Proof:

Let $\hat{f}_k : \mathbb{R}^n \to \mathbb{R}^n$ be a sequence of elementary functions, i.e. $|\hat{f}_k(\mathbb{R}^n)| < \infty$. Then let $B_i := \{x \in \mathbb{R}^n \mid \hat{f}_k(x) = \alpha_i\} \in \mathcal{B}(\mathbb{R}^n)$ such that:

$$(1) \quad \hat{f}_k(x) = \sum_{i=0}^{n} \alpha_i \mathbb{I}_{B_i}(x)$$

$$(2) \quad \lim_{x \to \infty} \hat{f}_k = f$$

Note that $\int_\Omega \mathbb{I}_{B_i}(\mathbf{x}) p(\mathbf{x}) d^n\mathbf{x} = P(\mathbf{x} \in B_i)$. By replacing $\mathbf{z}$ with $g(\boldsymbol{\epsilon}, \mathbf{x}; \psi))$ we obtain the following:

$$\begin{aligned}
\mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[\hat{f}_k(\mathbf{z})] &= \int_\Omega \hat{f}_k(\mathbf{z}) q_\psi(\mathbf{z}|x^l) d^n\mathbf{z} && (1)\\
&= \int_\Omega \sum_{i=0}^{n} \alpha_i \mathbb{I}_{B_i}(\mathbf{z}) q_\psi(\mathbf{z}|x^l) d^n\mathbf{z} && (2)\\
&= \sum_{i=0}^{n} \alpha_i \int_\Omega \mathbb{I}_{B_i}(\mathbf{z}) q_\psi(\mathbf{z}|x^l) d^n\mathbf{z} && (3)\\
&= \sum_{i=0}^{n} \alpha_i P(\mathbf{z} \in B_i \mid \mathbf{x} = x^l) && (4)\\
&= \sum_{i=0}^{n} \alpha_i P(g(\boldsymbol{\epsilon}, x^l; \psi) \in B_i) && (5)\\
&= \int_\Omega \sum_{i=0}^{n} \alpha_i \mathbb{I}_{B_i}(g(\boldsymbol{\epsilon}, x^l; \psi)) p(\boldsymbol{\epsilon}) d^n\boldsymbol{\epsilon} && (6)\\
&= \mathbb{E}_{p(\boldsymbol{\epsilon})}[\hat{f}_k(g(\boldsymbol{\epsilon}, x^l; \psi))] && (7)
\end{aligned}$$