

Ints Muraans 18.03.2019

possible alternatives to the event E . Namely, the quantity of information in the message m is given by the simple formula

$$I(n) = \log_2 n \quad (3.2.2)$$

When all possible outcomes of this experiment or all possible alternatives to the event E have the same probability, then the probability p of E (of D) is equal to $1/n$ (cf. Appendix E). Thus, as $\log_2 n = -\log_2 p$, it is possible to rewrite the formula (3.2.2) in the following form:

$$I(n) = -p \cdot \log_2 p \quad (3.2.3)$$

However, equal probabilities of events are a relatively rare event and it is more realistic to assume that the probabilities of experiment outcomes or of alternatives to the event, in general, are different. Thus, the answer of Shannon (1948) about information in the message m is subtler and more realistic because it takes into account the individual probability of each outcome or of each event alternative. Thus, it is assumed that initially n events $E_1, E_2, E_3, \dots, E_n$, one of which is E (or n outcomes $D_1, D_2, D_3, \dots, D_n$ of the experiment H , one of which is D), were possible and had probabilities $p_1, p_2, p_3, \dots, p_n$, correspondingly. Then the *entropy* of the message m is equal to

$$H(m) = H(p_1, p_2, \dots, p_n) = -\sum_{i=1}^n p_i \cdot \log_2 p_i \quad (3.2.4)$$

Probabilities used in the formula (3.2.4) are obtained from experiments and observation, using relative frequencies. For instance, let us consider such events as encountering a given letter, say "a", in an English text. In this case, it is possible to analyze many English texts to find relative frequency of occurrence for this letter "a" in different texts, that is, to calculate the number m of occurrences of "a" in a text T and to divide it by the number n of letters in this text T . The ratio m/n is the relative frequency of occurrence of "a" in the text T . For instance, the relative frequency of occurrence for this letter "e" in the sentence "Information is related to structures as energy is related to matter" is equal to $8/57$ because the sentence has 57 letters and "e" is used 8 times. Usually the average of such relative frequencies is used as the probability of occurrences and thus, of receiving this letter. If we know probabilities p_1, p_2, \dots, p_{26} for all letters a, b, c, \dots, z from the English alphabet, we can easily find information in one English letter calculating the

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\hspace{3 cm}\textit{Statistical Information Theory} \hfill 269\\

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$$\begin{aligned} & \text{\small} \\ & \text{\hspace{1.5cm}} H(m) = H(p_1, p_2, \dots, p_n) = - \sum_{i=1}^n p_i \cdot \log_2 p_i \quad \text{\hspace{1cm}} (3.2.4) \end{aligned}$$

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