

Problem 5.1

After every usage of the method functions *getSize*, *push* and *pop*, the private element *size* is only updated correctly to reflect the current size of the stack in the functions *push* and *pop* where size is actually changed. Therefore *length(elements)* will always equal *size*, as *size* is always correctly updated as a pointer to the last element of the stack (which happens to be the length of it as well).

$\therefore \text{length}(\text{elements}) == \text{size}$ is a good class invariant.

Problem 5.2

See next page.

Problem 5.3

Proof p:

Step case:

$$\text{zero} + n == n + \text{zero}$$

Due to *zero_right* and *zero_left*, $\text{zero} + n \rightsquigarrow n$ and $n + \text{zero} \rightsquigarrow n$

$\rightarrow m == m \rightsquigarrow \text{true}$

$\therefore \text{true}$

Induction step:

Goal to prove:

$$\text{succ}(m) + n == n + \text{succ}(m)$$

$$\begin{array}{l|l} \text{succ}(m) + n \rightsquigarrow \text{succ}(m + n) & \text{succ_left} \\ \text{succ}(m + n) \rightsquigarrow \text{succ}(n + m) & \text{Induction hypothesis} \\ \text{succ}(n + m) \rightsquigarrow n + \text{succ}(m) & \text{succ_right} \end{array}$$

$\therefore \text{succ}(m) + n == n + \text{succ}(m)$ (same vice versa).

Problem 5.2

```
class Date {
private:
    int year;
    int month;
    int day;

public:
    Date(int year, int month, int day) {
        this->year = year;
        this->month = month;
        this->day = day;
    }
    ~Date(){}

    Date yesterday() {
        int year;
        int month;
        int day;
        int r_month = 0;
        int r_year = 0;

        if(this->day == 1) {
            r_month = 1;
            day = getDay(this->month);
            if(this->month == 3) {
                day = 28;
            } else if(this->month == 5 || this->month == 7
                || this->month == 10 || this->month == 12) {
                day = 30;
            } else {
                day = 31;
            }
        } else {
            day = this->day - 1;
        }

        if(r_month == 1) {
            if(month == 1) {
                r_year = 1;
            } else {
                month = this->month - 1;
            }
        } else {
            month = this->month;
        }

        if(r_year == 1) {
            year = this->year - 1;
        } else {
            year = this->year;
        }

        Date yesterday = new Date(year, month, day);
        return yesterday;
    }

    Date tomorrow() {
        int year;
        int month;
        int day;

        if(this->day == 31) {
            day = 1;
            if(this->month == 12) {
                month = 1;
                year = this->year + 1;
            } else {
                month = this->month + 1;
                year = this->year;
            }
        } else {
            day = this->day + 1;
            month = this->month;
            year = this->year;
        }
        Date tomorrow = new Date(year, month, day);
        return tomorrow;
    }
};
```

Class invariant:

$$12 \leq \text{month} \leq 1 \mid [31, 30, 28]_{\perp \text{month}} \geq \text{day} \geq 1$$

Problem 5.4

The screenshot shows the Coq IDE with the following components:

- Main Editor:** Contains Coq code for an inductive type `seq`, a fixpoint `length`, and a theorem `length_corr` proven by induction. The code is as follows:


```

Inductive seq : nat -> Set :=
| niln : seq 0
| consn : forall n : nat, nat -> seq n -> seq (S n).

Fixpoint length (n : nat) (s : seq n) {struct s} : nat :=
match s with
| niln => 0
| consn i _ s' => S (length i s')
end.

Theorem length_corr : forall (n : nat) (s : seq n), length n s =
Proof.
  intros n s.

  (* reasoning by induction over s. Then, we have two new goals
  corresponding on the case analysis about s (either it is
  niln or some consn *)
  induction s.

  (* We are in the case where s is void. We can reduce the
  term: length 0 niln *)
  simpl..

  (* We obtain the goal 0 = 0. *)
  trivial.

  (* now, we treat the case s = consn n e s with induction.
  hypothesis IHs *)
  simpl..

  (* The induction hypothesis has type length n s = n..
  So we can use it to perform some rewriting in the goal: *)
  rewrite IHs..

  (* Now the goal is the trivial equality: S n = S n *)
  trivial.

  (* Now all sub cases are closed, we perform the ultimate
  step: typing the term built using tactics and save it as
  a witness of the theorem. *)
  Qed.
      
```
- Messages ("scratch"):** Displays the message "All proof terms checked by the kernel".
- Errors ("scratch"):** Empty window for error messages.
- Jobs ("scratch"):** A table showing the status of workers.

Worker	Job name
proofworker:0	Idle

The status bar at the bottom indicates "Ready", "Line: 41 Char: 8", "Coq is ready", and "0 / 0".