Gradient Sliding for SP with one composite

By Antyshev Tikhon
4^{th} year MIPT DCAM IS student

2023

Establishing goals

- We consider following saddle point problem with one composite:

$$\min_x \max_y F(x, y) + f(x)$$

where functions $f(x)$ and $F(x, y)$ are correspondingly L_f and L_F Lipshitz continuous

- We also consider the sum of the two functions to be μ_x and μ_y strongly convex

Gradient Sliding

- The first step is to use the gradient sliding algorithm (Kovalev et al. [2022]).
- The oracle of $\nabla f(x)$ will be called $\tilde{\mathcal{O}}\left(\sqrt{\frac{L_f}{\mu_x}}\right)$

Problem: we need to solve a suboptimization task.

FOAM

The gradient sliding produces the following suboptimization task:

$$\min_{x} \max_{y} \left(\langle \nabla f(x_g^k), x - x_g^k \rangle + \|x - x_g^k\|^2 + F(x, y) \right)$$

- The complexity of this task via FOAM (Kovalev and Gasnikov [2022]) is $\tilde{\mathcal{O}}\left(\frac{L_F}{\sqrt{(L_f+\mu_x)\cdot\mu_y}}\right)$

Final complexity

The final oracle complexity for ∇F can now be calculated:

$$\tilde{\mathcal{O}}\left(\sqrt{\frac{L_f}{\mu_x}}\right) \cdot \tilde{\mathcal{O}}\left(\frac{L_F}{\sqrt{(L_f + \mu_x) \cdot \mu_y}}\right) = \tilde{\mathcal{O}}\left(\frac{L_f}{\mu_x \mu_y}\right)$$

The Experiment

The test is performed on a Bilinear Quadratic SP.

$$\min_x \max_y \left(\underbrace{p(x)} + < y, Ax > - g(y) \right)$$

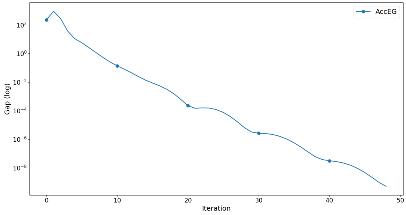

Figure: Visualized results

Conclusion

- A comparison with other methods is necessary
- Run method on more SP tasks
- The theory needs to be finished

References

Dmitry Kovalev, Aleksandr Beznosikov, Ekaterina Borodich, Alexander Gasnikov, and Gesualdo Scutari. Optimal gradient sliding and its application to distributed optimization under similarity. 2022. doi: 10.48550/ARXIV.2205.15136. URL https://arxiv.org/abs/2205.15136.

Dmitry Kovalev and Alexander Gasnikov. The first optimal algorithm for smooth and strongly-convex-strongly-concave minimax optimization. 2022. doi: 10.48550/ARXIV.2205.05653. URL https://arxiv.org/abs/2205.05653.