Problem x

Let a(0), a(1), a(2), … be an infinite sequence of integers that is not eventually periodic, which means there do not exist integers N and d > 0 such that a(n + d) = a(n) for all integers n ≥ N. Let S be the set of 2025-tuples containing (a(n + 1), …, a(n + 2025)) for n = 0, 1, 2, …. Find the minimum number of elements of S.

Solution Prefix $y_{<t^*}$

…

So if the sequence is not eventually periodic, then the set S cannot eventually become periodic. But S can still have finitely many distinct elements — we just can't have that the values eventually repeat in a periodic fashion.

But here's the crucial point:

Mistake y_{t^*}

If the set S has finitely many elements, then the sequence is eventually periodic.

$\sim\pi(\cdot \mid x, y_{<t^*}, y_{t^*})$

…

Final Answer: ∞

❌ Incorrect

Intervention $\tilde{y}_{t^*}$

Consider a sequence that takes only finitely many values (e.g., only 0 and 1). Even if the sequence is not eventually periodic, it might still have only finitely many distinct 2025-tuples. For example, the Fibonacci word over {0,1} is not eventually periodic, but every block of length k appears exactly once in a certain pattern — in fact, it has exactly k+1distinct blocks of length k. Can you think of a way to construct such a sequence? Try to understand how the number of distinct blocks relates to the periodicity.

$\sim\pi(\cdot \mid x, y_{<t^*}, \tilde{y}_{t^*})$

…

Final Answer: 2026

✅ Correct

Problem x

Find the number of polynomials

$$f(t) = A0 + A1\ t + A2\ t^2 + A3\ t^3 + t^4$$

such that A0, … , A3 are integers and all roots of f in the complex plane have absolute value ≤ 1.

Solution Prefix $y_{<t^*}$

…

Therefore, no root can have |z| < 1 unless its reciprocal is also a root — but then |z| > 1, contradiction.

Therefore, all roots must have |z| = 1.

Mistake y_{t^*}

Wait — is that true?

Yes! This is a key point.

Key Theorem (in complex analysis):

$\sim\pi(\cdot \mid x, y_{<t^*}, y_{t^*})$

…

Final Answer: 21

❌ Incorrect

Intervention $\tilde{y}_{t^*}$

Wait — is that true? Let's test this conclusion with a simple example. What about the polynomial f(t) = t^4? The coefficients are all integers (A3 = A2 = A1 = A0 = 0), and all its roots are z = 0. The absolute value is |0| ≤ 1, so this polynomial should be counted. However, its roots are not on the unit circle.

This means my conclusion that all roots must have |z| = 1 is incorrect.

$\sim\pi(\cdot \mid x, y_{<t^*}, \tilde{y}_{t^*})$

…

Final Answer: 2026

✅ Correct