SaDS HW 3

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Problem 3.1

See src folder

Problem 3.2

in C++:
int dec$(x)\{$
    $if(x == 0)$ *return* $x;$
    return dec$(x-1);$
$\}$

$$\dfrac{\dfrac{\checkmark}{\vdash int : type}\ \dfrac{\checkmark}{\vdash int : type}\ \text{(rule for base types)}\checkmark}{\dfrac{\vdash int \to int : type}{\vdash (dec : int \to int = (x : int)) \mapsto if(x = 0)\{0\}\ else\ \{dec(x-1)\} : int \to int}}\ \dfrac{\dfrac{\dfrac{\checkmark}{x : int \in \Gamma}\ \dfrac{\checkmark}{\Gamma \vdash x : int}\ \dfrac{\checkmark}{\Gamma \vdash 0 : int}}{\Gamma \vdash x == 0 : bool}\ \dfrac{\checkmark}{\Gamma \vdash 1 : int}}{\dfrac{dec : int \to int \Rightarrow int : type}{\Gamma\ \vdash if(x = 0)\{0\}else\{dec(x-1)\}}}$$

in C++
intdec$(x)\{$
    while$(x > 0)\{$
        $x = x - 1;$
    $\}$
    return $0;$
$\}$

$$\dfrac{\dfrac{\checkmark}{\vdash int : type}\ \dfrac{\checkmark}{\vdash int : type}\ \text{(rule for base types)}\checkmark}{\dfrac{\vdash int \to int : type}{((\vdash valdec : int \to int) = \Gamma) = (x : int) \mapsto while(x > 0)\{x = x - 1}}\ \dfrac{\dfrac{\dfrac{\checkmark}{x : int \in \Gamma}\ \dfrac{\checkmark}{\Gamma \vdash x : int}\ \dfrac{\checkmark}{0 : int \in \Gamma}}{\Gamma \vdash x > 0 : bool}\ \dfrac{\dfrac{\checkmark}{x : int \in \Gamma}\ \dfrac{\checkmark}{\Gamma \vdash x : int}\ \dfrac{\checkmark}{1 : int \in \Gamma}}{\Gamma \vdash x = x - 1 : int}}{\dfrac{\vdash dec : int \to int \Rightarrow int : type}{\Gamma \vdash while(x > 0)\{x = x - 1\} : int}}$$

Problem 3.3

C := A + B

$$\frac{\Gamma \vdash A : type \quad \Gamma \vdash B : type \quad \Gamma \vdash A + B : type}{\Gamma \vdash A + B : C}$$

$$\frac{\Gamma \vdash f(A) \rightsquigarrow inj1(A) \qquad \Gamma \vdash g(B) \rightsquigarrow inj2(B) \qquad \Gamma \vdash (u, f(A), f(B)) \rightsquigarrow C}{\Gamma \vdash A + B \rightsquigarrow (u, f(A), f(B))}$$