

Let us calculate first the estimate of our simplified version of expectation:

$$\mathbb{E}_{q_\psi(\mathbf{z}|x^l)}[f(\mathbf{z})] = \mathbb{E}_{p(\epsilon)}[f(g(\epsilon, x^l; \psi))] \approx \frac{1}{n} \sum_{i=1}^n f(g(\epsilon^i, x^l; \psi)) \quad (1)$$

Then we obtain for our loss function the following approximation:

$$\begin{aligned} ELBO(q_\psi(\mathbf{z}|x^l)) &= \mathbb{E}_{q_\psi(\mathbf{z}|x^l)}[\log(p_\theta(x^l|\mathbf{z}))] - D_{KL}(q_\psi(\mathbf{z}|x^l)||p_\theta(\mathbf{z})) \\ &\approx \frac{1}{n} \sum_{i=1}^n \log(p_{\hat{\theta}}(x^l|g(\epsilon^i, x^l; \psi))) - D_{KL}(q_\psi(\mathbf{z}|x^l)||p_\theta(\mathbf{z})) \end{aligned} \quad (2)$$

If in some cases the Kullback-Leibner Divergence is not computable we can take the Monte Carlo Estimate of the entire loss function:

$$\begin{aligned} ELBO(q_\psi(\mathbf{z}|x^l)) &= \mathbb{E}_{q_\psi(\mathbf{z}|x^l)}[\log(p_\theta(\mathbf{z}, x^l))] - \mathbb{E}_{q_\psi(\mathbf{z}|x^l)}[\log(q_\psi(\mathbf{z}|x^l))] \\ &\approx \frac{1}{n} \sum_{i=1}^n \log(p_{\hat{\theta}}(x^l, g(\epsilon^i, x^l; \psi))) - \log(q_\psi(g(\epsilon^i, x^l; \psi)|x^l)) \end{aligned}$$