupancy cost energy. In practice, this penalizes multi-occupation configurations and thus discourages spatial overlap.

E. Contact spin interaction

When $r_1 = r_2$, the excitations interact via a singlet–triplet structure:

$$H_{\text{contact}} = J_{\text{exch}}\, \vec{S}_1 \cdot \vec{S}_2 + \lambda_S\, \mathbb{1}_{\text{singlet}} + \lambda_T\, \mathbb{1}_{\text{triplet},\parallel}. \tag{7}$$

Here $\vec{S} \cdot \vec{S}$ reproduces the usual $-3/4$ (singlet) and $+1/4$ (triplet) splitting, while λ_S and λ_T adjust the relative preference.

IV. GROUND STATE RESULTS

We diagonalized H on the full 64-dimensional Hilbert space. Parameters were:

$$\begin{aligned} L_x &= L_y = 2, \\ J_{\text{hop}} &= 1.0, \\ m &= 0.1, \\ g_{\text{defrag}} &= 1.0, \quad \sigma_{\text{defrag}} = 1.0, \\ \lambda_G &= 5.0, \\ \lambda_S &= -1.0, \quad \lambda_T = 0.0, \quad J_{\text{exch}} = 1.0. \end{aligned}$$

A. Exchange antisymmetry

Define the diagnostic:

$$A = 1 - \frac{\|\psi + P_{\text{ex}}\psi\|^2}{\|\psi\|^2}. \tag{8}$$

$A = 1$ means perfectly antisymmetric; $A = 0$ means no antisymmetric component.

Result:

$$A = 1.000000, \qquad \text{antisymmetry violation} \approx 5 \times 10^{-28}.$$

Thus the ground state lies entirely in the antisymmetric sector.

B. Spatial overlap and spin structure

Probability that $r_1 = r_2$:

$$P_{\text{overlap}} = 0.151583.$$

Within this region:

$$P_{\text{singlet}} = 0.151583, \qquad P_{\text{triplet}} = 0.$$

The overlap region is *purely* singlet.

C. Gauss expectation

$$\langle H_{\text{Gauss}} \rangle = 3.257917,$$

indicating the Gauss sector plays an active role.

D. Interpretation

This is a controlled, finite-dimensional demonstration that a combination of:

- local information-flux balancing,
- defragmentation geometry,
- contact spin structure,

can bias a substrate toward a fermion-like antisymmetric ground state.

No fermionic creation/annihilation operators were introduced. No antisymmetrization was imposed. The symmetry arises as an energetic preference.

We do *not* claim this explains real fermions.

V. CHSH QUANTUM CORE

As a calibration of the quantum engine, we include a two-qubit Heisenberg dimer. With the Bell state as initial condition, the CHSH value

$$S(t) = E(A, B) + E(A, B') + E(A', B) - E(A', B')$$

achieves

$$S_{\max} \approx 2.828427,$$

the Tsirelson bound. This confirms that the quantum portion of the code behaves properly.

VI. FERMION TOY: SWAP OPERATOR EIGENSTRUCTURE

A two-qubit toy subsystem verifies the swap operator:

$$\langle\psi_{\rm sym}|P_{\rm ex}|\psi_{\rm sym}\rangle = +1, \qquad \langle\psi_{\rm asym}|P_{\rm ex}|\psi_{\rm asym}\rangle = -1.$$

This validates the operator used in the substrate diagnostics.

VII. CAVEATS

We emphasize several limitations:

- The model is finite-dimensional and not relativistic.
- No claim is made about reproducing Pauli exclusion in the continuum.
- No Standard Model structure is invoked.
- The results are specific to parameter choices.

Everything shown is a property of the specific Hamiltonian used.

VIII. FUTURE WORK

Potential extensions include:

- Scaling to larger lattices (e.g. 3×3).
- Adding explicit gauge-field Hilbert spaces.
- Studying phase diagrams across $(\lambda_G, \lambda_S, J_{\rm exch})$.
- Embedding CHSH correlations inside substrate degrees of freedom.

IX. CONCLUSION

Hilbert Substrate 1.3 provides a coherent finite-Hilbert environment in which information-flux constraints, defragmentation potentials, and spin-structured contact interactions combine to yield an emergent antisymmetric ground state. The framework is deliberately modest: we observe only what the model shows. Nevertheless, the result demonstrates that fermion-like structure can arise from local rules without being explicitly imposed.

[1] J. S. Bell, "On the Einstein Podolsky Rosen paradox," *Physics Physique Fizika* **1**, 195 (1964).
[2] B. S. Tsirelson, "Quantum generalizations of Bell's inequality," *Lett. Math. Phys.* **4**, 93–100 (1980).