TREE-WIDTH DRIVEN SDP FOR THE MAX-CUT PROBLEM

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ABSTRACT

This paper addresses the well studied graph theoretic Max-Cut problem, which has applications in Machine learning, Theoretical physics(Ising model) and VLSI design. Whilst original problem is NP-hard, in 1995 Goemans and Williamson proposed approximation technique using convex optimization (particularly with Semi-Definite Programming) with guarantee 0.878 of optimal cut. This is the best known approximation guarantee for the Max-Cut problem today. We introduce a new approach with better approximation in particular bounded treewidth case, providing a number of heuristics.

Keywords Max-Cut · SDP · Treewidth

1 Introduction

In this article we work with Max-Cut problem where one is interested in finding the cut of largest value in a given graph. It is well-known in common case Max-Cut is NP-hard problem[1]. Obviously, there is an obvious randomized 0.5-approximation algorithm: for each vertex flip a coin to decide to which half of the graph partition to assign it. In 1995 Goemans and Williamson introduced novel technique, which guarantee 0.878 of optimal cut, this is the best known approximation today. Interesting point here is if the unique games conjecture[2] is true, this is the best possible approximation ratio for maximum cut[3].

We focus on particular case, when graphs have bounded treewidth. Using Semi-Definite Programming and a bunch of heuristics we provide better approximation approach to Max-Cut problem.

2 Problem statement

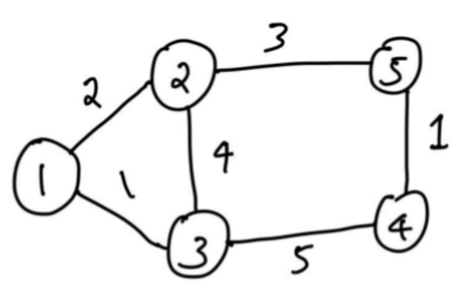
Given an undirected graph $G = (V, E)$ where every edge (u, v) has a weight $w_{uv} = w_{vu}$. We split V into two parts S and $\bar{S} := V \setminus S$ and count the weight of this cut as follows

$$W(S) := \sum_{(u,v) \,\in\, S\times\bar{S}} w_{uv}$$

The goal is to find the cut with maximal possible weight.

Example

1. For $S = \{1, 2, 3\},\ W(S) = 8$
2. For $S = \{1, 2, 3, 4\},\ W(S) = 4$
3. For $S = \varnothing,\ W(S) = 0$
4. For $S = \{1, 3, 5\},\ W(S) = 15$



The maxcut is the cut $S = \{1, 3, 5\}$.

[1] Reduction from 3 SAT to MAX CUT

[2] Unique games conjecture

[3] Max Cut

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