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A multi-parent genetic algorithm for solving longitude–latitude-based 4D traveling salesman problems under uncertainty



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ABSTRACT

In this study, we propose a mathematical model of a 4D clustered traveling salesman problem (CTSP) to address the cost-effective security and risk-related difficulties associated with the TSP. We used a multiparent-based memetic genetic algorithm to optimize paths between all clusters and proposed unique heuristic approaches to create clusters and reconnect them. We constructed a 4D CTSP considering multiple routes between two locations and multiple available vehicles on each route. Travel expenses and risks impact every itinerary; however, the behaviors of these costs and risks are always uncertain. We inspected various standard benchmark problems from (TSPLIB) using the proposed calculations. Real-life problems in the tourism industry motivate a longitude–latitude-based CTSP with risk constraints. Thus, we determined the risk of each path based on longitude and latitude. The contributions of this study are twofold: developing a genetic algorithm and heuristics based on mathematical modeling of a real problem.

1. Introduction

In real life, we face different (NP)-hard issues, such as vehicle routing problem (VRP), traveling salesman problem (TSP), traveling purchaser problem (TPP) and facility location problem. Heuristic methodologies such as genetic algorithm (GA) and tree growth algorithm and swarm optimization procedures such as ant colony optimization (ACO) and particle swarm optimization (PSO) have been proven efficient for these genuine NP-hard issues [1]. In this study, we propose a unique clustering and re-linking technique to construct a 4D clustered TSP (CTSP) and applied memetic GA to optimize each cluster.

Technological advances have led to the globalization of the exchange market from all perspectives. This allows a sales rep or agent to venture to different places quickly to satisfy their business perspectives. Itineraries for business or other travel plans are prepared without considering any parameters such as cost, comfort, chance, and time. These parameters are typically uncertain. The tourism industry is profitable everywhere in the world [2]. Researchers have studied different aspects of tourism. According to Silva et al. [3], study on low-cost tourism, low-cost carriers impact tourists' behaviors. Ayhan et al. [4] focused on rural tourism activities and demonstrated the importance of rural

tourism for rural development. Climate and weather act are important factors for the tourism industry. Perry [5] focused on the significant role of climate in tourism and how it is an equally important component that should be included in tourism grades. According to Kubo et al. [6] different geographical positions base coastal tourism on Japanese beaches. Researchers have not focused on region-wise tourism based on geographical positions. The risks may appear from different angles, such as bad climate situations, travel time, accidental risk, tourist health conditions, and so on. Focusing on these points, Huang et al. [7] proposed significant theories to analyze health-related behaviors with respect to geographical locations.. Thus, there are no solutions where tourists will get the best comfort with minimum risk at a reasonable cost, and management can overcome undesirable situations with minimum loss. To fill the gap, this study uses a longitude–latitude-based cluster-wise routing design.

Contemporary communication systems have improved scientific advances. As there are multiple ways to reach a destination, there are also various communication systems. Therefore, money, time, comfort, danger, and so on should be kept in mind, in addition to the selected means of communication. Travel planning does not always depend on

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- $^{1}\ https://www.hotelmize.com/blog/understanding-the-important-role-of-technology-in-tourism-marketing/.$
- ² https://www.ems.gov/.

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travel costs; it is also affected by time, comfort, safety, and so on. When a person is in a medical emergency or another danger, an ambulance or another conveyance is required for smooth and fast transport. Factors such as time and safety take precedence over value.² The concept of clustering is considered when we operate a set of nodes but use a subset of nodes to minimize loss. When a natural disaster such as an earthquake or heavy cyclone causes damage on a large scale, quick rescue or relief operations are done cluster-wise. In contrast, if a large-scale travel plan is organized according to a category, it is beneficial from the perspectives of travel management as well as finance. Keeping these real issues in mind, this study is our innovative response. According to this study, each tour arrangement ought to be 3D (e.g., a route with more than one vehicle accessible for going between source and destination) or 4D TSP (e.g., a diverse way is available where more than one vehicle is affordable on each route). Maity et al. [8] proposed a hybrid algorithm to solve 4D TSP. To solve an unsealed, constrained multi-objective solid TSP, Maity et al. [9] proposed a rough multiobjective GA. In this study, the risk is mathematically formulated based on latitude and longitude. We considered 3D and 4D CTSPs with risk constraints. The proposed clustering technique and cluster re-linking methods were unique. This study focused on the following:

- (i) Planning a large-scale trip for a natural disaster rescue operation or other travel purposes is a complex problem. How might we structure a refined visit plan on an enormous scale inside a state or a nation?
- (ii) Various challenging situations such as accidents and robbery are likely to be encountered when we set a route to a destination. What might be a visit plan with an ideal expense while keeping up the standard risk/comfort factor?
- (iii) A problem is divided into various small groups to reduce its complexity. Getting those groups together as a solution is a complex job. What should be the cluster sequence and cluster re-linking technique to get the optimal travel plan?
- (iv) Management can design a tour plan using the longitude and latitude of each place to minimize risk or loss and maximize comfort and reliability. What is the administrative method to deal with issues such as risk factors arising as an effect of different longitudes and latitudes?

Questions (i)–(iv) are the key issues that we attempted to address in this study.

This paper is presented as follows: Section 1 gives a brief introduction. A short literature review is presented in Section 2. In Section 3, we describe mathematical prerequisites. The formulation of CTSP showed in Section 4. In Section 5, the proposed CTSP algorithm is presented. Section 6 presents some empirical experiments. In Section 7, a brief discussion is given. Section 8 presents the practical implementation, and Section 9 focuses on managerial insights. Finally, in Section 10, the conclusion and future scope are discussed.

2. Literature review

The TSP is a well-known combinatorial optimization problem [10]. Initially, it takes a set of nodes from a complete weighted graph and finds a concise path by visiting every node except the origin node, just once. Generalization of the TSP as the family TSP was proposed by Bernardino and Paias [11]. Here, given cities were distributed within several families, and the shortest route that visits a given number of cities in each family was identified. Like TSP, different types of shortest-path problems have been studied by scholars. A gaussianvalued neutrosophic shortest path problem was studied by Kumar et al. [12] and Kumar et al. [13]. Different types of TSPs have been proposed based on various realistic phenomena [14,15]. The CTSP, the most popular form of the classical TSP was first observed by Chisman [16]. One of the most significant highlights of CTSP is that nodes inside a cluster are visited. In the CTSP, a traveler visits different cities with just a single movement. This study is an initial step for designing a multi-route and multi-vehicle CTSP as a 4D CTSP. Recently, Maity et al.

[9] studied risk as a constraint on TSP. We determined risk based on the distances between cities and the related longitudes and latitudes. Determining the risk of a visit depends upon the condition of the streets, types of vehicles, peace conditions, climate conditions, and so on. Thus risk can be considered an uncertain factor. This model has a fuzzy incentive for the risk and cost of a journey. The CTSP has extensive real-life applications. Laporte and Palekar [17] studied warehousing problems, Ozgur and Brown [18] considered job scheduling and sequence scheduling problems, Lokin [19] focused on manufacturing problems, Pop et al. [20] considered vehicle routing problems and Batsyn et al. [21] and Nasiri et al. [22] considered disk defragmentation problems. This study fills the gap of longitude–latitude-based risk and routing in terms of 4D (multi-path and multi-vehicle) constraint (risk) CTSP.

Uncertainty plays an important role in modeling real-life problems [23–25]. Any traveling or transportation between two locations is always affected or controlled by parameters such as cost, time, risk, comfort, demand, and supply, and these parameters are uncertain or fuzzy. Kumar et al. [26] studied the fuzzy Pythagorean transportation problem and considered both, the Pythagorean fuzzy arithmetic and numerical conditions in three different models in a Pythagorean fuzzy environment. A recent study considered uncertainty using triangular fuzzy numbers for machine performance with ranking [27].

In this study, a model of latitude-longitude-based CTSP with risk constraints is considered where clusters are created based on the latitude-longitude of geographical positions against each node or city. We assumed that there is more than one route and multiple vehicles available on each way to move from one town to a different city and considered a risk factor for each path. The proposed formula measured risk dynamically, with each city's or node's extreme weather and geographic location influencing the measurements.

Due to their limited computational power and enormous time, Chisman [16] applied exact solution methods for the CTSP, which were sufficient for minimal-size instances. Helsgaun [28] solved extensive cases, up to 85,900 cities, dividing the problem up to 17,180 clusters with an optimal solution, but it is also a time-consuming, laborious process. At the same time, most researchers have focused on getting satisfactory solutions within a reasonable time. Phuong et al. [29] proposed a priority-oriented constraint, a rule called d-relaxed priority, constructing an optimum delivery route based on the preference of each delivery point. This algorithm formed the mixed integer programming model for small and medium instances with up to 50 nodes. To solve up to 200 nodes, the proposed method is a metaheuristic approach following the ideas of the greedy randomized adaptive search procedure. A tabu-clustered TSP is a metaheuristics approach where the initial node is divided into two types of sub-clusters and a set of tabu nodes [30]. They suggested that the algorithm may be able to solve telemetry tracking and bidding resourcefulness scheduling problems. Laporte et al. [31] also introduced an inquiring method to unfold CTSP through a taboo search program to access clusters in a prespecified order. Another example of an algorithmic rule to resolve the optimization problem is the harmony search algorithm. In 2010, Yildiz and Öztürk [32] proposed a hybridized algorithm for the vehicle manufacturing industry to resolve shape optimization problems. This algorithm is a mix of Taguchi's method and the harmony search procedure. Assuming a large-scale TSP, the computational time within a limited period is not easily achievable every time. This study addresses the research gap of designing and solving large-scale problems in the CTSP. One of the most discussed evolutionary algorithms is the GA developed by Goldberg [33]. GA conducts a heuristic search, including exploration and exploitation. It has proven its efficiency and effectiveness in solving real-life NP-hard problems in reasonable computational time [34,35]. Several researchers have focused on GA to solve different optimization problems and their variants [36,37]. Ahmed [38] proposed a heuristic way to deal with the CTSP. It considers a pre-determined number of clusters, which are accessed in a prespecified order. A Lexi search calculation was created to get specific ideal answers for the CTSP. Binh et al. [39] developed a CTSP to address the clustered briefest-way tree issue. This problem is a combination of two sub-problems. One is an H-Problem, which is used to identify an arc set as a connector within the clusters, whereas the other is the L-Problem, which is used to construct a spanning tree for the sub-diagram in every group. The specified H-Problem is a bunch of new evolutionary operators. They are unique beginning populations, crossovers, and mutation operators. One of the most important aims of this algorithm is to reduce the search space of an evolutionary algorithm that applies to solving an optimization problem. Roy et al. [40] developed a new modified GA with a new multiparent crossover to solve medium and large-scale TSPs. Except for GA, other evolutionary algorithms such as the Ant Colony System studied by Bianchi et al. [41], the ACO studied by Mandala et al. [42], and PSO [43,44] have also been applied to optimize different optimization problems such as TSP as well as CTSP. A recent study by Sevedan et al. [45] proposed demand forecasting using a k-means algorithm with four layered hierarchies for supply chain networks. Though various developments have been made within the field of optimization problems, few shortcomings and limitations remain associated with the formulation of the matter and improving solution procedures. This study presents a novel cluster ordering and re-linking methodologies for the proposed 4D constraint-based CTSP. The features of the proposed work are as follows:

- The proposed CTSP is more realistic.
- In this problem, GA is memetic with multiparent (four parents) crossover, probabilistic selection, and random mutation.
- The proposed GA was tested with various informational indexes from (TSPLIB) as well as informative speculative collections. We accomplished a contextual analysis of the territory of West Bengal in India with the assistance of Google Maps.
- The proposed algorithm efficiently solved large-scale multi-route and multi-vehicle routing problems.

We formulated different risk factors depending on the geographical position or location of the city for different routes considering different vehicle availabilities. The prescribed models exhibited improved crisp cost and risk as well as fuzzy cost and risk. Cluster generation and relinking were successfully performed using a novel heuristic technique. The developed GA was a combination of probabilistic selection based on the analogous parameter P_s (say, probabilistic selection), multi parents crossover, and simple random mutation. The proposed multiparent crossover technique was based on social mimicry such as child adoption. When parents adopt a child, the child picks up behavior from both the biological and adopted parents. Initially, we randomly selected four solutions (say, parents) for the crossover operation. Then, offspring were created using the multiparent crossover method by comparing the values between the nodes of all four parents. Table 3 proves the efficiency of our applied GA over classical GA.

3. Mathematical preliminaries

3.1. Fuzzy possibility and necessity approach

Say \tilde{a} is a fuzzy number with membership function $\mu_{\tilde{a}}(x)$ and another one is \tilde{b} with membership function $\mu_{\tilde{b}}(x)$. At that point, as indicated by Zadeh [46],

$$pos(\tilde{a} * \tilde{b}) = sup\{min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)), x, y \in \Re, x * y\}$$

$$\tag{1}$$

here the multitude pos describes the possibility, * is any one of the relations >, <, =, \le , \ge and \Re represents set of real numbers.

$$nes(\tilde{a} * \tilde{b}) = 1 - pos(\tilde{a} * \tilde{b})$$
(2)

where the multitude nes represents necessity.

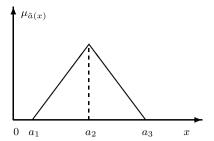


Fig. 1. Triangular fuzzy number $a = (a_1, a_2, a_3)$.

If \tilde{a} , $\tilde{b} \subseteq \Re$ and $\tilde{c} = f(\tilde{a}, \tilde{b})$ where $f : \Re \times \Re \to \Re$ is a binary operation then the membership function $\mu_{\tilde{c}}$ of \tilde{c} is defined as

For each
$$z \in \Re$$
, $\mu_{\bar{c}}(z) = \sup\{\min(\mu_{\bar{d}}(x), \mu_{\bar{b}}(y)), x, y \in \Re \text{ and } z = f(x, y)\}$
(3)

Triangular Fuzzy Number (TFN) [47]:

If \tilde{a} is a TFN and $\tilde{a}=(a_1,a_2,a_3)$ (cf. Fig. 1) then a_1,a_2 and a_3 are three parameters such as $a_1 < a_2 < a_3$. Now, we can characterize the TFN through the membership function $\mu_{\tilde{a}}$ such as

$$\mu_{\bar{a}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & \text{for } a_1 \le x \le a_2 \\ \frac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \le x \le a_3 \\ 0 & \text{otherwise.} \end{cases}$$
 (4)

We can derive the following lemmas based on the above definitions.

Lemma 3.1a. If $\tilde{a}=(a_1,a_2,a_3)$ be a TFN with $0< a_1$ and b is a crisp number then $pos(\tilde{a}< b) \geq \alpha$ iff $\frac{b-a_1}{a_2-a_1} \geq \alpha$.

Lemma 3.1b. If $\tilde{a}=(a_1,a_2,a_3)$ be a TFN with $0< a_1$ and b is a crisp number then $nes(\tilde{a}< b) \geq \alpha$ iff $\frac{a_3-b}{a_3-a_2} \leq 1-\alpha$.

Lemma 3.1c. If $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be TFNs with $0 < a_1$ and $0 < b_1$ then $pos(\tilde{a} < \tilde{b}) \ge \alpha$ iff $\frac{b_3 - a_1}{b_3 - b_2 + a_2 - a_1} \ge \alpha$.

Lemma 3.1d. If $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be TFNs with $0 < a_1$ and $0 < b_1$ then $nes(\tilde{a} < \tilde{b}) \ge \alpha$ iff $\frac{a_3 - b_1}{b_2 - b_1 + a_3 - a_2} \le 1 - \alpha$.

4. Formulation of multi-path multi-vehicle clustered TSP (4D CTSP) with risk constraints

4.1. Nomenclature

Some basic notations are available in Table 1.

4.2. Classical TSP with risk constraints

The TSP optimization problem involves creating an ideal tour with a finite set of unique cities, excluding the starting town or node. Say G=(V,A) is a graph with V as a set of N vertices and A as a set of arcs. Let c(i,j) be the traveling cost and r(i,j) be the traveling risk factor level in traveling from ith city to jth city. Then we can mathematically

Table 1
Notations and descriptions.

Notation	Description
N	Number of nodes/city (1, 2, 3,, N)
i, j, k	Index set
c(i, j)	Traveling cost from ith city to jth city
r(i, j)	Risk from ith city to jth city
c(i, j, l)	Traveling cost from ith city to jth city using Ith vehicle
r(i, j, l)	Risk from ith city to jth city using Ith vehicle
c(i, j, l, g)	Traveling cost from ith city to jth city using Ith vehicle and gth route
r(i, j, l, g)	Risk from ith city to jth city using Ith vehicle and gth route
$\tilde{c}(i, j, l)$	Fuzzy traveling cost from ith city to jth city using Ith vehicle
$\tilde{r}(i, j, l)$	Fuzzy risk from ith city to jth city using Ith vehicle
$\tilde{c}(i, j, l, g)$	Fuzzy traveling cost from ith city to jth city using Ith vehicle and gth route
$\tilde{r}(i, j, l, g)$	Fuzzy risk from ith city to jth city using Ith vehicle and gth route
x_{ij}	Decision variable
r_{max}	Highest allowable risk
m	Number of clusters
Q	Set of nodes {1, 2, 3,, N}
G	A graph
V	Set of N vertices
A	Set of arcs of graph G
r_{k-max}	Maximum allowable risk of kth cluster
$r_k(i,j)$	Risk between ith node of kth cluster and jth node of k th + 1 cluster
$r_m(i,j)$	Risk between ith node of mth cluster and jth node of 1st cluster

 Table 2

 Research questions, solutions and managerial insight.

Research questions	Solutions (with reference)	Managerial insight
Q1: How might we structure a refined visit plan on an enormous scale like inside a state or a nation?	Tables 10, 11 and Fig. 5	Management can take appropriate decision to draw profitable cluster wise tour plan.
Q2: What might be a visit plan with an ideal expense keeping up the standard risk or comfort factor?	Tables 3 to 11 and Fig. 5	Management can design cluster wise tour plan with standard risk factor to maintain the profit.
Q3: What should be the cluster sequence and cluster re-linking technique to be get the optimal travel plan?	Algorithm 5	Management should try to maximize profit while maintaining the risk of each cluster.
Q4: What is the administrative method to deal with such issues like as risk factors arising as an effect of different longitude and latitude?	Tables 10 and 11, and Fig. 5 Section 9	Management can design clusters based on longitude and latitude of each place.

formulate the problem as:

Minimize
$$Z = \sum_{i \neq j} c(i, j) x_{ij}$$

subject to $\sum_{i=1}^{N} x_{ij} = 1$ for $j = 1, 2, ..., N$

$$\sum_{j=1}^{N} x_{ij} = 1$$
 for $i = 1, 2, ..., N$

$$\sum_{i=1}^{N} \sum_{j=1}^{N} r(i, j) x_{ij} \le r_{max}$$
where $x_i \ne x_j, i, j = 1, 2, ..., N$.

where x_{ij} is the decision variable,

$$x_{ij} = \begin{cases} 1 & \text{if the salesman visits from } i\text{th} \ \ to \ \ j\text{th} \ \ city \\ 0 & \text{otherwise.} \end{cases}$$

and r_{max} is the maximum allowable risk factor. Then the above TSP reduces to determine a complete tour $(x_1, x_2, \dots, x_N, x_1)$

to minimize
$$Z = \sum_{i=1}^{N-1} c(x_i, x_{i+1}) + c(x_N, x_1)$$

subject to $\sum_{i=1}^{N-1} r(x_i, x_{i+1}) + r(x_N, x_1) \le r_{max}$
where $x_i \ne x_j, i, j = 1, 2, ..., N$ (7)

along with sub-tour elimination criteria

$$\sum_{i \in S} \sum_{j \in S}^{N} x_{ij} \le |S| - 1, \forall S \subset Q$$
where $x_{ij} \in \{0, 1\}, i, j = 1, 2, ..., N$

$$Q = \{1, 2, 3, ..., N\} \text{ set of nodes.}$$
(8)

(5)

(6)

4.3. Clustered TSP (2D CTSP) with risk constraint

The number of nodes is N, and it is divided into m clusters. The size of the clusters is $|v_k|$, $\forall k \in m$. Each cluster has a unique set of nodes, and $V = \{\bigcup_{j=1}^m v_i : v_i \cap v_j = \phi, \ i \neq j, \ i,j \in \{1,2,\ldots,m\}\}$. There is a decision variable $x_{ij}, x_{ij} = 1$ iff a tour is completed between ith node to jth node; otherwise, $x_{ij} = 0$; $i,j \in V$. Let c(i,j) be the cost of traveling from ith city to jth city. The mathematical formulation of CTSP can be represented as follows:

where m denotes the number of clusters. r_{k-max} is the kth cluster's maximum allowable risk. and r_{max} represents the maximum risk of the entire tour. Where, $r_{max} \leq \{\sum_{k=1}^{m-1} r_{k-max} + r_k(i,j) + r_{m-max} + r_m(i,j) : i,j \in \{v_i,v_j\}, \ v_i \cap v_j = \phi, \ i \neq j, \ r_k(i,j) \ \text{represents}$ the risk between ith node of kth cluster, and jth node of kth cluster. $r_m(i,j)$ stands for risk between ith node of mth cluster and jth node of 1st cluster}. Then the salesman has to determine a tour inside a cluster based on $(x_i, x_j \in v_k)$, $i,j \in \{1,2,\ldots,N\}$. Now, each cluster is optimized according to the following equation.

$$Minimize \ Z_{k} = \sum_{i=1}^{|v_{k}|} c(x_{i}, x_{i+1})$$

$$subject \ to \ \sum_{i=1}^{|v_{k}|} r(x_{i}, x_{i+1}) \leq r_{k-max}$$

$$where \ x_{i} \neq x_{j}, \ i, j = \{1, 2 \cdots, N\}, k \in \{1, 2 \cdots, m\}.$$
 (10)

Finally, a complete $tour(x_1, x_2, \dots, x_N, x_1)$ will be discovered by appending all clusters together. Then the above CTSP reduces to determine a complete tour $(x_1, x_2, \dots, x_N, x_1)$

$$to \ \ minimize \ \ Z = \sum_{i=1}^{N-1} c(x_i, x_{i+1}) + c(x_N, x_1)$$

$$subject \ \ to \ \ \sum_{i=1}^{N-1} r(x_i, x_{i+1}) + r(x_N, x_1) \le r_{max}$$

$$where \ \ x_i \ne x_j, \ \ i, j = 1, 2, \dots, N.$$
 (11)

along with Eq. (8).

4.4. Multi vehicle clustered TSP (3D CTSP) with risk constraint

Let c(i,j,t) be the cost for traveling from ith city to jth city using tth vehicle and r(i,j,t) be the risk factor in traveling from ith city to jth city using tth vehicle. The framing of 3D CTSP with risk constraint can be represented as follows:

$$Minimize \ Z = \sum_{i \neq j} c(i, j, t) x_{ijt}$$

$$subject \ to \ \sum_{i=1}^{N} x_{ijt} = 1 \ for \ j = 1, 2, ..., N,$$

$$\forall t \in \{1, 2, ..., w\}$$

$$\sum_{j=1}^{N} x_{ijt} = 1 \ for \ i = 1, 2, ..., N, \ \forall t \in \{1, 2, ..., w\}$$

$$\sum_{i \in v_k} \sum_{j \in v_k} x_{ijt} = |v_k| - 1, \ \forall |v_k| \in V, |v_k| \ge 1,$$

$$\sum_{i \in v_k} \sum_{j \in v_k} r(i, j, t) x_{ijt} \le r_{k-max}, \ \forall t \in \{1, 2, ..., w\},$$

$$k \in \{1, 2, ..., m\}$$

$$(12)$$

where m is the highest number of clusters and w is the highest number of vehicles, r_{k-max} is the maximum allowable risk of kth cluster and r_{max} is the maximum risk of a complete tour. Now the salesman has to determine a tour inside a cluster based on $(x_i, x_j \in v_k)$ with corresponding available vehicle types (p_1, p_2, \ldots, p_w) , where $x_i \in \{1, 2, \ldots, N\}$ and $p_i \in \{1, 2, \ldots, or\ w\}$. The following equation is used to minimize the cost and risk of each cluster.

$$Minimize \ Z_{k} = \sum_{i=1}^{|v_{k}|} c(x_{i}, x_{i+1}, p_{i})$$

$$subject \ to \ \sum_{i=1}^{|v_{k}|} r(x_{i}, x_{i+1}, p_{i}) \le r_{k-max}$$

$$where \ x_{i} \ne x_{j}, \ i, j = \{1, 2 \cdots, N\}, \ k \in \{1, 2 \cdots, m\}, \ and$$

$$p_{i} \in \{1, 2, 3, \dots, \ or \ w\}$$

$$(13)$$

Finally, a complete $tour(x_1, x_2, ..., x_N, x_1)$ will be discovered by appending all clusters together. Then the above 3D CTSP reduces to determine a complete tour $(x_1, x_2, ..., x_N, x_1)$

4.5. Multi vehicle clustered TSP (3D CTSP) in fuzzy environment

If costs and risk factors are considered fuzzy numbers for the above problem in Eq. (14), then $\tilde{c}(i,j,t)$ is the fuzzy cost and $\tilde{r}(i,j,t)$ is the fuzzy risk value, where \tilde{r}_{max} is also a fuzzy number. Now the above problem reduces to electing a complete tour (x_1,x_2,\ldots,x_N,x_1) choosing anyone available corresponding conveyances in each journey from the vehicle types (p_1,p_2,\ldots,p_w) so as

$$to \quad minimize \quad Z = \sum_{i=1}^{N-1} \tilde{c}(x_i, x_{i+1}, p_i) + \tilde{c}(x_N, x_1, p_l)$$

$$subject \quad to \quad \sum_{i=1}^{N-1} \tilde{r}(x_i, x_{i+1}, p_i) + \tilde{r}(x_N, x_1, p_1) \leq \tilde{r}_{max}$$

$$where \quad x_i \neq x_j, \quad i, j = 1, 2, \dots, N. \quad and \quad p_i, p_l \in \{1, 2, 3, \dots, or \ w\}$$

$$(15)$$

4.5.1. Possibility approaches (Optimistic)

By writing the fuzzy objective and constraints in an optimistic sense using Eq. (1), we have the following:

Determine a complete tour $(x_1, x_2, \dots, x_N, x_1)$ using any one available corresponding conveyances in each step from the vehicle types (p_1, p_2, \dots, p_w)

to minimize
$$F$$
 subject to $Pos(\sum_{i=1}^{N-1} \tilde{c}(x_i, x_{i+1}, p_i) + \tilde{c}(x_N, x_1, p_l) < F) \ge \alpha_3$ $Pos(\sum_{i=1}^{N-1} \tilde{r}(x_i, x_{i+1}, p_i) + \tilde{r}(x_N, x_1, p_l) \le \tilde{r}_{max}) \ge \beta_3$ where $x_i \ne x_j$, $i, j = 1, 2, ..., N$, $p_i, p_l \in \{1, 2, ..., or w\}$

where α_3 , β_3 are predefined levels of possibility respectively which are entirely determined by the salesman. If we consider the fuzzy numbers as TFNs, $\tilde{c}(i,j,k)=(c(i,j,k)_1,c(i,j,k)_2,c(i,j,k)_3)$, $\tilde{r}(i,j,k)=(r(i,j,k)_1,r(i,j,k)_2,r(i,j,k)_3)$, $\tilde{r}_{max}=(r_1,r_2,r_3)$. Then the above problems can be reduced accordingly Lemmas 3.1a and 3.1c as:

Determine a complete tour $(x_1, x_2, \dots, x_N, x_1)$ using any one available corresponding conveyance in each step from the vehicle types (p_1, p_2, \dots, p_w) so as

to minimize
$$F$$

subject to $\frac{F - F_1}{F_2 - F_1} \ge \alpha_3$
 $\frac{r_3 - R_1}{r_3 - r_2 + R_2 - R_1} \ge \beta_3$ (17)

where
$$F_{j} = \sum_{i=1}^{N-1} c(x_{i}, x_{i+1}, p_{i})_{j} + c(x_{N}, x_{1}, p_{l})_{j}, \ j = 1, 2, 3.$$

and $R_{j} = \sum_{i=1}^{N-1} r(x_{i}, x_{i+1}, p_{i})_{j} + r(x_{N}, x_{1}, p_{l})_{j}, \ j = 1, 2, 3.$ (18)

where
$$x_i \neq x_j$$
, $i, j = 1, 2, ..., N$. $p_i, p_l \in \{1, 2, ..., or w\}$

The objective function in Eq. (17) changed to

minimize
$$F_1 + \alpha_3(F_2 - F_1)$$

subject to $\frac{r_3 - R_1}{r_3 - r_2 + R_2 - R_1} \ge \beta_3$ (19)

Here α_3 , β_3 are predefined possibility levels.

4.5.2. Necessity approaches (Pessimistic)

Similarly, converting the fuzzy expression in Eq. (15) in a pessimistic sense using Eq. (2), we get as follows:

Using necessity measure, we have

where α_4 , β_4 are predefined levels of necessity, respectively, which are entirely determined by the salesman. Then the above problems can be reduced accordingly Lemmas 3.1b and 3.1d as:

Determine a complete tour $(x_1, x_2, \dots, x_N, x_1)$ using any one available corresponding conveyance in each step from the vehicle types (p_1, p_2, \dots, p_w) so as

minimize
$$F$$

subject to $\frac{F_3 - F}{F_3 - F_2} \le 1 - \alpha_4$
 $\frac{R_3 - r_1}{r_2 - r_1 + R_3 - R_2} \le 1 - \beta_4$ (21)

The objective function in Eq. (21) changed to

to minimize
$$F_3 - (1 - \alpha_4)(F_3 - F_2)$$
 subject to
$$\frac{R_3 - r_1}{r_2 - r_1 + R_3 - R_2} \le 1 - \beta_4$$
 (22)

Here α_4 and β_4 are predefined necessity levels.

4.6. Multi vehicle multi path clustered TSP (4D CTSP) with risk constraint

Let c(i, j, t, g) be the cost of traveling from ith city to jth city using tth vehicle and gth route and r(i, j, t, g) be the risk factor for traveling from ith city to jth city using tth vehicle and gth route.

The mathematical formulation of 4D CTSP with risk constraint can be represented as follows:

$$Minimize \ Z = \sum_{i \neq j} c(i, j, t, g) x_{ijtg}$$

$$subject \ to \ \sum_{i=1}^{N} x_{ijtg} = 1 \ for \ j = 1, 2, ..., N,$$

$$\forall t \in \{1, 2, ..., w\}, \ \forall g \in \{1, 2, ..., h\}$$

$$\sum_{j=1}^{N} x_{ijtg} = 1 \ for \ i = 1, 2, ..., N,$$

$$\forall t \in \{1, 2, ..., w\}, \ \forall g \in \{1, 2, ..., h\}$$

$$\sum_{i \in V_k} \sum_{j \in V_k} x_{ijtg} = |V_k| - 1, \ \forall |V_k| \subset V, \ |V_k| \ge 1, \ i \ne j,$$

$$k = 1, 2, 3, ..., m, \ \forall t \in \{1, 2, ..., w\}, \ \forall g \in \{1, 2, ..., h\}$$

$$\sum_{i \in V_k} \sum_{j \in V_k} r(i, j, t, g) x_{ijtg} \le r_{k-max}, \ i \ne j,$$

$$\forall t \in \{1, 2, ..., w\}, \ \forall g \in \{1, 2, ..., h\}, \ k \in \{1, 2, ..., m\}$$

where r_{k-max} is the maximum allowable risk of kth cluster and r_{max} is the maximum risk of a complete tour. Now the salesman has to determine a tour inside a cluster based on $(x_i, x_j \in v_k)$ with corresponding available vehicle types (p_1, p_2, \dots, p_w) with route types (s_1, s_2, \dots, s_h) , where $x_i \in \{1, 2, \dots, N\}$, $p_i \in \{1, 2, \dots, or\ w\}$ and $s_i \in \{1, 2, \dots, or\ h\}$. The following equation is used to minimize the cost and risk of each cluster.

$$\begin{aligned} & \textit{Minimize} & Z_k = \sum_{i=1}^{|v_k|} c(x_i, x_i + 1, p_i, s_i) \\ & \textit{subject to} & \sum_{i=1}^{|v_k|} r(x_i, x_i + 1, p_i, s_i) \leq r_{k-max} \\ & \textit{where} & x_i \neq x_j, i, j = 1, 2, \dots, N, \ k \in \{1, 2, \dots, m\} \\ & \textit{and} & p_i \in \{1, 2, 3, \dots, w\}, \quad s_i \in \{1, 2, 3, \dots, h\} \end{aligned}$$

Finally, a complete $tour(x_1, x_2, \dots, x_N, x_1)$ will be discovered by appending all clusters together. Then the above 4D CTSP reduces to determine a complete tour $(x_1, x_2, \dots, x_N, x_1)$

$$to \ \ minimize \ \ Z = \sum_{i=1}^{N-1} c(x_i, x_{i+1}, p_i, s_i) + c(x_N, x_1, p_1, s_1)$$

$$subject \ \ to \ \ \sum_{i=1}^{N-1} r(x_i, x_{i+1}, p_i, s_i) + r(x_N, x_1, p_l, s_1) \le r_{max}$$

$$where \ \ x_i \ne x_j, \quad i, j = 1, 2, \dots, N,$$

$$and \ \ p_i, p_l \in \{1, 2, 3, \dots, or \ w\} \ \ s_i, s_l \in \{1, 2, 3, \dots, or \ h\}$$
along with Eq. (8).

4.7. Multi vehicle multi path clustered TSP (4D CTSP) in fuzzy environment

If costs and risk factors are considered fuzzy numbers for the above problem, then $\tilde{c}(i,j,t,g)$ is the fuzzy cost, and $\tilde{r}(i,j,t,g)$ is the fuzzy risk value, where \tilde{r}_{max} is also a fuzzy number. Now the above problem reduces to electing a complete tour (x_1,x_2,\ldots,x_N,x_1) choosing anyone available corresponding conveyances in each journey from the vehicle

types (p_1, p_2, \dots, p_w) and route types $(s_1, s_2, \dots, or \ s_h)$ so as

4.7.1. Possibility approaches (Optimistic)

By writing the fuzzy objective and constraints in an optimistic sense using Eq. (1), we have the following:

Determine a complete tour $(x_1, x_2, \dots, x_N, x_1)$ using any one of the available conveyances from the vehicle types (p_1, p_2, \dots, p_w) and from the available route types $(s_1, s_2, \dots, or s_h)$.

$$\text{to minimize} \quad F \\ \text{subject to} \quad Pos(\sum_{i=1}^{N-1} \tilde{c}(x_i, x_{i+1}, p_i, s_i) + \tilde{c}(x_N, x_1, p_l, s_l) < F) \geq \alpha_3 \\ Pos(\sum_{i=1}^{N-1} \tilde{r}(x_i, x_{i+1}, p_i, s_i) + \tilde{r}(x_N, x_1, p_l, s_l) \leq \tilde{r}_{max}) \geq \beta_3 \\ \quad where \quad x_i \neq x_j, \quad i, j = 1, 2, \dots, N, \\ and \quad p_i, p_l \in \{1, 2, \dots, or \ w\}, \quad s_i, s_l \in \{1, 2, 3, \dots, or \ h\} \\ \end{cases}$$

where α_3 , β_3 are predefined levels of possibility respectively which are entirely determined by the salesman. If we consider the fuzzy numbers as TFNs,

$$\tilde{c}(i, j, k, l) = (c(i, j, k, l)_1, c(i, j, k, l)_2, c(i, j, k, l)_3),$$

$$\tilde{r}(i, j, k, l) = (r(i, j, k, l)_1, r(i, j, k, l)_2, r(i, j, k, l)_3),$$

$$\tilde{r}_{max} = (r_1, r_2, r_3).$$

Then the above problems can be reduced accordingly Lemmas 3.1a and 3.1c as:

Determine a complete tour (x_1,x_2,\ldots,x_N,x_1) using any one available conveyance in each journey from the vehicle types $(p_1,p_2,\ldots,or\ p_w)$ and available route types $(s_1,s_2,\ldots,or\ s_h)$ so as

to minimize
$$F$$

subject to $\frac{F - F_1}{F_2 - F_1} \ge \alpha_3$
 $\frac{r_3 - R_1}{r_3 - r_2 + R_2 - R_1} \ge \beta_3$ (28)

where
$$F_{j} = \sum_{i=1}^{N-1} c(x_{i}, x_{i+1}, p_{i}, s_{i})_{j} + c(x_{N}, x_{1}, p_{l}, s_{l})_{j}, \ j = 1, 2, 3.$$
and $R_{j} = \sum_{i=1}^{N-1} r(x_{i}, x_{i+1}, p_{i}, s_{i})_{j} + r(x_{N}, x_{1}, p_{l}, s_{l})_{j}, \ j = 1, 2, 3.$

$$where \ x_{i} \neq x_{j}, \ i, j = 1, 2, \dots, N,$$

$$and \ s_{i}, s_{l} \in \{1, 2, \dots, or \ h\}, \ p_{i}, p_{l} \in \{1, 2, \dots, or \ w\}$$

The objective function in Eq. (28) changed to

minimize
$$F_1 + \alpha_3(F_2 - F_1)$$

subject to $\frac{r_3 - R_1}{r_3 - r_2 + R_2 - R_1} \ge \beta_3$ (30)

Here α_3 , β_3 are predefined possibility levels.

4.7.2. Necessity approaches (Pessimistic)

Similarly, converting the fuzzy expression in Eq. (26) in a pessimistic sense using Eq. (2), we get as follows:

Using necessity measure, we have

$$\begin{array}{c} \text{minimize} \quad F \\ \text{subject to} \quad Nes(\sum_{i=1}^{N-1} \tilde{c}(x_i, x_{i+1}, p_i, s_i) + \tilde{c}(x_N, x_1, p_l, s_l) < F) \geq \alpha_4 \\ \\ Nes(\sum_{i=1}^{N-1} \tilde{r}(x_i, x_{i+1}, p_i, s_i) + \tilde{r}(x_N, x_1, p_l, s_l) \leq \tilde{r}_{max}) \geq \beta_4 \\ \\ where \quad x_i \neq x_j, i, j = 1, 2, \dots, N, \\ and \quad s_i, s_l \in \{1, 2, \dots, \text{ or } h\}, \quad p_i, p_l \in \{1, 2, \dots, \text{ or } w\} \end{array} \right)$$

where α_4 , β_4 are predefined levels of necessity respectively which are entirely determined by the salesman. Then the above problems can be reduced accordingly Lemmas 3.1b and 3.1d as:

Determine a complete tour $(x_1, x_2, \ldots, x_N, x_1)$ using any one of the available conveyance in each journey from the vehicle types $(p_1, p_2, \ldots, or p_w)$ and available route types $(s_1, s_2, \ldots, or s_h)$ so as

minimize
$$F$$

subject to $\frac{F_3 - F}{F_3 - F_2} \le 1 - \alpha_4$
 $\frac{R_3 - r_1}{r_2 - r_1 + R_3 - R_2} \le 1 - \beta_4$ (32)

The objective function in Eq. (32) changed to

to minimize
$$F_3 - (1 - \alpha_4)(F_3 - F_2)$$

subject to $\frac{R_3 - r_1}{r_2 - r_1 + R_3 - R_2} \le 1 - \beta_4$ (33)

Here α_4 and β_4 are predefined necessity levels.

5. Proposed genetic algorithm

5.1. Novel memetic genetic algorithm

We used a GA named novel memetic genetic algorithm (NMGA) [40]. It is a combination of probabilistic selection (Boltzmann probability), four parents' memetic crossover, and random mutation. NMGA is applied to find a new set of solutions among a set of potential solutions. It proceeds until the ending conditions are reached. The NMGA procedures are as follows:

- Representation: Considering N cities are available to make a complete tour, which comprises a solution. An integer vector X_i of N dimension is assumed, where $X_i = (x_{i1}, x_{i2}, ..., x_{iN})$ were cities, and $x_{i1}, x_{i2}, ..., x_N$ were N successive cities in a tour. In the beginning, a group of paths (tours) is required for a salesman. These paths were randomly generated for the GA. These initial paths were a group of possible solutions for the GA part of this algorithm.
- Probabilistic Selection: The main objective of TSP was to minimize the path cost and distance. Thus, the minimum fitness value (f_{min}) of a chromosome plays a vital role. The Matting pool was formed using the Boltzmann-Probability of all chromosomes in the initial population.
- Now, $p_B = e^{((f_{min} f(X_i))/T)}$, $T = T_0(1-a)^k$, k = (1 + 100*(g/G)), g = ongoing generation number, G = maximum generation, $T_0 =$ rand[10,150], $f(X_i)$ is fitness/objectives of chromosome corresponding to X_i , a = rand[0,1], i = chromosome number.
- Multiparent Crossover: Child adoption is a contemporary prevalent matter due to different practical circumstances. Here, aside from unique parents (father and mother), one additional parent (father and mother) is considered as a section. Based on this practical instance, we considered using a methodology with four parents (the initial two are unique parents and the other two are new parents) to deliver offspring. This methodology urged the crossover method to choose four individuals or parents in an ergodic manner to create offspring. The journey started with one node and then onto the next node, keeping the lowest traveling

cost depending on TSP conditions. Following the above origination, we performed the crossover procedure in the accompanying state. From the outset, four people (parents) from the mating pool were randomly selected. PR_1 , PR_2 , PR_3 , and PR_4 were the parents and CH_1 and CH_2 were the offspring.

• Random Mutation: An ergodic number r was created for every solution of P(t). Here, r was generated from the range [0,1], with the condition $r < p_m$; if the condition was true, then the solution was selected for mutation. Two nodes were selected in an ergodic manner from each chromosome, and they interchanged their positions and were replaced in the offspring set.

5.2. Proposed ripple clustering for cluster creation and re-linking

The objective of a TSP is for a salesman to find a perfect way or route. An ideal way implies that the shortest possible route. At the same time, the salesman finishes his visit in a limited number of urban areas, visiting every city just once and reaching the starting city. The CTSP has a challenge in creating the clusters and re-linking them to construct an optimized route. The proposed algorithm has three sections as a CTSP namely, cluster creation (CC), cluster optimization (CO), and cluster re-linking (CR), as algorithms 4 and 5. We developed two approaches to CC, namely, CC: Algorithm 1 (CC₁) based on Euclidean distance and Algorithm $2(CC_2)$ based on the longitude and latitude of the respective city or node. The CC algorithm named "Ripple Clustering" mimicked the natural phenomenon of ripples on water. If a drop of water falls in the middle of still water, then waves were created originating from the center of the drop. Each circular wave was a cluster. The proposed heuristic for cluster creation and re-linking was based on the distance determined from the centroid and the centroids were randomly updated. A local search was used to find the nearest points to the cluster. In each cluster, NMGA works to find the optimal path. Again for re-linking the clusters, a heuristic is designed. The main advantage of this algorithm is that CC, path generation and re-linking of the clusters are simultaneous. The proposed algorithms are described in the next subsection.

5.2.1. Proposed CC_1 algorithm

N is the set of given nodes, and K is the total number of prespecified clusters. The proposed algorithm is as follows:

Algorithm 1: Cluster Initialization (CC_1)

```
Data: Set of the given city N
```

Result: A prespecified number of clusters with a unique set of nodes that belong to N

- 1 Initial the number of clusters K;
- 2 Store the size of each cluster;
- 3 Select a random number r (node) as the centroid between 0 and N-1;
- 4 i = 1:

```
5 while i \neq K do
```

```
6 | count = 1;
7 | while count \neq |i| do
```

store the nearest nodes (Euclidean distance) from *r*;
selected nodes for one cluster would be ignored and not selected for another cluster;

end

11 end

10

5.2.2. Proposed CC_2 algorithm

N is the set of given nodes, and K is the total number of prespecified clusters. We consider all cities with their respective longitude and latitude.

Algorithm 2: Cluster Initialization (CC_2)

```
Data: Set of the given city N
```

Result: A prespecified number of clusters with a unique set of nodes that belong to N

- 1 Initialize the number of clusters, *K*;
- 2 Store the size of each cluster;
- 3 Sorted the latitude or longitude of each node;
- 4 Select a random node r as the centroid from N;
- 5 i = 1:

6 while $i \neq K$ do

```
count = 1;
```

while $count \neq |i|$ do

select the nearest node from r based on its latitude and

selected nodes for one cluster would be ignored and not selected for another cluster;

11 end

12 **end**

8

10

Algorithm 3: Cluster Initialization

Data: A set of given cities or nodes N

Result: A specified number of clusters, each with a unique set of nodes belonging to N

- 1 Initialize the number of clusters *K*;
- 2 Initialize the predefined size of each cluster;
- 3 Choose a random node from the total nodes as the centroid of the initial cluster;
- 4 Build the initial cluster consisting of the nearest nodes from the centroid up to the predefined size;

5 while $i \neq K$ do

 Randomly select a node from the remaining nodes as a centroid for the next cluster;

7 Create the next cluster based on the centroid;

while centroids \neq unchanged do

Calculate the centroids of all cluster centroids using the following formula:

 $\bar{x} = \sum x_i / K,$

 $\bar{y} = \sum y_i / K$

Calculate the middle point from (\bar{x}, \bar{y}) to the centroid of each cluster as the new centroid of each corresponding cluster;

From these new centroids, modify all clusters using step 6:

Find the longest edge between the centroid and a node of each cluster;

Update the centroids of each cluster by considering the middle point of the longest edge of each cluster;

15 end

16 end

10

11

12

13

14

5.2.3. Proposed CO algorithm

To optimize a cluster, we used NMGA, which is a memetic GA. It is a combination of Boltzmann probabilistic selection, four parent memetic crossover, and a random mutation. The proposed *CO* Algorithm is given in Algorithm 4.

Algorithm 4: Cluster Optimization (CO)

Data: A set of initial nodes within a cluster

Result: Nodes are in a sequence to produce a Hamiltonian path

- 1 Initialize the total nodes and size of a cluster;
- 2 Produce initial solutions ergodic manner;
- 3 Judge fitness of these initial solutions;
- 4 i = 1;
- 5 while $i \neq max_gen$ do
- Apply the Probabilistic selection procedure to prepare a mating pool;
- 7 Apply Multi parents crossover (see [40]);
- 8 Apply the random mutation;
- 9 Store the best solution from the population;
- 10 **end**
- 11 Store the best solution among all generations, as the final solution:

5.2.4. Proposed CR algorithm

To construct a complete tour, the proposed CR Algorithm, i.e., Algorithm 5, was applied to assemble all optimized clusters together. The proposed CR Algorithm was as follows:

Algorithm 5: Cluster re-linking (CR)

Data: A set of optimized clusters where each cluster is a unique set of nodes

Result: A Hamiltonian tour

- 1 Put the number of clusters (K);
- i = 1;
- 3 while $i \neq maximum$ iteration do
- 4 Randomly generate a cluster number between 1 and K;
 - Concatenate the clusters by joining the sub-path contiguously;
- 6 end
- 7 Store the best solution among all solutions as the final result;

5.3. Proposed algorithm

Two types of cluster creation methods were developed. The NMGA was applied to each cluster to optimize it. The proposed algorithm is given in Algorithm 6 to solve the CTSP. Fig. 2 presents three flowcharts for each of our three proposed algorithms.

Algorithm 6: Proposed CTSP

Data: For Crossover procedure (p_c) , $Maximum_{gen}(S_0)$, (pop_size) or noc and for Mutation procedure (p_m)

Result: The best solution

- 1 Put the number of clusters (*K*);
- 2 Call Algorithm 1 or Algorithm 2;
- 3 for steps = 1 to K do
- 4 Call Algorithm 4;
- 5 end
- 6 Call Algorithm 5;
- 7 **for** steps = 1 to noc **do**
- 8 | Evaluate fitness;
- 9 end
- 10 Store the minimum fitness;

5.4. Time complexity

The genetic algorithm has three operators: the selection operator, the crossover operator, and the mutation operator. O(SN) is the time complexity of the selection operator, and $O(S_{p_c}N^2)$ is for the crossover operator. The time complexity of the mutation operator is $O(S_{p_m}N^2)$, where S is the size of the population. Usually $p_c > p_m$, then $O(S_{p_c}N^2) > O(S_{p_m}N^2) > O(SN)$. If we consider g_0 as the maximum number of generations, then the time complexity would be $O(g_0SN^2)$. O(SN) is the time complexity to construct the initial population, and $O(g_0SN^2)$ is for fitness function calculation. Now $O(SN) < O(g_0SN^2)$. Thus, the time complexity of the GA is $O(g_0SN^2)$.

6. Empirical test

6.1. Test results for TSPLIB instances

To prove the efficiency of the proposed algorithm, standard TSPLIB [48] benchmark problems were studied. Three separate data sets were considered for the empirical test. The first dataset considered 11 benchmark instances with 29-575 vertices or nodes from TSPLIB. The second dataset was a hypothetical dataset of 10 nodes [49,50], and the third dataset was created using Google Maps. Here is the latitude and longitude of 38 places situated in the province of West Bengal, India. The results are presented in Table 3. Here, classical GA stands for the combination of RW selection, cyclic crossover, and random mutation. The results show the comparison between NMGA and classical GA. The rest of the result sections are divided into 10 cities (2D, 3D, and 4D CTSP) and another section for uncertain 2D, 3D, and 4D CTSP. The parameters were tuned for a different number of clusters (2–80 clusters), optimum solutions were taken from 100 independent runs, and maximum 500 generations were considered to determine the optimum path in each cluster. The value of the risk factor for each edge was taken randomly between 0 and 1.

Based on information in Table 3, we can make the following observations: First, the performance of NMGA was better than that of GA in all instances. Second, for all the cases, changing the number of clusters affected the optimal objective value and the corresponding absolute risk. When the optimal objective value increased, the risk factor decreased. However, it was not valid in all cases. In some cases, the number of clusters increased, but the optimum value as well as and risk decreased. In one instance, such as the rat –575 instance, the number of clusters increased from 60 to 70, but the objective value decreased from 17,185 to 17,155 with risk factor decline from 288.28 to 283.35. Thus, the final observation from Table 3 is that the selection of the number of clusters plays a vital role for the traveler concerning optimal cost and risk. Consideration of the risk in the proposed model has significance for the traveler, as shown in Table 3.

6.2. Test results for a 10-city problem

Herein we considered a problem of 10 cities based on hypothetical data in a crisp and fuzzy environment (see supplementary section), considering three routes between each node and four vehicles in each route. The results were found using NMGA in Tables 4 to 6 for 2D, 3D, and 4D CTSPs respectively. Here, a path (8 5 0 3 2 9 4 6 7 1) meant that a journey started from node 8 then, the next destination was node 5, and so on, finally coming back to node 8 after visiting node 1. The path, vehicle, and route sequence were indicators of the 4D CTSP. For example, in Table 6, path (4 5 3 6 2 8 1 9 7 0), vehicle (1 1 2 1 1 2 3 2 1 3), and route (2 1 1 2 1 2 1 2 2 2) implied traveling from node 4 to node 5 through vehicle 1 and route 2 similar interpretation can be used for other cases too.

As shown in Table 4, in the 2D CTSP, when there were two clusters, the minimum objective value was 187 and the risk was 6.47. However,

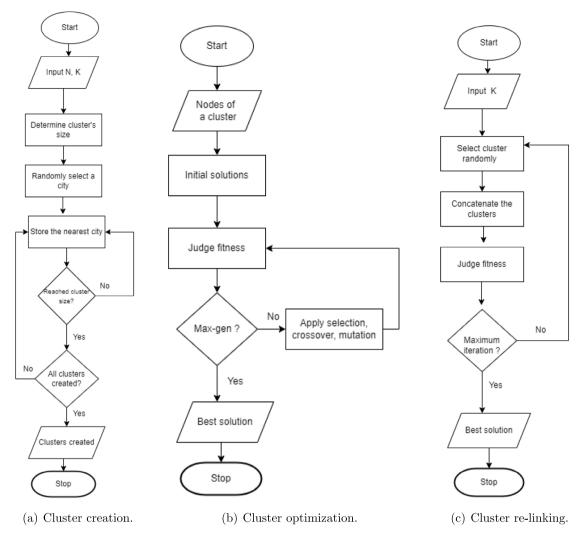


Fig. 2. Proposed clusterization, optimization, and re-linking

when three clusters were considered, the risk was 6.40 and the objective value was 165. Thus, considering approximately the same risk, the three-cluster concept is better than two clusters. Data presented in Table 5 are similar to those in Table 4. We get one of the best solutions (objective value of 159 and risk of 5.52), considering three clusters. The 3D CTSP performed better than the 2D CTSP when considering four vehicles between each node. For the sake of simplicity, the routes and vehicles were uniform.

Table 6 shows the benefits of the 4D CTSP over 2D and 3D CTSPs, considering three routes between each node and a maximum of four vehicles per way. A better solution was obtained when an objective value of 116 and a risk of 3.34 were considered. Tables 4 to 6 have a common scenario with the conflicting measure that the minimum risk should not produce the maximum objective and vice versa.

6.3. Test results of a 10-city problem with fuzzy data

Tables 7 to 9 show the results based on fuzzy data presenting the real phenomenon of a tour. Here, the objective value was the same for more than one solution, but with different risks because we considered a standard distance between two nodes along with the fuzzy traveling cost and risk between two nodes. For example, in Table 7, the objective value was 200 for multiple solutions, but their paths showed different risks. Table 8 shows test results for the 3D CTSP in fuzzy values. Here, cluster 2 performed better (objective value of 137 and risk of 7.42) than cluster 3 (objective value of 152 and risk of 7.63) demonstrating

the benefits of the 3D CTSP over the 2D CTSP. Table 9 presents the test results for the 4D CTSP. The 4D CTSP is superior to the 2D and 3D CTSPs with the objective value of 100 and risk of 3.06.

7. Discussion

The overall empirical test was conducted using the Ripple clustering technique. Furthermore, a comparison between the proposed Ripple clustering technique and k-means was performed (Fig. 3). The results are shown in Tables 4–9 based on the hypothetical data considering clusters 2 and 3. In most cases, in a crisp and fuzzy environment, the 4D CTSP identified the minimum cost and risk incurred. Introducing multiple vehicles and paths, i.e., 3D and 4D CTSPs with risk constraints helped measure real-life observations. The results shown in a fuzzy environment were better than those in a crisp environment. Our empirical studies aimed to answer the following questions:

- 1. How do the applied NMGA and GA act on TSPLIB instances as CTSPs with risk constraints?
- 2. How does the NMGA act with 3D and 4D CTSPs with risk constraints in crisp and fuzzy environments?
- 3. How does the NMGA act with a realistic dataset of 38 nodes based on their latitude and longitude?
- 4. How do we consider a managerial implementation in the tourism industry and routing in the supply chain?

Table 3
Performance of benchmarks from TSPLIB.

Instances	Algorithm	Clusters	Cluster wise risk	Total cost	Total ris
		2	8.52, 7.11	33 675	15.48
	NIMCA	3	5.63 3.19 4.73	44 653	14.32
wi-29	NMGA	4	3.86 4.14 1.99 4.18	44713	14.92
		5	1.57 3.69 2.62 3.37 4.38	48 835	15.73
	GA	4	4.71 3.64 4.33 3.22	65737	17.14
		5	4.58 3.50 3.87 3.23 5.03	716	20.83
eil-51	NMGA	10	2.66 3.79 1.43 3.48 1.86 1.5 3.13 2.6 3.25 1.99	804	24.05
		15	2.49 0.39 0.89 2.66 1.75 1.35 0.77 1.4 1.26 1.93 1.36 1.01 1.54 1.45 10.34	863	25.21
	GA	10	2.08 1.93 2.66 1.63 2.50 3.09 2.14 2.70 3.72 3.06	904	26.45
		5	5.03 5.8 4.2 5.05 6.95	11 093	25.91
erlin-52	NMGA	10	1.7 2.43 1.29 3.37 2.68 3.3 3.61 3.18 2.72 3.99	14 290	29.27
		15	2.16 0.93 1.7 1.77 0.16 1.18 2.38 1.65 2.12 0.68 0.89 0.81 0.93 1.42 4.66	13 026	24.08
	GA	10	1.79 2.15 2.60 3.39 2.19 3.21 2.64 3.04 3.20 5.14	19383	28.59
		10	2.26 2.93 3.42 3.75 2.67 2.11 4.9 3.45 3.99 6.97	885	36.23
		15	1.22 2.81 1.54 1.58 1.81	1116	33.30
	NMGA		1.68 2.33 2.14 2.67 2.32 1.71 1.8 0.86 3.49 3.4	-	
il-76	11111011	20	2.36 1.15 1.97 1.82 1.27 1.09 0.7 2.09 1.66 1.61 1.69 1.8 1.83 1.2 0.96 0.78 0.45 2.18 0.79 10.59	1127	37.14
			2.36 1.15 1.96 1.82 1.27		
		25	1.09 0.7 2.09 1.66 1.61 1.69 1.8 1.83 1.2 0.96	1294	38.06
			0.78 0.44 2.18 0.79 1.62 1.06 1.63 1.7 1.2 1.45		
	GA	20	1.66 1.74 1.61 0.34 0.40 1.44 2.14 1.54 1.89 1.59 1.93 0.41 1.11 1.45 0.84 0.99 1.90 1.31 1.64 8.86	1489	36.36
		15	4.24 3.11 3.68 3.03 3.2	47 980	53.09
			3.72 2.25 2.69 2.05 4.09 3.63 1.89 4.33 3.06 8.73		
		20	2.78 3.72 2.05 2.06 2.92 3.03 2.89 3.82 2.72 2.39 2.47 2.99 2.32 2.35 2.88 1.98 1.89 3.54 3.06 2.19	53 992	52.41
	NMGA	-	1.03 1.29 2.68 1.67 2.41		
roA-100		25	2.29 1.35 3.13 1.86 1.46 1.64 2.19 1.75 1.81 2.07	60800	53.04
		20	1.88 1.22 0.83 2.31 1.35 1.59 2.44 1.98 2.17 3.28	00000	00.01
			0.89 2.04 1.25 0.75 1.17 2.19 1.16 1.72 2.24 0.94		
		30	1.35 1.85 1.78 2.24 1.63 1.19 1.97 0.82 1.53 1.44	64 441	48.53
			0.41 1.47 2.06 1.35 0.90 1.14 2.48 0.73 1.27 5.49		
	-		2.82 2.48 2.34 1.63 2.21 2.24 3.07 1.55 2.60 3.71		
	GA	20	1.78 2.59 3.38 2.80 2.21 1.66 2.06 2.00 3.63 3.41	67 103	48.18
		15	2.14 3.13 3.05 2.25 2.32	1193	46.11
			3.36 3.35 1.64 3.41 4.39 2.91 1.81 3.77 2.63 7.21		
		20	2.26 2.80 2.47 1.95 2.13 1.89 2.19 2.71 2.02 2.17	1326	48.94
	NMGA		1.52 2.53 2.39 2.03 4.00 1.72 2.85 1.77 2.55 3.77		
	Mindri	0.5	2.12 1.80 1.69 1.87 2.16	1500	F0 10
il-101		25	2.78 1.54 2.12 1.65 1.77 1.89 1.21 1.16 1.85 2.87 2.14 1.87 1.31 1.75 2.35 2.93 1.93 2.59 1.16 2.07	1522	53.18
		-			
		30	1.64 1.57 1.38 2.33 0.75 1.15 1.85 1.54 1.82 0.62 1.54 0.50 2.16 0.39 2.51 1.98 1.49 1.13 0.71 1.25	1578	50.49
		30	1.21 2.01 1.61 1.24 1.83 2.05 2.63 1.04 2.23 6.83	1376	30.49
	-		2.68 2.53 1.38 2.03 2.55		
	GA	25	2.44 1.71 1.24 1.88 0.94 0.83 2.10 1.24 1.85 2.77	1702	49.13
			1.34 2.10 1.69 1.72 1.57 1.92 2.40 2.63 1.75 1.80		
		20	3.73 2.96 3.24 2.14 2.80 3.25 3.72 2.53 3.21 2.80	224000	C4.55
		20	2.56 3.38 4.85 4.39 3.11 2.74 2.43 2.47 1.95 6.55	224 006	64.75
			3.62 2.49 1.23 1.13 2.04		
		25	2.06 1.96 1.59 1.73 2.43 2.19 2.03 2.50 2.40 3.11	241 503	62.94
			2.75 2.22 3.44 2.32 2.23 2.28 2.89 1.52 3.28 3.06		
	NMGA		2.56 0.96 2.42 3.08 2.46 1.21 1.09 1.94 1.19 2.90		
ier-127		30	1.73 2.85 1.79 1.81 1.69 3.63 1.38 3.02 2.71 2.88	254 888	59.58
/			1.14 3.06 1.85 1.83 1.55 2.06 2.23 1.87 2.15 6.29		
		-	0.47 2.34 0.51 1.24 0.96 2.44 2.09 0.89 1.33 1.39		
		40	1.31 1.69 0.86 1.31 1.92 1.06 1.03 0.91 1.41 2.00	000.000	c= a :
		40	0.64 1.26 1.82 1.94 1.51 1.55 1.48 1.89 1.22 1.69	292 883	65.04

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Tah	le 3 i	(continued	١

Table 3 (conti	nued).				
	GA	30	2.59 1.87 1.26 2.32 1.67 2.35 1.41 2.64 2.42 0.79 2.00 1.84 1.70 0.89 2.34 2.39 1.71 1.70 1.99 1.36	279 861	64.58
			2.59 2.82 1.92 2.41 1.21 2.44 2.68 1.63 2.48 7.33		
		25	3.33 2.99 3.48 2.63 3.69 3.76 2.25 3.00 1.81 1.52 2.94 2.96 4.49 2.43 1.95	15 362	73.68
		23	3.51 3.09 3.30 4.19 3.39 2.62 2.86 2.85 3.13 10.91	15 302	73.00
			2.76 3.34 1.32 3.21 2.52 2.39 3.93 1.87 2.60 3.66		
		30	1.43 2.94 2.62 2.36 2.34 1.80 2.74 1.94 2.56 2.33	17 216	80.25
			2.79 3.32 2.82 2.43 2.91 4.49 2.65 2.56 3.07 2.87		
	NMGA		1.78 0.68 1.12 2.12 1.24 2.21 1.79 2.25 1.81 2.34		
ch-150		40	1.24 1.52 2.63 1.11 0.96 0.44 1.37 0.95 0.94 0.92	16 591	74.89
CII-130			2.23 1.50 1.77 1.15 0.88 0.42 1.82 1.91 0.75 0.87 1.56 0.77 1.22 2.65 2.22 1.58 2.35 1.06 1.76 16.86		
			1.77 0.68 1.12 2.12 1.24 2.21 1.79 2.25 1.81 2.34 1.24 1.52 2.63 1.11 0.96 0.44 1.37 0.95 0.94 0.92		
		50	2.23 1.50 1.77 1.15 0.89 0.42 1.82 1.91 0.75 0.87	19668	74.44
			1.56 0.77 1.22 2.65 2.21 1.58 2.35 1.24 1.76 2.07		
			2.33 1.04 0.95 1.53 2.20 1.24 1.68 0.63 1.72 1.94		
			1.87 2.51 2.71 2.68 2.09 1.27 2.50 0.68 2.44 2.14		
	GA	30	3.27 3.25 3.32 3.07 1.97 2.01 2.25 1.63 3.11 2.49	20 908	74.56
			2.33 2.87 2.46 2.87 2.14 2.96 2.51 1.99 3.51 3.20		
		00	3.55 3.01 4.95 4.61 4.11 3.85 4.50 2.92 4.59 3.56	5000	100 77
		30	2.17 3.93 5.26 3.29 3.49 4.05 5.48 4.71 3.49 3.83 4.30 4.08 4.49 3.31 4.20 4.18 4.67 5.62 4.35 13.90	5669	132.77
			2.72 2.19 2.30 3.67 3.11 3.27 2.71 3.37 2.55 2.92 2.39 2.49 2.39 1.73 2.40 2.47 3.70 3.51 2.60 1.94		
	NMGA	40	3.62 3.14 2.76 3.65 3.55 2.30 2.91 1.62 2.72 4.02	6452	133.65
ail 262			2.69 2.33 3.24 3.21 3.16 2.27 3.61 2.70 3.24 16.72		
gil-262			2.79 2.67 1.41 3.17 1.82 3.32 1.87 2.16 1.97 2.03		
			2.26 3.43 2.18 2.23 1.64 1.24 2.61 2.48 2.66 2.04		
		50	1.86 2.43 2.07 2.48 4.41 2.37 2.51 3.49 2.62 1.89	7870	126.62
			2.72 1.46 2.17 3.20 3.36 2.70 1.75 1.17 2.24 1.43 2.57 1.75 2.32 3.91 3.27 2.51 2.95 2.32 3.42 7.97		
	GA	30	5.41 5.66 4.11 4.17 3.96 4.12 2.59 4.47 3.83 3.13 3.88 3.44 3.89 5.22 3.43 3.97 3.00 5.57 2.91 2.76	9107	132.63
	GH	30	3.12 4.72 4.26 3.78 4.12 4.90 4.33 2.90 4.23 15.73	3107	102.00
			4.13 5.96 4.08 6.38 6.21 4.96 4.68 5.84 3.61 4.01		
		40	5.09 4.42 4.44 4.97 5.23 4.13 5.24 4.83 4.36 8.18	39 637	204.06
		40	5.23 5.02 4.03 5.25 4.22 5.63 5.39 5.21 5.61 5.93	39037	204.00
			6.12 5.52 5.91 2.84 5.98 4.29 5.60 5.24 5.70 5.32		
			3.07 4.02 3.96 3.42 3.79 2.61 1.59 2.51 3.41 2.95		
			3.72 2.44 2.44 3.80 2.49 3.06 3.52 2.67 3.36 3.49 2.10 3.03 2.61 4.27 2.27 3.71 2.09 2.69 3.31 2.57		
		60	2.87 2.23 3.42 2.05 2.96 2.75 2.73 3.67 3.44 3.65	47 972	201.83
	NMGA		3.81 3.63 3.70 3.04 1.18 3.14 2.56 3.88 2.59 4.55		
			1.83 3.84 3.46 2.03 2.41 1.55 3.31 3.75 3.12 22.99		
rd-400			2.99 3.19 3.16 2.55 1.61 2.21 2.87 1.84 2.16 2.54		
			2.25 2.80 3.20 2.85 1.79 3.93 1.78 3.03 2.59 1.77		
			2.69 2.87 2.35 3.48 1.74 3.15 2.42 3.13 3.25 1.74 3.15 2.42 3.13 1.61 3.38 1.98 1.88 2.68 3.25 2.28		
		80	1.43 3.84 1.98 1.61 3.53 1.27 2.49 1.18 2.05 2.38	57 869	194.68
			2.31 1.88 2.99 2.14 2.03 2.21 2.86 2.93 2.48 2.15		
			2.74 2.43 2.48 3.75 2.74 2.37 2.19 1.73 3.34 1.94		
			2.36 1.46 2.08 1.92 3.89 1.85 2.60 2.22 2.33 2.93		
			5.80 5.40 3.62 3.77 5.34 5.57 4.53 4.06 4.59 5.70		
	GA	40	4.76 6.07 5.47 4.29 5.37 4.04 3.92 4.41 5.35 3.66 5.24 3.98 6.24 5.63 6.56 5.75 5.28 4.46 4.44 5.60	52 341	202.78
			6.11 5.65 4.91 4.20 4.88 4.56 5.21 4.1 5.91 5.31		
			2.67 3.80 3.76 4.63 3.70 5.86 3.79 4.75 3.91 3.60		
			4.17 5.23 5.19 5.74 4.04 4.01 4.34 5.24 5.59 4.37		
		60	3.16 5.11 5.20 4.64 3.98 5.10 2.66 4.57 3.59 4.51	17105	288.28
		oo	4.79 5.16 5.00 4.70 4.08 4.19 3.90 4.32 3.97 4.77	17 185	200.28
			5.17 6.69 4.84 4.31 3.51 4.16 3.34 4.19 4.07 3.69		
			3.27 4.11 3.21 3.65 2.98 5.32 5.64 4.23 4.45 21.78		
			1.92 3.71 3.09 4.82 5.72 3.19 3.56 4.95 3.63 4.06		
			2.37 5.06 2.68 3.61 4.13 2.46 4.31 3.93 3.51 2.18 6.04 4.44 3.27 3.70 4.02 4.11 4.13 3.92 4.40 4.75		
		=0	4.76 4.26 3.84 4.52 4.00 4.34 3.77 2.77 3.38 4.76	17 155	283.35
		70			
	NMGA	70	2.24 4.79 4.21 3.57 4.90 2.85 4.07 4.47 3.35 3.37	-,	
rat-575	NMGA	70		-,	

(continued on next page)

3 2 4 7 1 9 0 6 8 5

3 2 7 1 8 5 4 6 9 0

Table 3 (continued).

Table 3 (contin	iuea).				
			2.09 3.23 3.82 3.90 1.69 3.23 2.41 2.49 4.62 3.17		
			3.39 2.47 3.52 2.94 3.72 4.38 3.66 3.60 3.39 4.37		
			3.45 4.82 3.28 4.56 4.72 2.52 2.91 3.21 3.92 2.43		
		80	3.67 3.97 3.77 2.67 4.61 4.01 3.26 2.56 2.83 4.75	18827	285.07
		80	1.94 3.71 3.78 2.99 4.42 4.11 3.26 4.33 3.04 4.10	1002/	285.07
			2.62 4.17 4.28 4.26 3.21 3.08 3.87 3.41 2.91 2.47		
			3.45 2.99 4.19 1.99 3.53 3.63 4.95 2.98 2.89 2.82		
			2.47 3.53 4.09 2.73 3.74 3.90 5.11 4.82 3.59 10.71		
			3.29 3.62 4.10 6.40 4.23 2.93 4.21 5.55 4.54 4.04		
			4.91 4.25 4.01 5.07 5.26 4.17 4.28 4.94 4.25 3.70		
	GA	60	4.08 3.99 4.05 3.78 2.97 4.37 4.20 3.87 4.41 5.61	20194	282.80
	G21 00	3.21 4.05 3.63 5.85 5.40 3.15 3.90 3.66 5.89 5.00	20194	202.00	
			4.07 3.29 5.03 4.90 4.42 4.77 3.75 6.63 4.40 3.40		
			3.68 4.21 3.42 4.71 5.38 4.97 4.74 4.26 3.67 21.60		

Table 4
Test results for 2D CTSP in crisp.

Node	Clusters	Cluster wise risk	Total cost	Total risk	Path
		2.91 2.63	187	6.47	8 5 0 3 2 9 4 6 7 1
		2.21 2.83	197	6.00	4903712685
10	2	2.21 2.83	199	5.58	1 2 4 8 5 6 9 0 3 7
10	2	2.91 2.63	206	5.57	2903854671
		2.91 3.84	249	5.39	8 9 0 3 2 4 5 6 7 1
		2.91 3.83	222	5.34	1 2 6 8 5 4 9 0 3 7
		2.21 1.76 2.59	205	6.91	4690327185
		2.21 1.76 2.59	169	6.78	6853247190
		2.21 2.13 2.55	195	6.41	0 3 2 7 6 9 4 1 8 5
10	3	2.21 1.76 2.59	165	6.40	7190685324
		2.21 1.76 2.59	225	6.26	$1\; 8\; 5\; 3\; 2\; 4\; 7\; 6\; 9\; 0$

190

198

6.20

6.06

2.21 1.76 2.59

2.21 1.76 2.59

Table 5
Test results for 3D CTSP in crisp.

Node	Clusters	Cluster wise risk	Total cost	Total risk	Path	Vehicle
		2.63 3.69	193	5.70	7045326198	3 3 2 2 1 4 3 4 3 3
		2.67 2.92	194	5.59	7053482619	4 4 2 4 2 3 2 4 4 2
10	2	2.67 4.47	206	5.58	7053426198	4 4 4 4 1 4 2 4 2 1
		4.09 2.77	185	5.31	0524867193	3 3 1 2 2 4 4 2 3 2
		2.65 3.95	245	4.94	8 7 5 1 2 6 9 0 3 4	1 2 4 4 2 4 4 4 4 2
		1.72 1.55 3.71	200	5.54	5 3 4 2 6 7 0 1 9 8	2 1 3 2 2 4 4 4 4 4
		1.62 1.45 3.22	161	5.52	4532670198	1 2 1 2 3 1 4 3 2 4
10	3	1.65 1.55 3.72	159	5.52	4532670198	1 1 2 3 2 1 3 4 2 2
		1.65 1.55 3.72	217	5.17	1098345267	3 4 2 4 1 1 2 2 3 1
		1.66 1.54 3.77	196	5.08	0148953267	3 2 1 2 4 2 2 2 2 3

Table 6
Test results for 4D CTSP in crisp.

Node	Clusters	Cluster wise risk	Total cost	Total risk	Path	Vehicle	Route
		3.13 3.22	141	5.45	4536281970	1 1 2 1 1 2 3 2 1 3	2112121222
		3.13 3.22	134	5.38	4536271980	$1\; 1\; 2\; 1\; 1\; 1\; 1\; 2\; 2\; 3$	$2\;1\;1\;2\;2\;1\;2\;2\;2\;1$
	0	3.13 3.22	142	5.38	4536271980	$1\;1\;2\;1\;1\;3\;2\;2\;2\;2$	$2\;1\;1\;2\;2\;1\;2\;2\;2\;1$
10	2	2.05 3.22	165	4.17	8 1 9 7 0 5 4 6 3 2	2122311211	$2\;1\;2\;2\;1\;1\;2\;1\;2\;2$
		1.29 2.84	124	3.67	4532619780	1 1 3 1 1 1 2 3 1 1	2121112221
		1.29 3.22	120	3.63	$0\; 1\; 9\; 7\; 8\; 4\; 5\; 3\; 2\; 6$	$2\; 3\; 2\; 3\; 2\; 1\; 1\; 3\; 1\; 2$	$1\; 2\; 2\; 1\; 2\; 2\; 1\; 2\; 1\; 2$
		1.30 1.44 1.69	141	4.51	0 9 4 8 3 5 1 2 6 7	3 2 2 3 2 1 3 1 1 1	2112122112
		1.43 0.49 2.02	136	3.98	5 3 2 1 4 6 0 9 7 8	$1\ 3\ 1\ 2\ 2\ 2\ 1\ 2\ 3\ 2$	$1\; 2\; 2\; 2\; 2\; 2\; 2\; 2\; 2\; 1$
10	3	1.43 0.49 2.02	143	3.49	1 4 6 0 9 7 8 5 3 2	2 2 2 1 2 3 2 1 3 3	2 2 2 2 2 2 2 1 2 2
		0.55 1.23 0.73	116	3.34	7190548326	1 1 1 1 2 2 2 3 1 3	2 1 2 1 2 2 2 2 1 2
		1.3 1.75 1.65	149	2.92	3 5 4 7 1 2 6 0 9 8	$2\; 2\; 3\; 1\; 2\; 2\; 2\; 1\; 2\; 1\\$	$1\; 2\; 2\; 1\; 2\; 2\; 2\; 2\; 1\; 1$

Table 7
Test results for 2D CTSP in fuzzy.

Node	Clusters	Cluster wise risk	Total cost	Total risk	Path
		3.75 3.65	200	7.56	4903712685
		3.43 3.65	204	7.22	1 2 4 8 5 6 9 0 3 7
10	2	3.75 3.65	200	7.22	1 2 6 8 5 4 9 0 3 7
		3.75 3.65	202	6.89	6203719485
		3.75 3.65	211	6.80	$1\; 2\; 4\; 8\; 5\; 6\; 9\; 3\; 0\; 7$
		2.04 2.53 3.11	205	7.62	3 2 7 1 8 5 4 6 9 0
		2.04 2.53 3.11	178	7.21	6853247190
10	3	2.04 2.53 3.11	178	7.16	7190685324
		2.04 2.53 3.11	188	7.13	3 2 4 6 8 5 7 1 9 0
		2.04 2.53 3.11	178	7.00	3 2 4 7 1 9 0 6 8 5

Table 8
Test results for 3D CTSP in fuzzy.

Node	Clusters	Cluster wise risk	Total cost	Total risk	Path	Vehicle
		3.63 3.41	137	7.42	0536721948	1122221111
		3.69 3.41	144	7.26	6053489712	$2\;1\;1\;2\;2\;1\;1\;1\;1\;2$
10	2	3.69 3.41	161	7.07	2197480536	$2\;1\;1\;2\;2\;1\;1\;1\;2\;1$
		3.63 3.41	160	6.65	8 9 7 1 6 0 5 3 2 4	$1\;1\;1\;1\;2\;1\;1\;2\;2\;3$
		3.63 3.41	180	6.38	$0\; 5\; 3\; 6\; 1\; 2\; 4\; 9\; 7\; 8$	$1\;1\;2\;2\;1\;2\;1\;1\;2\;3$
		2.40 2.38 2.79	152	7.63	7132605948	1 2 2 1 2 1 2 2 2 1
		2.40 2.38 2.87	152	7.53	2605948713	$1\; 2\; 1\; 3\; 2\; 2\; 2\; 1\; 2\; 3$
10	3	2.11 2.33 2.65	173	7.51	6 3 5 4 7 1 9 8 0 2	$1\; 2\; 2\; 3\; 1\; 1\; 2\; 2\; 1\; 3$
		2.40 2.38 2.87	155	7.34	7532601948	$1\; 1\; 3\; 1\; 2\; 1\; 2\; 2\; 2\; 2$
		2.24 2.38 2.40	167	7.27	7130548926	$1\; 2\; 2\; 2\; 2\; 2\; 3\; 2\; 1\; 2$

Table 9
Test results for 4D CTSP in fuzzy.

Node	Clusters	Cluster wise risk	Total cost	Total risk	Path	Vehicle	Route
		3.14 1.60	104	4.79	0549781362	1 1 1 2 1 2 1 2 3 2	1 1 2 2 1 2 2 1 2 2
		3.14 1.60	124	4.79	8 1 3 6 2 0 5 4 9 7	2123311121	$2\; 2\; 1\; 2\; 2\; 1\; 1\; 2\; 2\; 1$
		2.20 1.60	144	4.17	0549632781	1 1 1 2 2 1 2 1 2 1	1 1 2 2 2 1 2 1 1 2
10	2	2.20 1.60	148	4.17	1 2 6 8 3 0 5 4 9 7	1 3 2 3 3 1 1 1 2 1	2121211221
		1.41 1.60	100	3.06	8132605497	1 1 3 1 2 1 1 1 2 1	1 2 2 1 2 1 1 2 2 1
		1.41 1.60	100	3.06	0549781326	1112111312	1 1 2 2 1 1 2 2 1 2
		1.41 1.60	101	3.06	8 1 3 2 6 0 5 4 9 7	$2\; 1\; 3\; 1\; 2\; 1\; 1\; 1\; 2\; 1$	2 2 2 1 1 1 1 2 2 1
		1.15 1.88 0.96	158	4.67	9 1 8 0 2 6 4 7 3 5	1 1 3 2 1 2 2 3 2 1	1 1 2 2 1 2 2 2 1 2
		0.37 1.06 1.75	141	3.86	4 2 6 0 7 8 9 1 3 5	$1\; 1\; 2\; 2\; 1\; 1\; 2\; 2\; 2\; 1$	$1\;1\;1\;2\;1\;2\;2\;1\;1\;2$
		0.37 1.88 1.15	132	3.71	8 1 9 0 7 3 5 4 2 6	1 1 3 2 3 2 1 1 1 3	$1\;1\;2\;1\;2\;1\;2\;1\;1\;2$
10	3	0.76 1.73 1.15	105	3.38	3678190542	2131121112	1111111111
		0.76 1.73 1.15	117	3.38	5 4 2 3 6 7 8 1 9 0	1 1 3 2 1 2 1 1 3 3	1121121122
		0.76 1.46 0.87	113	2.44	5 4 2 6 7 8 3 9 1 0	1111112123	1111122122
		0.76 1.46 0.87	113	2.44	6783910542	1112123111	$1\;1\;2\;2\;1\;2\;2\;1\;1\;1$

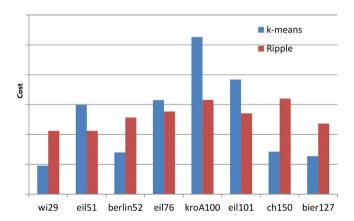


Fig. 3. Comparison of performance between k-means and Ripple clustering technique.

In Tables 3–11, we found a common trend where the risk decreased but the travel cost increased, and vice versa. However, it was not true for all cases as proved by our computational studies. A contradictory incident was observed in each table for all three data sets. For example, Table 5 showed that the minimum cost for two clusters increased

with a cumulative risk of 7.79 and traveling cost of 147. Here, 7.79 was not the minimum risk, but 147 was one of the best objective values. From Tables 10 and 11, we can conclude that our proposed algorithm can deal with real-life problems where each node physically exists with corresponding longitude and latitude values. Fig. 5 shows a sample optimized complete tour, where 38 nodes are distributed between multiple clusters and presented with the help of Google Maps. Fig. 4 shows the map of West Bengal, a province of India, as well as the map of India.

8. Practical implementation

8.1. Data analysis on the province of West Bengal in India

In this case, we selected West Bengal(WB), an Indian province, to gather the necessary data. The longitude and latitude of each well-known tourist destination in this province were taken from Google Maps (data available in the supplementary section). We used our clustering techniques to group related tourist destinations together based on their longitude and latitude. According to their geographic locations, we divided the entire travel region into three sectors, namely- hill, plateau, and plain. These three sectors were in the WB state. Latitudes greater than 26°N of WB defined the hill region, whereas the Bay of Bengal Sea defined the plain region. Greater than 88°E longitude were

Table 10
Performance for 38 nodes with real data (2D CTSP).

Node	Clusters	Cluster wise risk	Total cost	Total risk	Path
38	2	5.28 7.93	3054.669	13.404	13 21 10 25 30 35 0 15 33 4 14 12 2 24 31 18 17 22 27 19 34 32 29 16 28 5 3 20 6 1 8 9 37 23 7 26 11 36
	3	2.93 5.06 5.60	3347.069	13.30	25 21 18 17 22 27 36 13 26 7 20 34 19 32 29 16 5 28 6 1 8 9 3 37 23 11 15 33 4 14 12 2 24 31 35 0 30 10
	4	2.32 2.93 3.56 4.77	3732.17	13.70	34 32 29 20 23 7 26 13 36 27 22 17 18 21 25 30 33 24 15 4 14 12 2 31 35 0 10 11 37 3 9 8 1 6 5 28 16 19
	5	1.90 2.04 2.73 2.77 3.82	4042.369	13.88	27 17 36 26 13 21 25 7 23 20 29 32 34 19 6 1 8 9 37 3 5 28 16 11 22 18 30 35 0 33 24 15 4 14 12 2 31 10
	6	1.68 1.60 2.03 2.69 2.71 3.67	4513.10	15.06	10 4 15 14 12 2 24 31 35 30 0 33 27 22 17 18 21 25 36 13 26 7 20 34 19 32 29 28 37 23 6 1 8 9 3 5 16 11
	7	1.31 1.35 1.79 2.13 2.41 2.33 3.67	5014.40	15.37	15 4 14 12 2 10 35 0 33 31 24 30 21 18 22 25 13 17 36 2 7 26 20 29 34 19 32 28 37 23 6 1 8 9 3 5 16 11
	8	1.12 1.11 1.06 1.37 1.71 1.99 1.62 3.82	4855.87	14.192	6 1 8 9 37 3 5 28 16 11 31 14 0 10 21 18 17 22 15 4 12 2 24 33 35 30 25 13 36 27 7 26 20 34 23 29 32 19



Fig. 4. West Bengal, A province of INDIA.

plateaus. Tables 10 and 11 show the outcomes of 2D and 3D CTSPs based on real-world data for 38 tourist destinations. We considered the number of clusters from 2 to 8 and presented the cumulative risk with the lowest possible travel cost. In Table 10, the minimum cost covered two clusters, while the minimum risk covered three clusters. However, the acceptable risk was in the range of 2–5 for the various clusters discovered in the four clusters (Fig. 5). This is the most physically and financially feasible option for the traveler. If the traveler has to identify the various risks associated with their travel plans. It can be found in Table 10. Table 11 shows that 3D CTSP outperformed 2D CTSP in clusters 2–8. The risk in the 3D CTSP was much lower than that in the

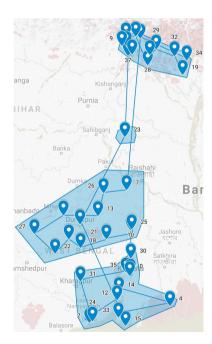


Fig. 5. A sample optimized route presentation in Map of West Bengal.

2D CTSP. The plain region had a low risk, whereas the plateau and hill regions had a higher risk due to sharp bends, heavy rain, snowfall, and other factors.

To calculate the travel risk for the mentioned three parts $(R_p, p \in 1,2,3)$ of a tour dynamically, we propose a realistic formulation as follows: $R_p = \frac{\log(d_{ij})}{\alpha_p K}$, $p \in (1,2,3)$ as the risk of the plain, plateau, and hill sectors, respectively. Here, K and α_p are parameters, and d_{ij} is the distance between ith city and jth city. The value of K depends upon the maximum and minimum lengths of (d_{ij}) , and the calculated risk

Table 11
Performance for 38 nodes with real data (3D CTSP).

Node	Clusters	Cluster wise risk	Result	Total risk	Path	Vehicle
	2	4.11 6.54	3770.99	10.94	11 36 26 7 23 6 20 3 16 29 34 19 32 28 37 8 1 9 5 10 25 30 33 15 4 14 12 35 24 2 31 22 18 27 17 13 21 0	1 2 4 3 2 4 3 4 1 4 1 1 4 2 2 4 1 4 3 2 4 3 1 2 1 3 3 3 1 2 2 2 4 2 4 4 3 4
	3	2.54 3.13 4.90	3471.40	10.46	36 27 22 17 18 21 25 13 26 7 20 34 19 32 29 16 28 5 37 6 1 8 9 3 23 11 10 30 35 0 14 33 15 4 12 24 2 31	$1\;1\;2\;1\;1\;2\;1\;4\;2\;2\;2\;3\;1\;1\;3\;4\;4\;2\;3\;4\;2\;1\;2\;2\;4\;1\;3\;2\;4\;2\;1\;3\;4\;2\;1\;1\;3\;2$
	4	2.00 2.23 2.39 4.15	3702.10	10.92	19 16 28 5 6 1 8 9 37 3 11 10 35 0 15 4 14 12 2 31 24 33 30 25 21 18 17 22 27 36 13 26 7 23 20 29 32 34	2 2 1 1 1 4 4 2 1 3 2 2 1 1 4 1 4 2 2 2 1 1 2 1 1 2 1 4 2 2 4 2 1 1 1 2 3 3
38	5	1.64 1.63 1.78 1.77 3.55	4674.90	10.70	6 1 8 9 37 3 28 5 16 11 22 18 30 35 0 33 24 15 4 14 12 2 31 10 25 21 13 26 36 17 27 19 34 32 29 20 23 7	2 1 1 1 1 4 2 2 2 1 2 1 1 1 1 1 1 4 2 1 1 1 1
	6	1.44 1.37 1.39 1.73 1.99 2.92	4310.70	10.88	36 13 26 7 20 34 19 32 29 28 37 23 6 1 8 9 3 5 16 11 27 22 17 18 21 25 10 4 15 14 12 2 24 31 35 30 0 33	$1\; 2\; 1\; 1\; 1\; 1\; 4\; 1\; 1\; 2\; 1\; 1\; 1\; 2\; 1\; 1\; 2\; 1\; 1\; 2\; 1\; 1\; 2\; 1\; 1\; 2\; 1\; 1\; 1\; 1\; 1\; 1\; 1\; 1\; 1\; 1\; 1\; 1\; 1\;$
	7	1.13 1.16 1.27 1.25 1.49 1.78 2.92	4704.40	11.17	10 35 0 33 31 15 4 14 12 2 24 30 21 18 22 25 13 17 36 27 7 26 20 29 34 19 32 28 37 23 6 1 8 9 3 5 16 11	1 2 1 1 1 2 1 4 1 1 2 1 1 1 3 1 1 1 1 1 1 2 1 1 1 4 1 1 1 1 1 1 1 1 2 1 1 2 1 4
	8	0.96 0.95 0.91 0.92 1.07 1.25 1.11 3.55	4667.30	10.79	6 1 8 9 37 3 28 5 16 11 31 14 0 10 25 13 36 27 22 17 18 21 15 4 12 2 24 33 35 30 7 26 20 34 23 29 32 19	1 1 2 1 1 2 1 4 2 2 1 1 1 1 1 2 1 1 1 1

 $(R_p, p \in (1, 2, 3))$ should be between 0 and 1. For the empirical test, $\alpha_p \in [2, 4]$, where $p \in \{1, 2, 3\}$.

9. Managerial insights

This section presents the managerial decision. When a predefined tour plan suddenly decreased the size of the complete tour, what will be the optimal decision to save the management from loss? Here, automated system is needed to build a new optimized tour plan. Finally, the system produced tour divided into a different number of clusters, where clusters were non-homogeneous. The complete tour may be shorter based on the requisition of any valuable customer due to any unavoidable situation such as extreme heat, heavy snowfall, and continuous heavy rain. In this case, management can redesign the tour depending on the different risk factors to satisfy the customer. The manager's dynamic tour allocation was based on various risk factors for the destinations. Hence, both the company and the tourist benefited. Managerial implementations corresponding to the research questions are addressed in Table 2.

10. Conclusion

The findings of this study provide three major advancements. First, a constraint multi-path multi-vehicle CTSP (4D CTSP) under imprecision was first mathematically formulated. Second, a novel clustering and re-linking approach using multiparent GA for the best route design within each cluster was included in the methodological development. Finally, it illustrated how the suggested model and methods can be applied to the tourism industry in any region of the world. This study is an excellent addition to the body of knowledge in the CTSP. Based on the latitude and longitude of the locations, the risk of the entire tour was calculated. TSPLIB instances were tested with a different number of clusters to determine the effectiveness of the suggested algorithm. We planned to consider a variety of risks as well as the visitor's realistic and pessimistic senses of the world. The limitation of this study is that it did not consider stochastic behavior-related uncertainty. A better generalization that takes into account geographic and meteorological parameters necessitates a more elegant design. The risk also depends on the state of the roads and the vehicles; comfort and time are possible additional restrictions. In the future, researchers can overcome these restrictions by using real-time weather parameters to calculate risk.

CRediT authorship contribution statement

Apurba Manna: Conceptualization, Methodology, Software, Writing – original draft. Arindam Roy: Conceptualization, Methodology, Writing – original draft. Samir Maity: Conceptualization, Methodology, Supervision, Validation, Review – original draft. Sukumar Mondal: Supervision, Validation, Writing – review & editing. Izabela Ewa Nielsen: Supervision, Validation, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Funding

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Ethics approval

This article does not contain any studies with human participants or animals performed by any authors.

Informed consent

Informed consent was obtained from all individual participants included in the study.

Consent for publication

The manuscript has not been sent to any other journal for publication.

Data availability

Data will be made available on request

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.dajour.2023.100287.

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