Neural Networks Loss Landscape Convergence in Different Low-Dimensional Spaces

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Problem Statement

- Neural networks achieve higher accuracy with larger datasets.
- However, computational resources limit training possibilities.
- Key question: **When does additional data stop reshaping the loss landscape?**

Proposed Method

- Analyze loss landscape via Hessian eigenvectors.
- Project high-dimensional parameter space onto lower-dimensional space.
- Utilize Monte-Carlo methods to approximate landscape changes.

Main Idea

- **Hessian-based projections** identify primary curvature directions.
- Calculate a Δ-function to quantify changes when increasing dataset size:

$$\Delta_k = \int (L_k(w) - L_{k-1}(w))^2 p(w) dw$$

- Identify the threshold at which further data does not significantly alter landscape.

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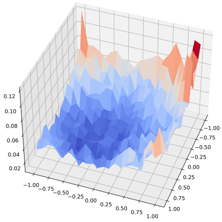
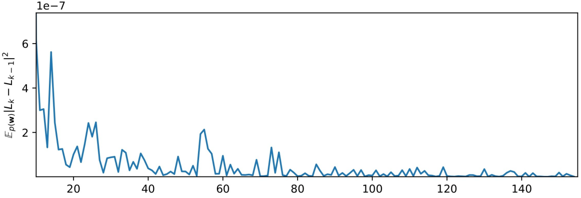
Goal: Measure how the loss function changes as the training set size grows:

$$\Delta_k = \mathbb{E}_{p(\mathbf{w})}\Big(\mathcal{L}_k(\mathbf{w}) - \mathcal{L}_{k-1}(\mathbf{w})\Big)^2.$$

Method:

- **Monte Carlo:** Generate points near the minimum according to $p(\mathbf{w})$ and average the differences.
- **Hessian Eigenvectors:** Use directions with the largest eigenvalues to focus on key curvature components.





Δ_k *function*

Experiment Results

- Experiments conducted on MNIST and Fashion-MNIST datasets.
- Low-dimensional visualization clearly illustrates stabilization.
- Verified theoretical bounds empirically.

Practical Implications

- Enables identification of minimal viable dataset size.
- Reduces computational cost significantly.
- Provides guidelines for efficient data collection.

References

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