

ELBO

The ELBO of $q_\psi(\mathbf{z})$ is a lower bound for $\log(p_{\hat{\theta}}(\mathbf{x}))$:

$$ELBO(q_\psi(\mathbf{z})) \leq \log(p_{\hat{\theta}}(\mathbf{x})) \quad (1)$$

We can see that in our definition of the ELBO:

$$\begin{aligned} ELBO(q_\psi(\mathbf{z})) &= \phi(q_\psi(\mathbf{z})) \\ &= \log(p_\theta(\mathbf{x})) - D_{KL}(q_\psi(\mathbf{z}|\mathbf{x})||p_\theta(\mathbf{z}|\mathbf{x})) \\ &\leq \log(p_{\hat{\theta}}(\mathbf{x})) \end{aligned}$$

Since the Kullback-Leibner Divergence is strictly non-negative.