

I denoted \mathbb{H} as the cross-entropy. Then we can rewrite the ELBO as:

$$\begin{aligned} ELBO(q_\psi(\mathbf{z}|\mathbf{x})) &= \mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[\log(p_\theta(\mathbf{z}, \mathbf{x}))] - \mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[\log(q_\psi(\mathbf{z}|\mathbf{x}))] \\ &= \mathbb{E}_{q_\psi(\mathbf{z}|\mathbf{x})}[\log(p_\theta(\mathbf{z}, \mathbf{x}))] + \mathbb{H}(q_\psi(\mathbf{z}|\mathbf{x})) \end{aligned} \quad (1)$$

The G-REP is represented as the following identity:

$$\nabla_\psi ELBO(q_\psi(\mathbf{z}|\mathbf{x})) = \mathbf{g}^{rep} + \mathbf{g}^{corr} + \nabla_\psi \mathbb{H}(q_\psi(\mathbf{z}|\mathbf{x})) \quad (2)$$

The terms \mathbf{g}^{rep} is the gradient of the reparameterized term and \mathbf{g}^{corr} a correction term that is needed when $p(\epsilon)$ does depend on ψ