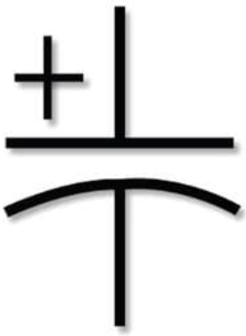
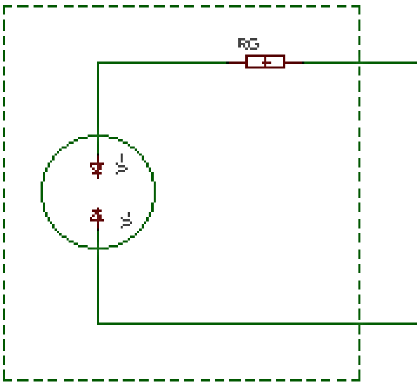
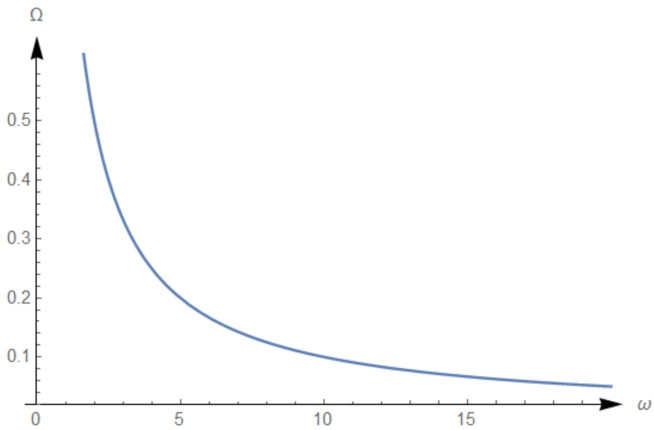
$$V_C(t) = \frac{Q(t)}{C}$$

$$X_C = \frac{1}{j\,\omega\,C}$$







$$V_{Out} = \frac{Z_2}{Z_1 + Z_2} V_{in} \Rightarrow \frac{V_{Out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$$

$$\underbrace{\frac{V_{Out}}{V_{in}}}_{K} = \frac{\frac{1}{j\,\omega\,C}}{R_G + \frac{1}{j\,\omega\,C}} \Rightarrow K = \frac{\frac{1}{j\,\omega\,C}}{\left(R_G + \frac{1}{j\,\omega\,C}\right)} \frac{\left(R_G - \frac{1}{j\,\omega\,C}\right)}{\left(R_G - \frac{1}{j\,\omega\,C}\right)}$$

$$K = \frac{\frac{R_G}{j\,\omega\,C} - \left(\frac{1}{j\,\omega\,C}\right)^2}{R_G^2 - \left(\frac{1}{j\,\omega\,C}\right)^2} \Rightarrow$$

Recordar que por definición: $j^2 = -1$

$$K = \frac{\frac{1}{(\omega\,C)^2} + \frac{R_G}{j\,\omega\,C}}{R_G^2 + \frac{1}{(\omega\,C)^2}} \Rightarrow K = \frac{\frac{1}{(\omega\,C)^2}}{\frac{(\omega\,C)^2\,R_G^2 + 1}{(\omega\,C)^2}} + j\,\frac{\frac{R_G}{\omega\,C}}{\frac{(\omega\,C)^2\,R_G^2 + 1}{(\omega\,C)^2}} \Rightarrow$$

$$K = \frac{(\omega\,C)^2}{(\omega\,C)^2\,[(\omega\,C)^2\,R_G^2 + 1]} + j\,\frac{R_G\,(\omega\,C)^2}{(\omega\,C)\,[(\omega\,C)^2\,R_G^2 + 1]} \Rightarrow$$

$$K = \frac{1}{(\omega\,C)^2\,R_G^2 + 1} + j\,\frac{R_G\,(\omega\,C)}{(\omega\,C)^2\,R_G^2 + 1} \Rightarrow$$

$$|K| = \sqrt{\left(\frac{1}{(\omega\,C)^2\,R_G^2 + 1}\right)^2 + \left(\frac{R_G\,(\omega\,C)}{(\omega\,C)^2\,R_G^2 + 1}\right)^2} \Rightarrow$$

$$|K| = \sqrt{\frac{1}{[(\omega\,C)^2\,R_G^2 + 1]^2} + \frac{[R_G\,(\omega\,C)]^2}{[(\omega\,C)^2\,R_G^2 + 1]^2}} \Rightarrow$$

$$|K| = \sqrt{\frac{1 + [R_G\,(\omega\,C)]^2}{[(\omega\,C)^2\,R_G^2 + 1]^2}} \Rightarrow |K| = \sqrt{\frac{(\omega\,C)^2\,R_G^2 + 1}{[(\omega\,C)^2\,R_G^2 + 1]^2}}$$

$$|K| = \sqrt{\frac{1}{(\omega\ C)^2\ R_G^2 + 1}} \Rightarrow |K| = \frac{\sqrt{1}}{\sqrt{(\omega\ C)^2\ R_G^2 + 1}}$$

$$\therefore\ |K| = \frac{1}{\sqrt{(\omega\ C)^2\ R_G^2 + 1}}$$

Por otro lado:

$$|K| = \left|\frac{V_{Out}}{V_{in}}\right| = \left|\frac{1}{2}\right| = \frac{1}{2}$$

$$\frac{1}{\sqrt{(\omega\ C)^2\ R_G^2 + 1}} = \frac{1}{2} \Rightarrow \frac{1^2}{\left(\sqrt{(\omega\ C)^2\ R_G^2 + 1}\right)^2} = \left(\frac{1}{2}\right)^2$$

$$\frac{1}{\omega^2\ C^2\ R_G^2 + 1} = \frac{1}{4} \Rightarrow 4 = \omega^2\ C^2\ R_G^2 + 1$$

$$3 = \omega^2\ C^2\ R_G^2$$

$$C^2 = \frac{3}{\omega^2\ R_G^2}$$

$$\Rightarrow\ C = \sqrt{\frac{3}{\omega^2\ R_G^2}} = \frac{\sqrt{3}}{\sqrt{\omega^2\ R_G^2}}$$

$$\therefore\ C = \frac{\sqrt{3}}{\omega\ R_G}$$

Relacionando: $\omega = 2\ \pi\ f$

$$\boxed{C\ (f) = \frac{\sqrt{3}}{2\ R_G\ \pi\ f}}$$