Image Analysis Tutorial

DUSTBUSTERS Summer School

Jonathan Williams, May 2022

Background and Motivation

One of the design goals of ALMA was to image the theoretically expected rings and structures in planet-forming disks. As witnessed from the spectacular image of HL Tau in 2014 and subsequently with the DSHARP large program, it certainly met that goal. Indeed, it has transformed our field and we now know that disk structure is almost ubiquitous. The focus has now shifted from detection to characterization and using these structures to learn about disk physics and planet formation. The goal of this tutorial is to analyze high resolution ALMA continuum images of disks from the DSHARP survey so that you can measure basic structural properties.

You will rotate and deproject a disk image and then determine the azimuthally averaged emission. This allows the radial profile to be determined with high sensitivity to faint emission at large radii and quantify deviations from azimuthal symmetry, such as spirals. As designed at the summer school, the tutorial was intended to teach this analysis from first principles but the instructor then learned (at the school) that there now is a very nice package, `gofish`, that carries this out in one command! Nevertheless, it is still useful to understand the underlying principles as this provides flexibility for nuanced analysis that may not be so simply packaged. You can then use the software to check your results, confident that you now know what goes on "under the hood".

First, we describe the mathematics to convert the sky image to the disk frame. We will then download ALMA data from the DSHARP website and apply these equations in two cases, with and without azimuthal symmetry. We will then produce polar plots and radial profiles. Finally, we will check our results with `gofish` and explore extensions to this work.

Disk deprojection

Figure 1 shows a schematic of a inclined disk at a given position angle (PA) on the sky. The convention is that PA is defined as zero toward the north and increases toward the east. However, we plot images in equatorial coordinates with east pointing toward the left so mentally consider the mirror image of this Figure such that α increases toward the right and you will see that the position angle increases in a clockwise sense on a conventional cartesian grid. You can then use a rotation matrix to relate the coordinate system along the minor and major axes of the disk to the observed equatorial offsets,

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos PA & -\sin PA \\ \sin PA & \cos PA \end{pmatrix} \begin{pmatrix} \Delta\alpha \\ \Delta\delta \end{pmatrix}.$$

The final step is to convert to the disk frame by taking out the inclination factor that compresses the image along the minor axis,

$$x = x'/\cos i$$
$$y = y'.$$

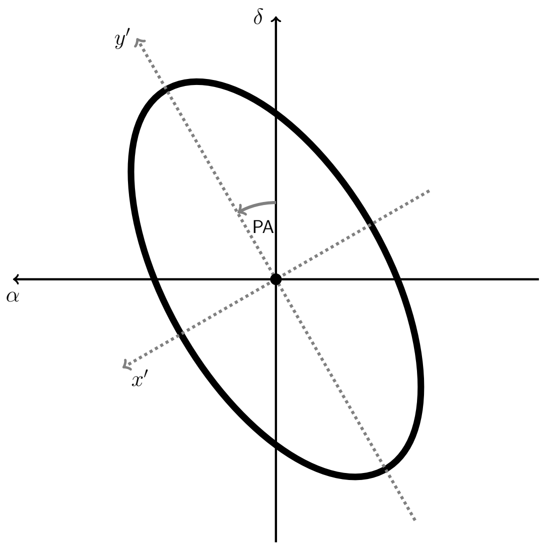

Figure 1: *Schematic of an inclined disk rotated at a position angle, PA, relative to the equatorial coordinate system* (α, δ)*. The coordinates* (x', y') *are along the disk minor and major axes respectively.*

Coding exercise 1: Download the fits image files for AS 209 and IM Lup from the DSHARP website, plot the sky image, and fit a gaussian to determine the disk center, position angle, and axis ratio (which is the cosine of the inclination). See the starter notebook `image_analysis_gaussfit.ipynb`.

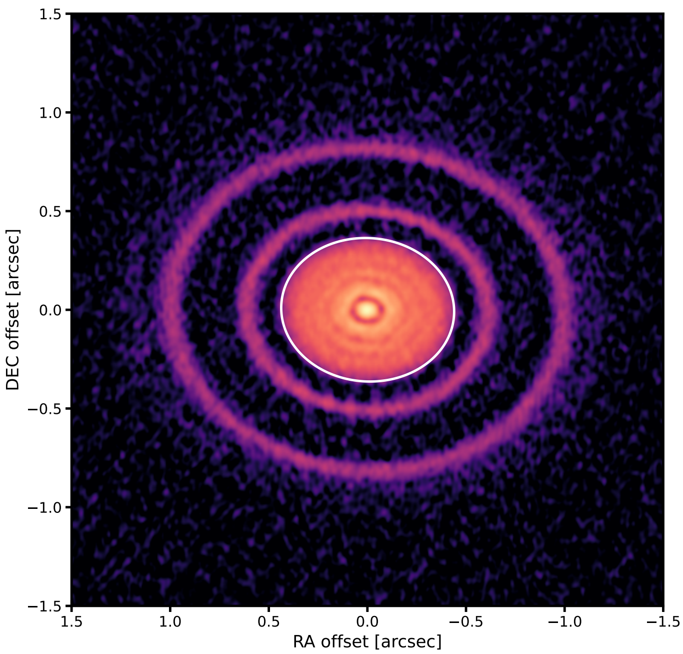

Figure 2: *Output of coding exercise 1: a gaussian fit (in white) to AS 209.*

Coding exercise 2: Using the results from the first exercise (or use the values in `image_analysis_AS209.ipynb`), apply the rotation and stretching transform to deproject the image. Next "unwrap" the deprojected map to calculate the radially averaged profile and make an azimuth-radius plot. Define polar coordinates in the disk frame,

$$r = (x^2 + y^2)^{1/2}$$
$$\theta = \tan^{-1}(y/x),$$

and then regrid into this new coordinate system. The rings appear as vertical lines, i.e., constant radius, and there are no obvious azimuthal asymmetries (however note that a different group at the summer school did the same analysis on the IM Lup disk to study its spiral structure which is an interesting alternative). Now stacking along the azimuth axis, you can calculate the radial profile of the surface brightness which we will compare to two other methods next.

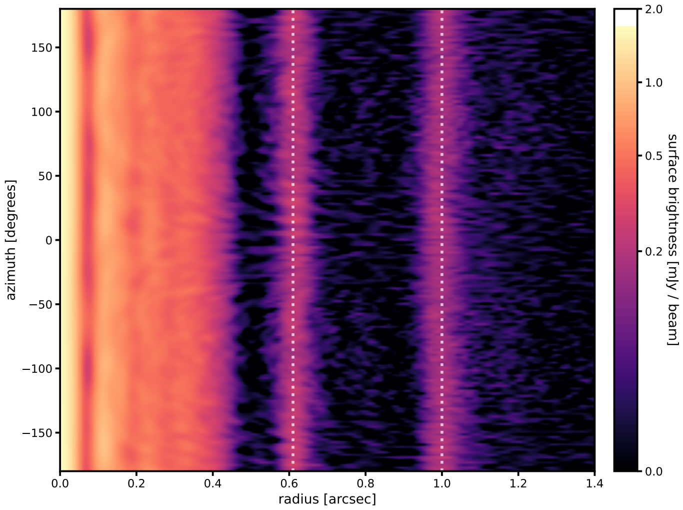

Figure 3: *Output of coding exercise 2: the surface brightness of AS 209 in disk frame polar coordinates. The vertical dotted lines show the two outer rings at $0\farcs6$ and $1''$.*

Coding exercise 3: The images produced from the ALMA data are the Fourier Transform of the measured visibilities that have then been "cleaned" through a non-linear iterative process and then convolved with a user-defined beam size. Though less intuitive, it is possible to determine finer scale structure by analyzing the visibilities directly. As an example, here we will use the `frank` code that fits Bessel functions (the FT of a ring) to study AS 209. The paper describing the math and application to ALMA data is in Jennings et al. 2020 and the code documentation is on github. A notebook that walks you through the steps for this particular case is `frank_visibility_modeling.ipynb`. You should find that the visibility fits reveals fine details in the disk structure, both in the contrast and the number of rings.

There are some discrepancies with the CLEAN profile at very low flux levels and, because `frank` does not provide errors (for good reasons that are documented in the links above), we can not say much more about this. The notebook also compares the CLEAN map with the `frank` results in the 2D image plane which can be a nice way to visualize results in talks and papers as well as search for azimuthal features, though for peer-reviewed work, this should be done more carefully by subtracting visibilities and cleaning the residuals as in, e.g., Andrews et al. 2021.

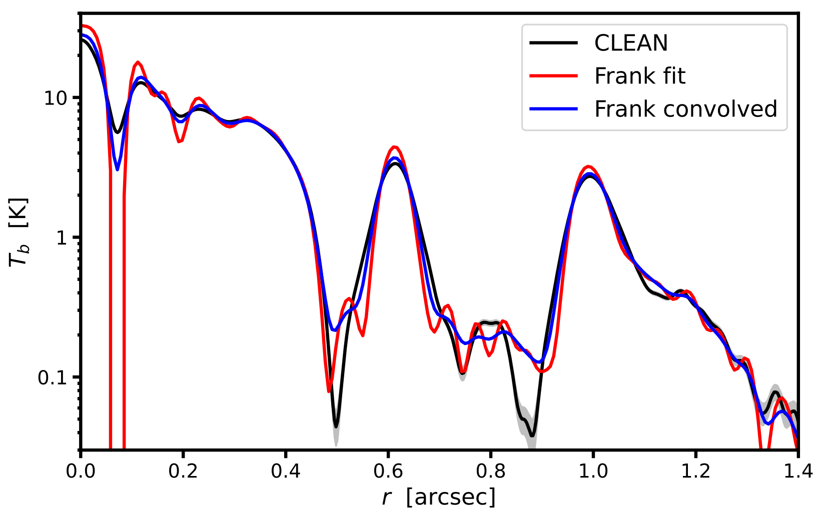

Figure 4: *Output of coding exercise 3: comparison of the brightness temperature radial profile derived from the CLEAN image with the results of the frank fitting to the visibilities.*

Coding exercise 4: We have worked with 2D continuum data only but ALMA has also produced gloriously detailed maps of the line emission in disks. Moreover, the ordered Keplerian profile can be used to recover the 3D structure (the subject of the DUSTBUSTERS tutorial by Stefano Facchini). There are new ALMA programs and new analytic techniques that are pushing this new area forward in exciting ways. And, as I learned at this workshop, there is a tool that can calculate radial profiles in a single command:

```
r, I, I_err = cube.radial_profile(x0=RA_offset, y0=DEC_offset, inc=inclination,
                                  PA=PA, mstar=stellar_mass, dist=distance)
```

To see this in action and comparison with our "manual" labor, see the notebook `gofish_AS209.ipynb` and gofish documentation.

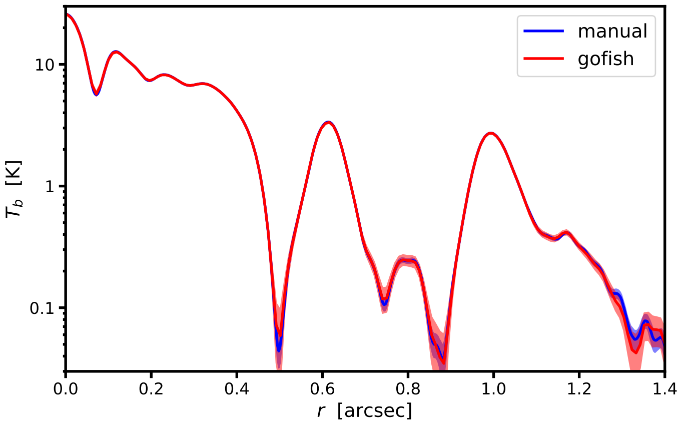

Figure 5: *Output of coding exercise 4: comparison of the brightness temperature radial profile derived from our coding work here with the* `gofish` *code. Could we have saved ourselves a bunch of work...?*

So was all our work up to this point for naught? No! Especially if you are new to disk studies, thinking about the geometry and coding it up is a worthwhile exercise. And you can now go further by, for example, using the radial profile to enhance the IM Lup spirals (see below from notebook `image_analysis_IMLup.ipynb`). Whatever your speciality, your own PhD research will push the field forward in part through your own customized coding and analysis so getting experience here and then seeing how it could be packaged into a much more versatile code is important. And ultimately, learning these sorts of analytical abilities is what enables new results... and leads to faculty positions or marketable skills in the wide world beyond academia ☺

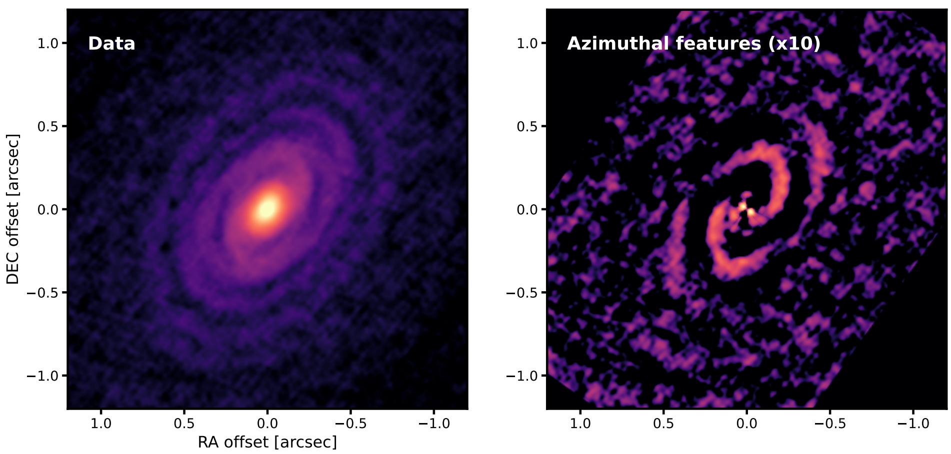

Figure 6: *DSHARP continuum image of IM Lup and the residuals after removing the radially symmetric component, bringing out the spiral features.*