

Let  $T : \mathbb{R}^n \times \Theta \rightarrow \mathbb{R}$ ,  $(\epsilon, \psi) \mapsto T(\epsilon, \psi)$  be a differentiable bijection s.t.  $T(\epsilon, \psi) = \mathbf{z}$ . Furthermore, let  $p_\psi(\epsilon)$  define the pdf of  $\epsilon$  where its distribution depends weakly on  $\psi$ .

(I) Then the **gradient of the reparameterized ELBO** is:

$$\nabla_\psi ELBO(q_\psi(\mathbf{z})) = \mathbb{E}_{p_\psi(\epsilon)} [\nabla_{\mathbf{z}} f(\mathbf{z})|_{\mathbf{z}=T(\epsilon, \psi)} \nabla_\psi T(\epsilon, \psi)] + \nabla_\psi \mathbb{H}(q_\psi(\mathbf{z}))$$

(II) Then the **BBVI gradient** is:

$$\nabla_\psi ELBO(q_\psi(\mathbf{z})) = \mathbb{E}_{q_\psi(\mathbf{z})} [f(\mathbf{z}) \nabla_\psi \log(q_\psi(\mathbf{z}))] + \nabla_\psi \mathbb{H}(q_\psi(\mathbf{z}))$$