The score function method or also known as the log-derivative trick:

$$\nabla_x log(f(x)) = \frac{\nabla_x f(x)}{x}$$

Proof.

$$\begin{aligned}
\nabla_\psi \mathbb{E}_{q_\psi(\mathbf{z})}[f(\mathbf{z})] &= \nabla_\psi \mathbb{E}_{p_\psi(\boldsymbol{\epsilon})}[f(T(\boldsymbol{\epsilon}, \psi))] \\
&= \nabla_\psi \int_\Omega p_\psi(\boldsymbol{\epsilon}) f(T(\boldsymbol{\epsilon}, \psi)) d^n \boldsymbol{\epsilon} \\
&= \int_\Omega p_\psi(\boldsymbol{\epsilon}) \nabla_\psi f(T(\boldsymbol{\epsilon}, \psi)) d^n \boldsymbol{\epsilon} + \int_\Omega \nabla_\psi p_\psi(\boldsymbol{\epsilon}) f(T(\boldsymbol{\epsilon}, \psi)) d^n \boldsymbol{\epsilon} \\
&= \mathbf{g}^{rep} + \int_\Omega \nabla_\psi p_\psi(\boldsymbol{\epsilon}) f(T(\boldsymbol{\epsilon}, \psi)) d^n \boldsymbol{\epsilon} \\
&= \mathbf{g}^{rep} + \int_\Omega \frac{\nabla_\psi p_\psi(\boldsymbol{\epsilon})}{p_\psi(\boldsymbol{\epsilon})} p_\psi(\boldsymbol{\epsilon}) f(T(\boldsymbol{\epsilon}, \psi)) d^n \boldsymbol{\epsilon} d^n \boldsymbol{\epsilon} \\
&= \mathbf{g}^{rep} + \int_\Omega p_\psi(\boldsymbol{\epsilon}) f(T(\boldsymbol{\epsilon}, \psi)) \nabla_\psi log(p_\psi(\boldsymbol{\epsilon})) d^n \boldsymbol{\epsilon} \\
&= \mathbf{g}^{rep} + \mathbf{g}^{corr}
\end{aligned}$$

$\square$