MATH 301: INTRODUCTION TO PROOFS
HOMEWORK 3

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Problems.
 §2.E (Chapter 2 Exercises) ‖ 2.6, 2.14, 2.15, 2.16

Problem 1. Let X be $\mathbb{Z}$ or $\mathbb{Q}$ and define a sentence p by
$$\forall x \in X, \exists y \in X, (x < y \wedge [\forall z \in X, \neg(x < z \wedge z < y)]).$$
Describe what p asserts about the set X. Find the maximally negated logical formula equivalent to $\neg p$. Prove that p is true when $X = \mathbb{Z}$ and false when $X = \mathbb{Q}$.

Problem 2. Let X, Y be classical sets. Prove that $X \setminus (X \setminus Y) = X \cap Y$. Can we drop the condition of being *classical* about either X or Y and have the same conclusion? If so, which one?

Problem 3. Prove the De Morgan's laws for classical sets A, X, Y and a family of classical sets $\{X_i | i \in I\}$.

 (1) $A \setminus (X \cup Y) = (A \setminus X) \cap (A \setminus Y)$
 (2) $A \setminus (X \cap Y) = (A \setminus X) \cup (A \setminus Y)$
 (3) $A \setminus \bigcup_{i \in I} X_i = \bigcap_{i \in I} (A \setminus X_i)$
 (4) $A \setminus \bigcap_{i \in I} X_i = \bigcup_{i \in I} (A \setminus X_i)$