

MATH 301: INTRODUCTION TO PROOFS
HOMEWORK 6

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Problem 1.

- (i) Show that if $f: X \rightarrow X$ is idempotent, then $\text{Fix}(f) \cong \text{Im}(f)$.
- (ii) For an idempotent function $f: X \rightarrow X$, show that

$$X / \sim_f \cong \text{Fix}(f) \cong \text{Im}(f)$$

- (iii) Using the previous parts, construct an idempotent function $f: \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N} \times \mathbb{N}$ such that $\text{Fix}(f)$ is in bijection with the set of integers. Describe the elements of $\text{Fix}(f)$.

Problem 2. Suppose $r: A \rightarrow B$ is a retraction. Show that

$$B \cong A / \sim_r .$$

Problem 3. In the previous lecture, we defined the set \mathbb{Q} of rational numbers to be the quotient set by the equivalence relation

$$(u, a) \approx (v, b) =_{\text{def}} (u(b+1) = v(a+1))$$

on the set $\mathbb{Z} \times \mathbb{N}$. Define the relation

$$(u, a) \approx' (v, b) \Leftrightarrow ub = va$$

on the set $\mathbb{Z} \times \mathbb{Z} \setminus \{0\}$.

- (i) Show that this relation is an equivalence relation.
- (ii) Write \mathbb{Q}' for the quotient set obtained by the relation \approx' . Show that \mathbb{Q}' is in bijection with \mathbb{Q} , and conclude that it either way we define the same set of rational numbers.