

Let (Ω, \mathcal{A}, P) be a probability space and $\mathbf{x}_1, \dots, \mathbf{x}_n$ be iid. random variables with values in the measurable space $(\mathbb{R}^n, \mathcal{B}(\mathbb{R}^n))$. Then for any Borel measurable function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ we have:

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^n f(\mathbf{x}_i) \tag{1}$$

is an unbiased and consistent estimate of $\mathbb{E}[f(\mathbf{x})]$.