Weighted Random Search for Hyperparameter Optimization

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Motivation

Main idea

The motivation is to modernize the generally accepted approaches with an improvement in the quality of work. A weighted search method is proposed, which suggests that a value that has already led to a good result is a good candidate for a new test and should be tested in new combinations of hyperparameter values.

The WRS Method

Algorithm 1 A WRS Step - Objective Function Maximization

```
Input: F; (X^k, F(X^k)); p_i, k_i, P_i(x), i = 1, ..., d
Output: (X^{k+1}, F(X^{k+1}))
Randomly generate p, uniform in (0,1)
for i = 1 to d do
    if (p_i ≥ p or k ≤ k_i) then
        // either the probability condition is met or more samples are needed
        Generate x_i^{k+1} according to P_i(x)
    else
        x_i^{k+1} = x_i^k
    end if
end for
// usually this is the most time consuming step
Compute F(X^{k+1})
if F(X^{k+1}) ≥ F(X^k) then
    return (X^{k+1}, F(X^{k+1}))
else
    return (X^k, F(X^k))
end if
```

The WRS Method

Algorithm 2 WRS - Objective Function Maximization

Input: F; N; $P_i(x), i = 1, \dots, d$
Output: $(X^N, F(X^N))$

// Phase 1 - Run RS
for $k = 1$ to $N_0 < N$ **do**
    Perform RS step, compute $(X^k, F(X^k))$
end for
// Intermediate phase, determine input for WRS
Determine the probability of change $p_i, i = 1, \dots, d$
Determine the minimum number of required values $k_i, i = 1, \dots, d$
// Phase 2 - Run WRS
for $k = N_0 + 1$ to N **do**
    Perform WRS Step described in Algorithm 1, compute $(X^k, F(X^k))$
end for
return $(X^N, F(X^N))$

Theoretical Aspects and Convergence

Multi-dimensional case

For the general case of optimizing a function $F : S_1 \times S_2 \cdots \times S_d \to R$, with $Si, i = 1, \ldots, d$ countable sets and under the same assumption that the variables are not statistically correlated, P_{RS} and P_{WRS} are defined as:

$$p_{RS} = \prod_{i=1}^{d} \frac{1}{|S_i|}, p_{WRS} = \frac{1}{|S_1|} \prod_{i=2}^{d} \left(p_i \frac{1}{|S_i|} + (1 p_i) \frac{1}{|S_i| - m_i + 1} \right)$$

where m_i is the number of distinct values already generated for x_i.

Theorem

For any function $F : S_1 \times S_2 \cdots \times S_d \to R$ there exist $k_i, i = 1, \ldots, d$, so that $p_{WRS:n} \geq p_{RS:n}$.

An Example: Griewank Function Optimization

Grievank function

$$G_d = 1 + \frac{1}{4000}\sum_{i=1}^{d} x_i^2 - \prod_{i=1}^{d} \cos\frac{x_i}{\sqrt{i}}$$

We use a slightly modified version of G_6, given by:

$$G_6^* = 1 + \frac{i-1}{4000}\sum_{i=1}^{6} x_i^2 - \prod_{i=1}^{6} \cos\frac{x_i}{\sqrt{i}}$$

An Example: Griewank Function Optimization

Parameter	x_1	x_2	x_3	x_4	x_5	x_6
Weight	0.07	0.18	1.24	7.77	23.52	43.96
Probability	0.002	0.004	0.028	0.177	0.535	1.00

Figure: Parameter weights and probabilities for G_6^*.

Optimizer	Best Found Value	Average Value	SD
RS	-1.50	-33.10	14.06
WRS	**-1.28**	**-14.58**	**10.63**

Figure: WRS vs. RS results for G_6^* - values for 1000 runs.

An Example: Griewank Function Optimization

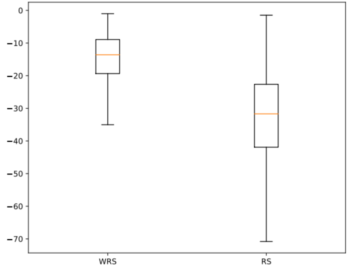


Figure 1: Performance of WRS vs. RS for the G_6^* optimization

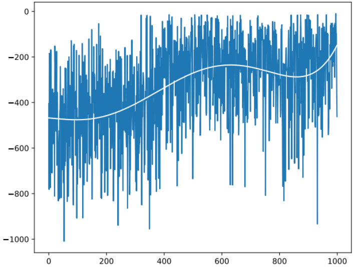


Figure 2: Convergence of WRS for the G_6^* function

CNN Hyperparameter Optimization

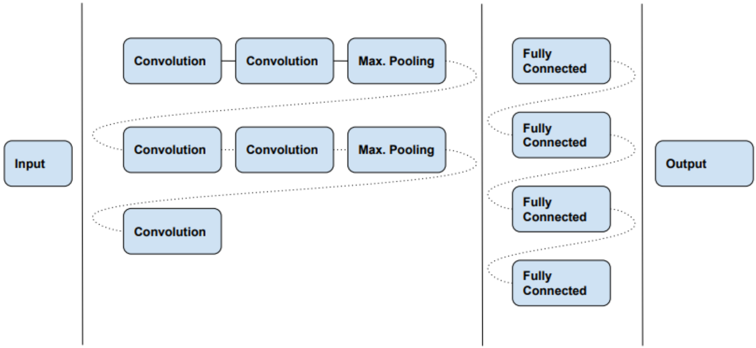
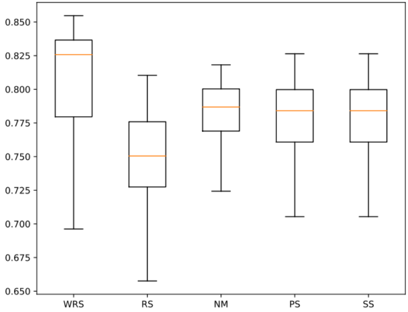


Figure: CNN architecture.

CNN Hyperparameter Optimization



Optimizer	Best Result	Average	SD
WRS	**0.85**	**0.79**	0.09
RS	0.81	0.75	0.04
NM	0.81	0.77	0.03
PS	0.83	0.78	0.03
SS	0.82	0.75	0.05

Figure: 1. Performance of WRS, RS, NM, PS and SS for CNN optimization. 2. Algorithms' results for CNN accuracy on CIFAR-10.

Literature

1. **Main article** Weighted Random Search for Hyperparameter Optimization.