Now we rewrite the term $\mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[log(p(\mathbf{z},\mathbf{x}))]$. Applying Baye's theorem once again we obtain:

$$log(p_\theta(\mathbf{z},\mathbf{x})) = \quad log(p_\theta(\mathbf{x}|\mathbf{z})p_\theta(\mathbf{z})) \quad = log(p_\theta(\mathbf{x}|\mathbf{z})) + log(p_\theta(\mathbf{z}))$$

$$\Rightarrow \quad \mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[log(p_\theta(\mathbf{z},\mathbf{x}))] = \mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[log(p_\theta(\mathbf{x}|\mathbf{z}))] + \mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[log(p_\theta(\mathbf{z}))]$$

This gives us our final objective that we would like to maximize:

$$\begin{aligned}
\phi(q_\psi(\mathbf{z})) &= -D_{KL}(q_\psi(\mathbf{z}|\mathbf{x})||p_\theta(\mathbf{z}|\mathbf{x})) + log(p_\theta(\mathbf{x})) \\
&= \mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[log(p_\theta(\mathbf{z},\mathbf{x}))] - \mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[log(q_\psi(\mathbf{z}|\mathbf{x}))] \\
&= \mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[log(p_\theta(\mathbf{x}|\mathbf{z}))] + \mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[log(p_\theta(\mathbf{z}))] - \mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[log(q_\psi(\mathbf{z}|\mathbf{x}))] \\
&= \mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[log(p_\theta(\mathbf{x}|\mathbf{z}))] - (\mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[log(p_\theta(\mathbf{z}))] - \mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[log(q_\psi(\mathbf{z}|\mathbf{x}))]) \\
&= \mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[log(p_\theta(\mathbf{x}|\mathbf{z}))] - D_{KL}(q_\psi(\mathbf{z}|\mathbf{x})||p_\theta(\mathbf{z}))
\end{aligned}$$

We call $\phi(q_\psi(\mathbf{z}))$ the ELBO of $q_\psi(\mathbf{z})$.