METHODS WITH PRECONDITIONING WITH WEIGHT DECAY REGULARIZATION

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ABSTRACT

This paper investigates the convergence behavior of optimization methods that utilize weight decay regularization, specifically focusing on popular variants such as AdamW. We explore different alternatives to these method, with the goal of investigating their convergence speed and accuracy. Also we conduct experiments on benchmark datasets and models in order to compare them on practice. Overall, our study provides insights into the design of regularization techniques methods with preconditioning.

Keywords Adam · AdamW · OASIS · Regularization · Weight Decay · Optimization

1 Introduction

In machine learning we consider unconstrained optimization problem

$$\min_{w \in \mathbb{R}^d} f(w). \tag{1}$$

Problems of the form (1) cover a plethora of applications, including empirical risk minimization, deep learning [LeCun et al., 2015], and supervised learning [Cunningham et al., 2008] tasks such as regularized least squares [Rifkin and Lippert, 2007] or logistic regression [Shalev-Shwartz and Ben-David, 2014].

The classic base method for solving the optimization is gradient descent, but this minimization problem can be difficult to solve, particularly when the number of training samples n, or problem dimension d, is large. In modern machine learning especially large problems represent the greatest interest. For such cases stochastic gradient descent [Bottou, 2010] became popular solution. Despite its simplicity, it proved itself to be an efficient and effective optimization method. For a long time first-ordered methods were most popular approach of solving optimization problems.

Other way of solving the problem are methods with adaptive gradient [Wilson et al., 2017]. These methods posses several superiority over first-ordered methods. Firstly, they have bigger potential of distributed solving, because first ordered methods spend majority of time on "communication". Secondly, they are less sensitive to the choice of hyperparameters up to the point that hyperparameters can be set equal to one. Lastly, this methods often simply show faster convergence on modern large optimization problems, especially this methods became applicable in neural networks solving. Nowadays it is known that preconditioning methods often outperform other methods on modern large optimization problems [Zhang et al., 2018, Yao et al., 2021, Kingma and Ba, 2014, Goldberg et al., 2011].

Preconditioning methods refer to techniques that involve scaling the gradient of a problem by a specific matrix D_t, which enables the gradient to take into account the geometry of the problem. In the classical case $D_t = (\nabla^2 f)^{-1}$, which corresponds newton's method, however hessian is difficult to calculate and even more difficult to reverse, because of that some heuristics are used to replace the reversed hessian [Dennis and Moré, 1977]. In OASIS [Goldberg et al., 2011] or AdaHessian [Yao et al., 2021] hessian is assumed to have diagonal dominance. In Adam [Kingma and Ba, 2014] gradient is simply normalized, etc. This heuristics were proved to be effective and efficient. General scheme of methods with preconditioning can be framed in the following algorithm

Algorithm 1 General scheme for preconditions methods

Require: η, f
  while w not converged **do**
    $t = t + 1$
    $g_t \leftarrow \nabla f(w_{t-1})$
    $D_t \leftarrow$ pseudo-hessian, precondtioning, scaling
    $w_t = w_{t-1} - \eta \cdot g_t D_t^{-1}$
  end while

Regularization is a powerful technique in machine learning that aims to prevent overfitting by adding additional constraints to the model. It has been widely applied to various machine learning problems, including image classification, speech recognition, and natural language processing, and has shown its effectiveness in improving the generalization capability of neural networks [Poggio et al., 1987].

In methods with preconditioning appears to be several ways to include regularization. We can include regularizer r in g_t calculation so it will be taken into consideration while calculating D_t. This method is equal to considering optimization problem

$$\min_{w \in \mathbb{R}^d} f(w) + r(w). \tag{2}$$

Or we can include regularizer only on last step, decreasing norm of w [Loshchilov and Hutter, 2017]. This way of regularization is called weight decay and surprisingly turns out to be more efficient in practical problems. There are few other ways of considering regularizer which will be discussed further in the paper.

In general, our paper provides insight into comparison of different consideration ways of regularization is methods with preconditioning. Here, we provide a brief summary of our main contributions:

- **Proof of preconditioned methods' with weight decay convergence**. We derive convergence guarantees for preconditioned methods considering assumptions of smoothness, strongly convex and PL-condition.
- **Research of the loss function** Comparison of accuracy and loss function for AdamW and AdamL2. As a result we saw that AdamW asymptotically converges to a non-zero value
- **Competitive Numerical Results** We investigate the empirical performance of Adam's variation including new one on a variety of standard machine learning tasks, including logistic regression.

2 Problem statement

We want to investigate the convergence speed of the AdamW method and the newly proposed MyAdamW method in machine learning problems, and we also plan to prove the convergence of these methods and investigate the obtained solution.

We consider the unconstrained optimization problem

$$\min_{w \in \mathbb{R}^d} f(w)$$

But we can add regulirazation $r(w)$ – regularization, and solve the unconstrained optimization problem.

$$\min_{w \in \mathbb{R}^d} F(w) = f(w) + r(w)$$

In the convergence algorithm that we study, we will investigate algorithms of the following two kinds. In the first one, the regularization function is taken out separately in the recalculation of model weights, and in the second one, the function is dominated by the inverse "hessian".

It's general scheme of all algorithms with preconditioning.

1

Algorithm 2 General scheme for preconditions methods

Require: η, ϵ, f, r
  while w not converged **do**
    $t = t + 1$
    $g_t = \nabla f_t(w_{t-1})$ что за операция?
    $D_t = \text{diag}(\sqrt{g_t \odot g_t} + \varepsilon)$ AdamW
    $D_t = \mathbb{E}[z^T \nabla^2 f(w_{t-1}) z]$ cyan - OASIS, where z in Rademacher distribution
    $w_t = w_{t-1} - \eta \cdot g_t D_t^{-1}$
  end while

The OASIS algorithm differs from Adam in that it updates the learning rate and calculates the pseudo-Hessian matrix at each step using the expectation and a random variable distributed according to the Rademacher distribution.

This is OASIS, the main difference from Adam is how the Hessian is considered, but when proving convergence, this will not play any role.

Algorithm 3 OASIS

Require: $w_0, \eta_0, D_0, \theta_0 = +\infty$
  $w_1 = w_0 - \eta \hat{D}_0^{-1} \nabla F(w_0)$
  for $k = 1, 2, \ldots$ **do**
    $D_k = \beta D_{k-1} + (1 - \beta_2) \cdot diag\left(z_k \odot \nabla^2 F(w_k) z_k\right)$
    $(\hat{D}_k)_{ii} = max\{|D_k|_{i,i}; \alpha\}, \forall i = \overline{1, d}$
    $\eta_k = min\{\sqrt{1 + \theta_{k-1}} \cdot \eta_{k-1}; \frac{||w_k - w_{k-1}||_{\hat{D_k}}}{2||\nabla F(w_k) - \nabla F(w_{k-1})||^*_{\hat{D_k}}}\}$
    $\theta_k = \frac{\eta_k}{\eta_{k-1}}$
  end for

Now we will show the main differences between the methods of AdamL2, AdamW and MyAdamW – the algorithm that we propose in this article

Algorithm 4 Adam(λ)

Require: $\eta, \beta_1, \beta_2, \epsilon, f, r$
  while θ not converged **do**
    $t = t + 1$
    $g_t = \nabla f(w_{t-1}) + \nabla r(w_{t-1})$ blue – AdamL2
    $m_t = \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t$
    $v_t = \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2$
    $\hat{m}_t = \frac{m_t}{1 - \beta_1^t} + \nabla r(w_{t-1})$ yellow – MyAdamW
    $\hat{v}_t = \frac{v_t}{1 - \beta_2^t}$
    $w_t = w_{t-1} - \eta \cdot \frac{\hat{m}_t}{\sqrt{\hat{v}_t} + \epsilon} - \eta \nabla r(w_{t-1})$ red - AdamW
  end while

может быть зелёный;
+ жёлтый плохо читаем

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3 Our evaluations

We proof convergence for preconditioned methods with weight decay.

Throughout this work we assume that each f : $\mathbb{R}^d \to \mathbb{R}$ is twice differentiable and also L-smooth. This is formalized in the following assumption.

Assumption 1 (Convex). *The function f is convex, i.e.* $\forall y, x \in \mathbb{R}^d$

$$f(y) \geq f(x) + \langle \nabla f(x), y - x \rangle$$

не приведён отдельно ваш алгоритм

3

of f

4

Assumption 2 (L-smoothness). *The gradients of F are L-Lipschitz continuous* $\forall w \in \mathbb{R}^d$*, i.e. there exists a constant* $L > 0$ *such that* $\forall x, y \in \mathbb{R}^d$,

$$f(x) \leq f(y) + \langle \nabla f(y), x - y \rangle + \frac{L}{2}||x - y||^2$$

Assumption 3 (μ - strongly convex). *The function f is* μ*-strongly convex, i.e., there exists a constant* $\mu > 0$ *such that* $\forall x, y \in \mathbb{R}^d$

$$f(x) \geq f(y) + \langle f(y), x - y \rangle + \frac{\mu}{2}||x - y||^2$$

лучше ввести раньше обозначения

Assumption 4 (PL–condition). *If there exists* $\mu > 0$*, such that* $||\nabla f(w)|| \geq 2\mu(f(w) - f^*)$*,* $\forall w \in \mathbb{R}^d$

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Assumption 5 (l-smoothness). *The gradient of r is l-Lipschitz continuous* $\forall w \in \mathbb{R}^d$*, i.e. there exists a constant* $l > 0$ *such that* $\forall x, y \in \mathbb{R}^d$,

$$r(x) \leq r(y) + \langle \nabla r(y), x - y \rangle + \frac{l}{2}||x - y||^2$$

3.1 Proof of convergence

We proof convergence of algorithms with preconditioning 2

Theorem 1. *Suppose the Assumption 1-Convex, 2-L-smoothness, 4-PL–condition and let* $\varepsilon > 0$ *and let the step-size satisfy*

$$\eta < \frac{2\alpha}{L + l \cdot \alpha}$$

что такое α ?

добавь ссылку

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Then, the number of iterations performed by AdamW algorithm, starting from an initial point $w_0 \in \mathbb{R}^d$ *with* $\Delta_0 = \tilde{F}(w_0) - \tilde{F}^*$*, required to obtain and* ε*-approximate solution of the convex problem (link here to problem 1) can be bounded by* что такое $\tilde{F}$?

$$T = \mathcal{O}\left(\frac{2\Delta_0 \Gamma \alpha}{(2\alpha - \tilde{L}\eta)\eta\varepsilon}\right)$$

не понятно, что такое $\tilde{L}$

Theorem 2. *Suppose the Assumption 1-Convex, 2-L-smoothness, 4-PL–condition, 5-l-smoothness and let* $\varepsilon > 0$ *and let the step-size satisfy*

$$\eta \leq \frac{2\alpha}{\tilde{L}}$$

аналогично 6

Then, the number of iterations performed by AdamW algorithm, starting from an initial point $w_0 \in \mathbb{R}^d$ *with* $\Delta_0 = \tilde{F}(w_0) - \tilde{F}^*$*, required to obtain and* ε*-approximate solution of the convex problem (link here to problem 1) can be bounded by*

$$T = \mathcal{O}\left(\frac{\ln \frac{\Delta_0}{\epsilon}}{2\mu\eta^2(\frac{1}{\eta} - \frac{\tilde{L}}{2\alpha})}\right)$$

может быть ε

comparison: с чем сравниваете ?

$$||\nabla f + D_t \nabla r|| = ||\nabla f + D_t \nabla r|| < \epsilon$$

$$||\nabla f + \nabla r|| = ||\nabla f + \nabla r \pm D_t \nabla r|| < \epsilon + ||D_t \nabla r - \nabla r|| \leq \epsilon + ||D_t - I||||\nabla r||$$

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4 Numerical experiments

надо добавить цели эксперимента, не понятно, что вы делаете

Experimental conditions

- Model: ResNet18 (100 epoch, batch size 128),

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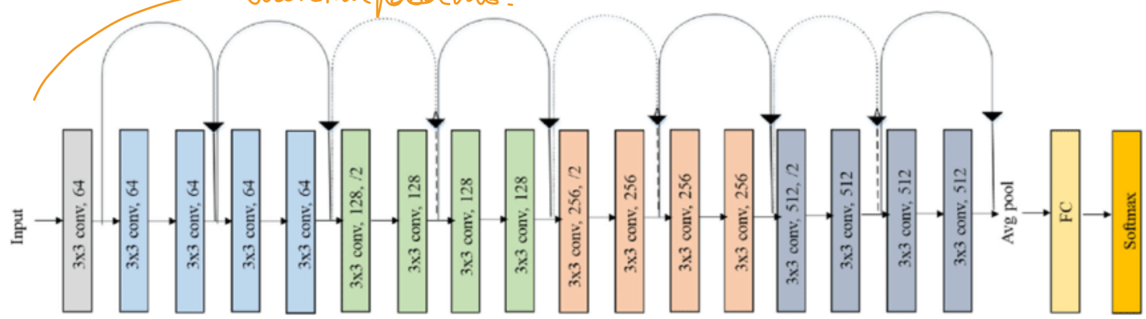
что это за картинка, наверное, стоит её проко-ментировать.



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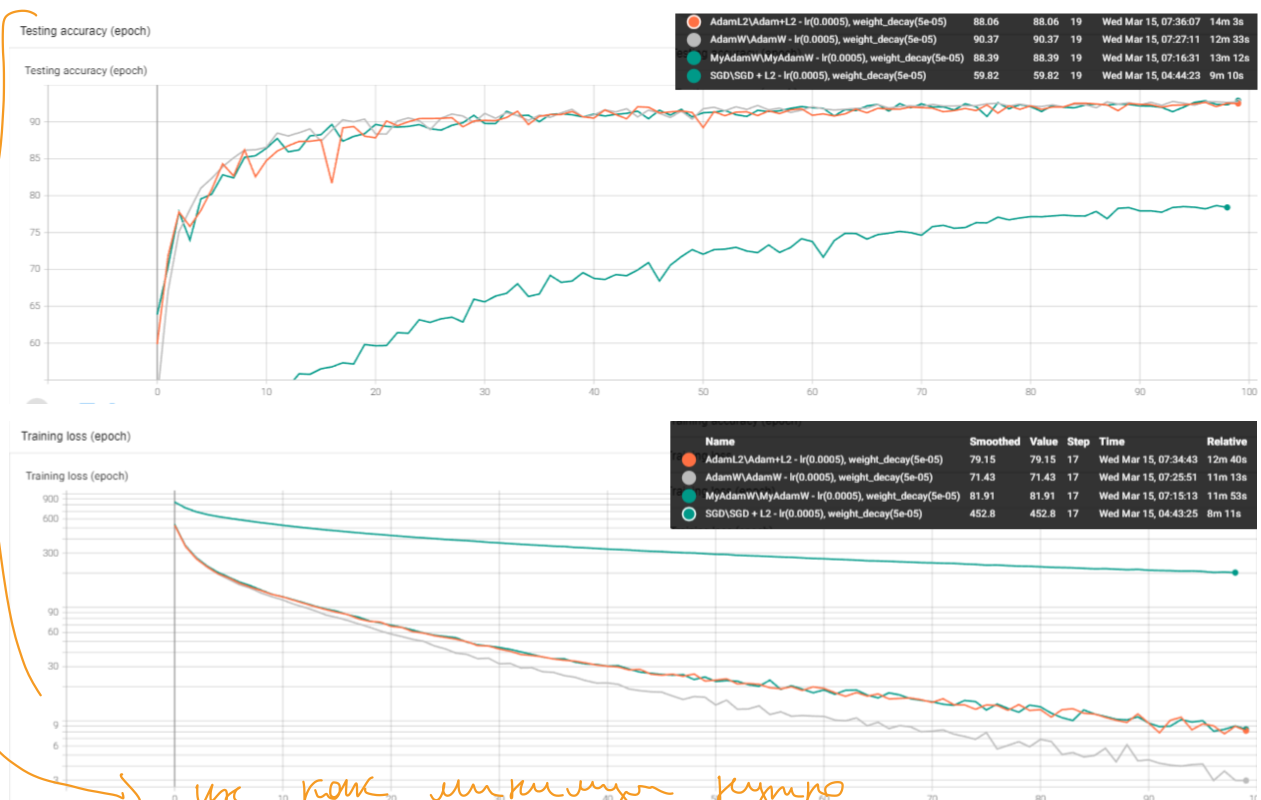
- CosineAnnealingLR scheduler: $\eta_t = \eta_{min} + \frac{1}{2}(\eta_{min} + \eta_{max})\left(1 + cos\left(\frac{T_{cur}}{T_{max}\pi}\right)\right)$
- Grid of learning rates = [0.01, 0.005, 0.0005], weight decays = [0.005, 0.0005, 0.00005]

надо добавить описание параметров

- Data set: CIFAR10. The CIFAR-10 dataset consists of 60000 32x32 colour images in 10 classes, with 6000 images per class. There are 50000 training images and 10000 test images.

графики плохо читаемы



их как минимум нужно прокомментировать.

- Model: Logistic regression
- Optimizers: AdamW, AdamL2, OASIS, GD, MyAdamW. } ссылки на методы
- Grid of learning rates = [0.1, 0.05, 0.01], weight decays = [0.1, 0.05, 0.005]
- Data set: mushrooms. описание где?!?

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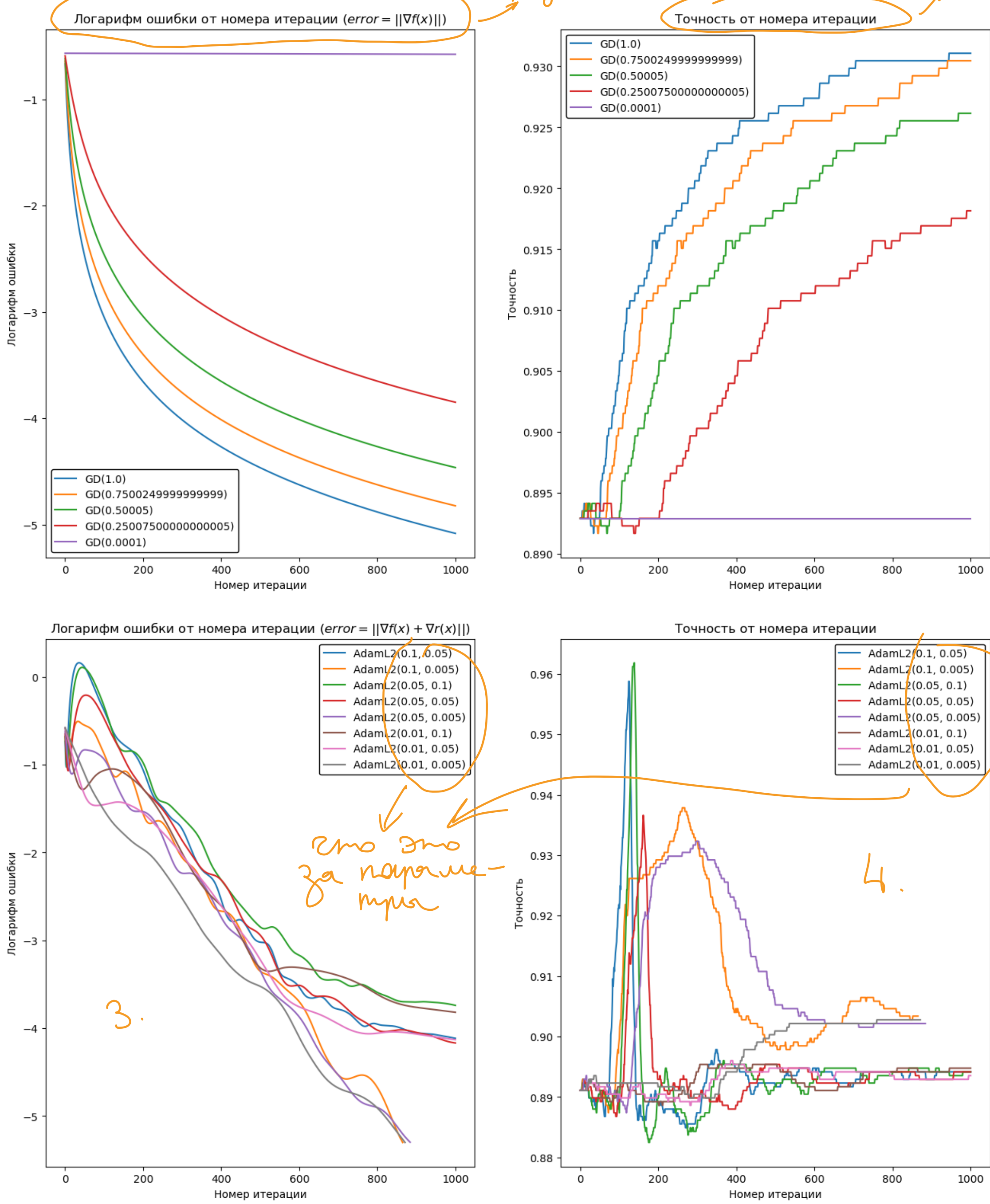
эм... вроде бы статья на английском



что это за параметры

3.

4.

1) Подписи на русском

2) Графики 3. и 4. тяжело читаются, особенно четвертый

3) Нет комментариев к графикам.

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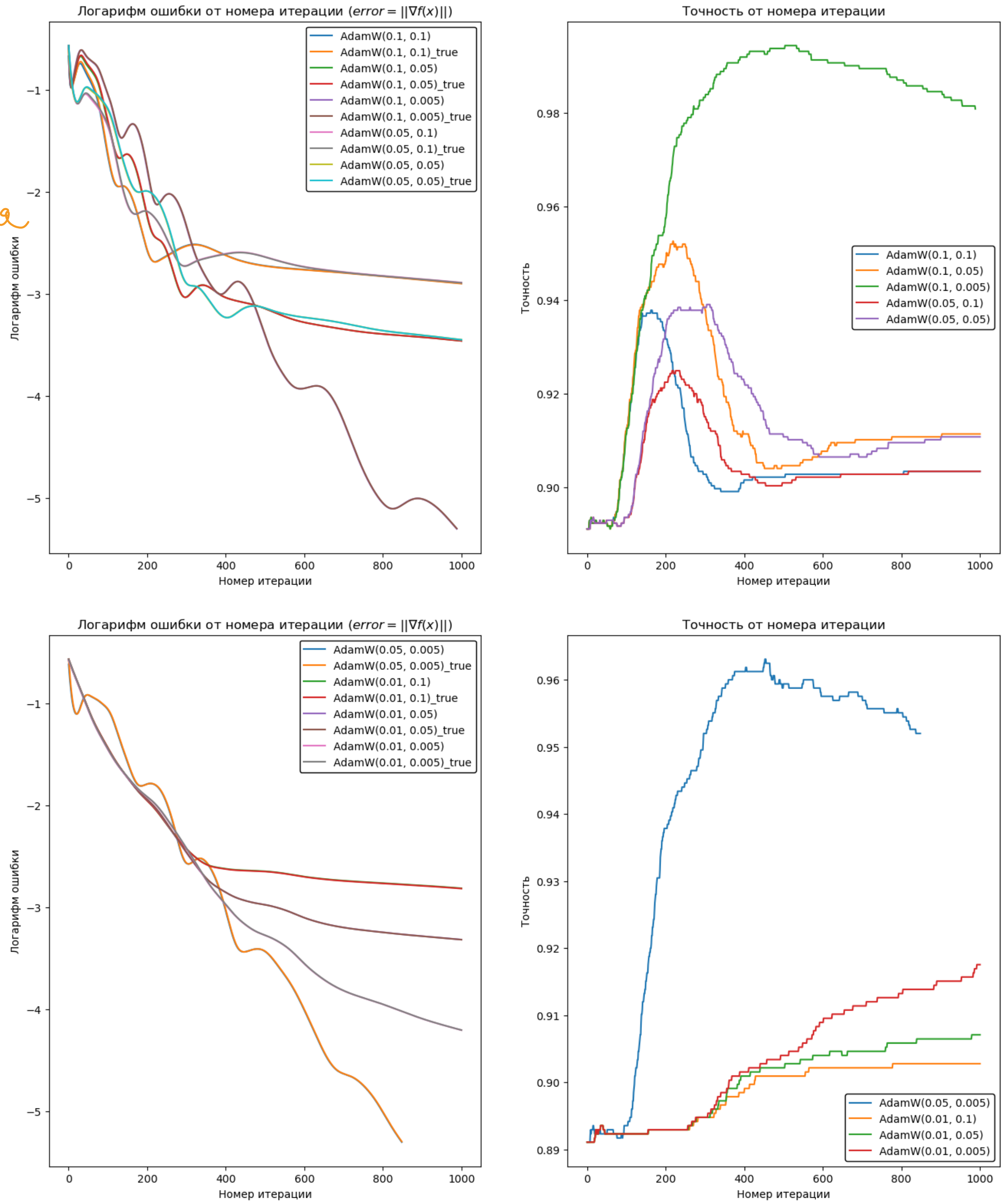
замечания аналогичны пункту 11



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замечание касаемо всей работы: необходимо выделить жирным вектора и матрицы

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5 Appendix

Theorem 3. *Suppose the Assumption 1-Convex, 2-L-smoothness, 4-PL–condition and let $\varepsilon > 0$ and let the step-size satisfy*

$$\eta < \frac{2\alpha}{L + l \cdot \alpha}$$

Then, the number of iterations performed by AdamW algorithm, starting from an initial point $w_0 \in \mathbb{R}^d$ with $\Delta_0 = \tilde{F}(w_0) - \tilde{F}^*$, required to obtain and ε-approximate solution of the convex problem (link here to problem 1) can be bounded by

$$T = \mathcal{O}\left(\frac{2\Delta_0 \Gamma \alpha}{(2\alpha - \tilde{L}\eta)\eta\varepsilon}\right)$$

Proof. Let's write ~~first~~ second assumption for step t and $t+1$:

$$f(w_{t+1}) \leq f(w_t) + \langle \nabla f(w_t), w_{t+1} - w_t \rangle + \frac{L}{2} ||w_{t+1} - w_t||^2$$

Okay, by definition for our algorithm we have:

это не окей.. нужно писать формально

$$w_{t+1} - w_t = -\eta D_t^{-1} \nabla f(w_t) - \eta \nabla r(w_t)$$

and

что такое D_t

$$\nabla f(w_t) = \frac{1}{\eta} D^t (w_t - w_{t+1}) - D^t \nabla r(w_t)$$

Okay, now let's replace $\nabla f(w_t)$ and $I \leq \frac{D_t}{\alpha}$

$$f(w_{t+1}) \leq f(w_t) + \langle \frac{1}{\eta} D_t (w_t - w_{t+1}) - D_t \nabla r(w_t), w_{t+1} - w_t \rangle + \frac{L}{2\alpha} ||w_{t+1} - w_t||^2_{D_t}$$

$$f(w_{t+1}) \leq f(w_t) + \left(\frac{L}{2\alpha} - \frac{1}{\eta}\right) ||w_{t+1} - w_t||^2_{D_t} - \langle D_t \nabla r(w_t), w_{t+1} - w_t \rangle$$

Lets define new variable $\tilde{r}: \nabla \tilde{r} = D_t \nabla r(w_t)$. Then rewrite step using the variable and 5-th assumption.

$$\tilde{r}(w_{t+1}) \leq \tilde{r}(w_t) + \langle \tilde{r}(w_t), w_{t+1} - w_t \rangle + \frac{l}{2}(w_{t+1} - w_t)^T D_t (w_{t+1} - w_t)$$

$$f(w_{t+1}) \leq f(w_t) + \left(\frac{L}{2\alpha} - \frac{1}{\eta}\right) ||w_{t+1} - w_t||^2_{D_t} + \tilde{r}(w_t) - \tilde{r}(w_{t+1}) + \frac{l}{2}||w_{t+1} - w_t||^2_{D_t}$$

$\tilde{F}(w) = f(w) + \tilde{r}(w)$, $F(w) = f(w) + r(w)$, $(\tilde{L} = L + l\alpha)$, we get:

$$\tilde{F}(w_{t+1}) \leq \tilde{F}(w_t) + \left(\frac{\tilde{L}}{2\alpha} - \frac{1}{\eta}\right) ||w_{t+1} - w_t||^2_{D_t}$$

$$\left(\frac{1}{\eta} - \frac{\tilde{L}}{2\alpha}\right) ||w_{t+1} - w_t||^2_{D_t} \leq \tilde{F}(w_t) - \tilde{F}(w_{t+1})$$

$$\frac{\eta^2(T+1)}{\Gamma}\left(\frac{1}{\eta} - \frac{\tilde{L}}{2\alpha}\right) \cdot \min_{k=0,T} ||\nabla f(w_t) + \nabla \tilde{r}(w_t)||^2 \leq \frac{\eta^2}{\Gamma}\left(\frac{1}{\eta} - \frac{\tilde{L}}{2\alpha}\right) \cdot \sum_{t=0}^{T} ||\nabla f(w_t) + \nabla \tilde{r}(w_t)||^2 \leq \tilde{F}(w_0) - \tilde{F}(w_*)$$

$$\min_{t=0,T} ||\nabla f(w_t) + \nabla \tilde{r}(w_t)||^2 \leq \frac{(\tilde{F}(w_0) - \tilde{F}(w_*))\Gamma}{(\frac{1}{\eta} - \frac{\tilde{L}}{2\alpha})\eta^2(T+1)} = \varepsilon$$

$$T + 1 \geq \frac{\Delta_0 \Gamma}{(\frac{1}{\eta} - \frac{\tilde{L}}{2\alpha})\eta^2 \varepsilon}$$

Then:

$$T = \mathcal{O}\left(\frac{2\Delta_0 \Gamma \alpha}{(2\alpha - \tilde{L}\eta)\eta\varepsilon}\right)$$

□

Theorem 4. *Suppose the Assumption 1-Convex, 2-L-smoothness, 4-PL–condition, 5-l-smoothness and let $\varepsilon > 0$ and let the step-size satisfy*

$$\eta \leq \frac{2\alpha}{\tilde{L}}$$

Then, the number of iterations performed by AdamW algorithm, starting from an initial point $w_0 \in \mathbb{R}^d$ with $\Delta_0 = \tilde{F}(w_0) - \tilde{F}^*$, required to obtain and ε-approximate solution of the convex problem (link here to problem 1) can be bounded by

$$T = \mathcal{O}\left(\frac{\ln \frac{\Delta_0}{\epsilon}}{2\mu\eta^2(\frac{1}{\eta} - \frac{\tilde{L}}{2\alpha})}\right)$$

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Proof. Assume

$$\nabla \tilde{F} = \nabla f + \nabla \tilde{r} \tag{3}$$

это надо бы
в основную часть

$$L + ||D_t|| l = \tilde{L} \tag{4}$$

$$w_{t+1} - w_t = -\eta D_t^{-1} \nabla r(w_t) - \eta \nabla r(w_t) = -\eta D_t^{-1}(\nabla f + \nabla \tilde{r})(w_t) = -\eta D_t^{-1} \nabla \tilde{F}(w_t)$$

Then we write $\tilde{L}$-smoothness for $\tilde{F}$

$$\tilde{F}(w_{t+1}) - \tilde{F}(w_t) \leq \langle \nabla \tilde{F}(w_t), w_{t+1} - w_t \rangle + \frac{\tilde{L}}{2}||w_{t+1} - w_t||^2$$

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$$\tilde{F}(w_{t+1}) - \tilde{F}(w_t) \leq -\langle \frac{1}{\eta} D_t(w_{t+1} - w_t), w_{t+1} - w_t \rangle + \frac{\tilde{L}}{2}||w_{t+1} - w_t||^2 = (\frac{\tilde{L}}{2\alpha} - \frac{1}{\eta})||w_{t+1} - w_t||^2_{D_t}$$

что за норма ?

$$= (\frac{\tilde{L}}{2\alpha} - \frac{1}{\eta})||w_{t+1} - w_t||^2_{D_t} = (\frac{\tilde{L}}{2\alpha} - \frac{1}{\eta})|| - \eta D_t^{-1} \nabla \tilde{F}(w_t)||^2_{D_t} \leq (\frac{\tilde{L}}{2\alpha} - \frac{1}{\eta})\eta^2 ||\nabla \tilde{F}(w_t)||^2_{D_t^{-1}}$$

Then we use PL-condition for the function $\tilde{F}$:

$$||\nabla \tilde{F}(w_t)||^2_{D_t^{-1}} \geq 2\mu(\tilde{F}(w_t) - \tilde{F}^*)$$

$$\tilde{F}(w_t) - F^* \geq \tilde{F}(w_{t+1}) - \tilde{F}^* + (\frac{1}{\eta} - \frac{\tilde{L}}{2\alpha})\eta^2 2\mu(\tilde{F}(w_t) - \tilde{F}^*) = \left(1 + 2\mu\eta^2(\frac{1}{\eta} - \frac{\tilde{L}}{2\alpha})\right)(\tilde{F}(w_{t+1}) - \tilde{F}^*)$$

$$\epsilon \geq \Delta_0 \left(1 + 2\mu\eta^2(\frac{1}{\eta} - \frac{\tilde{L}}{2\alpha})\right)^{-T} \geq (\tilde{F}(w_T) - \tilde{F}^*)$$

???

$$T = \frac{\ln \frac{\Delta_0}{\epsilon}}{\ln(1 + 2\mu\eta^2(\frac{1}{\eta} - \frac{\tilde{L}}{2\alpha}))} \approx \frac{\ln \frac{\Delta_0}{\epsilon}}{2\mu\eta^2(\frac{1}{\eta} - \frac{\tilde{L}}{2\alpha})}$$

Then:

$$T = \mathcal{O}\left(\frac{\ln \frac{\Delta_0}{\epsilon}}{2\mu\eta^2(\frac{1}{\eta} - \frac{\tilde{L}}{2\alpha})}\right)$$

□

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6 Notes

Почему это тут? Нет ссылок на эти заметки в основной части и в теоремах.

But we need to remember that: $D_t = diag(\nabla f(w_t))$

$||\nabla f(w_t)+\nabla\tilde{r}(w_t)||^2 = ||\nabla f(w_t)+D_t\nabla r(w_t)||^2 = ||\nabla f(w_t)\cdot(I+\lambda w_t\nabla f(w_t))||^2 \leq ||\nabla f(w_t)||^2\cdot||I+\lambda w_t\nabla f(w_t)||^2$

Some additional information: $2\langle\nabla f(w_t)^2, \lambda w_t\rangle = \langle\nabla f(w_t)^2, \lambda\nabla f(w_t)w_t\rangle$

это
это ?

$$||\nabla f(w_t)+\nabla\tilde{r}(w_t)||^2 = ||\nabla f(w_t)||^2 + 2\langle\nabla f(w_t)^2, \lambda w_t\rangle + ||\lambda\nabla f(w_t)w_t||^2$$

? ?

$$... = \langle\nabla f(w_t), \nabla f(w_t)\rangle + 2\langle\nabla f(w_t), \lambda\nabla f(w_t)w_t\rangle + \lambda^2\langle\nabla f(w_t)w_t, \nabla f(w_t)w_t\rangle = \langle\nabla f(w_t)^2 + 2\lambda\nabla f(w_t)w_t + ...\rangle$$

If life was a dream, then it will right:

$$\lambda\langle\nabla f(w_t)^2(2+\lambda w_t), w_t\rangle \geq 0$$

Assumption Function activation (B). *ut it's right for Neural Networks with function activation: sigmoid and tanh.*

From beginning:

$$||w_{t+1}-w_t||^2_{D_t} = ||\eta D_t^{-1}\nabla f(w_t)+\eta\nabla r(w_t)||^2_{D_t} = \eta^2||\nabla f(w_t)+\nabla\tilde{r}(w_t)||^2_{D_t^{-1}} \geq \frac{\eta^2}{\Gamma}||\nabla f(w_t)+\nabla\tilde{r}(w_t)||^2$$

To collapse this inequality, you need to choose a step so that the coefficient at the norm of the difference is negative, but we get nothing.

момент we

$$\tilde{F}(w_{t+1}) \leq \tilde{F}(w_t)$$

Now consider when the solution to the new problem will be the solution to the original problem.

$$||\nabla f(\tilde{w}_*)+\nabla\tilde{r}(\tilde{w}_*)|| \leq ||\nabla f(\tilde{w}_*)|| + ||\nabla\tilde{r}(\tilde{w}_*)|| \leq ||\nabla f(\tilde{w}_*)|| + \Gamma||\nabla r(w_*)|| \leq \varepsilon$$

If $\Gamma > 1$, then we get:

$$||\nabla f(\tilde{w}_*)+\nabla r(\tilde{w}_*)|| \leq ||\nabla f(\tilde{w}_*)|| + ||\nabla r(w_*)|| \leq ||\nabla f(\tilde{w}_*)|| + \Gamma||\nabla r(w_*)|| \leq \varepsilon$$

Our next proposal, it's changing in AdamW algorithm: consider, that $r(w) = \frac{\lambda}{2}||w||^2$

Algorithm 5 MyAdamW(λ)

Наверное стоит перенести в начало

Require: $\alpha, \beta_1, \beta_2, \epsilon, f$
  $m_0 = 0$ – 1-st moment vector
  $v_0 = 0$ – 2-nd moment vector
  $t = 0$ – timestep
  while θ not converged **do**
    $t = t + 1$
    $g_t = \nabla_\theta f_t(\theta_{t-1})$
    $m_t = \beta_1 \cdot m_{t-1} + (1-\beta_1)\cdot g_t$
    $v_t = \beta_2 \cdot v_{t-1} + (1-\beta_2)\cdot g_t^2$
    $v_{t,i} = \max(\lambda, v_{t,i})$, for each i
    $\hat{m}_t = \frac{m_t}{1-\beta_1^t}$
    $\hat{v}_t = \frac{v_t}{1-\beta_2^t}$
    $\theta_t = \theta_{t-1} - \alpha\cdot\frac{\hat{m}_t}{\sqrt{v_t}+\epsilon} - \alpha\theta_{t-1}$
  end while

ere I will write out the full standard algorithms in machine learning that are used:

Algorithm 6 Adam

Require: $\alpha, \beta_1, \beta_2, \epsilon, f$
$m_0 = 0$ – 1-st moment vector
$v_0 = 0$ – 2-nd moment vector
$t = 0$ – timestep
while θ not converged **do**
$t = t + 1$
$g_t = \nabla_\theta f_t(\theta_{t-1})$
$m_t = \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t$
$v_t = \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2$
$\hat{m}_t = \frac{m_t}{1-\beta_1^t}$
$\hat{v}_t = \frac{v_t}{1-\beta_2^t}$
$\theta_t = \theta_{t-1} - \alpha \cdot \frac{\hat{m}_t}{\sqrt{v_t}+\epsilon}$
end while

This is a modified version of the Adam algorithm, where the weight decay is taken out from under the Hessian

Algorithm 7 AdamW

Require: $\alpha, \beta_1, \beta_2, \epsilon, f$
$m_0 = 0$ – 1-st moment vector
$v_0 = 0$ – 2-nd moment vector
$t = 0$ – timestep
while θ not converged **do**
$t = t + 1$
$g_t = \nabla_\theta f_t(\theta_{t-1})$
$m_t = \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t$
$v_t = \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2$
$\hat{m}_t = \frac{m_t}{1-\beta_1^t}$
$\hat{v}_t = \frac{v_t}{1-\beta_2^t}$
$\theta_t = \theta_{t-1} - \alpha \cdot \frac{\hat{m}_t}{\sqrt{v_t}+\epsilon} - \lambda\theta_t$
end while

Our modified version of the Adam algorithm:

Algorithm 8 MyAdamW

Require: $\alpha, \beta_1, \beta_2, \epsilon, f$
$m_0 = 0$ – 1-st moment vector
$v_0 = 0$ – 2-nd moment vector
$t = 0$ – timestep
while θ not converged **do**
$t = t + 1$
$g_t = \nabla_\theta f_t(\theta_{t-1})$
$m_t = \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t$
$v_t = \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2$
$v_{t,i} = \max(\lambda, v_{t,i})$, for each i
$\hat{m}_t = \frac{m_t}{1-\beta_1^t}$
$\hat{v}_t = \frac{v_t}{1-\beta_2^t}$
$\theta_t = \theta_{t-1} - \alpha \cdot \frac{\hat{m}_t}{\sqrt{v_t}+\epsilon} - \alpha\theta_{t-1}$
end while