The Mr. Magnetic Constraint: Why Maxwell Emerges from the Substrate

This note summarizes a key result from our recent simulations and theoretical filtering. When we attempted to build a gauge sector directly from gradient energy terms like "|\nabla A|^2", we discovered that the system produced no propagating photon modes. Energy pooled in static configurations instead of matching a relativistic dispersion relation. This failure is not random—it reflects a deep structure imposed by our axioms.

1. Starting assumptions

We work from three substrate axioms: 1. **Hilbert Space Realism** – the world-state is a vector in a Hilbert space. 2. **Unitary Evolution** – dynamics arise from a Hermitian Hamiltonian generating reversible time evolution. 3. **Emergent Classicality** – at coarse scales, some degrees of freedom behave like classical fields or particles.

We also impose emergent-world desiderata: - **Locality** - **Isotropy and translation symmetry** - **Local phase redundancy** (promoting global U(1) to U(1)-gauge) - **Finite signal speed** - **Linear behavior in the small-amplitude limit**

These constraints determine what forms of gauge energy are allowed.

2. Why a gauge field appears at all

When local phase redundancy is imposed, a connection field $A_i(x)$ naturally arises. The derivative acting on $\psi(x)$ must become the covariant derivative:

$$D_i\psi = (\partial_i - iqA_i)\psi.$$

Up to this point, no specific dynamics for A_i are fixed; the Hamiltonian still needs a gauge-sector energy.

3. The most general local quadratic energy

The gauge sector can, in principle, contain the following rotationally invariant quadratic terms:

$$H_{\text{gauge}} = \int d^dx \left[\tfrac{1}{2}E_iE_i + \tfrac{\alpha}{2}(\nabla\cdot A)^2 + \tfrac{\beta}{2}(\nabla\times A)^2 + \dots\right].$$

Here, E_i is the canonical momentum conjugate to A_i. Only terms consistent with all axioms and constraints are allowed.

4. Filtering by gauge redundancy

Local phase redundancy requires the transformation:

$$A_i \to A_i + \partial_i\chi.$$

Under this transformation: - **The divergence term** $(\nabla \cdot A)^2$ changes; it is *not gauge-invariant*. - **The curl term** $(\nabla \times A)^2$ is unchanged; curl(gradient) = 0.

Therefore, the divergence term violates the gauge redundancy implied by emergent classicality, and is strictly forbidden. Only the curl-based magnetic energy survives.

5. Consequence: Mr. Magnetic

After filtering, the unique admissible gauge Hamiltonian density is:

$$H_{\text{gauge}} = \frac{1}{2}E^2 + \frac{c^2}{2}B^2,$$

with the magnetic field defined as:

$$B = \nabla \times A.$$

This is exactly the magnetic component of Maxwell theory. It isn't optional—it's the only term compatible with: - Hilbert space realism, - unitary local evolution, - emergent gauge redundancy, - and classical wave propagation.

6. Why this produces real photons

The surviving Hamiltonian yields the well-known equations:

$$\dot{A} = E,$$

$$\dot{E} = c^2(\nabla \times B) - j,$$

which combine to give the wave equation for transverse components:

$$\ddot{A} - c^2\nabla^2 A = -j.$$

This implies the photon dispersion relation:

$$\omega = c|k|.$$

Simulations confirm that without the B^2 term, no propagating modes appear; adding the term produces a clean light cone.

7. Summary

Maxwell's magnetic energy term is not an extra axiom. It is the *unique* gauge-invariant, local, isotropic quadratic term compatible with our substrate axioms. In this sense:

> **Emergent classicality in a Hilbert-space substrate requires Maxwell-like gauge dynamics.**

This is the "Mr. Magnetic" constraint we discovered numerically and then derived analytically.