

$$\begin{array}{ccccc}
\mathbb{K}^{n,1} & \xrightarrow{\quad [f]_{B_1, \hat{B}_1} \quad} & & \mathbb{K}^{m,1} & \\
\downarrow [Id_{\mathcal{V}}]_{B_1, B_2} & \swarrow \Phi_{B_1} & & \nearrow \Phi_{\hat{B}_1} & \downarrow [Id_{\mathcal{V}}]_{\hat{B}_2, \hat{B}_1} \\
& & \mathcal{V} \xrightarrow{\quad f \quad} \mathcal{W} & & \\
& \searrow \Phi_{B_2} & & \swarrow \Phi_{\hat{B}_2} & \\
\mathbb{K}^{n,1} & \xrightarrow{\quad [f]_{B_2, \hat{B}_2} \quad} & & \mathbb{K}^{m,1} &
\end{array}$$

The diagram illustrates a commutative structure involving vector spaces and linear maps. At the center, two vector spaces \mathcal{V} and \mathcal{W} are connected by a map $f: \mathcal{V} \rightarrow \mathcal{W}$. Each space has an identity map, $Id_{\mathcal{V}}$ and $Id_{\mathcal{W}}$, represented by blue curved arrows. Surrounding these are four vector spaces: $\mathbb{K}^{n,1}$ (top-left and bottom-left) and $\mathbb{K}^{m,1}$ (top-right and bottom-right). Red arrows labeled $\Phi_{B_1}, \Phi_{B_2}, \Phi_{\hat{B}_1}, \Phi_{\hat{B}_2}$ connect the central spaces to the surrounding ones. Dotted red arrows labeled $[f]_{B_1, \hat{B}_1}$ and $[f]_{B_2, \hat{B}_2}$ connect the top and bottom $\mathbb{K}^{n,1}$ spaces to the corresponding $\mathbb{K}^{m,1}$ spaces. Vertical dotted blue arrows labeled $[Id_{\mathcal{V}}]_{B_1, B_2}$ and $[Id_{\mathcal{V}}]_{\hat{B}_2, \hat{B}_1}$ connect the top and bottom $\mathbb{K}^{n,1}$ spaces, and the top and bottom $\mathbb{K}^{m,1}$ spaces.