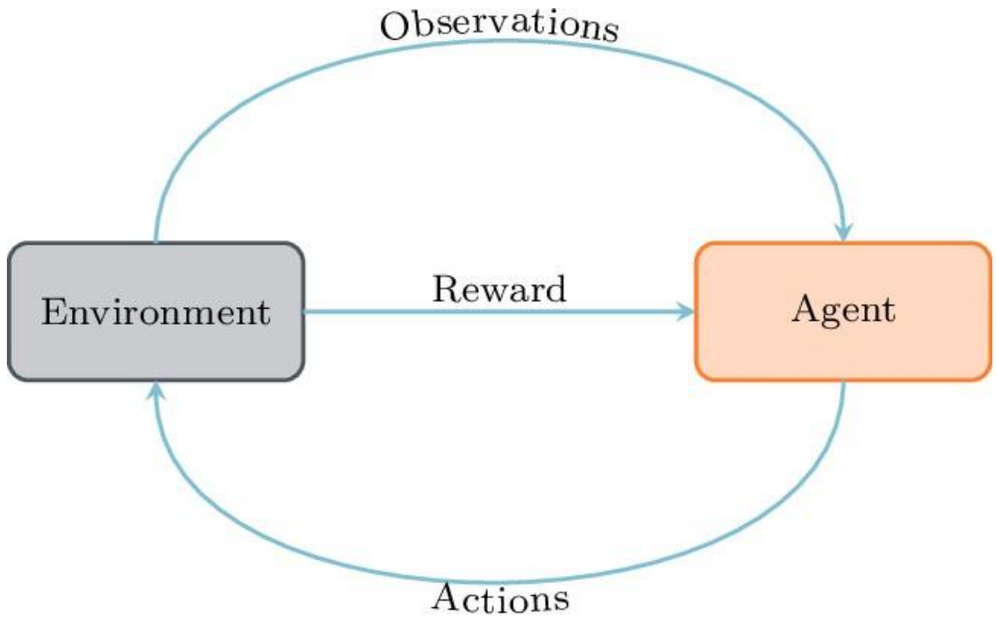
Deep Learning

Lecture 9

from really good course in AI masters (https://ozonmasters.ru/reinforcementlearning).

Reinforcement Learning: previous lecture



Recap: Value Functions

Total Discounted Return:

$$R_t = r(s_t, a_t) + \gamma r(s_{t+1}, a_{t+1}) + \gamma^2 r(s_{t+2}, a_{t+2}) + \ldots = \sum_{\tau=t}^{\infty} \gamma^{\tau - t}\, r(s_{t+\tau}, a_{t+\tau})$$

State Value Function / V-Function:

$$V_\pi(s) = \mathbb{E}_\pi[R_t | s_t = s] = \mathbb{E}_\pi[\sum_{i=0}^{\infty} \gamma^i r_{t+i} | s_t = s]$$

State-Action Value Function / Q-Function:

$$Q_\pi(s, a) = \mathbb{E}_\pi[R_t | s_t = s, a_t = a] = \mathbb{E}_\pi[\sum_{i=0}^{\infty} \gamma^i r_{t+i} | s_t = s, a_t = a]$$

Recap: Bellman Equations

Bellman Expectation Equations for policy π:

$$V_\pi(s) = \sum_a \pi(a|s) \sum_{s'} p(s'|s, a)[r + \gamma V_\pi(s')]$$

$$Q_\pi(s, a) = \sum_{s'} p(s'|s, a)[r + \gamma \sum_{a'} \pi(a'|s') Q_\pi(s', a')]$$

Bellman Optimality Equations:

$$V_*(s) = max_a \sum_{s'} p(s'|s, a)[r + \gamma V_*(s')] = max_a Q_*(s, a)$$

$$Q_*(s, a) = \sum_{s'} p(s'|s, a) [r + \gamma max_{a'} Q_*(s', a')]$$

Recap: Policy Iteration

Policy Iteration (using iterative policy evaluation) for estimating $\pi \approx \pi_*$

1. Initialization
   $V(s) \in \mathbb{R}$ and $\pi(s) \in \mathcal{A}(s)$ arbitrarily for all $s \in \mathcal{S}$

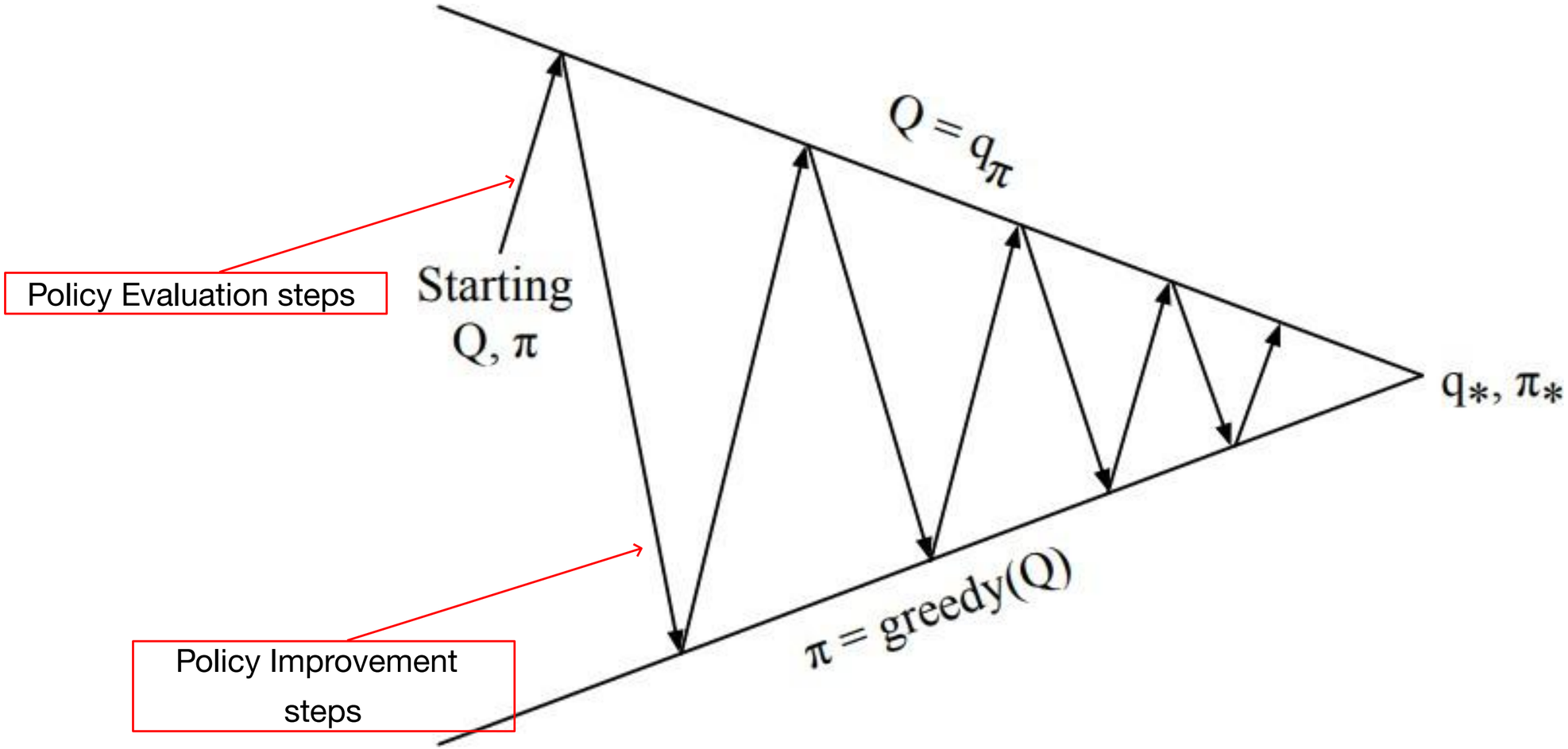
2. Policy Evaluation
   Loop:
      $\Delta \leftarrow 0$
      Loop for each $s \in \mathcal{S}$:
         $v \leftarrow V(s)$
         $V(s) \leftarrow \sum_{s',r} p(s', r \mid s, \pi(s)) \left[r + \gamma V(s') \right]$
         $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
   until $\Delta < \theta$ (a small positive number determining the accuracy of estimation)

3. Policy Improvement
   policy-stable $\leftarrow$ *true*
   For each $s \in \mathcal{S}$:
      old-action $\leftarrow \pi(s)$
      $\pi(s) \leftarrow \text{argmax}_a \sum_{s',r} p(s', r \mid s, a) \left[r + \gamma V(s') \right]$
      If *old-action* $\neq \pi(s)$, then *policy-stable* $\leftarrow$ *false*
   If *policy-stable*, then stop and return $V \approx v_*$ and $\pi \approx \pi_*$; else go to 2

Use Belman **Expectation** Equations to learn V/Q for current policy

Greedily Update policy w.r.t. V/Q-function

Recap: Policy Iteration



Recap: Value Iteration

Value Iteration, for estimating $\pi \approx \pi_*$

Algorithm parameter: a small threshold $\theta > 0$ determining accuracy of estimation
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(terminal) = 0$

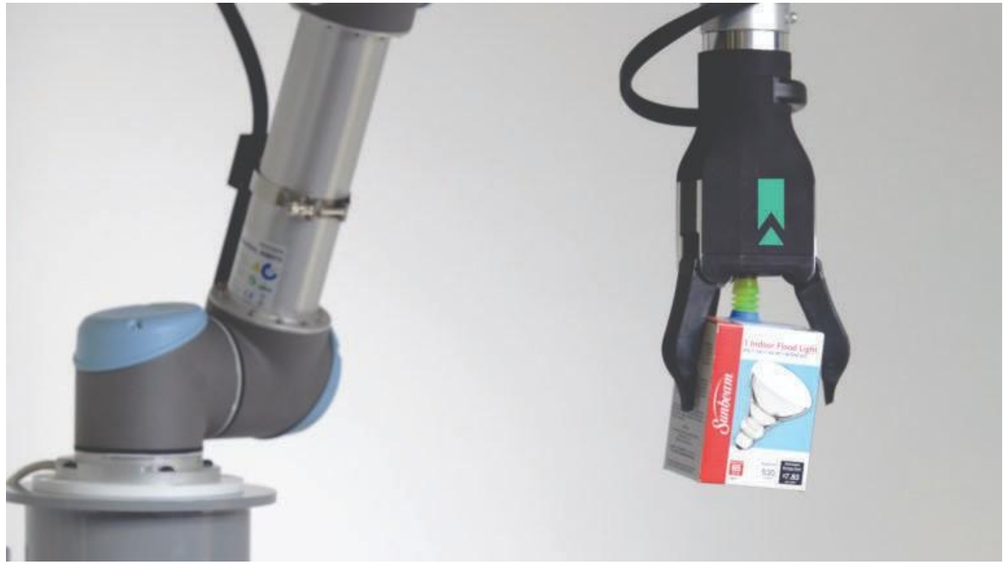
Loop:
| $\Delta \leftarrow 0$
| Loop for each $s \in \mathcal{S}$:
| $v \leftarrow V(s)$
| $V(s) \leftarrow \max_a \sum_{s',r} p(s', r \mid s, a)\left[r + \gamma V(s')\right]$
| $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
until $\Delta < \theta$

Output a deterministic policy, $\pi \approx \pi_*$, such that
$\pi(s) = \text{argmax}_a \sum_{s',r} p(s', r \mid s, a)\left[r + \gamma V(s')\right]$

Use Bellman **Optimality** Equations to learn V/Q for current policy

Policy Improvement is implicitly used here

But we don't know the model

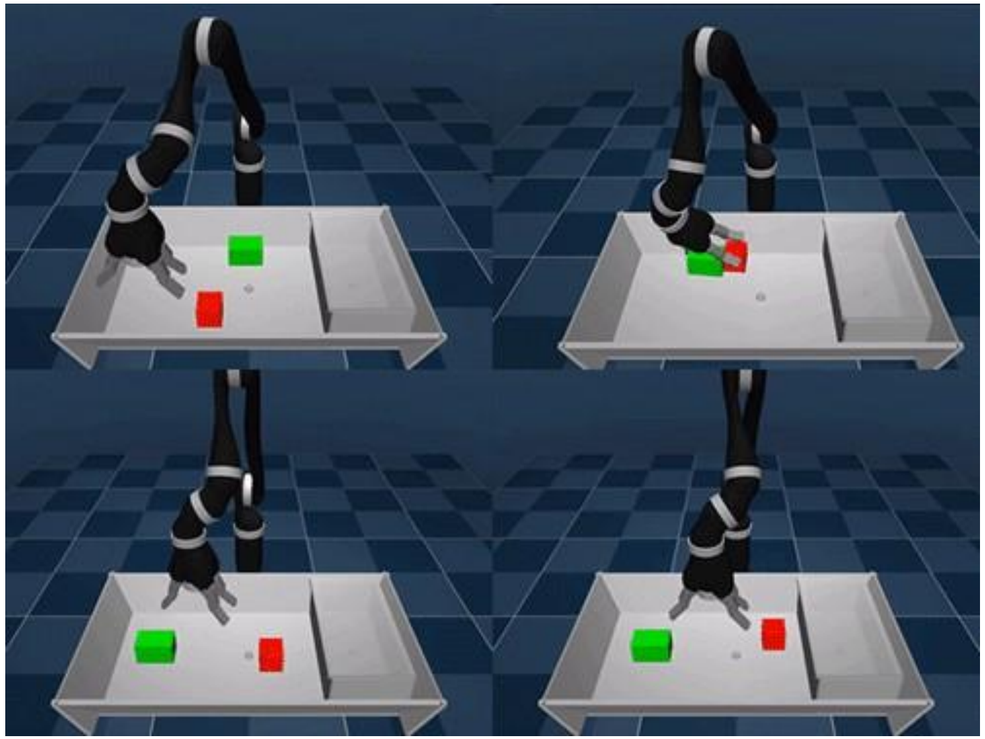


$$V_\pi(s) = \sum_a \pi(a|s) \sum_{s'} p(s'|s,a)[r + \gamma V_\pi(s')]$$

$$Q_\pi(s, a) = \sum_{s'} p(s'|s,a)[r + \gamma \sum_{a'} \pi(a'|s') Q_\pi(s', a')]$$

Solution: Use Sampling

- The model of the environment is unknown!
- :(But you still can interact with it!



Monte-Carlo Methods

GOAL: Learn value functions Q_π or V_π without knowing $p(s' \mid s, a)$ and $R(s, a)$

RECALL that value function is the expected return:

$$Q_\pi(s\ , a) = \mathsf{E}_\pi[\sum_{k=0}^{\infty} \gamma^k r_{t+k+1} \mid S_t = s, A_t = a]$$

First Visit Monte-Carlo

IDEA: Estimate expectation $Q_\pi(s, a)$ with empirical mean $q(s, a)$:

- Generate an episode with π
  The first time-step t that state-action (s, a) is visited
- Increment counter $N(s, a) = N(s, a) + 1$
- Increment total return $S(s, a) = S(s, a) + R_t$
- Value is estimated by mean return $q(s, a) = S(s, a)/N(s, a)$

By Law of Large Numbers, $q(s, a) \rightarrow Q_\pi(s, a)$ as $N(s, a) \rightarrow \infty$

Monte-Carlo Methods: Prediction

We can update mean values incrementally:

$$\mu_{k+1} = \frac{1}{k+1}\sum_{k=1}^{k+1} x_k = \mu_k + \frac{1}{k+1}(x_k - \mu_k)$$

Old estimate — Learning rate — Prediction error

Incremental Monte Carlo Update:

$$q(s_t, a_t) \leftarrow q(s_t, a_t) + \frac{1}{N(s_t, a_t)}(R_t - q(s_t, a_t))$$

In non-stationary problems we can fix learning rate:

$$q(s_t, a_t) \leftarrow q(s_t, a_t) + \alpha(R_t - q(s_t, a_t))$$

Monte-Carlo Methods

- MC methods learn directly from episodes of experience
- MC is model-free: no knowledge of MDP transitions / rewards
- MC learns from complete episodes: no bootstrapping
- MC uses the simplest possible idea: value = mean return
- Caveat: can only apply MC to episodic MDPs:
    - All episodes must terminate

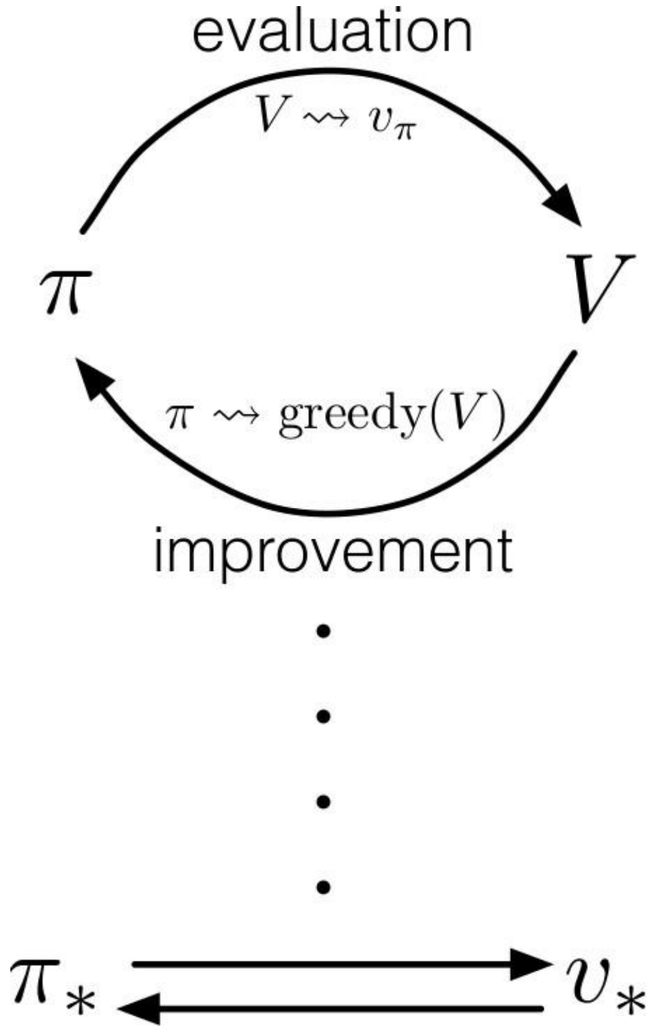
Monte-Carlo Methods: Control

Remember **Policy Iteration**?

How would look PI version with Monte-Carlo Policy Evaluation?

- **Policy Evaluation**: Monte-Carlo Evaluation, $q = Q_\pi$
- **Policy Improvement**: Greedy policy improvement,

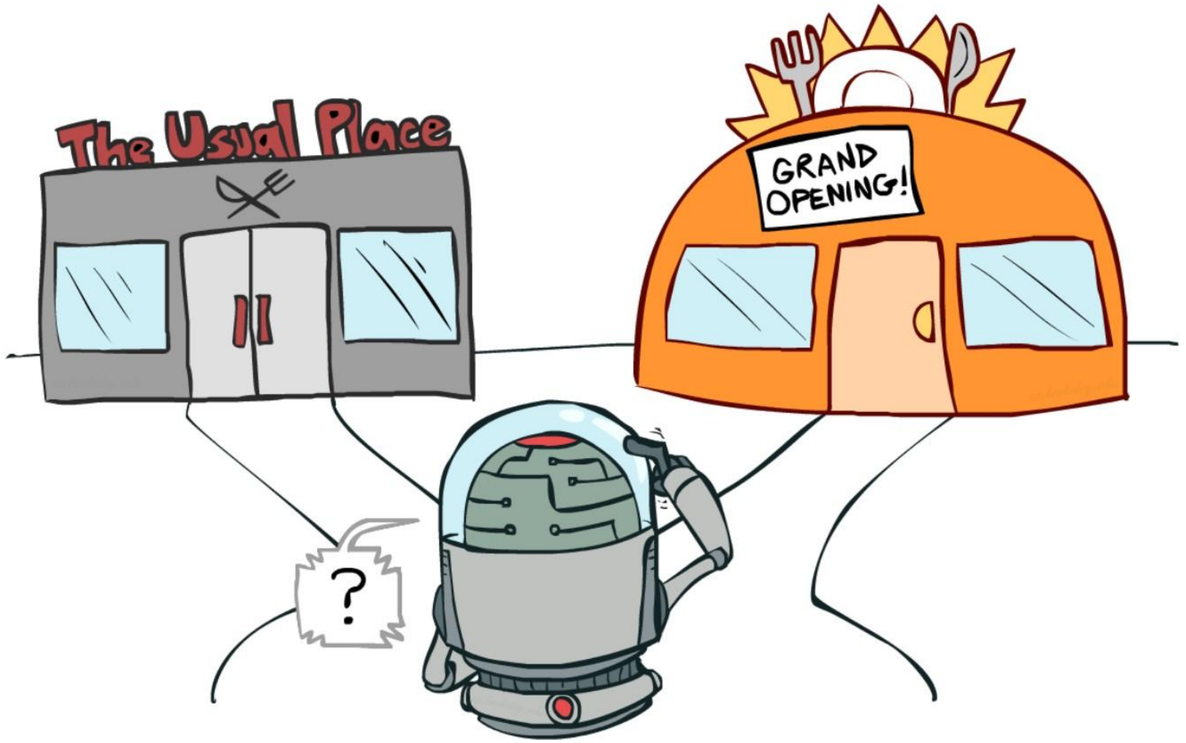
i.e. $\pi'(s) = argmax_a q(s, a)$



Questions:

- Why we estimate Q_π and not V_π?
- Do you see any problem with policy improvement step?

Exploration-Exploitation Problem



Agent can't visit every (s, a) with greedy policy!
Agent can't get correct $q(s, a)$ estimates without visiting (s, a) frequently!

(i.e. remember law of large numbers)

Monte-Carlo Methods: Control

Use ϵ-greedy policy:

- Simplest idea for ensuring continual exploration
- All m actions are tried with non-zero probability
- With probability $1-\epsilon$ choose the greedy action
- With probability ϵ choose an action at random

$$\pi(a|s) = \begin{cases} \epsilon/m + 1 - \epsilon & \text{if } a^* = \underset{a \in \mathcal{A}}{\text{argmax}}\ Q(s,a) \\ \epsilon/m & \text{otherwise} \end{cases}$$

Monte-Carlo Methods: Control

That any ε-greedy policy with respect to q_π is an improvement over any ε-soft policy π is assured by the policy improvement theorem. Let π' be the ε-greedy policy. The conditions of the policy improvement theorem apply because for any $s \in \mathcal{S}$:

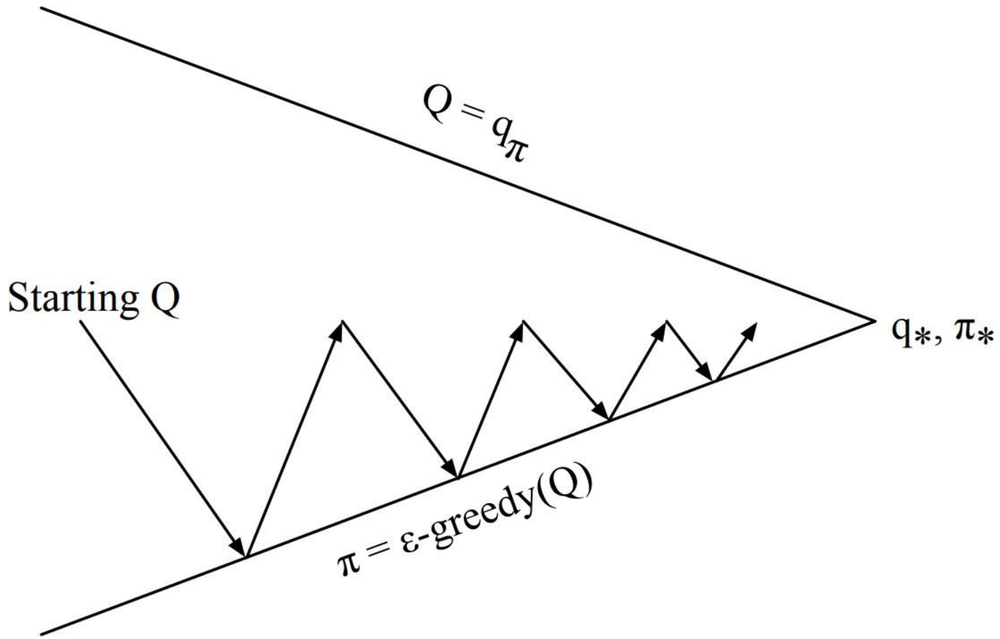
$$
\begin{aligned}
q_\pi(s, \pi'(s)) &= \sum_a \pi'(a|s) q_\pi(s,a) \\
&= \frac{\varepsilon}{|\mathcal{A}(s)|} \sum_a q_\pi(s,a) + (1-\varepsilon) \max_a q_\pi(s,a) \qquad (5.2) \\
&\geq \frac{\varepsilon}{|\mathcal{A}(s)|} \sum_a q_\pi(s,a) + (1-\varepsilon) \sum_a \frac{\pi(a|s) - \frac{\varepsilon}{|\mathcal{A}(s)|}}{1-\varepsilon} q_\pi(s,a) \\
&= \frac{\varepsilon}{|\mathcal{A}(s)|} \sum_a q_\pi(s,a) - \frac{\varepsilon}{|\mathcal{A}(s)|} \sum_a q_\pi(s,a) + \sum_a \pi(a|s) q_\pi(s,a) \\
&= v_\pi(s).
\end{aligned}
$$

Monte-Carlo Method

Policy Iteration with Monte-Carlo method:

For every episode:

- **Policy Evaluation**: Monte-Carlo Evaluation, $q = Q_\pi$
- **Policy Improvement**: ϵ-greedy policy improvement



Monte-Carlo Methods: GLIE

Definition

Greedy in the Limit with Infinite Exploration (GLIE)

- All state-action pairs are explored infinitely many times,

$$\lim_{k \to \infty} N_k(s, a) = \infty$$

- The policy converges on a greedy policy,

$$\lim_{k \to \infty} \pi_k(a|s) = \mathbf{1}(a = \operatorname*{argmax}_{a' \in \mathcal{A}} Q_k(s, a'))$$

- For example, ϵ-greedy is GLIE if ϵ reduces to zero at $\epsilon_k = \frac{1}{k}$

Temporal Difference Learning

Problems with Monte-Carlo method:

- Updates policy only once per episode, i.e. only episodic MDPs
- Doesn't use MDP properties

Solution:

- Recall Bellman equation:

$$Q_\pi(s, a) = \sum_{s'} p(s'|s, a)[r + \gamma \sum_{a'} \pi(a'|s')Q_\pi(s', a')]$$

- Use sampling instead of knowledge about the model:

$$q_\pi(s, a) = \mathbb{E}_{s'}[r + \gamma \mathbb{E}_{a'} q_\pi(s', a')] = \mathbb{E}_\tau[r + \gamma q_\pi(s', a')]$$

TD-learning: Prediction

Goal: learn Q_π online from experience

Incremental Monte-Carlo:

- Update value $q(s_t, a_t)$ toward actual return R_t

$$q(s_t, a_t) \leftarrow q(s_t, a_t) + \alpha(R_t - q(s_t, a_t))$$

Temporal-Difference learning:

- Update value $q(s_t, a_t)$ toward estimated return $r_{t+1} + \gamma q(s_{t+1}, a_{t+1})$

$$q(s_t, a_t) \leftarrow q(s_t, a_t) + \alpha(r_t + \gamma q(s_{t+1}, a_{t+1}) - q(s_t, a_t))$$

$r_{t+1} + \gamma q(s_{t+1}, a_{t+1})$ is called the TD target

$\delta_t = r_{t+1} + \gamma q(s_{t+1}, a_{t+1}) - q(s_t, a_t)$ is called the TD error

Temporal Difference Learning

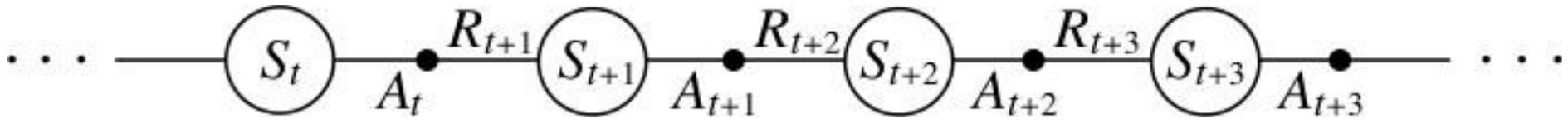
Temporal Difference Learning:

- TD methods learn directly from episodes of experience
- TD is model-free: no knowledge of MDP transitions / rewards
- TD learns from incomplete episodes, by bootstrapping
- TD updates a guess towards a guess

TD-learning: SARSA update

$$q(s_t, a_t) \leftarrow q(s_t, a_t) + \alpha(r_t + \gamma q(s_{t+1}, a_{t+1}) - q(s_t, a_t))$$

This update is called **SARSA**: **S**tate, **A**ction, **R**eward, next **S**tate, next **A**ction

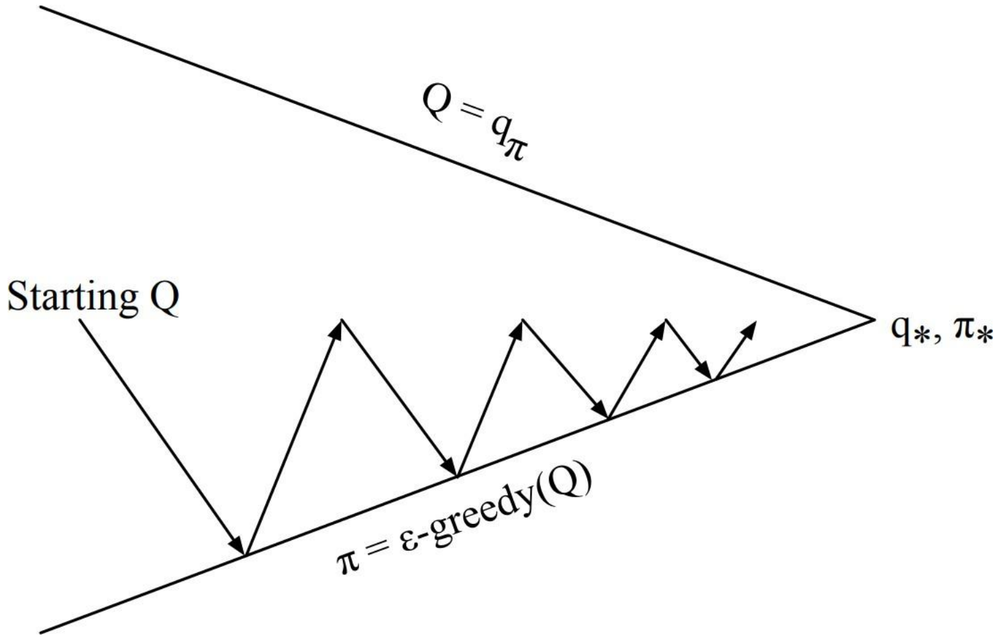


TD-learning: SARSA as Policy Iteration

Policy Iteration with Temporal Difference Learning:

For **every step**:

- **Policy Evaluation**: SARSA Evaluation, $q = Q_\pi$
- **Policy Improvement**: ϵ-greedy policy improvement



TD-learning: SARSA algorithm

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
    Initialize S
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Loop for each step of episode:
        Take action A, observe R, S'
        Choose A' from S' using policy derived from Q (e.g., ε-greedy)
        $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
        $S \leftarrow S'$; $A \leftarrow A'$;
    until S is terminal

TD-learning: SARSA

Theorem

Sarsa converges to the optimal action-value function, $Q(s,a) \rightarrow q_*(s,a)$*, under the following conditions:*

- *GLIE sequence of policies* $\pi_t(a|s)$
- *Robbins-Monro sequence of step-sizes* α_t

$$\sum_{t=1}^{\infty} \alpha_t = \infty$$

$$\sum_{t=1}^{\infty} \alpha_t^2 < \infty$$

TD-learning: Q-Learning

We approximate Bellman Expectation Equation with SARSA update:

$$Q_\pi(s,\, a) = \sum_{s'} p(s' \mid s,\, a)[r + \gamma \sum_{a'} \pi(a' \mid s') Q_\pi(s',\, a')]$$

Can we utilize Bellman Optimality Equation for TD-Learning?

$$Q_*(s,\, a) = \sum_{s'} p(s' \mid s,\, a)[r + \gamma \max_{a'} Q_*(s',\, a')]$$

TD-learning: Q-Learning

We approximate Bellman Expectation Equation with SARSA update:

$$Q_\pi(s,\ a) = \sum_{s'} p(s' \mid s,\ a)[r + \gamma \sum_{a'} \pi(a' \mid s') Q_\pi(s',\ a')]$$

Can we utilize Bellman Optimality Equation for TD-Learning?

$$Q_*(s,\ a) = \sum_{s'} p(s' \mid s,\ a)[r + \gamma \max_{a'} Q_*(s',\ a')]$$

Yes, of course:

$$q(s,\ a) = \mathrm{E}_{s'}[r + \gamma \max_{a'} q(s',\ a')]$$

TD-learning: Q-Learning vs SARSA

From Bellman Expectation Equation (SARSA) :

$$q(s, a) = \mathbb{E}_{s'}[r + \gamma \mathbb{E}_{a'} q(s', a')]$$

a' comes from the policy π that generated this experience!

From Bellman Optimality Equation (Q-Learning):

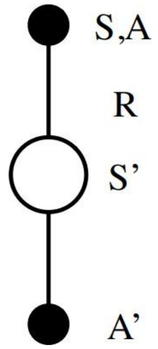
$$q(s, a) = \mathbb{E}_{s'}[r + \gamma \max_{a'} q(s', a')]$$

No connection to the actual policy π

TD-learning: Q-Learning vs SARSA

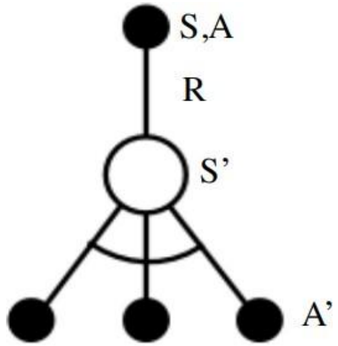
SARSA Update:

$$q(s_t, a_t) \leftarrow q(s_t, a_t) + \alpha(r_t + \gamma q(s_{t+1}, a_{t+1}) - q(s_t, a_t))$$



Q-Learning Update:

$$q(s_t, a_t) \leftarrow q(s_t, a_t) + \alpha(r_t + \gamma max_{a'} q(s_{t+1}, a') - q(s_t, a_t))$$



TD-learning: Q-Learning

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
    $S \leftarrow S'$
  until S is terminal

Theorem

Q-learning control converges to the optimal action-value function, $Q(s, a) \rightarrow q_*(s, a)$

On-policy vs Off-Policy Algorithms

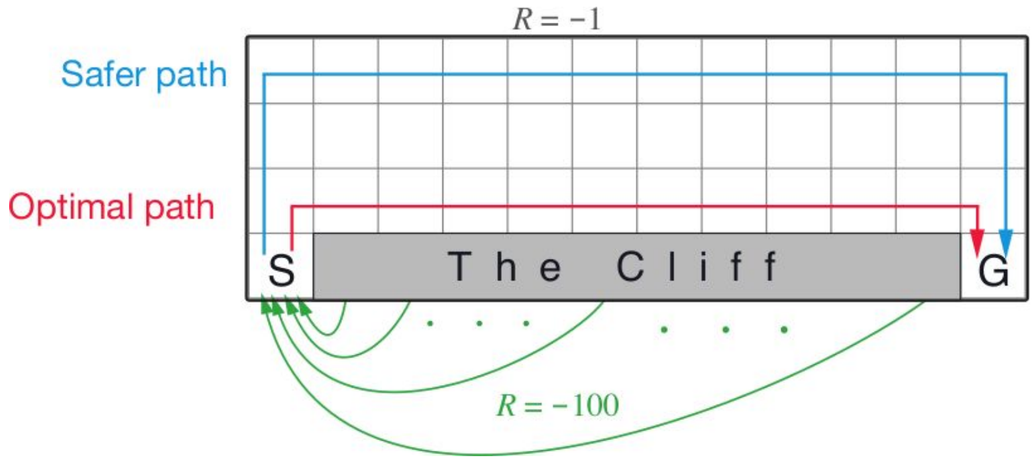
SARSA and Monte-Carlo are **on-policy** algorithms:

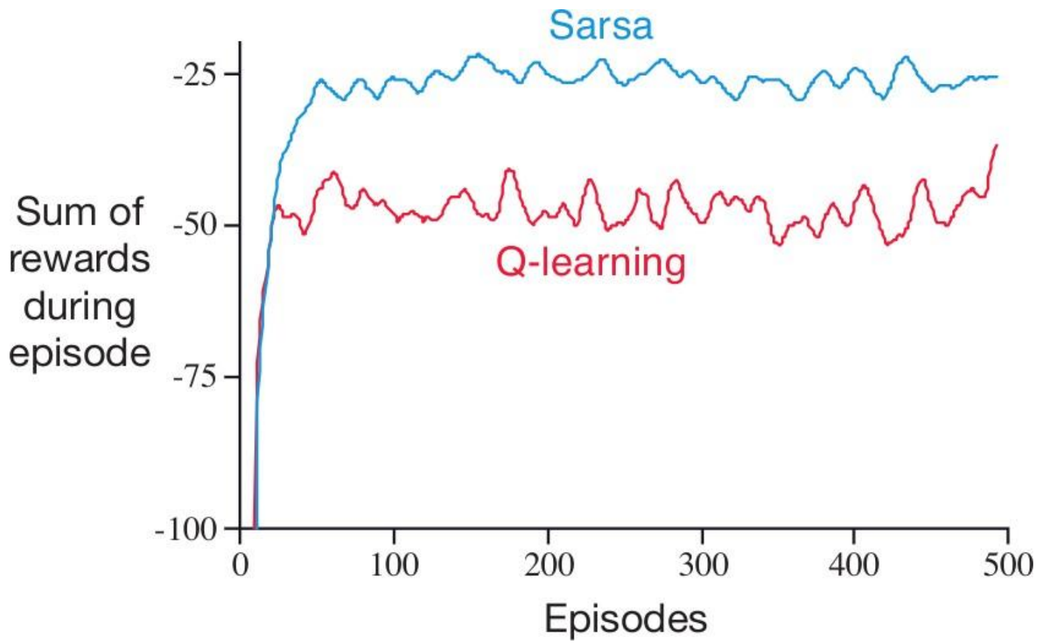
- Improve policy π_k only from experience sampled with this policy π_k
- Can't use old trajectories sampled with π_{k-i}

Q-Learning is **off-policy** algorithm:

- Can Learn policy π using experience generated with other policy μ
- Learn from observing humans or other agents
- Re-use experience generated from old policies
- Learn about optimal policy while following exploratory policy
- Learn about multiple policies while following one policy

TD-learning: Cliff Example



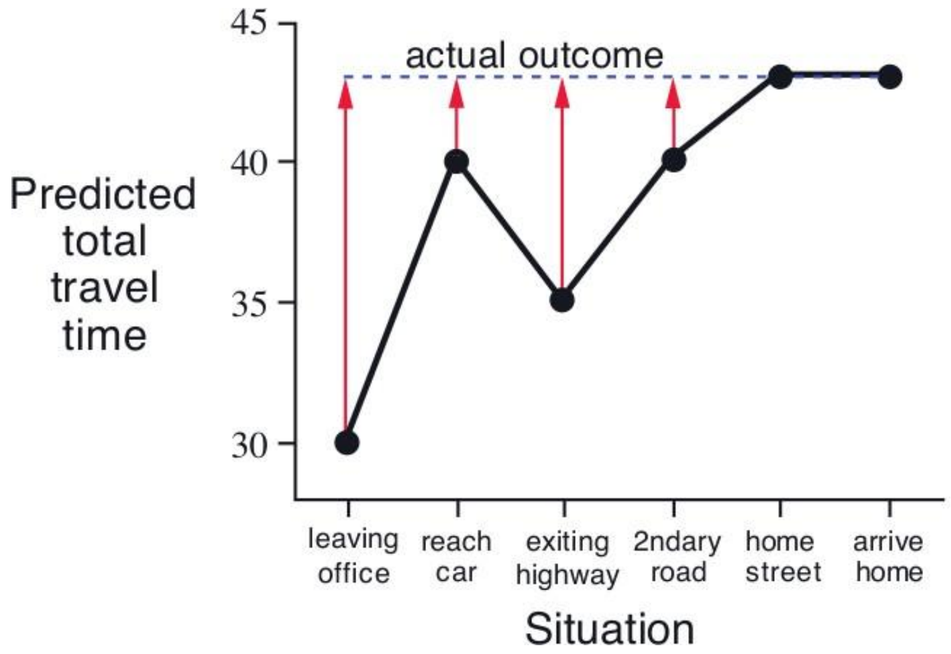


TD vs MC: Driving Home Example

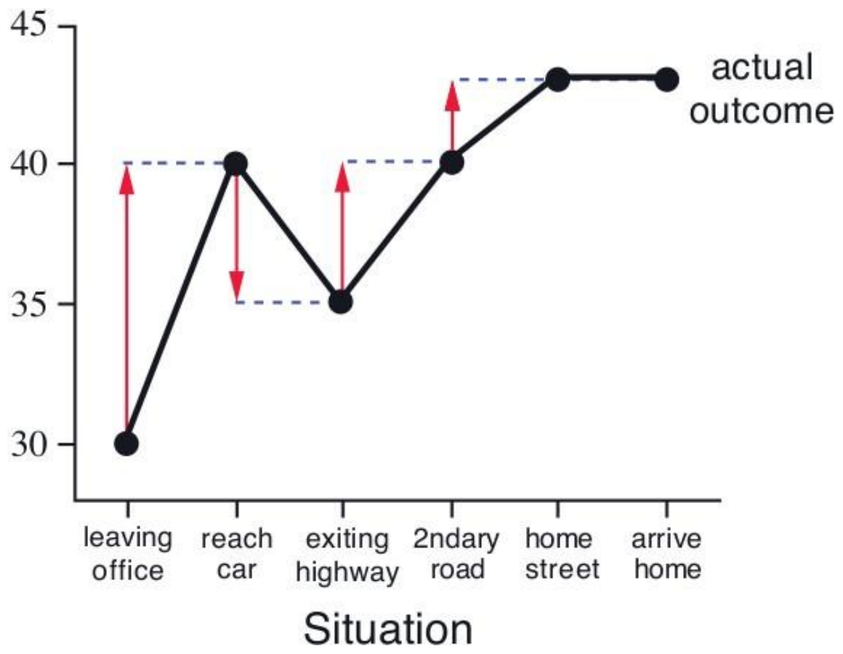
State	*Elapsed Time (minutes)*	*Predicted Time to Go*	*Predicted Total Time*
leaving office, friday at 6	0	30	30
reach car, raining	5	35	40
exiting highway	20	15	35
2ndary road, behind truck	30	10	40
entering home street	40	3	43
arrive home	43	0	43

TD vs MC: Driving Home Example

Monte Carlo



Temporal Difference



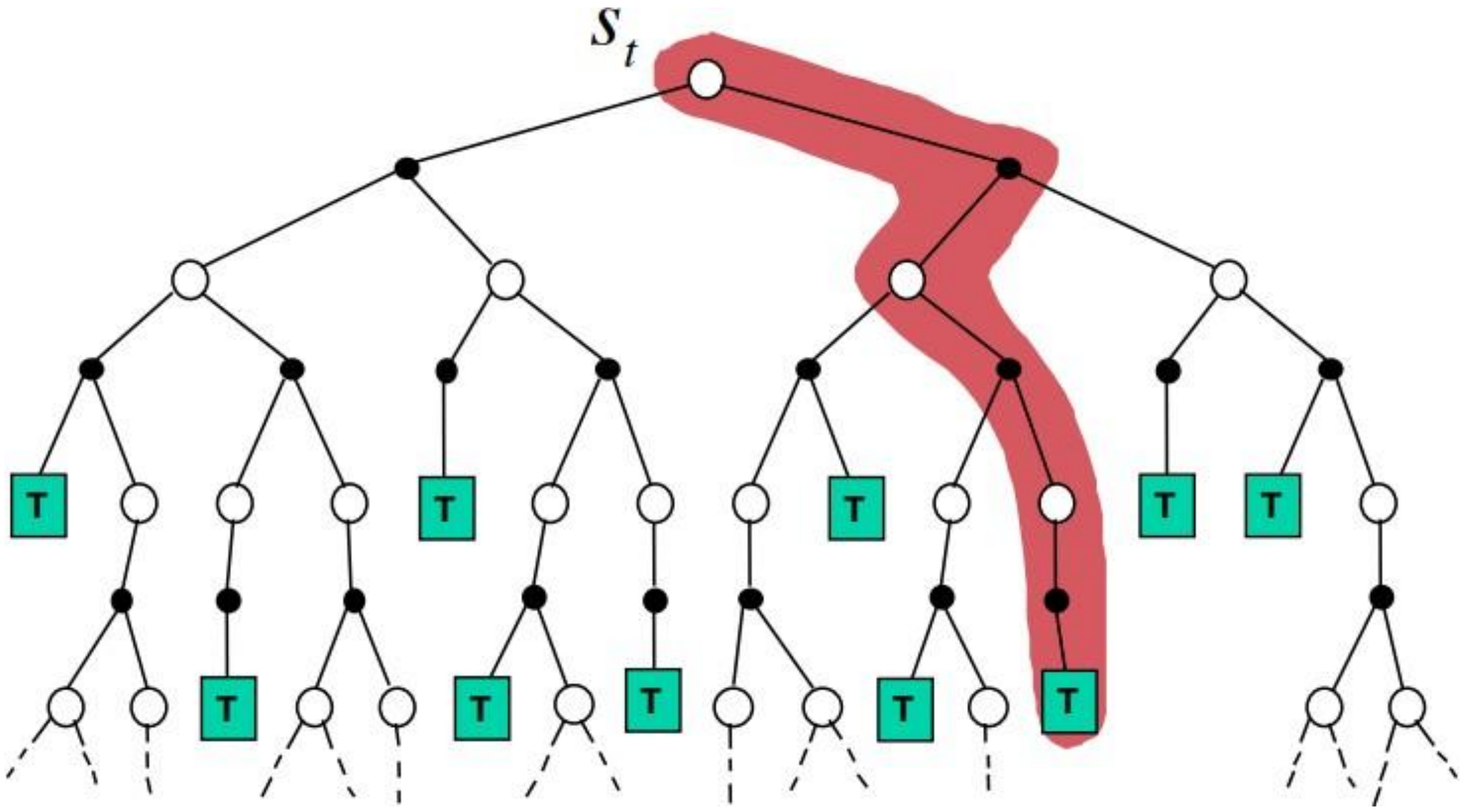
TD vs MC: Bias-Variance Tradeoff

- TD target is much lower variance than the return:
- Return $G_t = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-t-1} R_T$ is unbiased estimate of $Q_\pi(s_t, a_t)$
- True TD target $R_{t+1} + \gamma Q_\pi(s_{t+1}, a_{t+1})$ is unbiased estimate of $Q_\pi(s_t, a_t)$
- TD target $R_{t+1} + \gamma q(s_{t+1}, a_{t+1})$ is biased estimate of $Q_\pi(s_t, a_t)$
- TD target is much lower variance than the return:
  - **Return** depends on many random actions, transitions, rewards
  - **TD target** depends on one random action, transition, reward

- Monte-Carlo Methods: **high variance, no bias**
- TD-Обучение: **low variance, has bias**

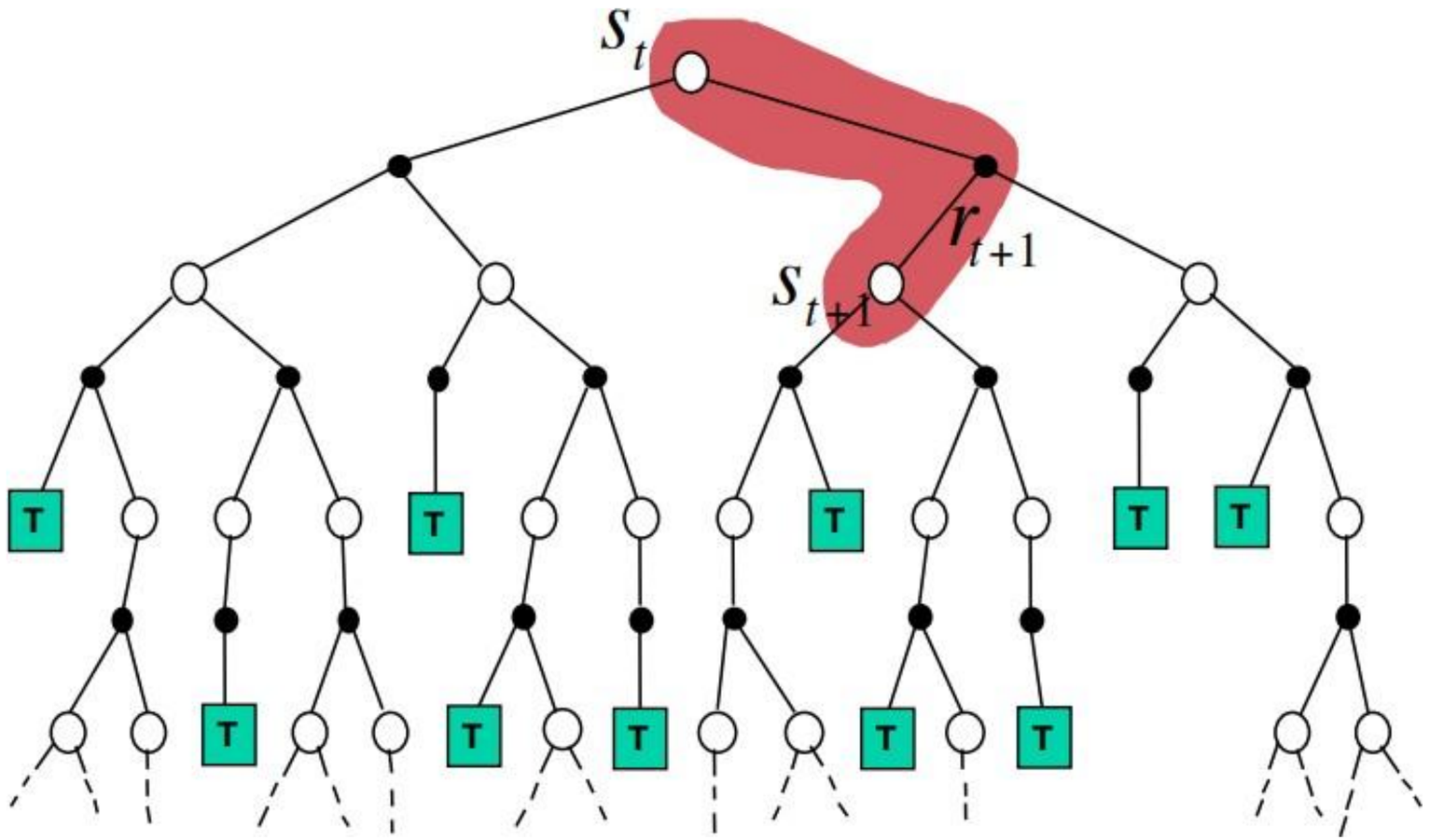
Monte-Carlo Backup

$$q(s_t, a_t) \leftarrow q(s_t, a_t) + \alpha(G_t - q(s_t, a_t))$$



Temporal-Difference Backup

$$q(s_t, a_t) \leftarrow q(s_t, a_t) + \alpha(r_t + \gamma q(s_{t+1}, a_{t+1}) - q(s_t, a_t))$$



N-step Returns

Consider the following n-step returns for n = 1, 2, ...:

(TD: SARSA) $n = 1 \quad G_t^{(1)} = R_{t+1} + \gamma Q(S_{t+1}, A_{t+1})$

$n = 2 \quad G_t^{(2)} = R_{t+1} + \gamma R_{t+2} + \gamma^2 Q(S_{t+2}, A_{t+2})$

$\vdots \qquad \vdots$

(MC) $n = \infty \quad G_t^{(\infty)} = R_{t+1} + \gamma R_{t+2} + ... + \gamma^{T-t-1} R_T$

n-step Temporal Difference Learning:

$$q(s_t, a_t) \leftarrow q(s_t, a_t) + \alpha(G_t^{(n)} - q(s_t, a_t))$$

Recap: Q-learning

Bellman optimality equation for Q^*: $\forall s, a$:

$$Q^*(s,a) := r(s,a) + \gamma \mathbb{E}_{s' \sim p(s'|s,a)} \max_{a'} Q^*(s',a')$$

Optimal policy:

$$\pi^*(s) := \operatorname*{argmax}_a Q^*(s,a)$$

Q-learning can learn Q^* from trial and error in *finite* MDPs: $|\mathcal{S}| \ll \infty, |\mathcal{A}| \ll \infty$

Important!

Q-learning is off-policy

Recap: Q-learning update intuition

For transition (s, a, r, s'):

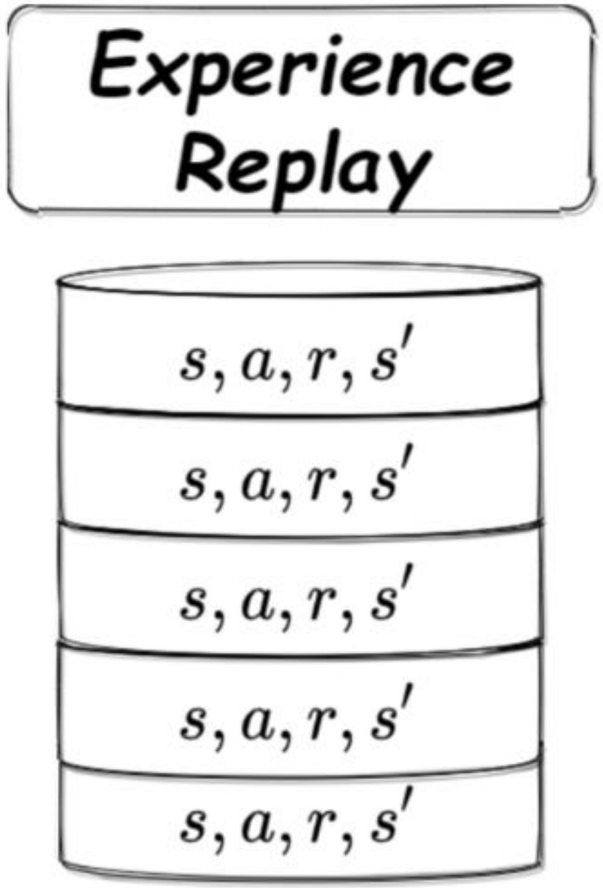
$$Q_{k+1}(s,a) \leftarrow Q_k(s,a) + \alpha_k \underbrace{\left(\overbrace{r + \gamma \max_{a'} Q_k(s', a')}^{\text{Bellman target}} - Q_k(s,a) \right)}_{\text{temporal difference}}$$

s, a — considered **fixed**, can be chosen arbitrarily;

(!) *each pair must be updated infinite times;*

$s' \sim p(s' \mid s, a)$ — **random variable** from interaction experience.

Experience Replay



Collect all experienced transitions in **replay buffer**!

On each training step sample **uniformly** random transition from experience replay and perform update using it.

✓ allows reuse of experience;

✓ allows training on expert data;

× sample s' comes from empirical approximation:

$$s' \sim \hat{p}(s' \mid s, a) \approx p(s' \mid s, a)$$

✓ smoothes out with size of experience replay!

Q-learning with experience replay

Q-learning (online)

Initialize $Q(s, a)$ arbitrarily;

observe s_0;

for $k = 0, 1, 2 \ldots$

- take action $a_k \sim \varepsilon$-greedy$(Q(s_k, a))$;
- observe r_k, s_{k+1};
- $y := r_k + \gamma \max_{a_{k+1}} Q(s_{k+1}, a_{k+1})$;
- $Q(s_k, a_k) \leftarrow (1 - \alpha_k) Q(s_k, a_k) + \alpha_k y$

Q-learning (with replay buffer)

Initialize $Q(s, a)$ arbitrarily, $\mathcal{D} = \varnothing$;

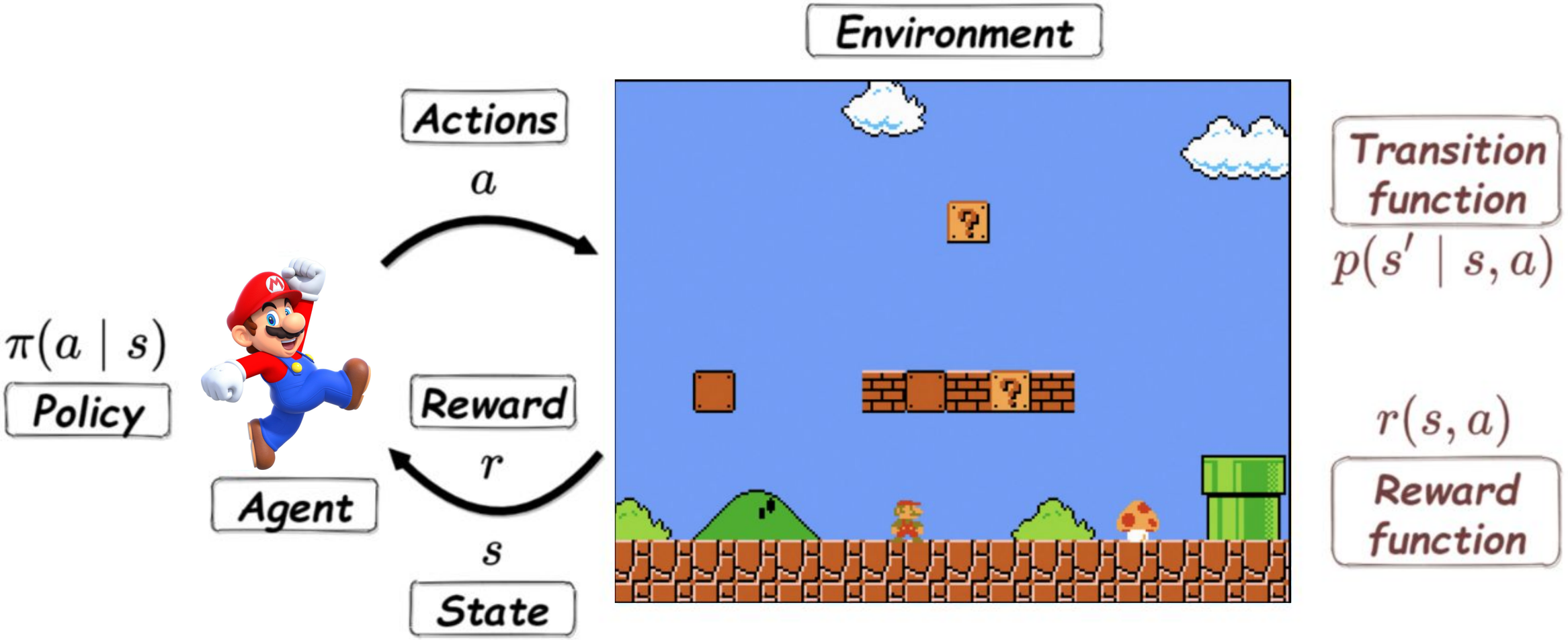
observe s_0;

for $k = 0, 1, 2 \ldots$

- take action $a_k \sim \varepsilon$-greedy$(Q(s_k, a))$;
- observe r_k, s_{k+1};
- store (s_k, a_k, r_k, s_{k+1}) in $\mathcal{D}$;
- sample (s, a, r, s') from $\mathcal{D}$;
- $y := r + \gamma \max_{a'} Q(s', a')$;
- $Q(s, a) \leftarrow (1 - \alpha_k) Q(s, a) + \alpha_k y$

Deep RL

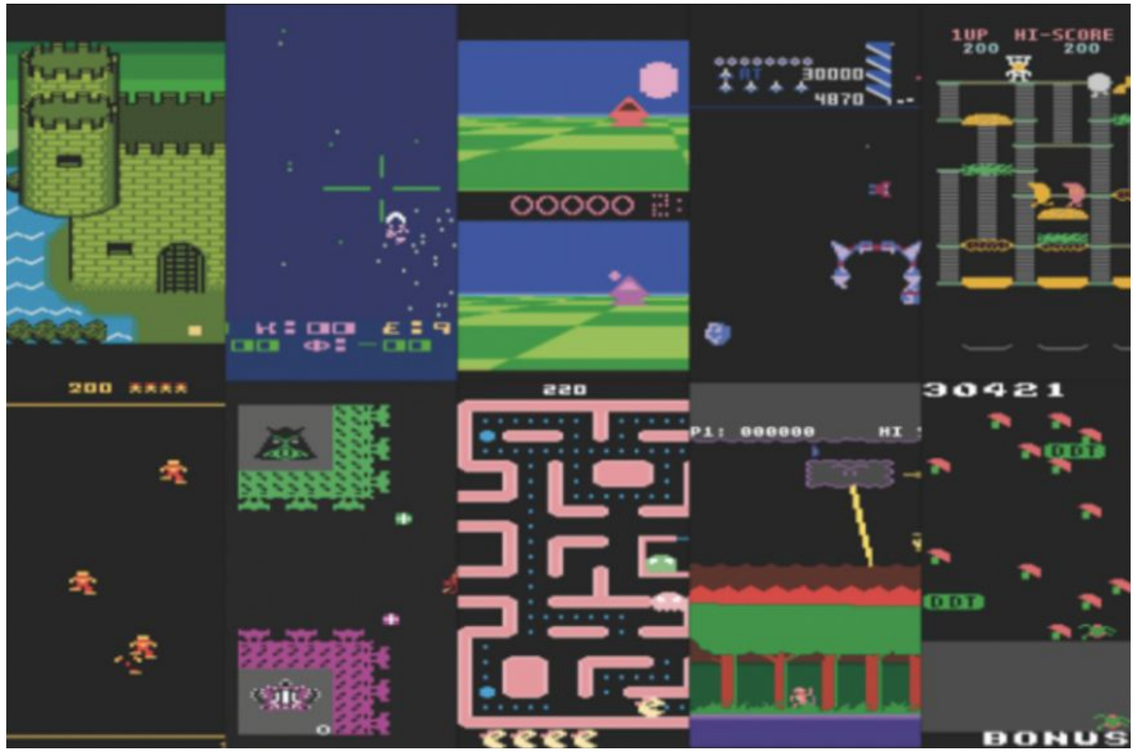
Benchmarks: Video Games



Atari Games

Setup: $|\mathcal{S}| \ll \infty, |\mathcal{A}| \ll \infty$

- 57 various games
- Only screen image as input.
- No game-specific features.
- Finite-state case... not quite finite.
- $|\mathcal{A}| \leqslant 18$



Approximate $Q^*(s, a)$ with neural network!

Is Atari game a MDP?

Practical notes: preprocessing

Action selection frequency:

- Framestack;
- Frameskip;
- MaxAndSkip;
- (sometimes) Sticky actions;

Atari-specific preprocessing:

- EpisodicLife;
- FireReset;

Standard tricks:

- Crop image;
- Rescale (often to 84x84);
- Grayscale;

Reward preprocessing:

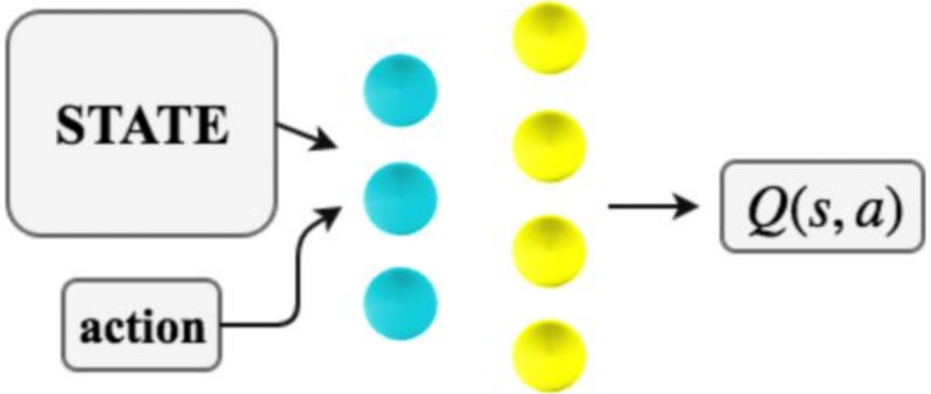
(!) Clip reward to $\{-1, 0, 1\}$;

Important!

No hyperparameter tuning for each specific game!

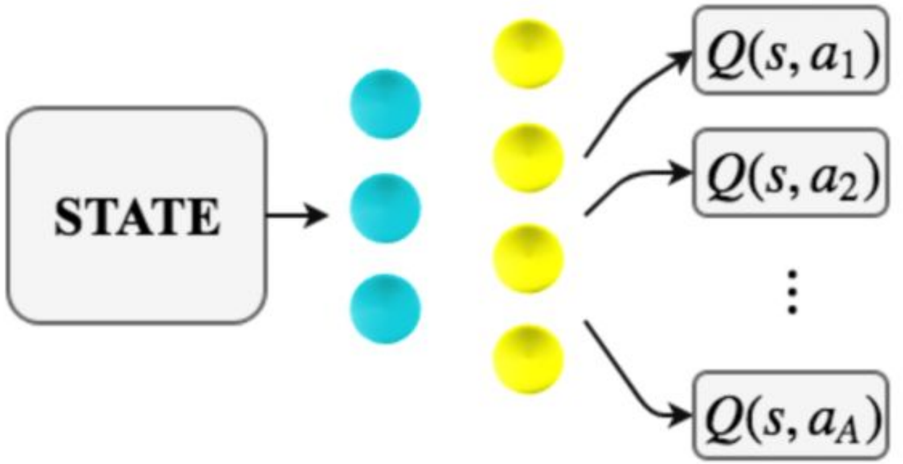
Deep Q-network

Option 1



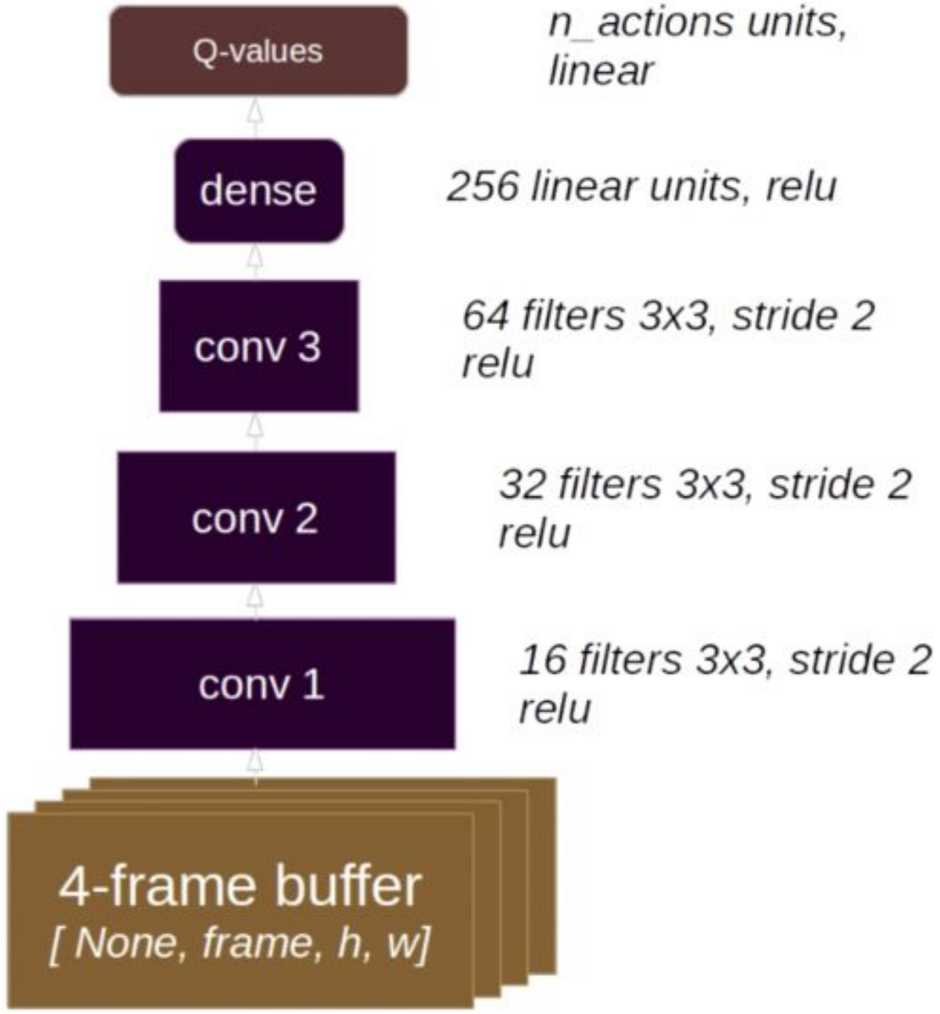
× $\underset{a}{\operatorname{argmax}}\, Q(s, a, \theta)$ — expensive!

Option 2



✓ $\underset{a}{\operatorname{argmax}}\, Q(s, a, \theta)$ — one forward pass.

Deep Q-network: architecture



- 3-4 convolutional layers followed by 1-2 dense layers;
- stride > 1 for size reduction; linear layers still wide;

Think twice before using:

- × max pooling
- × batch normalization
- × dropout

How to train your network?

Given:

- dataset (x, y), where x — input, y — output;
- loss function $\text{Loss}(y, \hat{y})$;
- parametric family $\hat{y}(x, \theta)$;

in **supervised learning** we optimized

$$\sum_{(x,y)} \text{Loss}(y, \hat{y}(x, \theta)) \to \min_{\theta}$$

using techniques based on stochastic gradient descent.

In **reinforcement learning**:

- we have to collect all data ourself
  - some exploration-exploitation technique is required, e. g. ε-greedy;
  - the data is not i.i.d.
- what is the **target** y?
- what is the loss function?

Important

Q-learning is SGD.

Idea: Approximate Dynamic Programming

Consider the following **regression task**:

- s, a is input;
- $y \in \mathbb{R}$ is target:

$$y(s,a) := r(s,a) + \gamma \mathbb{E}_{s'} \max_{a'} Q(s', a', \theta_k)$$

- MSE loss function: $\text{Loss}(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$

$$\frac{1}{2}\mathbb{E}_{(s,a,y)}(y - Q(s,a,\theta_{k+1}))^2 \to \min_{\theta_{k+1}}$$

Looking at the gradient

$$\nabla_\theta \frac{1}{2}\left(y - Q(s,a,\theta)\right)^2 = \underbrace{\left(y - Q(s,a,\theta)\right)}_{\text{temporal difference}} \overbrace{\nabla_\theta Q(s,a,\theta)}^{\substack{\text{how to increase} \\ \text{output value}}} =$$

$$= \left(r + \gamma \mathbb{E}_{s'} \max_{a'} Q(s', a', \theta_k) - Q(s,a,\theta)\right) \nabla_\theta Q(s,a,\theta) \approx$$

$$\approx \left(r + \gamma \max_{a'} Q(s', a', \theta_k) - Q(s,a,\theta)\right) \nabla_\theta Q(s,a,\theta),$$

where $s' \sim p(s' \mid s, a)$.

Deep Q-learning

Consider the following **regression task**:

- s, a is input;
- $y \in \mathbb{R}$ is target:

$$y(s, a) := r + \gamma \max_{a'} Q(s', a', \theta_k)$$

  where $r = r(s, a), s' \sim p(s' \mid s, a)$;
- MSE loss function: $\text{Loss}(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$

Important!

We can do the same with other Bellman equations (e.g. for V^π)!

Intuition: why this is Q-learning

Consider the following setting:

- *tabular* parametric family $Q(s, a, \theta) := \theta_{s,a}$;
- define Bellman target as $y := r(s, a) + \gamma \max_{a'} Q(s', a', \theta)$;

$$\theta \leftarrow \theta - \alpha \left(y - Q(s, a, \theta)\right) \nabla_\theta Q(s, a, \theta)$$

$$\theta \leftarrow \theta - \alpha \left(y - Q(s, a, \theta)\right) \mathrm{OHE}\left(s, a\right)$$

$$\theta_{s,a} \leftarrow \theta_{s,a} - \alpha \left(y - Q(s, a, \theta)\right)$$

Why stop gradients from target?

$$\mathbb{E}_{s,a,r,s'}\Big(\underbrace{r+\gamma\max_{a} Q(s',a')}_{\text{fixed; no gradient flow!}}-Q(s,a)\Big)^2\to\min_{Q}$$

This is a consequence of **causality**:

- ✓ match *past* estimations to *future*;
- ✗ do not match *future* estimations to *past*;

Note that unlike supervised learning:

- target «depends» on our own model;
- it is still *better* that we currently have; («provides direction»)

$Q^*(s,a)$ $\quad r+\gamma\max_{a'} Q^*(s'=A,a')$

Target network

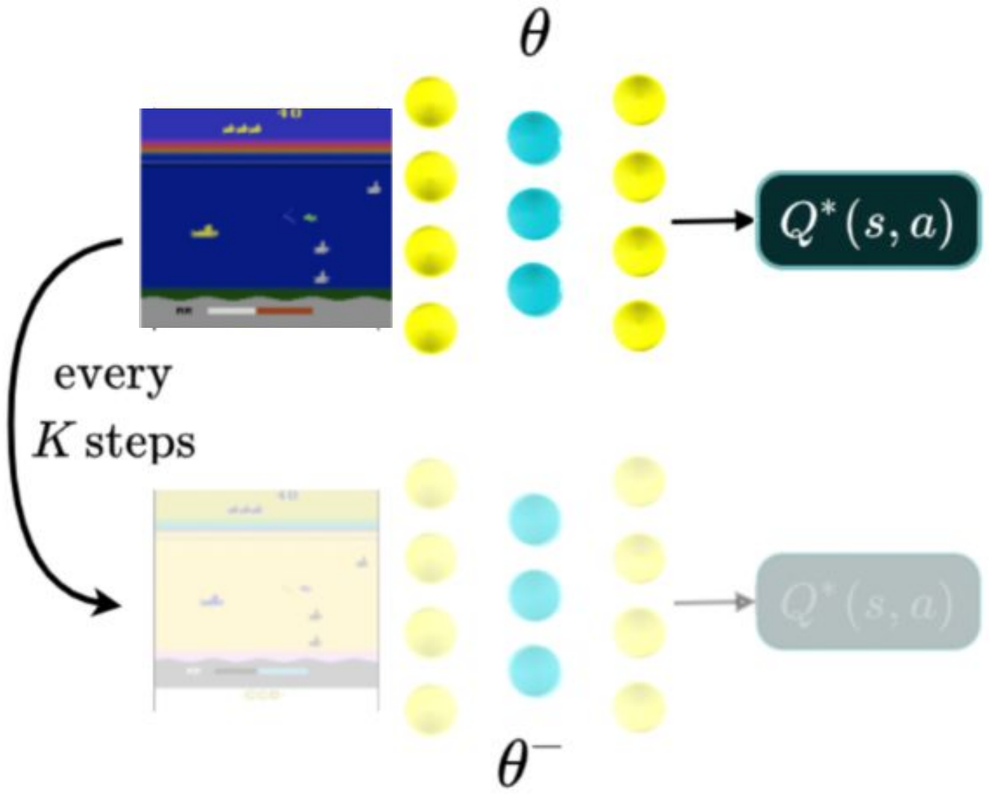
For how long to solve one regression task?

- Till convergence
  - × too long
- One SGD iteration
  - × completely unstable :(

✓ ≈1000 SGD iterations

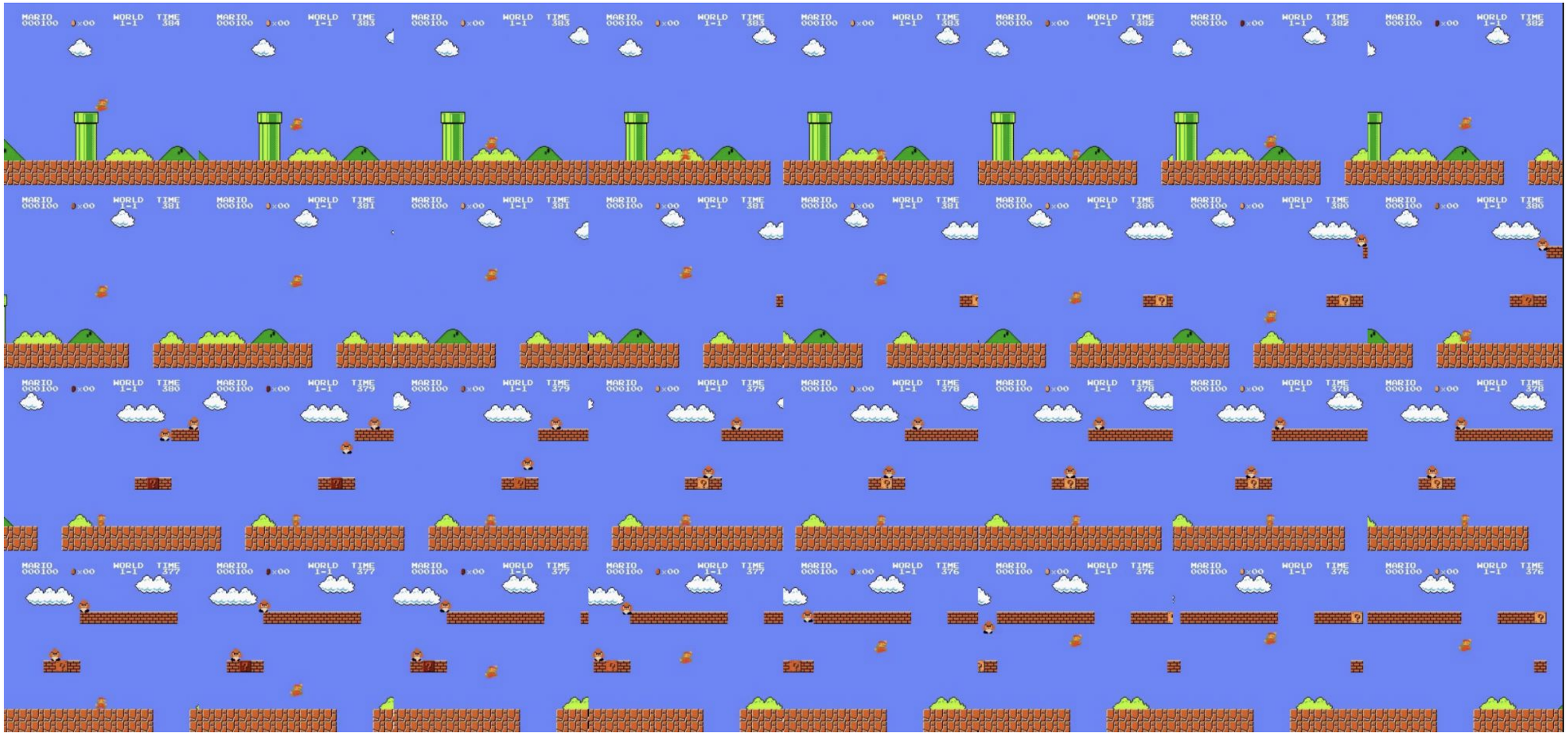
a) $\theta^- \leftarrow \theta$ every K SGD iterations

b) $\theta^- \leftarrow (1-\beta)\theta^- + \beta\theta$



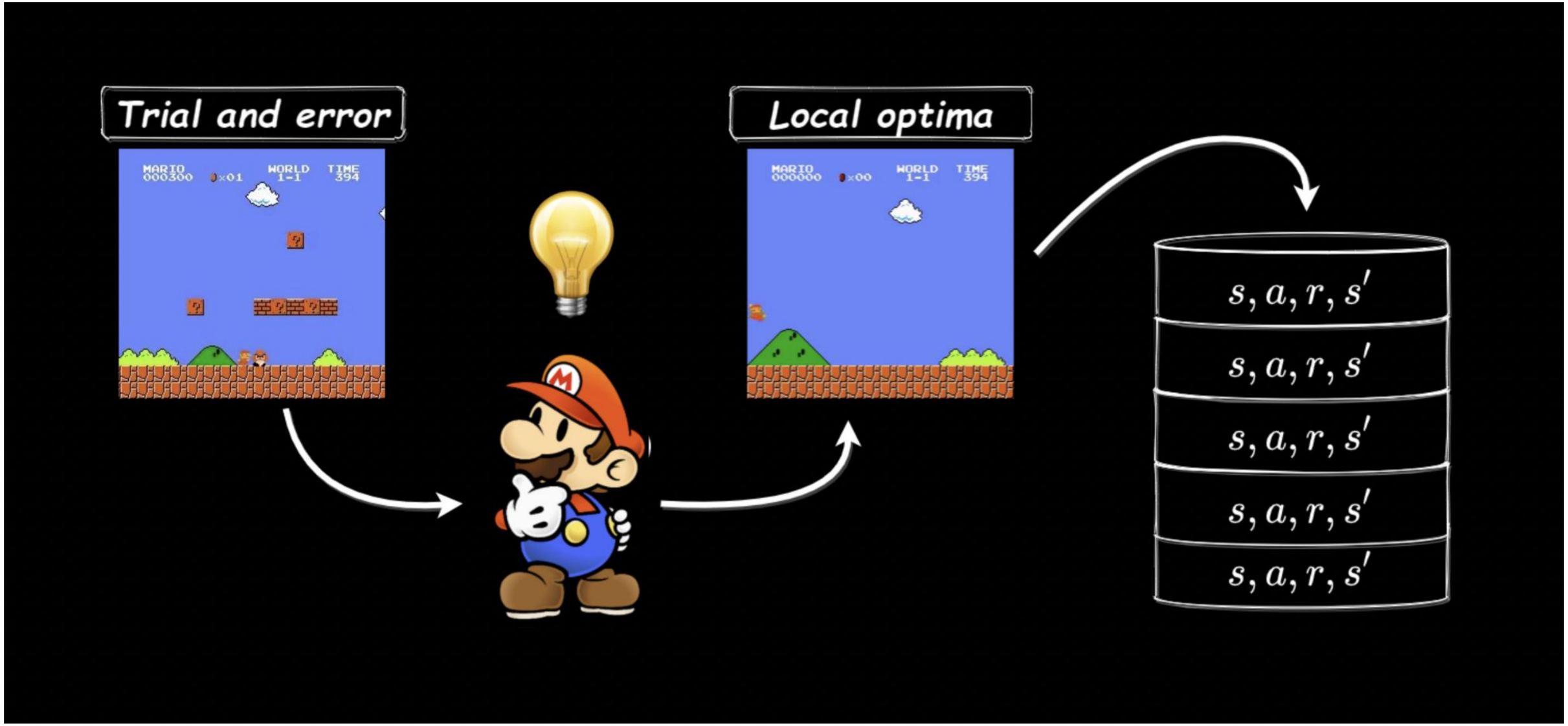
Store a copy of your old Q-network $Q(s, a, \theta^-)$

Distributional shift issue



Experience Replay

Do we still need exploration?



Full alrogithm

Deep Q-learning

Initialize $Q(s, a, \theta)$ arbitrarily, $\theta^- := \theta$, $\mathcal{D} = \varnothing$;

observe s_0;
for $k = 0, 1, 2 \ldots$

- take action $a_k \sim \varepsilon$-greedy$(Q(s_k, a, \theta))$;
- observe $r_k, s_{k+1}, \text{done}_{k+1}$, store $(s_k, a_k, r_k, s_{k+1}, \text{done}_{k+1})$ in $\mathcal{D}$;
- sample batch of transitions $\mathbb{T} := (s, a, r, s', \text{done})$ from $\mathcal{D}$;
- $y(\mathbb{T}) := r + \gamma(1 - \text{done}) \max_{a'} Q(s', a', \theta^-)$;
- perform a step of gradient descent:

$$\theta \leftarrow \theta - \frac{\alpha}{B} \sum_{\mathbb{T}} \nabla_\theta \left(Q(s, a, \theta) - y(\mathbb{T})\right)^2$$

- update target network: if $k \bmod K = 0$: $\theta^- \leftarrow \theta$

Recap

- Monte-Carlo methods (sample + epsilon greedy)
- Temporal difference learning (SARSA)
- Q-learning
- On-policy/off-policy
- Replay buffer
- DQN