

# Variational Canonical Correlation Analysis

## Week 1

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# Deep Variational Canonical Correlation Analysis<sup>1</sup>

**Challenge:** it is hard to satisfy the constraints set by Deep Canonical

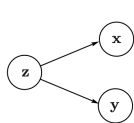
**Solution:** extends the latent variable interpretation to nonlinear observation models.

**VCCA-private:** the challenge is that large variations in the input space can not be explained by  $\mathbf{z}$ .

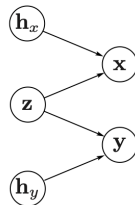
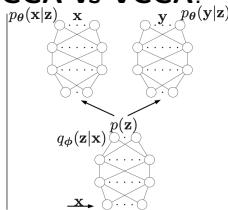
**DCCA:**

$$\begin{aligned} \max_{f, g, U, V} \quad & \text{tr}(U^\top f(X)g(Y)^\top V) \\ \text{s.t.} \quad & U^\top (f(X)f(X)^\top) U = \\ & V^\top (g(Y)g(Y)^\top) V = N \cdot I. \end{aligned}$$

**Probabilistic CCA vs VCCA:**



$$\begin{aligned} \mathbf{z} &\sim \mathcal{N}(\mathbf{0}, \mathbf{I}) \\ \mathbf{x}|\mathbf{z} &\sim \mathcal{N}(\mathbf{W}_x \mathbf{z}, \Phi_x) \\ \mathbf{y}|\mathbf{z} &\sim \mathcal{N}(\mathbf{W}_y \mathbf{z}, \Phi_y) \end{aligned}$$



**Results:** outstanding performance with reduced training efforts.

<sup>1</sup>Wang W. et. al, Deep Variational Canonical Correlation Analysis, 2017

# Variational Interpretable Deep Canonical Correlation Analysis<sup>2</sup>

**Challenge:** it is challenging to build an interpretable model understanding.

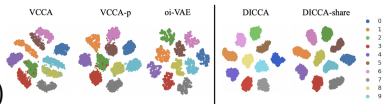
**Solution:** interpretable sparsity prior.

**Sparsity prior**

$$Z \sim N(0, I), Z^m \sim N(0, I), \\ X^m \sim N(f_m(\Lambda^m Z + W^m Z^m), \Psi^m)$$

$$\gamma_{mj}^2 \sim \Gamma((d_m + 1)/2, \lambda^2/2)$$

$$\Lambda_{\cdot,j}^{(m)}, W_{\cdot,j}^{(m)} \sim N(0, \gamma_{mj}^2 I)$$



The learned features of the images are well separated.

Method	View 1 MSE (STD)	View 2 MSE (STD)
oi-VAE	0.059 (0.009)	0.172 (0.009)
DPCCA	0.052 (0.012)	0.134 (0.003)
VCCA	0.023 (0.011)	0.088 (0.0042)
VCCA-p	0.024 (0.011)	0.084 (0.005)
DICCA (Ours)	<b>0.016 (0.005)</b>	<b>0.080 (0.005)</b>

The proposed method outperforms all existing baselines.

<sup>2</sup>Qui L. et. al, Variational Interpretable Deep Canonical Correlation Analysis, 2022