
Intrinsically Motivated Exploration for Automated Discovery of Patterns in Morphogenetic Systems

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Abstract

1 Exploration is a corner stone of machine learning algorithms used to discover
2 novel strategies or patterns. Intrinsically motivated goal exploration processes
3 (IMGEs) were shown to enable autonomous agents to efficiently explore and
4 map the diversity of the effects they can produce on their environment. With
5 IMGEs, agents self-define their own experiments by imagining goals, then try
6 to achieve them by leveraging their past discoveries. Progressively they learn
7 which goals are achievable and which are not. IMGEs were shown to enable
8 efficient discovery and learning of diverse repertoires of skills in high-dimensional
9 robots. In this article, we show that the IMGE framework can also be used in an
10 entirely different application area: automated discovery of self-organized patterns
11 in complex morphogenetic systems. We also introduce a new IMGE algorithm
12 where goal representations are learned online and incrementally (past approaches
13 used precollected training data with batch learning). For experimentation, we
14 use Lenia, a continuous game-of-life cellular automaton. We study how IMGEs
15 algorithms enable to discover a variety of complex self-organized visual patterns.
16 We compare random search and goal exploration methods with hand-defined,
17 pretrained and online learned goal spaces. The results show that goal exploration
18 methods identify more diverse patterns compared to random exploration. Moreover,
19 the online learned goal spaces allow to successfully discover interesting patterns
20 similar to the ones manually identified by human experts. Our results exemplify the
21 ability of IMGEs to aid the discovery of novel structures and patterns in complex
22 systems. We are optimistic that their application will aid the understanding and
23 discovery of new knowledge in various domains of science and engineering.

1 Introduction

25 Exploration is the process of searching new solutions for a problem or new information about a system.
26 It is a corner stone of many machine learning algorithms. For example, a robotic reinforcement
27 learning agent may have to explore for discovering new objects and effects it can produce on them.
28 Moreover, exploration is an important part of knowledge discovery in science and engineering.
29 In order to understand or optimize a system, one must explore to discover what are its potential
30 behaviours and how to represent them.

31 Intrinsically motivated goal exploration processes (IMGEs) have shown to be efficient exploration
32 strategies for autonomous agents to discover and map the diversity of effects they can produce in
33 their environment [3, 12, 26]. With IMGEs, agents self-define their own experiments by imagining
34 goals, then try to achieve them by leveraging their past discoveries (Fig. 1). Progressively they learn
35 which goals are achievable and which are not. The goals are defined in a space of representations

Code and additional videos at <https://intrinsically-motivated-discovery.github.io/>

36 which describe the important features of the raw observation space. For a robot that interacts with
37 objects the locations and properties of those objects could be such features [12]. With deep neural
38 networks, the goal representations can be directly learned from raw pixel perception by training the
39 latent layers of autoencoders with pre-collected data [29, 26].

40 So far, IMGEPs have been mainly used in the context of autonomous learning agents/robots. They
41 enabled an efficient exploration of diverse skill repertoires in high-dimensional robots [3, 12].
42 Nonetheless, their exploration capabilities are not constrained to this field and can be used in a
43 variety of application scenarios. In this paper we exemplify their application for automating the
44 discovery of complex behaviours and patterns of high-dimensional complex systems such as studied in
45 developmental (theoretical) biology, chemistry or physics. Based on our results, IMGEPs show a high
46 potential to be efficient tools for helping scientists to discover and analyze novel high-dimensional
47 self-organized structures in these complex systems. In a recent step in that direction, Grizou et al.
48 [17] showed that IMGEPs are capable of making autonomously discoveries in a chemical system
49 based on a simple low-dimensional hand engineered goal space. In this paper we show that the full
50 abilities of IMGEPs can be utilized for such environments by also learning in an unsupervised manner
51 the representations that define the goal space. We show this ability on the example of discovering
52 morphogenetic patterns in *Lenia* [6], a continuous game-of-life cellular automaton.

53 Moreover, we introduce with this paper a new IMGEP algorithm able to learn online the represen-
54 tations for its goal space. In previous IMGEP approaches, the goal representation either had to be
55 defined by hand [12] or learned by a batch process with prerecorded data [29]. This required either
56 expert knowledge about the system or existing data that might also bias the exploration. The online
57 learning approach can be directly applied without expert knowledge or preexisting data.

58 In summary, the paper provides the following new contributions:

- 59 • The application of IMGEPs in a new domain: The discovery of structures and patterns in
60 high-dimensional complex systems with autonomous learning of goal representations.
- 61 • A novel online representation learning algorithm for IMGEPs that does not rely on expert
62 knowledge or precollected data.

63 2 Related Work

64 Inspired from the way human children can self-develop a hierarchy of skills in order to make
65 sense of the world, *intrinsically-motivated* learning [2, 3] is a family of computational models that
66 autonomously organize an agent exploration curriculum in order to discover efficiently a maximally
67 diverse set of outcomes that can be produced in an unknown environment. Intrinsically Motivated
68 Goal Exploration Processes (IMGEPs) [3, 12] is a family of curiosity-driven algorithm which have
69 been developed in the context of high-dimensional complex real world systems. Population-based
70 versions of these algorithms, leveraging episodic memory and hindsight learning, have shown to
71 enable robots or artificial agents to acquire diverse repertoires of skills [35, 12] as to bootstrap
72 the exploration capacity for deep reinforcement learning problems with rare or deceptive rewards
73 [8]. Recent work [26, 29] studies how to automatically learn the goal representations with the use
74 deep variational autoencoders. However, the training is done in an early stage and passively on a
75 precollected set of available images. A related family of algorithms in evolutionary computation is
76 novelty search [27] and quality-diversity algorithms [30], which can be formalized as special kind of
77 population-based IMGEPs with a fixed random goal sampling policy.

78 Intrinsically motivated learning techniques have also been widely developed to handle exploration
79 in reinforcement learning, with diverse approaches ranging from estimating visitation counts [4],
80 measures of empowerment [16], to goal exploration approaches [11] with hindsight learning [1] and
81 automated curriculum [9], or related concepts such as auxiliary tasks [34] and general value functions
82 [38]. However, these approaches focused on the problem of sequential decisions in MDPs (incurring
83 a cost on sample efficiency), orthogonal to the automated discovery framework considered here with
84 independent experiments (allowing the use of memory-based sample efficient methods).

85 Active inquiry-based learning strategies have been used in biology [23, 22], chemistry [10] and
86 astrophysics [33] to autonomously query which set of experiments to perform in order to improve the
87 overall model of the system. These data-driven approaches considerably reduce the experimental costs
88 but still require a database background of representative experiments. Recently, machine learning

algorithms [31, 32, 36] have been integrated into the experimental laboratory and often combined to the use of robotics and automation platforms [15, 20]. These methods open a brand new perspective to the way scientific experiments are conducted, but most of them rely on *expert* knowledge and optimize specific target properties. Rather than trying to find the optimal physico-chemical model from a database of collected experiments, we are interested to automatically discover a diversity of unseen patterns without requiring prior knowledge of the system.

Representation learning aims at finding a set of low-dimensional explanatory factors representing high-dimensional input data [5] and is a key problem in a lot of areas in order to understand the underlying structure of complex observations. Deep variational *autoencoders* (VAE) [24] are one of the most popular approaches and many state-of-the-art methods [41, 18, 25, 19, 21, 7] build on top of it using varying objectives and network architectures. See Tschannen et al. [39] for an in-depth review.

3 Method

This section describes intrinsically motivated goal exploration and the online learning approach for representation of their goal spaces.

3.1 Intrinsically Motivated Goal Exploration Processes

An IMGEP (Fig. 1) is a sequence of experiments that explore the parameters of a system by targeting self-generated goals. It aims to maximize the diversity of observations from that system within a budget of N experiments.

The systems are defined by three components. A parameterization space Θ corresponding to their controllable parameters θ . An observation space O where an observation o is a vector representing all the signals captured from the system. For this paper, the observations are a time series of images which depict the morphogenesis of activity patterns. Finally, an unknown environment dynamic $D: \theta \rightarrow O$ which maps parameters to observations.

To explore a system, an IMGEP defines a goal space \mathcal{T} that represents relevant features of its observations. For a robot that has to manipulate objects and observes them with a video camera, those features could be the object positions. From this goal space a goal $g \sim G(\mathcal{H})$ is sampled by a goal sampling distribution. For the robot example this would correspond to a sampling of positions to where the robot should move the objects. Then, a parameter θ is chosen by a parameter sampling policy $\Pi = \Pr(\theta; g, \mathcal{H})$ that should be explored in order to reach the goal g . Usually the parameter sampling policy and in some cases the goal sampling distribution take into account previous explorations which are stored in a history \mathcal{H} . After a parameter is selected it is explored on the system and the outcome o observed. Based on the observation the actually reached goal $\mathcal{T}(o)$ is computed and together with its corresponding parameter and observation stored in history \mathcal{H} . The exploration process is repeated until a certain number of steps N or another constraint is reached. Because the sampling of goals and parameters depend on a history of explored parameters, an initial set of N_{init} parameters are randomly sampled and explored before the goal exploration

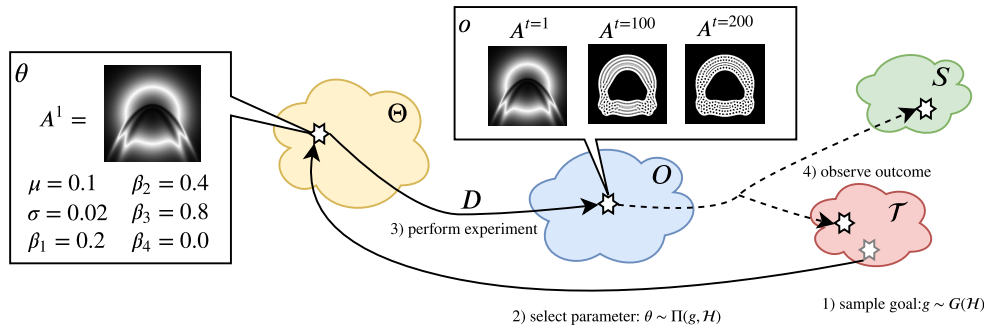


Figure 1: The intrinsically motivated goal exploration process (IMGEP) on the example of Lenia.

process starts. Optional statistics S about the collected observations and can be used to analyze and visualize exploration results, however they are unknown to the discovery algorithm.

Depending on the algorithmic instantiation of this architecture, different goal and parameter sampling mechanisms can be used [3, 13]. For this paper we chose for both simple approaches. Goals are sampled from a uniform distribution over the goal space. Parameters are chosen by selecting for a given goal the parameter from the history whose reached goal has the shortest distance in the goal space to the given goal. This parameter is then mutated to by a random process.

3.2 Learning of Goal Spaces via Online Representation Learning

For IMGEPs the definition of the goal space \mathcal{T} is a critical part, because it defines which behaviors and observations the process tries to find in the target system. A straightforward choice to define a goal space is by selecting features manually, such as by using computer vision algorithms to detect the positions of objects from the recorded video images [12, 17]. The problem of this approach is that expert knowledge is required to select helpful features. Moreover, for unknown and high-dimensional systems even experts might not know which features to use.

A more elaborated approach is to learn goal space features with unsupervised representation learning. Representation learning is able to learn a mapping $\tilde{R}_{\mathcal{T}}(o)$ from the raw sensor observations o to a compact latent vector $\mathbf{z} \in \mathbb{R}^d$. This latent mapping can then be used as a goal space where a latent vector $\mathbf{z} = g$ is interpreted as a goal.

Previous approaches applied already successfully the learning of goal spaces with variational autoencoders (VAE) [26, 29]. However, the goal spaces were learned before the start of the exploration and from a prerecorded dataset of observations from the target system. During the exploration the learned representations were kept fixed. A problem with this pretraining approach is that training samples may be limited and often biased towards the initial knowledge of the system.

In this paper we attempt to address this problem by continuously adapting the learned representation to the different observations of the system encountered during the exploration process. We believe it is crucial to capture the factors of variations of patterns unseen so far by the network to further enable the discovery of a diversity of similar patterns in the system. To address this challenge, we propose an online goal space learning IMGEP (IMGEP-OGL), which can learn the goal space in an incremental manner during the exploration process (Algorithm 1). We are using different variants of VAEs for the representation learning part of the algorithm. For details about the different variants please refer to Section in the Supplementary Materials.

Algorithm 1: IMGEP-OGL

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1 Initialize goal space representation  $R_{\mathcal{T}}$  with random weights
2 Initialize an empty training dataset  $D_{\mathcal{T}} = \emptyset$ 
3 for  $i \leftarrow 1$  to  $N$  do
4   if  $i < N_{init}$  then // Initial random iterations to populate  $\mathcal{H}$ 
5     Sample  $\theta \sim \mathcal{U}(\Theta)$ 
6   else // Intrinsically motivated iterations
7     Sample a goal  $g \sim \mathcal{G}(\mathcal{H})$  based on space represented by  $R_{\mathcal{T}}$ 
8     Choose  $\theta \sim \Pi(g, \mathcal{H})$ 
9     Perform an experiment with  $\theta$  and observe  $o$ 
10    Append  $(\mathcal{T}(o), \theta, o)$  to the history  $\mathcal{H}$ 
11    Append  $o$  to  $D_{\mathcal{T}}$ 
12    if  $i \bmod K == 0$  then // Periodically train the network
13      for  $E$  epochs do
14        Train  $R_{\mathcal{T}}$  on  $D_{\mathcal{T}}$  with importance sampling
15      for  $(g, \theta, o) \in \mathcal{H}$  do // Update the database of reached goals
16         $\mathcal{H}[g] \leftarrow R_{\mathcal{T}}(o)$ 

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157 Their training procedure is integrated in the goal sampling exploration process by first initializing
 158 the VAE network with random weights. After the exploration of each new parameter the resulting
 159 observation is added to a dataset of training examples $D_{\mathcal{T}}$. The VAE network is then trained every K
 160 explorations for E epochs on the collected training data. Importance sampling is used to give more
 161 weight to recently discovered patterns.

162 4 Experiments

163 The usefulness of IMGEPs for the discovery of novel patterns in complex system was evaluated
 164 on the Lenia system. The following sections introduce Lenia, the different algorithms that were
 165 compared and the experimental procedure.

166 4.1 Target System: Lenia

167 Lenia [6] is a continuous cellular automaton [40] similar to Conway’s Game of Life [14]. Game-
 168 of-life systems have been used many times as abstract models for theoretical understanding of how
 169 self-organized structures may form in natural morphogenetic systems. Lenia, in particular, represents
 170 a high-dimensional complex dynamical system where diverse visual structures can self-organize,
 171 including solitons, and yet are hard to find by manual exploration. It is therefore well suited to test
 172 the performance of exploration algorithms for unknown and complex systems.

173 Lenia consists of a two-dimensional grid of cells $A \in \mathbb{R}^{256 \times 256}$ where the state of each cell is a
 174 real-valued scalar activity $A_{x,y} \in [0, 1]$. The state of cells evolves over discrete time steps (Fig. 2,
 175 a). The activity change of a cell between time steps is computed by integrating the activity of
 176 neighbouring cells. Several parameters $\{R, T, \mu, \sigma, \beta_1, \dots, \beta_4\}$ control the process.

177 Lenia can be understood as a morphogenetic system where the parameters θ and the initial state A^1
 178 represent the genes of a developmental process. They control into which final activity pattern the
 179 initial pattern morphs (Fig. 2). Lenia can produce diverse patterns with different dynamics such as
 180 stable, non-stable or chaotic patterns. Most interesting, patterns that resemble microscopic animals
 181 can be produced [6]. We use Lenia to study if IMGEPs can autonomously discover such patterns.

182 We implemented different pattern classifiers to analyze the exploration results. We differentiate
 183 between dead and alive patterns. A pattern is dead if the activity of all cells are either 0 or 1. Alive
 184 patterns are classified into animals and non-animals. Animals are a connected areas of positive
 185 activity which are finite, i.e. which do not infinitely cross several borders. All other patterns are
 186 non-animals which usually spread over the whole state space.

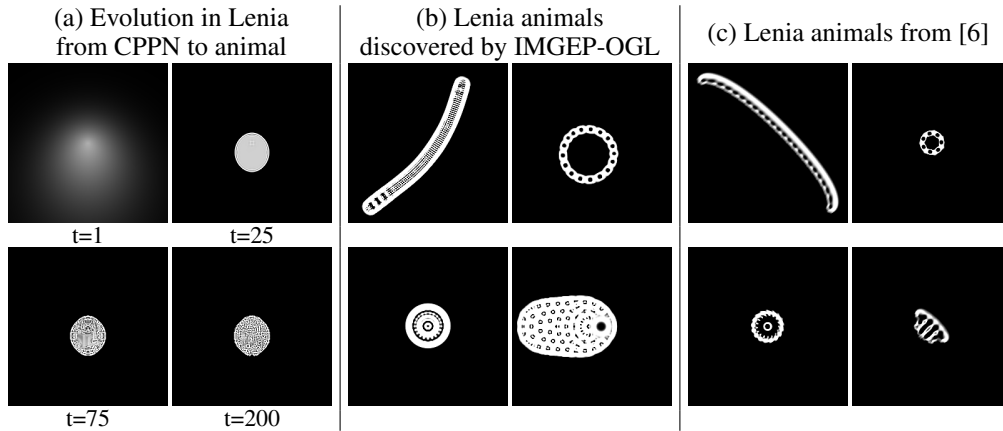


Figure 2: Example patterns produced by the Lenia system. Illustration of the dynamical morphing from an initial CPPN image to an animal (a). The automated discovery (b) is able to find similar complex animals as a human-expert manual search (c) by [6].

4.2 Compared Algorithms

The exploration behavior of different IMGEP algorithms were evaluated and compared to a random exploration. The IMGEP variants differ in their way how the goal space is defined or learned.

Random exploration: The IMGEP variants were compared to a random exploration that sampled randomly for each of the N exploration iterations the parameters θ and the initial state A^1 .

IMGEP-HGS - Goal exploration with hand-defined goal space: The first class of IMGEP uses a hand-defined goal space that is constructed by 5 features that were measured about the final pattern that developed in Lenia for each exploration. We tested different combinations of summary statistics and selected the one that was overall best. Its goal space consists of: 1) the sum over the activity of all cells, 2) the density of the activated cells, 3) the velocity of the activity center, 4) a asymmetry measure of the entity and 5) a distribution measure of the entity.

IMGEP-PGL - Goal exploration with a pretrained learned goal space: For this IMGEP variant the goal space was learned with one of several VAE approaches on training data before the exploration process started. The training set consisted of 558 Lenia patterns. Half of the patterns were animals that have been manually identified by [6]. The other half were randomly initialized patterns that were created with the same procedure as described in Section 4.3.

IMGEP-OGI - Goal exploration with online learning of the goal space: The final algorithm is the new online variant for IMGEPs (Algorithm 1).

Please refer to the Supplementary Materials for details about the parameterization of the algorithms.

4.3 Experimental Procedure

For each algorithm 10 exploration experiments were conducted to measure their average performance. Each experiment consisted of $N = 5000$ exploration iterations. For the IMGEP variants the first $N_{\text{init}} = 1000$ iterations were random explorations to populate their histories \mathcal{H} . For all algorithms an identical initial set of random explorations was used to allow a better comparison between them. For the following 4000 iterations each IMGEP approach sampled a goal g by a uniform distribution over its goal space. Then, the parameter θ_k from a previous exploration in \mathcal{H} was selected whose reached goal g_k had the minimum distance to the current goal g within the current goal space embedding. This parameter was then mutated by a random process to get the parameter θ that was explored.

The parameters consisted of a compositional pattern producing network (CPPN) [37] that generates the initial state A^1 for Lenia and the settings defining Lenia’s kernel function and update rules: $\theta = [\text{CPPN} \rightarrow A^1, R, T, \mu, \sigma, \beta_1, \dots, \beta_4]$. CPPNs are recurrent neural networks that were originally used to generate and evolve gray scale images, but that can be similarly used for the initial activity patterns of Lenia. The networks are randomly initialized and mutated by a random process that defines their structure and connection weights as done in [37]. The random initialization of the other Lenia settings was done by an uniform distribution and their mutation by a Gaussian distribution around the original values. The meta parameters to initialize and mutate parameters θ are equal for all algorithms (SM, Section 1.4). They were manually chosen without optimizing them for a specific algorithm. The parameters of the CPPN networks were set to initialize and mutate networks that generate similar images as in [37].

5 Results

We compared random explorations and different IMGEP algorithms on their ability to produce diverse Lenia patterns. Diversity is measured in a statistic space constructed by hand-defined features. Furthermore, we compared the goal spaces of hand-defined and learned IMGEP variants.

5.1 Diversity of Explored Lenia Patterns

Diversity is measured by the spread of the exploration in a statistic space S defined by 8 features. The features are: 1) sum over the activity of all cells, 2) number active cells, 3) density measure of the activated cells, 4) velocity of the activity center, 5) average movement angle of the activity center, 6) angular rotation of the activity center, 7) a asymmetry measure of the entity and 8) a distribution

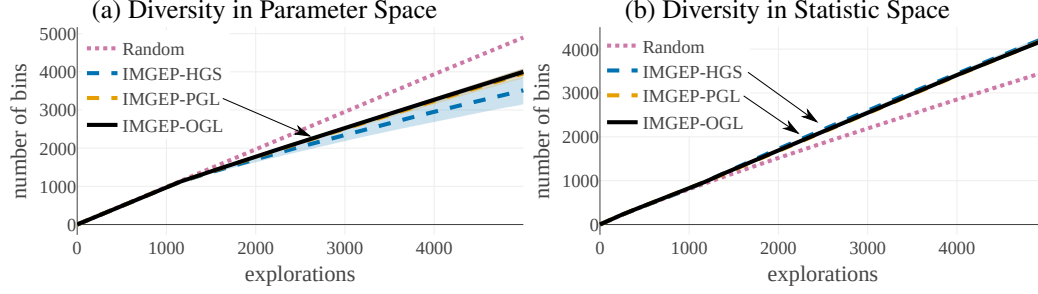


Figure 3: (a) Although random explorations reach the highest diversity in the space of parameters Θ , (b) IMGEPs reach a higher diversity in the statistic space. They discover a more diverse set of Lenia patterns. Depicted is the average diversity ($n = 10$) with the standard deviation as shaded area (for some not visible because it is too small).

235 measure of the entity. See the Supplementary Material for more details. Please note that the choice of
 236 this space was arbitrary and results might change depending on the used features.

237 For each experiment all explored patterns were projected into the statistic space. To measure the
 238 diversity of the found solutions we used a simple measure of the area that the exploration covered in
 239 the statistic space. The measure discretizes the statistic space into bins of equal size by splitting each
 240 dimension into 10 sections resulting in 10^8 bins. The number of bins in which at least one explored
 241 entity falls is counted and used as a measure for diversity.

242 We also measured the diversity in the space of parameters Θ . This measure excluded the high-
 243 dimensional initial state A^1 and used only the other parameters as dimensions: $R, T, \mu, \sigma, b_1, \dots, b_4$.
 244 Also for this space 10 bins per dimension were used to discretize the space.

245 Comparing the diversity between the parameter space and the statistic space reveals the advantage of
 246 IMGEPs over random explorations (Fig. 3). Although random explorations have reached the highest
 247 diversity in the space of parameters, they are outperformed in terms of diversity by the IMGEP
 248 approaches in the statistic space. Thus, the IMGEP approaches are better in the actual objective of
 249 our exploration, finding a diverse set of Lenia patterns.

250 All IMGEP variants reached a similar diversity measure if measured over all explored patterns (Fig. 3,
 251 b). This is not the case if the diversity within certain subgroups of patterns is considered. In the
 252 case of the diversity only over explored animals (Fig. 4, a) the new online approach IMGEP-OGL
 253 is finding the highest diversity of animals. It is closely followed by the pretrained IMGEP-PGL
 254 approach. The hand-defined goal space approach IMGEP-HGS can identify a diversity of 50%
 255 compared to IMGEP-OGL, and random explorations only less than 30%. In the case of diversity
 256 over non-animal patterns the IMGEP-HGS reached the highest diversity followed by the random

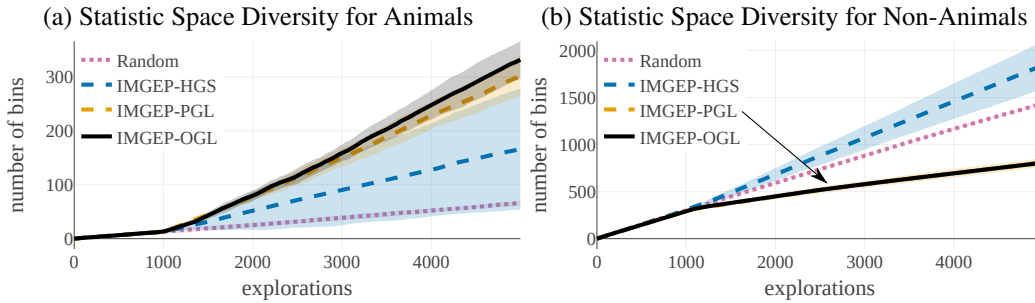


Figure 4: (a) IMGEPs with a learned goal space discovered a larger diversity of animals compared to the IMGEP with a hand-defined goal space. (b) For non-animals the hand-defined goal space discovered the highest diversity. Depicted is the average diversity ($n = 10$) with the standard deviation as shaded area (for some not visible because it is too small).

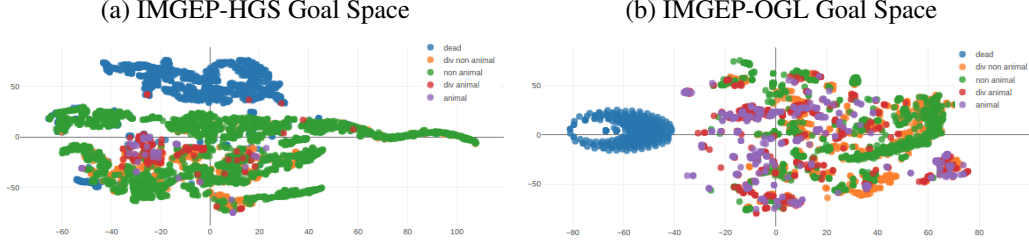


Figure 5: (a) Hand-defined and (b) learned goal space have major differences (shown here by a t-SNE visualization) which explain the difference in their resulting diversity of animals and non-animals (Fig. 4).

explorations (Fig. 4, b). The two approaches that learned the goal space (IMGEP-OGL, IMGEP-PGL) reached the lowest diversity for non-animal patterns.

5.2 Differences in Goal Spaces

We analyzed the goal spaces of the different IMGEP variants to understand their behavior by visualizing their reached goals in a two-dimensional space. t-SNE [28] was used to reduce the high-dimensional goal spaces. It puts points that were nearby in the high-dimensional space also close to each other in the two-dimensional visualization.

The goal spaces of IMGEP-HGS and IMGEP-OGL show strong differences between each other which we believe explain their different abilities to find either a high diversity of non-animals or animals (Fig. 4). The goal space of the IMGEP-HGS (Fig. 5, a) shows large areas and several clusters for non-animal patterns. Animals form only two small and nearby clusters. Thus, the hand-defined features seem poor to discriminate and describe animal patterns in Lenia. As a consequence, when goals are uniformly sampled within this goal space during the exploration process, then more goals are generated in regions that describe non-animals. This can explain why IMGEP-HGS explored a higher diversity of non-animal patterns.

In contrast, features learned by IMGEP-OGL capture better factors that differentiate animal patterns as can be observed by several clusters of animals that span a wide area in its goal space (Fig. 5, b). We attribute this effect to the difficulty of VAEs to capture sharp details [42]. They therefore can only represent the general form of the patterns. Non-animals usually span over the whole state space and differ in their fine-grained structures. VAEs have therefore difficulty to learn their important discriminators. Animals differ also often in their form and extension and can be easier differentiated. The VAE learned goal-spaces seem therefore better suited for exploration of diverse animal patterns.

6 Conclusion

We presented in this paper the application of intrinsically motivated exploration via IMGEPs towards a new and exciting application area: the discovery of patterns and structures in complex systems. All evaluated IMGEP variants were able to discover a diverse set of patterns for Lenia, a cellular automaton, by directly exploring its high-dimensional parameters and observing its high-dimensional output patterns. We could demonstrate that goal spaces for such systems can be successfully learned via deep VAEs which allow the identification of animal-like patterns similar to those identified by human experts (Fig. 2). Moreover, our new approach of learning goal spaces online via data collected during the exploration process could outperform a pretrained and fixed goal space in terms of identifying a diverse set of animal-like patterns.

We believe that IMGEPs are able to facilitate the study of similar complex and high-dimensional systems in different fields of engineering and science such as physics and chemistry. IMGEPs allow to explore unknown systems efficiently to discover their interesting behaviors and patterns which can help to understand the systems better or to find new solutions for problems in them.

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Supplementary Material for "Intrinsically Motivated Exploration for Automated Discovery of Patterns in Morphogenetic Systems"

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1 Lenia

1.1 System and Parameters

We used the Lenia implementation as described in [2] for all experiments. We based our code on the python implementation provided at <https://github.com/Chakazul/Lenia>. All Lenia experiments used the following configurations:

- State size: 256×256 ($A \in \mathbb{R}^{256 \times 256}$)
- Number of steps: $T = 200$
- Growth mapping: exponential with $G(u; \mu, \sigma) = 2 \exp\left(-\frac{(u-\mu)^2}{2\sigma^2}\right) - 1$
- Kernel function: exponential with $K_C(r) = \exp\left(\alpha - \frac{\alpha}{4r(1-r)}\right)$, with $\alpha = 4$
- Kernel shell: $K_S(r; \beta) = \beta_{\lfloor Br \rfloor} K_C(Br \bmod 1)$, with $\beta = (\beta_1, \beta_2, \beta_3, \beta_4)$

All other settings are part of the parameter space $\theta = [R, T, \mu, \sigma, \beta_1, \dots, \beta_4]$, where R is the radius of the circle around a cell whose cells influence its activity. T controls the growth strength update per time step. The form of the kernel function is controlled by $\mu, \sigma, \beta_1, \dots, \beta_4$. The cell grid of Lenia is similar to the surface of a ball, because the cells on the top border are neighbors to the bottom cells and the left and right border are also connected.

1.2 Classifier

We developed classifier for the patterns that are observed in Lenia. This section describes the classifiers in detail.

Dead Classifier: Dead patterns are all patterns where the activity of all cells are either 0 or 1.

Animal Classifier: The final pattern of a Lenia entity is classified as an animal if it is finite. In Lenia the borders of opposite sites are connected, so that the space is similar to a ball surface. We define an animal as a finite connected pattern of activity. Cells are connected as a pattern if they are active, i.e. their activity is larger than 0.1, and if they influence each other, i.e. if the activity of one cell is in the radius of the influence of the kernel function K_S which is controlled by the parameter R . Moreover, the connected pattern must be at least 80% of all the activation in the space and a pattern must exist for the last two time steps. These constraints are used to avoid that too small patterns or chaotic entities are classified as animals.

Non-Animal Classifier: We also classified non-animal patterns which are all entities that were not dead and not an animal. These are patterns that usually spread over the whole state space or are connected via borders and therefore not finite.

Diverging Classifier: We classified further if patterns are diverging and most probably would become dead if Lenia would run for more time steps. Diverging entities are classified by measuring the average sum over the activity over the middle third of all time steps and compared it to the last third of all time steps. If the two differed by 5%, the pattern was classified as diverging and most likely would be either go fully activated or fully non-activated:

$$0.95 < \frac{\bar{A}[\frac{1}{3}T, \frac{2}{3}T]}{\bar{A}[\frac{2}{3}T, T]} < 1.05 \text{ with } \bar{A}^{[t_1, t_2]} = \frac{1}{t_2 - t_1} \sum_{t=t_1}^{t_2} \bar{A}^t \text{ with } \bar{A}^t = \frac{1}{N} \sum_{i=1}^N A_i^t$$

1.3 CPPNs for the generation of Lenia start states

To define the initial activation pattern A^1 compositional pattern producing networks (CPPN) [15], which were originally used to evolve images, are used. CPPNs are recurrent neural networks with the x and y coordinate of a grid cell and its distance r to the grid center as input. Their output is the activation for the cell. Randomly initiated CPPNs can produce various patterns. Moreover, it is possible to mutate the networks to evolve their output.

We used the following package to implement CPPNs: <https://github.com/CodeReclaimers/neat-python>. We had to adjust the sigmoid and gaussian function from the ones used in [15] to replicate similar images.

1.4 Random initialization and mutation of parameters

Tables 1 and 2 list the parameters used to initialize and mutate the Lenia and CPPN parameters during an exploration. Please see [15] for details about the parameters of the CPPN evolution and mutation.

Table 1: Configuration for the initialization and mutation of Lenia system parameters θ for all experiments.

Parameter	Type	initial min	initial max	mutation σ	mutation min	mutation max
R	\mathbb{N}	2	20	0.5	2	20
T	\mathbb{N}	1	20	0.5	1	20
μ	\mathbb{R}	0	1	0.05	0	1
σ	\mathbb{R}	0.001	0.2	0.01	0.01	1
$b[1]$	\mathbb{R}	0	1	0.05	0	1
$b[2]$	\mathbb{R}	0	1	0.05	0	1
$b[3]$	\mathbb{R}	0	1	0.05	0	1
$b[4]$	\mathbb{R}	0	1	0.05	0	1

Table 2: Configuration for the initialization and mutation of CPPN networks that generate the initial state for the Lenia system.

Parameter	Value
Initial number of hidden neurons	4
Mutation neuron add probability	0.02
Mutation neuron delete probability	0.02
Initial connections	random connections with probability 0.6
Mutation connection add probability	0.05
Mutation connection delete probability	0.01
Initial activation functions	sigmoid, gauss
Mutation rate of activation functions	0.1
Min synapse weight	-3
Max synapse weight	3
Initial synapse weight	Gaussian Distribution with $\mu = 0, \sigma = 0.4$
Mutation rate of synapse weights	0.05
Mutation replace rate of synapse weights	0.01
Mutation power of synapse weights	1
Mutation enable/disable rate of synapse weights	0.02

2 Learning to represent goals with deep variational autoencoders

Variational Autoencoder (VAE) [10] is a popular approach for generative modelling. It uses a standard Gaussian prior $p(\mathbf{z}) = \mathcal{N}(0, I)$ and a diagonal Gaussian posterior $q(\mathbf{z}|\mathbf{x}, \chi) = \mathcal{N}(\mu, \sigma)$. The VAE framework couples a neural *encoder* $q(\mathbf{z}|\mathbf{x}, \chi)$ that, given a data point x , outputs the mean μ and variance σ of the representative distribution in the latent space with a neural *decoder* $p(\mathbf{x}|\mathbf{z}, \psi)$ that, given a sampled latent representation z from that distribution, tries to reconstruct the original data x . Under these assumptions, training is done by maximizing the computationally tractable evidence lower bound (with $\beta = 1$):

$$\mathcal{L}(\chi, \psi) = \underbrace{\mathbb{E}_{\mathbf{z} \sim q_{\chi}(\mathbf{z}|\mathbf{x})} [\log p_{\psi}(\mathbf{x}|\mathbf{z})]}_a - \beta \times \underbrace{\mathbb{D}_{KL}[q_{\chi}(\mathbf{z}|\mathbf{x}) \| p(\mathbf{z})]}_b \quad (1)$$

The first term (a) represents the expected reconstruction accuracy while the second (b) is the KL divergence of the approximate posterior from the prior.

$$b = \mathbb{D}_{KL}[\mathcal{N}(\mu(\mathbf{x}), \Sigma(\mathbf{x})) \| \mathcal{N}(0, I)] = \sum_{j=1}^d \underbrace{\mathbb{D}_{KL}[\mathcal{N}(\mu(\mathbf{x})_j, \sigma(\mathbf{x})_j) \| \mathcal{N}(0, 1)]}_{b_j} \quad (2)$$

Many current state-of-the-art approaches [6, 8, 3, 12] build on the VAE framework and augment the VAE objective to enhance interpretability and disentanglement of the latent variables. In this paper, we couple the VAE architecture with three different objectives: the classical VAE objective (1), the β -VAE objective [6] and an augmented β -VAE objective (3).

The β -VAE objective re-weights the b term by a factor $\beta > 1$, aiming to enhance the disentangling properties of the learned latent factors. We are interested in such properties as it has been shown that it can benefit exploration [13]. However, heavily penalizing b can result in the network learning to “sacrifice” one or more of the learned latent variables in order nullify their contribution b_j to this term (2). Those dimensions become completely uninformative and useless for further exploration in the learned latent space.

$$b_{aug} = \sum_{j=1}^d b_j + \underbrace{\text{Var}([b_1, \dots, b_d])}_c \quad (3)$$

To prevent this phenomenon to happen, we augmented b with a new term c (3) to force the network to decrease *together* the individual contributions of the different latent variables (by minimizing the variance of the individual contributions) and not only to minimize the averaged contribution (sum). Prior work also reports this phenomenon [1, 11, 4, 5, 16, 17] and other modifications of the training objective have been proposed [16, 17].

We tested these three variants in the context of IMGEP with learned goal spaces. The VAE networks were trained on the loss function given in Equation (4) with the hyper-parameters $\{\beta = 1, \gamma = false\}$, $\{\beta = 5, \gamma = false\}$ and $\{\beta = 5, \gamma = true\}$ corresponding to the three variants outlined above. For the last two variants, $\beta = 5$ was chosen manually and was not optimized.

$$L = -a + \beta(b + \mathbf{1}_\gamma \times c) \quad (4)$$

In the main paper paper, the results shown for the IMGEP-PGL and IMGEP-OGL approaches have been obtained with representations trained using the last variant (β -VAE augmented objective). We present this variant for clarity purpose but it does not seem to improve the results in the context of IMGEPs with learned goal spaces. Actually, when comparing the results we could not find significant trends promoting one method over the other.

2.1 Implementation Details

Network architecture

The VAE architecture used in this paper to learn the different goal spaces can be seen in Table 3.

Table 3: Variational autoencoder architecture for the pretrained and online experiments.

Encoder	Decoder
Input image x : $256 \times 256 \times 1$	Input latent vector z : 8×1
Conv layer: 32 kernels 4×4 , stride 2, 1-padding + ReLU	FC layers : $256 + \text{ReLU}$, $16 \times 16 \times 32 + \text{ReLU}$
Conv layer: 32 kernels 4×4 , stride 2, 1-padding + ReLU	TransposeConv layer: 32 kernels 4×4 , stride 2, 1-padding + ReLU
Conv layer: 32 kernels 4×4 , stride 2, 1-padding + ReLU	TransposeConv layer: 32 kernels 4×4 , stride 2, 1-padding + ReLU
Conv layer: 32 kernels 4×4 , stride 2, 1-padding + ReLU	TransposeConv layer: 32 kernels 4×4 , stride 2, 1-padding + ReLU
FC layers : $256 + \text{ReLU}$, $256 + \text{ReLU}$, FC: 2×8	TransposeConv layer: 32 kernels 4×4 , stride 2, 1-padding

Training procedure and hyper-parameters

The models are trained for 2000 epochs. The training objectives (three variants) are given in section 2 and binary cross entropy is used as reconstruction loss. We use Adam optimizer [9] ($lr = 1e-3$, $\beta_1 = 0.9$, $\beta_2 = 0.999$, $\epsilon = 1e-8$, weight decay= $1e-5$) with a batch size of 64.

For image data augmentation we use random x and y -translations (up to half the image size and with probability 0.3), rotation (up to 40 degrees and with probability 0.3), horizontal and vertical flipping (with probability 0.2). The translations and rotations are preceded by spherical padding to preserve Lenia spherical continuity.

In the IMGEP-PGL experiments, we keep the best model (with the higher accuracy on the validation set) that was obtained during the pretraining phase for the later exploration .

105 For the IMGEP-OGL models, the period of training is set to 100, meaning that every 100 runs we
106 train the network for 40 epochs (2000 epochs in total in maximum, less if there is not enough data
107 after the first T runs to start the training). For the importance sampling, we use a weighted random
108 sampler that samples that samples newly-discovered images from the training data set half of the time
109 (with a probability of $\frac{0.5}{N}$ where N is the number of new images coming from the last 100 runs).

110 Data sets

111 For the IMGEP-PGL experiments, the data set used to train the VAE has 558 images which are
112 distributed into a training (75%), validation (10%) and testing (15%) data sets. The images are Lenia
113 patterns, half of them (279) are the ones manually identified by [2] as *animals* and the other half
114 (279) are randomly initialized CPPN patterns (see Section 4.3 of the main paper).

115 For the IMGEP-OGL experiments, the data sets are constructed incrementally with the non-dead
116 observations o encountered during the goal sampling phase. One image every ten is added to the
117 validation set (10%) and the rest is used in the training set.

118 2.2 Visualization of the learned Goal Spaces

119 This section provides complementary material for the analysis made in Section 5.2 of the main
120 paper. The goal spaces learned in the IMGEP-OGL algorithm (Figure 1 (b)) are compared to the
121 hand-defined goal spaces from the IMGEP-HFS algorithm (Figure 1 (a)). The five-dimensional (a)
122 and eight-dimensional (b) goal spaces are visualized with t-SNE [14] and PCA [7] dimensionality
123 reduction techniques.

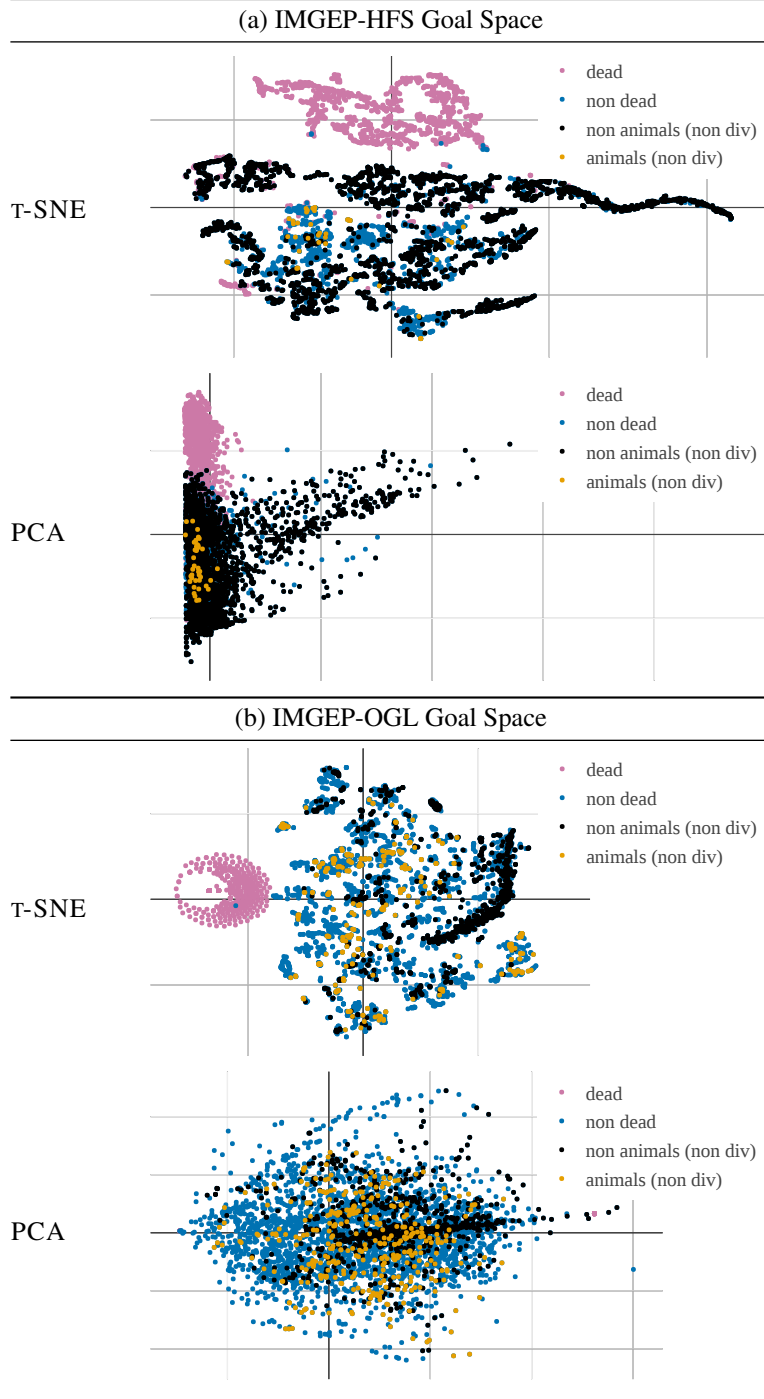


Figure 1: T-SNE and PCA visualization of the (a) hand-defined and (b) learned goal spaces

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