

SORTING ALGORITHMS ANALYSIS:

Input Array:

Sorted Elements:

{1,4,6,7,9,11,14,17,24,28,33,37,41,44,48,53,58,65,69,74,80,86,92,100,102,104,107,119,122,125,135,140};

Reverse Sorted Elements:

{140,135,125,122,119,107,104,102,100,92,86,80,74,69,65,58,53,48,44,41,37,33,28,24,17,14,11,9,7,6,4,1};

Random32 Elements:

{191,135,478,33,6,7234,78,56,23,2,2396,34557,65446,865,1245,8066,3456,743,66,2444,567,787,9189,1334,7766,876,8899,5167,7819,573,7788,1856};

Constant rate:

Merge Sort:

Sorted Array:

(n = 32)

$$\begin{aligned} O(n \log n) &= 32 * \log 32 \\ &= 32 * 5 = 160 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor

$$\text{Constant factor} = 80 / 160 = 0.5$$

Reverse Sorted Array:

(n = 32)

$$\begin{aligned} O(n \log n) &= 32 * \log 32 \\ &= 32 * 5 = 160 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor

$$\text{Constant factor} = 124 / 160 = 0.775$$

Random 32:

$$\begin{aligned} O(n \log n) &= 32 * \log 32 \\ &= 32 * 5 = 160 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor

$$\text{Constant factor} = 80 / 160 = 0.5$$

Random 2¹⁰ (1024):

$$\begin{aligned} O(n \log n) &= 1024 * \log 1024 \\ &= 1024 * 10 = 10240 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor

$$\text{Constant factor} = 5120 / 10240 = 0.5$$

Random 2¹⁵ (32768):

$$\begin{aligned} O(n \log n) &= 32768 * \log 32768 \\ &= 32768 * 15 = 491520 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor

$$\text{Constant factor} = 245820 / 491520 = 0.5001$$

Random 2²⁰ (1048576):

$$\begin{aligned} O(n \log n) &= 1048576 * \log 1048576 \\ &= 1048576 * 20 = 20971520 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor

$$\text{Constant factor} = 10538932 / 20971520 = 0.5025$$

Quick Sort:**Sorted Array:**

$$(n = 32)$$

$$\begin{aligned} O(n \log n) &= 32 * \log 32 \\ &= 32 * 5 = 160 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor

$$\text{Constant factor} = 496 / 160 = 3.1$$

Reverse Sorted Array:

$$(n = 32)$$

$$\begin{aligned} O(n \log n) &= 32 * \log 32 \\ &= 32 * 5 = 160 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor

$$\text{Constant factor} = 130 / 160 = 0.8125$$

Random 32:

$$\begin{aligned} O(n \log n) &= 32 * \log 32 \\ &= 32 * 5 = 160 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor
Constant factor = $496 / 160 = 3.1$

Random 2^{10} (1024):

$$\begin{aligned} O(n \log n) &= 1024 * \log 1024 \\ &= 1024 * 10 = 10240 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor
Constant factor = $145 / 10240 = 0.014$

Random 2^{15} (32768):

$$\begin{aligned} O(n \log n) &= 32768 * \log 32768 \\ &= 32768 * 15 = 491520 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor
Constant factor = $219 / 491520 = 0.0004$

Random 2^{20} (1048576):

$$\begin{aligned} O(n \log n) &= 1048576 * \log 1048576 \\ &= 1048576 * 20 = 20971520 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor
Constant factor = $342 / 20971520 = 0.000016$

Heap Sort:

Sorted Array:

$$(n = 32)$$

$$\begin{aligned} O(n \log n) &= 32 * \log 32 \\ &= 32 * 5 = 160 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor
Constant factor = $421 / 160 = 2.631$

Reverse Sorted Array:

$$(n = 32)$$

$$\begin{aligned} O(n \log n) &= 32 * \log 32 \\ &= 32 * 5 = 160 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor
Constant factor = $388 / 160 = 2.425$

Random 32:

$$\begin{aligned} O(n \log n) &= 32 * \log 32 \\ &= 32 * 5 = 160 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor

$$\text{Constant factor} = 339 / 160 = 2.118$$

Random 2¹⁰ (1024):

$$\begin{aligned} O(n \log n) &= 1024 * \log 1024 \\ &= 1024 * 10 = 10240 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor

$$\text{Constant factor} = 276005 / 10240 = 26.953$$

Random 2¹⁵ (32768):

$$\begin{aligned} O(n \log n) &= 32768 * \log 32768 \\ &= 32768 * 15 = 491520 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor

$$\text{Constant factor} = 267993330 / 491520 = 545.233$$

Random 2²⁰ (1048576):

$$\begin{aligned} O(n \log n) &= 1048576 * \log 1048576 \\ &= 1048576 * 20 = 20971520 \end{aligned}$$

Key comparisons / $n \log n$ = Constant factor

$$\text{Constant factor} = 348023325 / 20971520 = 16.595$$

	MergeSort		QuickSort		HeapSort	
	Number of Key Comparisons	Measured clock time (in ns)	Number of Key Comparisons	Measured clock time (in ns)	Number of Key Comparisons	Measured clock time (in ns)
Best Case (Sorted Array)	80	0.430284	496	0.169309	421	0.166951
Average Case (Random Array)	124	0.061052	130	0.04994	388	0.116406
Worst Case (Reverse Sorted Array)	80	0.07489	496	0.029349	339	0.131873
Random $2^{10} = 1024$	5120	5.617048	145	0.022394	276005	8.317122
Random $2^{15} = 32768$	245820	47.283714	219	0.029786	267993330	1413.270431
Random $2^{20} = 1048576$	10538932	508.501448	342	0.030268	348023325	2477566.877369