Methods comparison for graphs

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Main idea of Riemannian approach

Let we have EEG trial $X \in \mathbb{R}^{C \times T}$, a record of C electrodes with T time samples. And the record follows multivariate normal model, i.e. $X \sim \mathcal{N}(0, \Sigma)$, where $\Sigma \in \mathbb{R}^{C \times C}$.
Information theory gives us metric, the Fisher information, for probability distributions. As probability distributions belong on Riemann manifold, with such metric we can define Riemann distance δ_R between distributions. Let we have $X_1 \sim \mathcal{N}(0, \Sigma_1)$ and $X_2 \sim \mathcal{N}(0, \Sigma_2)$ the distance would be:

$$\delta_R(\Sigma_1, \Sigma_2) = || \log(\Sigma_1^{-1/2} \Sigma_2 \Sigma_1^{-1/2}) ||_F = \left[\sum_{c=1}^{C} \log^2 \lambda_c \right]^{1/2}, \quad (1)$$

where λ_c - all eigenvalues of $\Sigma_1^{-1/2} \Sigma_2 \Sigma_1^{-1/2}$

Minimum Distance to Riemannian Mean (MDM)

Let we have dataset $X_i \sim \mathcal{N}(0, \Sigma_i)$ and corresponding labels $y_i \in \{1 : T_c\}$. The training process is computing the covariance matrix for each T_c classes using Riemann geometric mean:

$$\overline{\Sigma}_K = \mathcal{G}(\Sigma_i | y_i = K) = \arg \min_{\Sigma \in P(n)} \sum_i \delta_R^2(\Sigma, \Sigma_i), \tag{2}$$

where $K \in [1 : T_c]$ - class label
The inference is finding of minimum distance, i.e.

$$\hat{y} = \arg \min_K \delta_R(\overline{\Sigma}_K, \Sigma) \tag{3}$$

Other methods[1] introduces more complex representation of covariance matrices.

[1] A Plug&Play P300 BCI Using Information Geometry

Comparison

Strengths	Weaknesses
Riemannian approach	
Less data to learn Easy to understand	Complexity depends on C No connection between electrodes Only classification?
Graph Laplacian approach	
Uses connection	Uses diff equations Complex inference

Table: Comparison of approaches