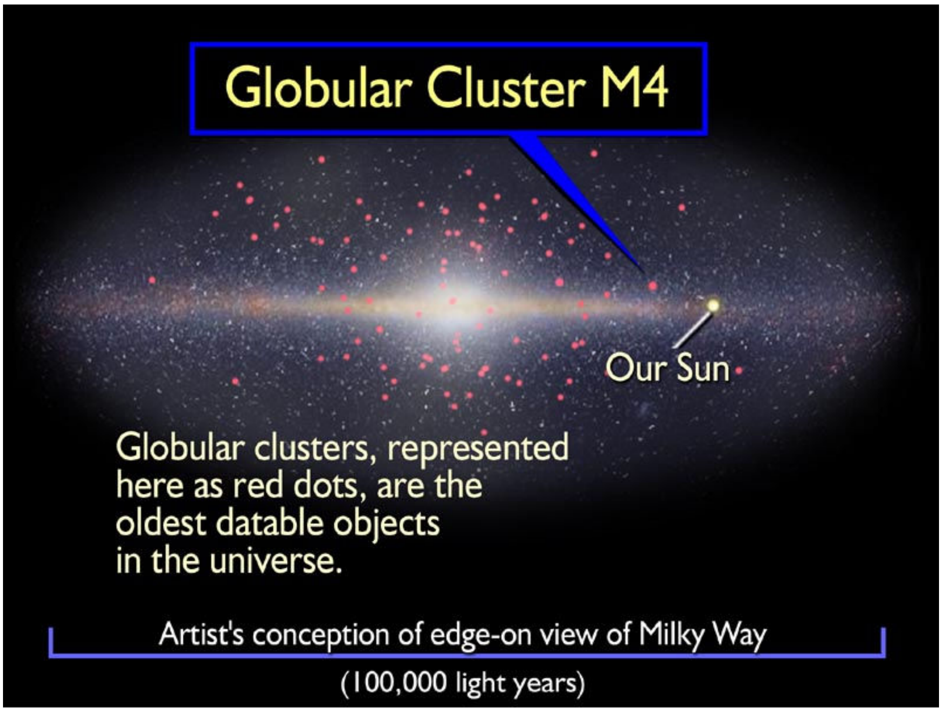
Relativistic Astrophysics and Cosmology
Michaelmas 2010 — Handout 21

AGES OF REAL OBJECTS IN UNIVERSE (CONTD.)

Globular clusters

Globular clusters are ‘metal’ poor and have a roughly spherical distribution, rather than flattened into plane of Galaxy.



- Suggests globular clusters are part of the oldest population in the Galaxy. Hence give lower bound on the age of the universe.
- They have $\sim 10^5$ stars born at the same time, presumably all with the same chemical composition. Thus their relative positions on the Hertzsprung-Russell

diagram should be just due to differences in mass.

- Usual version of this is to plot 'colour' (indicating temperature) versus magnitude (basically luminosity, since all at same distance)

- They do indeed have a very tight locus, and the 'turn off point' from the main sequence, where the hydrogen has been exhausted, is a sensitive indicator of the age of the cluster.

- The lower down (in luminosity or flux) this turn off occurs, the older the cluster must be. In conjunction with models of stellar evolution, we can thus read off the age of a cluster.

The globular cluster M92.

↑ Flux

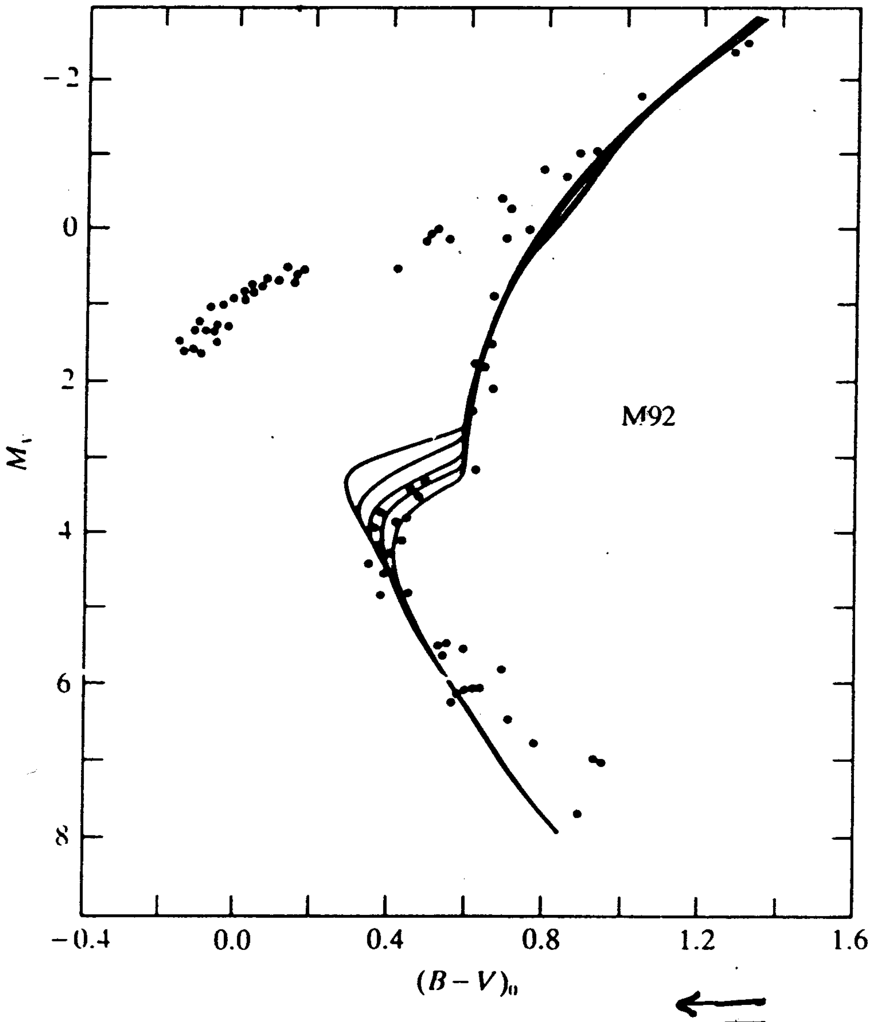


Figure 3-24. Observed CM diagram and theoretical isochrones for the globular cluster M92. The absolute-magnitude scale was fixed by setting M_V(RR) = +0.6. The isochrones are for models with $(X, Y, Z) = (0.80, 0.20, 1 \times 10^{-4})$ at ages (from top to bottom) of 10, 12, 14, 16, and 18 ($\times 10^9$) years. The turnoff point indicates an age of (14–16) $\times$ 10^9 years. [From (T2, 199).]

Age found = 14–16 × 10^9 yrs.

Example is shown for the cluster M92 — yields an age of 14 to 16×10^9 years. A minimum age averaged over many clusters and taking into account the uncertainties in stellar evolution models, is thought to be about $12.5 \pm 1.5 \times 10^9$ years.

Radioactive dating

Here we carry out dating of the elements themselves, rather than the rocks we find them in. As an example, we believe Uranium is formed in supernovae, with the isotopes being formed by a rapid process of neutron addition, called the 'r-process'.

- This process is predicted to give an initial abundance ratio of

$$\left[\frac{U^{235}}{U^{238}}\right]_{\text{initial}} = 1.65 \pm 0.15$$

  in the formation event which gives rise to the material.

- We say this happened in the very first generation of supernovae to give a lower bound on its age.

- Using the known decay rates of

$$\begin{aligned} \lambda(U^{235}) &= 0.971 \times 10^{-9} \text{ per year} \\ \lambda(U^{238}) &= 0.154 \times 10^{-9} \text{ per year} \end{aligned}$$

  together with the present abundance ratio of

$$\left[\frac{U^{235}}{U^{238}}\right]_{t_0} = 0.00723$$

we get

$$
\begin{aligned}
t_{\rm gal} \quad &\geq \quad \frac{\ln\left[U^{235}/U^{238}\right]_{\rm initial} - \ln\left[U^{235}/U^{238}\right]_{t_0}}{\lambda(U^{235}) - \lambda(U^{238})} \\
&\geq \quad 6.6 \times 10^{9} \quad \text{years.}
\end{aligned}
$$

- Can try this out with other ratios as well, and try to estimate over what period supernovae must have been going off and producing the elements incorporated in our rocks today. This yields estimates of 9 to 15×10^{9} years, similar to globular clusters.
- This work is not easy, and the results are still somewhat controversial.

Analytic results for a flat Λ universe

Cosmological models which are close to spatial flatness, but have a non-zero cosmological constant, are currently favoured by the data.

Although generally difficult to solve, one special case, that of exact spatial flatness, everything becomes reasonably simple again, and we can give explicit formulae for quantities of interest.

Dynamical equations for $\epsilon = 0$ (matter-dominated):

$$\frac{\ddot{R}}{R} + \frac{4\pi G\rho}{3} - \frac{\Lambda}{3} = 0, \qquad (A)$$

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8\pi G\rho}{3} - \frac{\Lambda}{3} = 0 \quad (B)$$

A convenient way of solving is to take $2 \times (A) + (B)$ and thereby eliminate ρ:

$$2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 - \Lambda = 0$$

- In the flat case there is no absolute definition of R – no curvature hence no scale (c.f. EdS).

- Use H (=$\dot{R}/R$) here, since this is insensitive to the absolute scale of R

Hence

$$H = \frac{\dot{R}}{R} \qquad \Longrightarrow \qquad \dot{H} = \frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2}$$

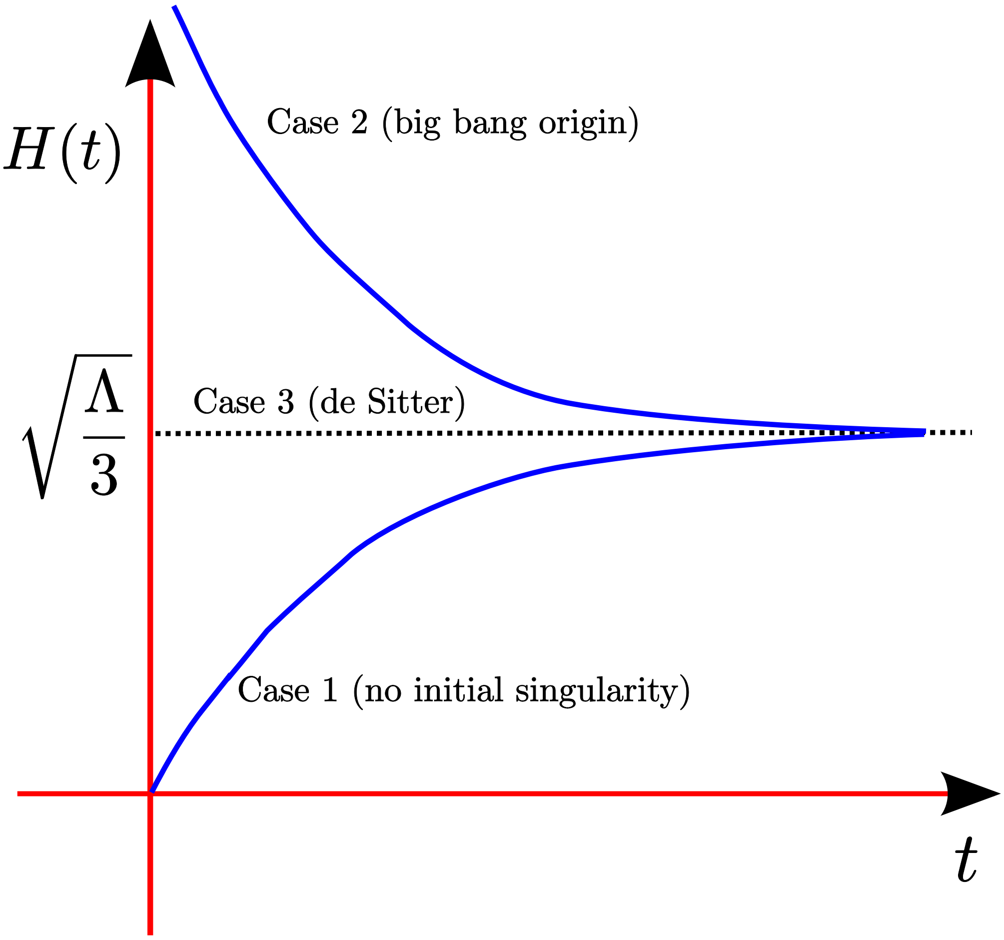
We therefore obtain

$$\boxed{2\dot{H} + 3H^2 = \Lambda}$$

Assuming $\Lambda > 0$, there are three basic solutions, the first two of which are in some sense *dual* to each other.

$$H(t) = \begin{cases} b\tanh(at) & \text{Case 1} \\ b/\tanh(at) & \text{Case 2} \\ b & \text{Case 3} \end{cases}$$

- $b = \sqrt{\Lambda/3}$, $a = 3/2\sqrt{\Lambda/3}$ in all cases.
- Case 1: H increases with time, starting from 0
- Case 2: H decreases, starting from infinity at $t = 0$
- In both these cases, H tends asymptotically to the value $\sqrt{\Lambda/3}$.
- Case 3: H remains constant at this value for all time. This case is called the *de Sitter universe*.



It may seem odd that we can have such drastically different behaviour from the same equation, but what distinguishes these cases is the behaviour of the density.

From the (B) equation:

$$\rho = \frac{1}{8\pi G}(3H^2 - \Lambda)$$

hence

$$8\pi G\rho(t) = \begin{cases} -\Lambda \operatorname{sech}^2(at) & \text{Case 1} \\ \Lambda \operatorname{cosech}^2(at) & \text{Case 2} \\ 0 & \text{Case 3,} \end{cases}$$

- Case 1 corresponds to *negative* density, and is unphysical.
- Case 3, the de Sitter universe, is empty of matter, and consists of pure 'dark energy' (the cosmological constant).
- Case 2 corresponds most closely with our current universe, having a positive, time varying ρ.

For this case, then

$$H(t) = \frac{\sqrt{\Lambda/3}}{\tanh\left(3/2\sqrt{\Lambda/3}\,t\right)}$$
$$\rho(t) = \frac{\Lambda}{8\pi G}\frac{1}{\sinh^2\left(3/2\sqrt{\Lambda/3}\,t\right)}$$

Inverting the first of these, we can obtain a useful relation for the age of the universe

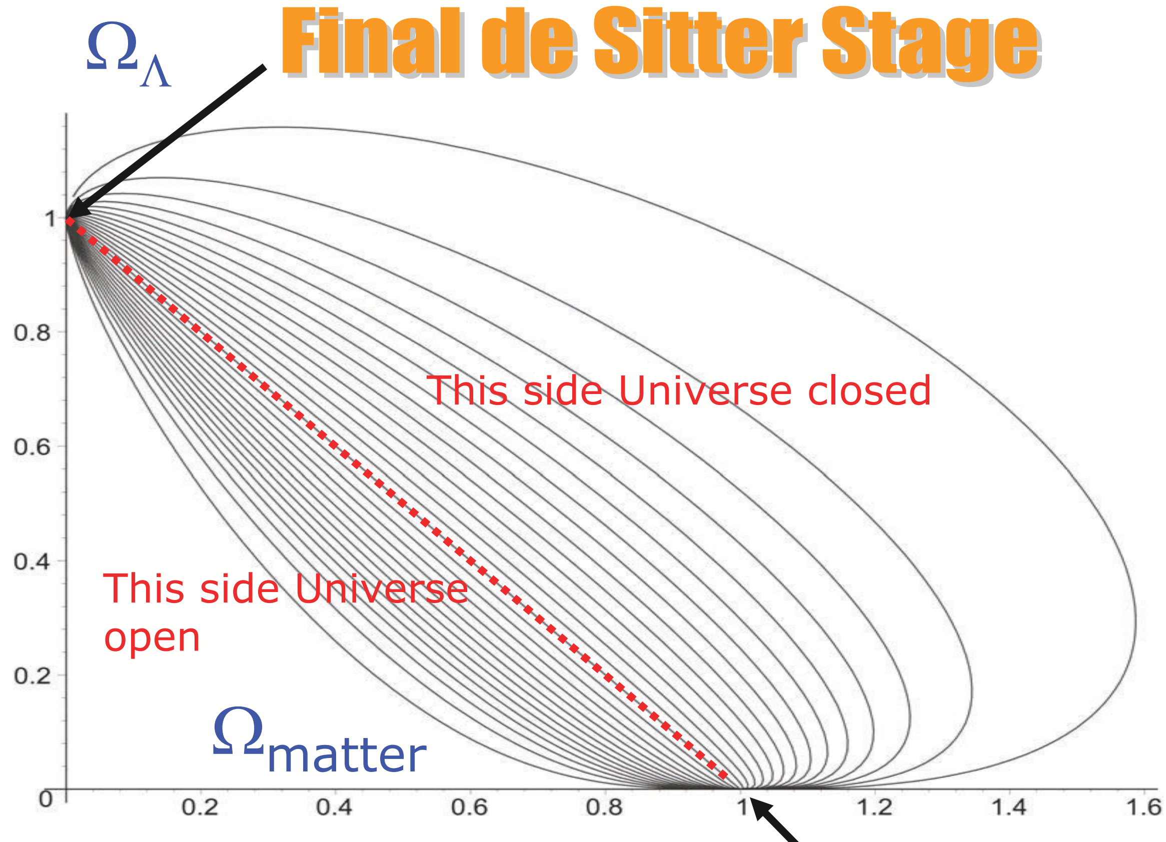
$$\boxed{t = \frac{2}{3H}\frac{\tanh^{-1}\sqrt{\Omega_\Lambda}}{\sqrt{\Omega_\Lambda}}}$$

- This goes over to the Einstein de Sitter relation $t = 2/(3H)$ as Ω_Λ tends to 0.
- Also, as $t \to \infty$, then $\Omega_\Lambda \to 1$. For this reason, our current universe is described as *asymptotically de Sitter* — it currently has a non-negligible matter content but as time progresses this gets diluted, and it tends towards

pure de Sitter.

- It is not just in a flat model that the universe is asymptotically de Sitter.
- Plot variation for a whole set of model universes, spanning both open and closed models with Λ.
- For virtually all the ways in which one could come out of the big bang, then Λ acts as a 'focussing' term, driving the universe towards an asymptotic end state of pure de Sitter.
- Note there are a few trajectories out of the big bang in which Λ is not large enough to overwhelm the attraction of matter, so that the universe re-collapses — these are not shown on the diagram, but would come out to the right, virtually parallel to the Ω_m axis.

Flow lines for the Universe

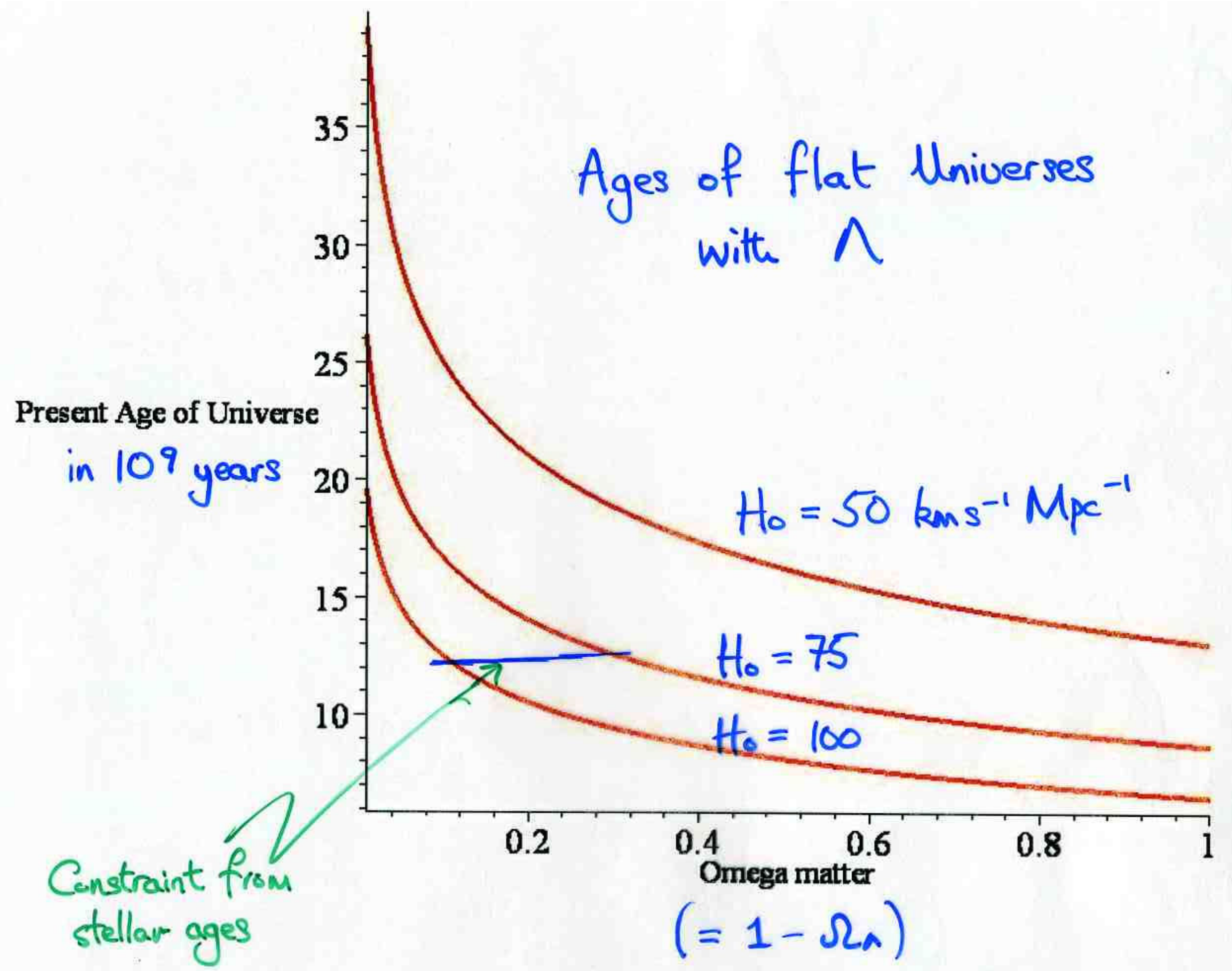


Universe starts at $(\Omega_{matter},\Omega_\Lambda)=(1,0)$ and moves to attractor point at (0,1) (de Sitter) – which curve are we on??

AGES

Can plot ages in a flat Λ universe, with different curves corresponding to different values of H_0.

Now it is clear that for H_0's near $70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, and $\Omega_{m0} \sim 0.3$, there is no longer a problem with the age constraint from globular clusters. Taking e.g. $H_0 = 72\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $\Omega_{\Lambda 0} = 0.7$ yields $t_0 = 13.1\,\mathrm{Gyr}$, which while not very much above the ages of the globular clusters, does at least allow some time for them to form.



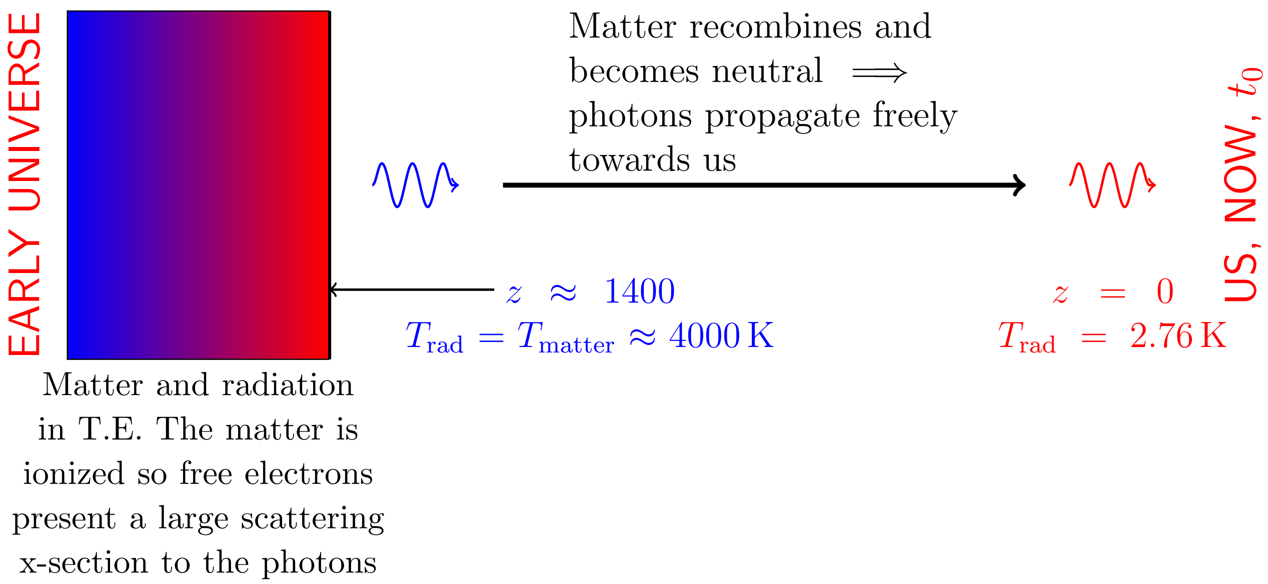
Thermal History of the Universe

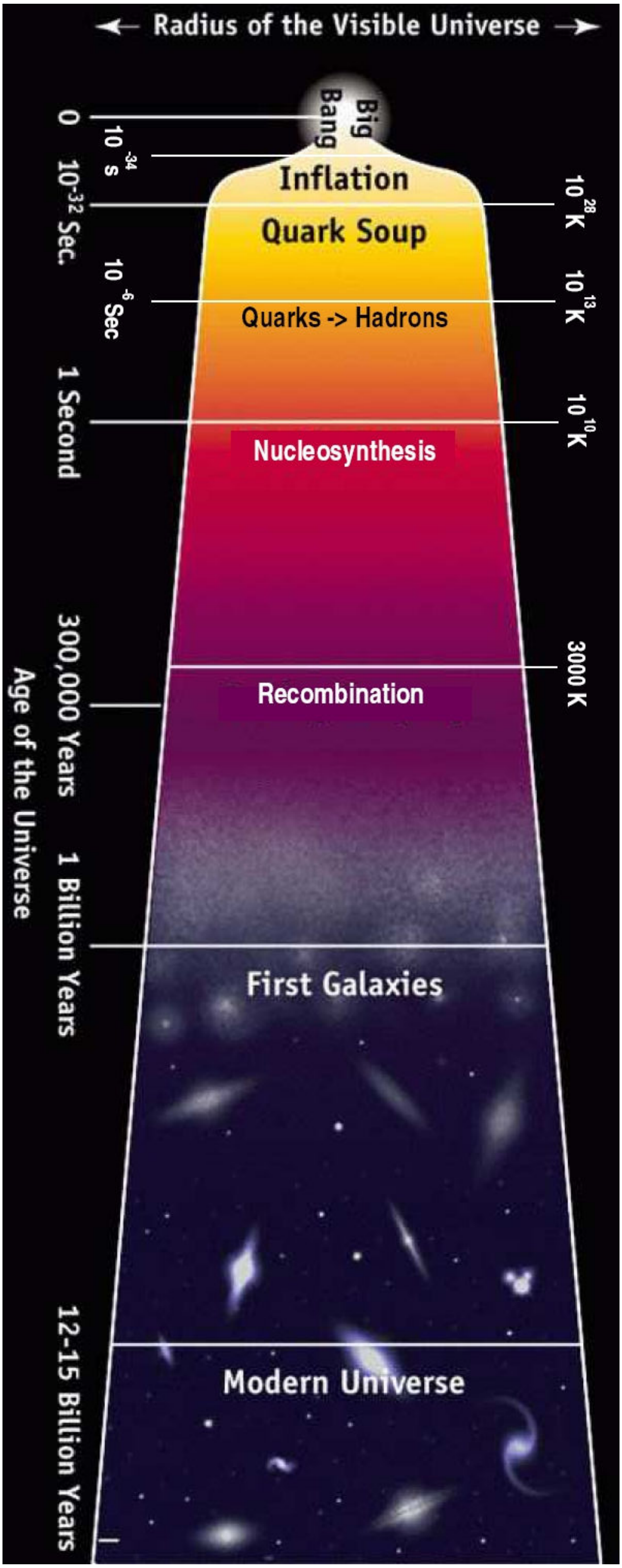
Next give a sketch of what we need to understand about the overall thermal history of the universe.

A sequence of important stages in the history of the universe are shown in the figure on the next page.

Let's examine more closely what happens at recombination.

The universe's energy density is dominated initially by the cosmic microwave background CMB, and this is in thermal equilibrium with matter through to recombination. At this point the universe suddenly becomes transparent, and the photons propagate freely towards us.





Radius of the Visible Universe
Big Bang
Inflation
Quark Soup
Quarks -> Hadrons
Nucleosynthesis
Recombination
First Galaxies
Modern Universe
0
10^{-34} s
10^{-32} Sec.
10^{-6} Sec
1 Second
300,000 Years
1 Billion Years
12-15 Billion Years
Age of the Universe
10^{28} K
10^{13} K
10^{10} K
3000 K

We want to understand where some of the numbers on the diagram p13 come from, and also the statement that initially the CMB energy density dominates. Why is this?

The energy density of black body radiation at temperature T is aT^4, where $a = 7.56 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$ is the reduced Stefan-Boltzmann constant (=$4\sigma/c$, where σ is the actual SB constant).

Also from the continuity equation, $\rho_r \propto R^{-4}$ while $\rho_m \propto R^{-3}$ and know that $1 + z = R_0/R$.

So for matter ρ scales as $(1+z)^3$ while for radiation it scales as $(1+z)^4$.

This shows that even if radiation is subdominant now, in terms of energy density, it must have been dominant in the past. (Will work out when, in a moment.)

Note in combination with $\rho_r \propto T^4$, we have shown that for black body radiation, then

$$\boxed{T \propto 1/R \propto (1+z)}$$

Thus can see where we are on the timeline of important stages of the universe shown above, by converting redshift to temperature of the CMB via

$$T_{\mathrm{CMB}}(z) = T_{\mathrm{CMB0}}(1+z)$$

Note our proof of this hinged on the blackbody nature of the radiation (since used energy density $\propto T^4$). We shall show shortly that the CMB remains blackbody even after recombination, when the radiation is no longer in thermal equilibrium with the matter.

WHERE ARE THE ENERGY DENSITIES EQUAL?

Characterising the current matter density in terms of the density parameter Ω_{m0}, the epoch of equality satisfies

$$\Omega_{m0}\frac{3H_0^2}{8\pi G}c^2 = (1+z)aT_0^4$$

$$\text{i.e.} \quad (1+z_{\rm eq}) = \frac{3\Omega_{m0}H_0^2c^2}{8\pi GaT_0^4}$$

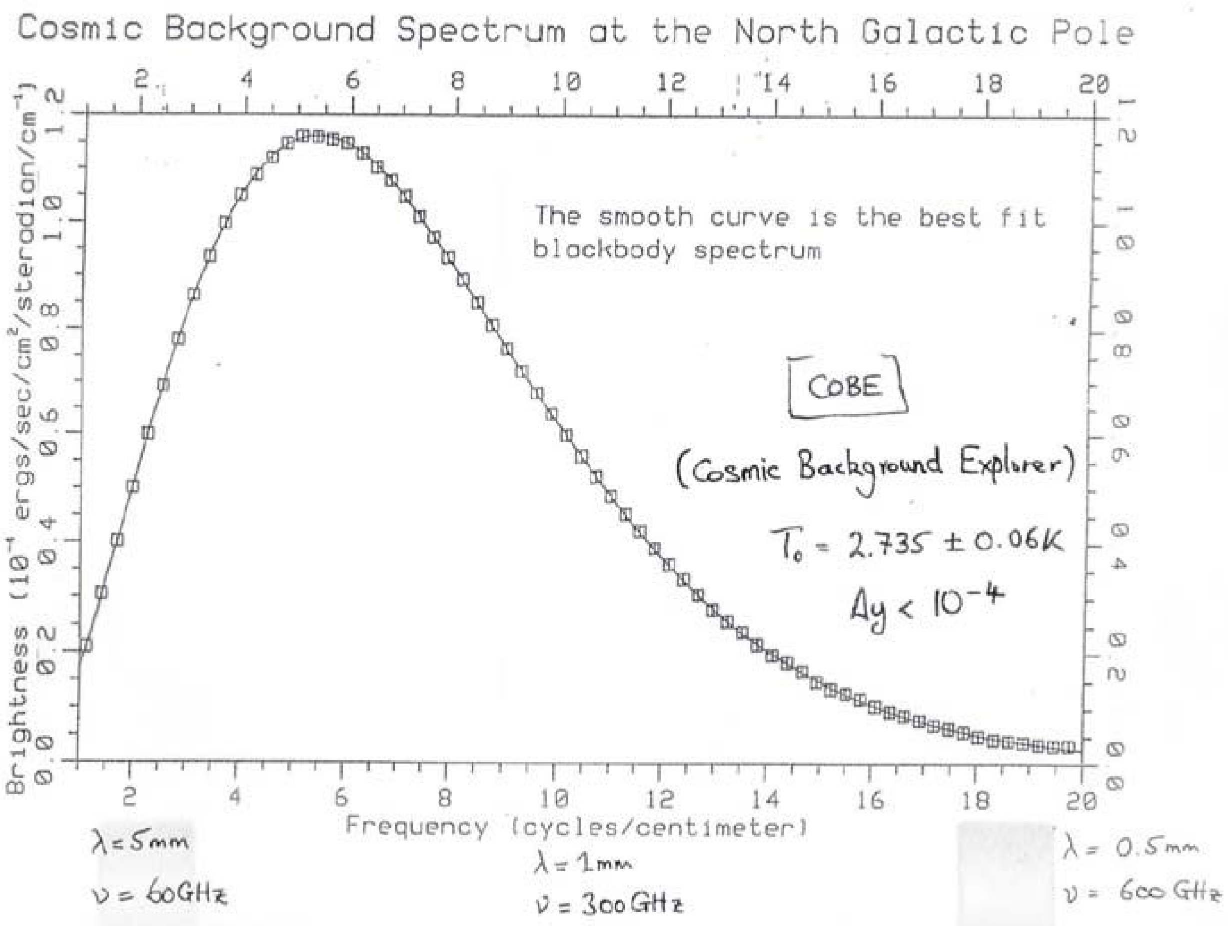
Best current values: $H_0 = 72\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $T_0 = 2.73\,\mathrm{K}$ and $\Omega_{m0} = 0.3$, $\Longrightarrow \; 1+z_{\rm eq} = 6248$

More generally, writing $H_0 = h/100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$:

$$\boxed{1+z_{\rm eq} = 4.0\times 10^4\,\Omega_{m0}\,h^2}$$

Now recombination occurs in the range $z = 1100$–1400 (see below) and so for all reasonable values of Ω_{m0} and h, the epoch of equality occurs *before* recombination (though after nucleosynthesis).

The Cosmic Microwave Backround



The spectrum of the CMB as observed by the COBE satellite

Why does the CMB remain blackbody?

- CMB is BB when *emitted* in the early universe, since it is then in thermal equilibrium with the matter.
- How does it maintain its BB-form as it propagates towards us, in extreme thermodynamic *dis*equilibrium with the matter?

Let $n_\nu d\nu$ be the (proper) number density of photons with frequencies in the range $(\nu, \nu + d\nu)$.

For blackbody radiation

$$n_\nu = \frac{8\pi\nu^2}{c^3\left(e^{h\nu/kT} - 1\right)},$$

since $B_\nu = I_\nu = (c/4\pi)u_\nu$, and $u_\nu = n_\nu h\nu$.

Consider a *comoving* volume $V(t)$, i.e. one where its proper volume grows as R^3. Let t_1 be the epoch immediately after recombination, and t_0 today. If photons travel to us completely unhindered from recombination, then the numbers of photons in comoving volume elements should be conserved:

$$n_{\nu_1}(t_1)d\nu_1(t_1)V(t_1) = n_{\nu_0}(t_0)d\nu_0(t_0)V(t_0).$$

The standard redshift argument gives: $\nu \propto 1/R$, and $d\nu \propto 1/R$:

$$\begin{aligned} n_{\nu_0}(t_0) &= n_{\nu_1}(t_1)\frac{d\nu_1(t_1)}{d\nu_0(t_0)}\frac{V(t_1)}{V(t_0)} \\ &= n_{\nu_1}(t_1)\frac{R_0}{R_1}\left(\frac{R_1}{R_0}\right)^3 = \left(\frac{R_1}{R_0}\right)^2 n_{\nu_1}(t_1). \end{aligned}$$

But $n_{\nu_1}(t_1)$ *does* have a blackbody form, at temperature T_1

say, and thus

$$n_{\nu_0}(t_0) = \left(\frac{R_1}{R_0}\right)^2 \frac{8\pi\nu_1^2}{c^3} \Bigg/ \left[\exp(\frac{h\nu_1}{kT_1}) - 1\right].$$

Finally, $\nu_1 = (R_0/R_1)\nu_0$, so that we can rewrite the last equation as

$$n_{\nu_0}(t_0) = \frac{8\pi\nu_0^2}{c^3} \Bigg/ \left[\exp(\frac{h\nu_0}{kT_1 R_1/R_0}) - 1\right].$$

Radiation still has a Planckian form, but temperature:

$$T_0 = \frac{T_1 R_1}{R_0} = \frac{T_1}{1+z}.$$

Note this is another proof that $T \propto 1/R$.

Recombination

Why does recombination happen at a temperature of $\approx 4000\,\mathrm{K}$?

Note using $T \propto (1+z)$ this value $\implies$ $z_{\mathrm{rec}} \approx 1400$ (for $T_0 \approx 2.7\,\mathrm{K}$)

- Ionization potential of hydrogen is $13.6\,\mathrm{eV}$, which corresponds to a temperature of approximately $160,000\,\mathrm{K}$.
- Recombination cannot be an instantaneous process – e.g. if an electron and proton recombine directly to the ground state get a a $13.6\,\mathrm{eV}$ ionizing photon.
- Need to use Saha equation plus consideration of all transitions to get a correct description of the ionization fraction with time.
- Can get approximate answer, however, by requiring that the universe cools to the point where there are no longer sufficient photons of energy $> 13.6\,\mathrm{eV}$.
- Show in a moment that there are about 10^9 photons per baryon, and that this number is the same all the way back through to recombination.

Thus can approximate the temperature of recombination by requiring that there be less than one photon of energy $> 13.6\,\mathrm{eV}$ per proton. Express this via a Boltzmann factor:

$$\exp\left(-\frac{13.6\,\mathrm{eV}}{kT_{\mathrm{rec}}}\right) \sim 10^{-9},$$

This gives $T_{\mathrm{rec}} \sim 7600\,\mathrm{K}$. This is not too bad an estimate — certainly much closer to the real value of about $4000\,\mathrm{K}$ than the $160,000\,\mathrm{K}$ we started with!

Obtaining the 'photon to baryon ratio'

The total number of photons in blackbody radiation at temperature T_0 is

$$n_\gamma = \frac{8\pi}{c^3}\int_0^\infty \frac{\nu^2\,d\nu}{e^{h\nu/kT}-1} = \frac{0.37aT_0^3}{k} = 4.2\times10^8\,\mathrm{m}^{-3}$$

Number density of protons today is

$$n_p = \frac{\rho_{0\,\mathrm{matter}}}{m_p} = \frac{3H_0^2\Omega_{m0}}{8\pi G m_p},$$

where we should use the contribution to Ω_{m0} from ordinary (as against dark) matter.

Will discuss this later, but probably $\Omega_{\mathrm{baryonic}} \sim 0.05$

Also use $H_0 = 72\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

Then $n_p = 0.29\,\mathrm{m}^{-3}$ and

$$\frac{n_\gamma}{n_p} = \left.\frac{\text{no. of photons}}{\text{no. of baryons}}\right|_{\text{today}} = 1.4 \times 10^9,$$

Since both n_γ and n_p scale as R^{-3} this ratio will be the same at recombination.

In fact one can show that this ratio is the same right back to very early times — e.g. its value predates nucleosynthesis.

Thought that it is associated with whatever it is that gave rise to matter/antimatter asymmetry — e.g. if equal amounts of matter and anti-matter existed originally, but there is a slight asymmetry in their annihilation, by about 1 part in 10^9, then this could explain observed photon/baryon ratio, and also current absence of anti-matter.

But all of this still pretty unclear.