Time series reconstruction

Skorik Sergey

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Graph Wavelet Neural Network

Motivation

In masters[a] thesis examined the problem of decoding brain signals. As a decoder model, a graph neural network (GNN) was proposed, in which the graph Fourier transform is used as convolution. The disadvantage of this convolution is that the kernel does not have the property of being localized in space.

The study[b] introduces the wavelet transform on graphs.

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[a] Varenik Natalia. *Construction of a connectivity map of functional groups in the task of decoding brain signals*. `http://www.machinelearning.ru/wiki/images/b/b2/Varenik2022master_thesis.pdf`. [Online; accessed 20-October-2023]. 2022.

[b] David K Hammond, Pierre Vandergheynst, and Rémi Gribonval. "Wavelets on graphs via spectral graph theory". In: *Applied and Computational Harmonic Analysis* 30.2 (2011), pp. 129–150.

Graph Wavelet Neural Network

Graph Wavelet Neural Network

Based on this transform, study[a] presents graph wavelet neural network (GWNN).

[a] Bingbing Xu et al. "Graph wavelet neural network". In: *arXiv preprint arXiv:1904.07785* (2019).

Analytic graph diffusion framework

Theory

Let's consider a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. To model the spread of pathological tau species along the brain's structural network, we represent the regional tau burden as a time-varying graph signal vector $\boldsymbol{x}(t) = [x(v_i, t), v_i \in \mathcal{V}]$, $\boldsymbol{x(t)} \in \mathbb{R}^N$, $|\mathcal{V}| = N$. $\boldsymbol{x}(t)$ is the solution to a first-order PDE, usually referred to as the network diffusion equation:

$$\frac{\partial \boldsymbol{x}(t)}{\partial t} = -\beta \boldsymbol{L}\boldsymbol{x}(t). \tag{1}$$

Where $\boldsymbol{L} \in \mathbb{R}^{N \times N}$ is the normalized graph Laplacian matrix. To model active generation or clearance alongside passive spread, a source term, $\boldsymbol{s}(t)$, can be added to (1) leading to an inhomogeneous PDE:

$$\frac{\partial \boldsymbol{x}(t)}{\partial t} = -\beta \boldsymbol{L}\boldsymbol{x}(t) + \boldsymbol{s}(t). \tag{2}$$

Analytic graph diffusion framework[1]

Theory

Modulate this source term $\boldsymbol{s}(t)$ we can build various models.

- Linear source model $\boldsymbol{s}(t) = \boldsymbol{r}t$ with criterion $\Phi_{\text{LIN}}(\beta, \boldsymbol{r}) = \frac{1}{2}\|\boldsymbol{f}(\boldsymbol{x}_0, \beta, \boldsymbol{r}) - \boldsymbol{x}_t\|_2^2$
- Exponential source model $\boldsymbol{s}(t) = \boldsymbol{\alpha}(e^{\sigma t} - 1)$ and criterion $\Phi_{\text{EXP}}(\beta, \boldsymbol{\alpha}, \sigma) = \frac{1}{2}\|\boldsymbol{f}(\boldsymbol{x}_0, \beta, \boldsymbol{\alpha}, \sigma) - \boldsymbol{x}_t\|_2^2$

All gradients, $\boldsymbol{x}_0$ and other can be calculated analytically.

[1] Fan Yang et al. "Longitudinal predictive modeling of tau progression along the structural connectome". In: *Neuroimage* 237 (2021), p. 118126.