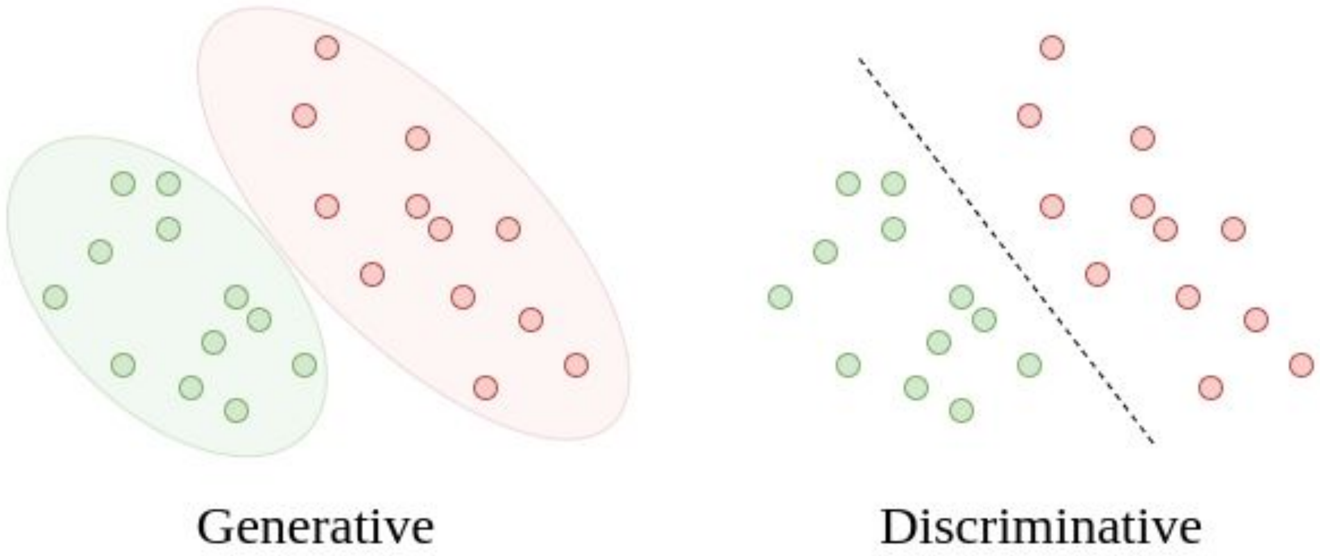
Deep Learning

Lecture 11

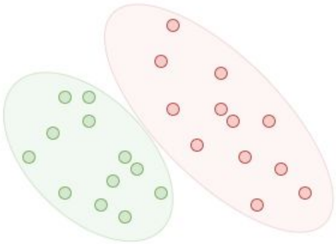
Recap

- What is RL?
- State, action, policy, reward, markovian property, MDP
- Why don’t we use it everywhere?
- V-function, Q-function
- Value Iteration, Policy iteration
- Monte-Carlo methods (sample + epsilon greedy)
- Temporal difference learning (SARSA)
- Q-learning
- On-policy/off-policy
- Replay buffer
- DQN
- Policy gradients (Reinforce + improvements)
- Actor-Critic algorithm and A2C

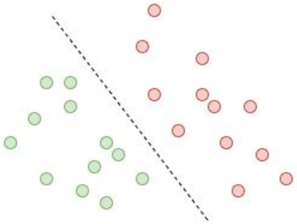
Discriminative vs Generative models



Discriminative vs Generative models



Generative



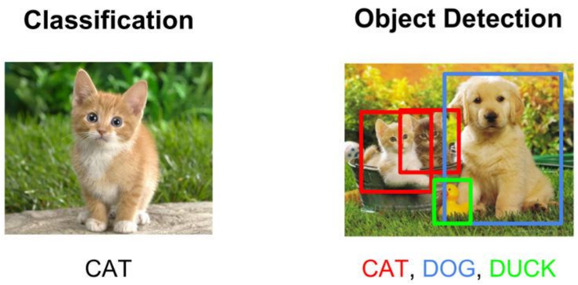
Discriminative

Generative models describe how data is generated using probabilistic models.

They predict P(y|x), the probability of y given x, calculating the P(x,y), the probability of x and y.

A discriminative model does not care how the data is generated. Here we just care about P(y|x).

Discriminative models



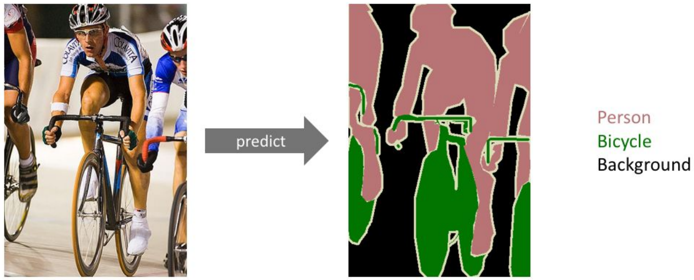


Image classification & Object detection

Semantic segmentation

Where is a discriminative model useful?

Generative model

Handwritten text generation

Kanye West

Animal image generation

Cat

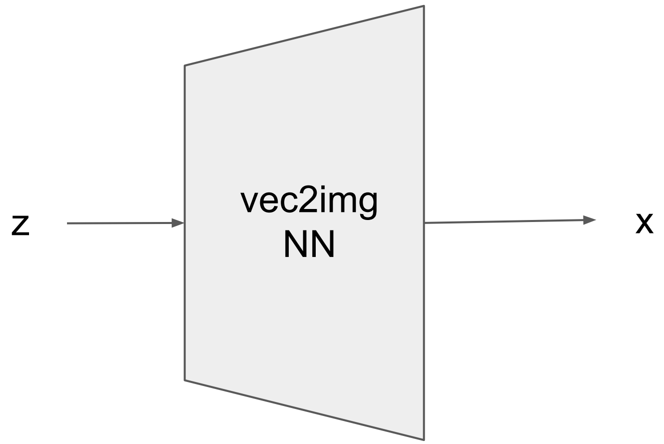
Where is a generative model useful?

We can solve it in discriminative way, but it will be lacking of diversity. For cat picture we will get only one picture

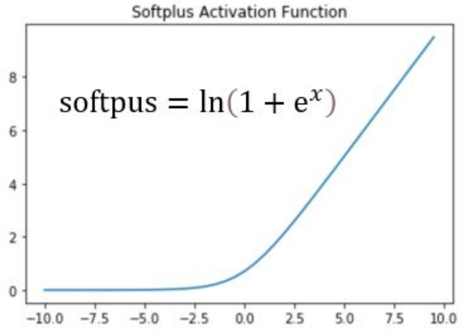
Let’s create generative models

Generative model

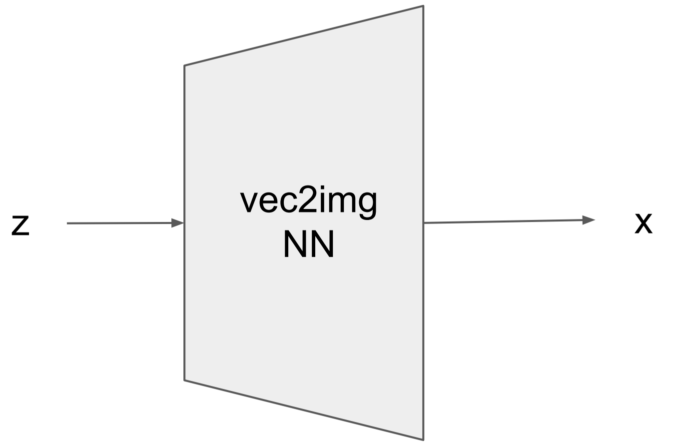
$$z \sim \mathcal{N}(z|0, I)$$
$$x \sim \mathcal{N}(x|\mu(z), diag(\sigma_i^2(z)))$$



1) How to make output for variance positive?



Generative model



$$z \sim \mathcal{N}(z|0, I)$$
$$x \sim \mathcal{N}(x|\mu(z), diag(\sigma_i^2(z)))$$

2) How to make output of distribution lay in [0, 1]

$$x_i \sim Beta(x|a(z, \theta), b(z, \theta))$$

Kinda works bad

Conditional optimization trick

Consider an conditional optimization problem

$$f(x) \to \min_{x \in Q}$$

How to apply a gradient descent?

$$\mathbf{x}_{k+1} = P_Q\left(\mathbf{x}_k - \alpha_k \nabla f\left(\mathbf{x}_k\right)\right)$$

So, we somehow should be able to project parameter space only to whose parameters which has output inside [0,1]. Computationally hard!!

But what if we have function mapping all space to Q?

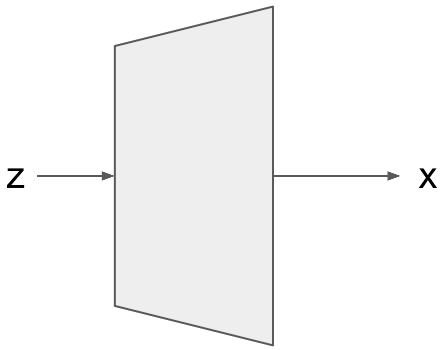
$$\bar{x} \in R^d, x = g(\bar{x}) \in Q$$

We can rewrite conditional optimization problem to unconditional

$$f(g(\bar{x})) \to \min_{\bar{x}}$$

What transformation we can apply?

Maximum likelihood



$$P(X, Z|\theta) = \prod_{i=1}^{N} p(z_i)p(x_i|z_i, \theta) \to \max$$

Unflexible

$$P(X|\theta) = \int P(X, Z|\theta)dZ \to \max$$

Computationally untractable

How to tackle the latter problem?

Find some lower bound of log-likelihood

$$\log P(X|\theta) \geqslant L(\theta, q) \to \max_{\theta, q}$$

Using Yensen inequality -> Evidence Lower Bound (ELBO)

$$\log p(X|\theta) \geqslant E_{q(z)} \log \frac{p(X, Z|\theta)}{q(Z)}$$

EM-algorithm

$$\log p(X|\theta) \geqslant E_{q(z)} \log \frac{p(X, Z|\theta)}{q(Z)}$$

Let’s fix parameter and optimize q. Then, fix q and optimize parameters

(E)xpectation-step:

$$q(z) = p(Z|X, \theta)$$

(M)aximization step:

$$\mathbb{E}_{q(z)} \log p(X, Z|\theta) \to \max_{\theta}$$

EM-algorithm tricks

Let’s parametrize q(\lambda) and optimize parameters of distribution $$q(z) = p(Z|X, \theta)$$

It’s not necessary to find optimal parameters. We can only one gradient step update

$$q(z) = p(Z|X, \theta)$$

$$\mathbb{E}_{q(z)} \log p(X, Z|\theta) \to \max_{\theta}$$

ELBO optimization

How to optimize ELBO? Gradient descent!

NN parameter optimization

$$\nabla_\theta ELBO(\theta, \lambda) = \nabla E_{q(z|\lambda)} f(x, z, \theta, \lambda) = E_{q(z|\lambda)} \nabla f(x, z, \theta, \lambda) \approx \frac{1}{M} \sum_{j=1}^{M} \nabla_\theta f(x, z_j)$$

Distribution parameter optimization

$$\nabla_\lambda ELBO(\theta, \lambda) \neq \nabla_\lambda E_{q(z|\lambda)} f(x, z, \theta, \lambda)$$

REINFORCE trick

$$\nabla_\lambda \mathbb{E}_{q(z|\lambda)} f(x,z,\theta,\lambda) = \nabla_\lambda \int q(z|\lambda) f(x,z,\theta,\lambda) dz = \int \nabla_\lambda q(z|\lambda) f(x,z,\theta,\lambda) + \int q(z|\lambda) \nabla_\lambda f(x,z,\theta,\lambda) dz$$

$$\int \nabla_\lambda q(z|\lambda) f(x,z,\theta,\lambda) = \int q(z|\lambda) \frac{\nabla_\lambda q(z|\lambda)}{q(z|\lambda)} f(x,z,\theta,\lambda) = \int q(z|\lambda) \nabla_\lambda \log q(z|\lambda) f(x,z,\theta,\lambda)$$

This type of gradient has a big variance. As we're building model by our own, can we solve this problem?

Reparametrization trick

Example:

$$z_0 \sim \mathcal{N}(z_0|0, I) \longrightarrow z = Az_0 + b$$

$$z \sim \mathcal{N}(b, AA^T)$$

$$z \sim \mathcal{N}(z|\mu, \Sigma) \longrightarrow z = Lz_0 + \mu$$

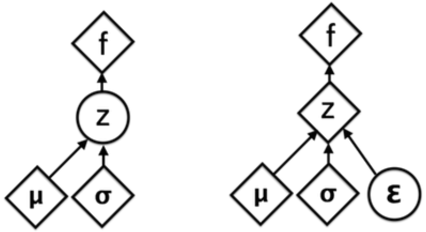
$$z_0 \sim \mathcal{N}(z_0|0, I)$$

Reparametrization trick

If we have a transformation $z = g(z_0, \lambda)$

We can calculate the gradient using the following form

$$\nabla_\lambda \mathbb{E}_{q(z|\lambda)} f(x, g(z_0, \lambda), \theta, \lambda) = \nabla_\lambda \mathbb{E}_{q_0(z_0)} f(x, g(z_0, \lambda), \theta, \lambda)$$



Original Reparametrized

$$g_\theta(\varepsilon) = \mu_\theta + \varepsilon\sigma_\theta$$

ELBO optimization

$$\log p(X|\theta) \geqslant E_{q(z)} \log \frac{p(X, Z|\theta)}{q(Z)}$$

$$\sum_{i=1}^{N} \mathbb{E}_{q(z_i|\lambda)} \log \frac{p(x_i, z_i|\theta, \lambda)}{q(z_i|\lambda)}$$

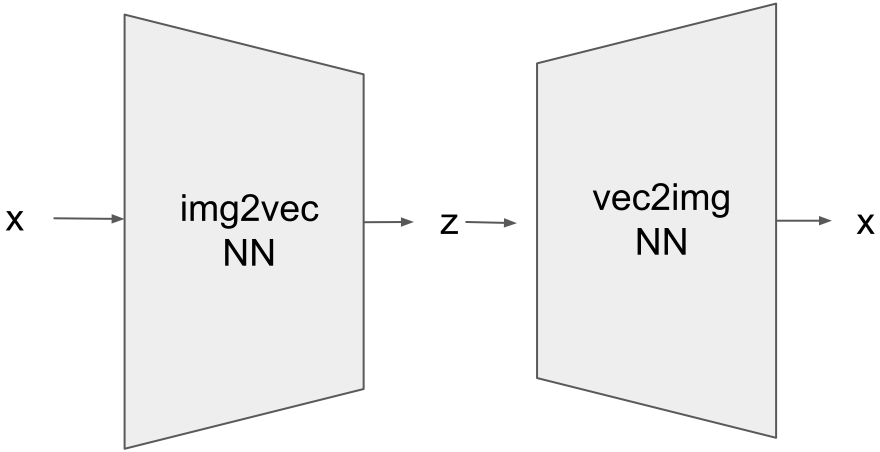
How to choose q-distribution?

$$q(z_i) = \mathcal{N}(z_i|\mu_i, \sigma_i^2)$$

Number of parameters grows with number of objects

What about test objects?

How to parametrize distribution?



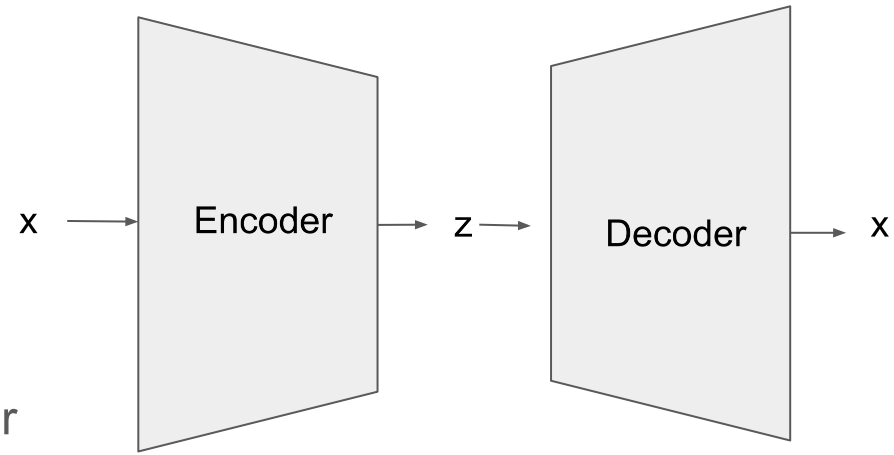
Let’s parametrize q-distribution parameters with input

$$q(z_i) = \mathcal{N}(z_i | \mu_\phi(x_i), \sigma^2(x_i))$$

Voila! VAE is done

Algorithm (Training)

- Process input with encoder
- Sample from latent distribution
- Apply reparametrization
- Process latent vector with decoder

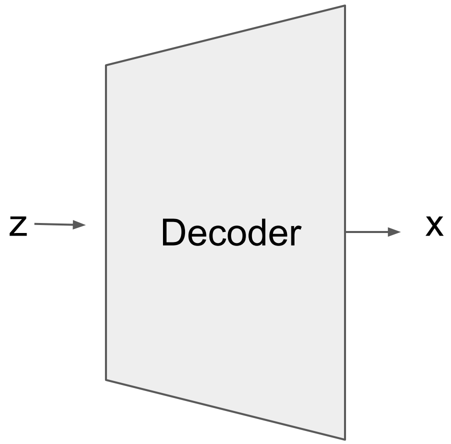


Gradient backpropagation

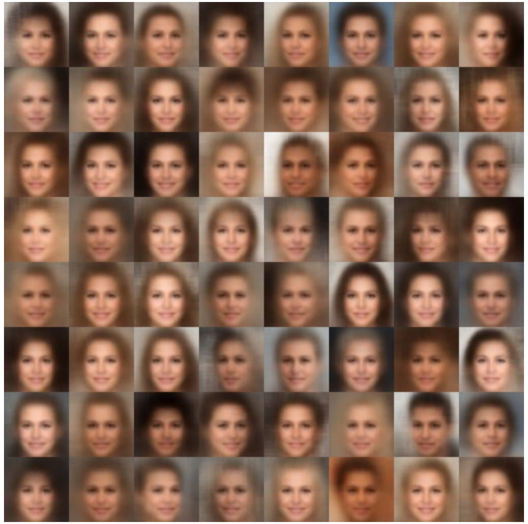
- Run loss.backward()
- optimizer.step()

Algorithm (Generation stage)

- Sample latent vector
- Apply decoder



Examples



Recap

- Discriminative vs Generative models
- Reparametrization tricks
- Variational Auto Encoder