Distributed Newton-Type Methods with Communication Compression and Bernoulli Aggregation

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Outline

1. The Problem
2. Brief Comparison with Related Works
3. The 3 Special Newton-type Methods
4. Federated Newton Learn (FedNL)
5. Extensions (PP, LS, CR, BC)
6. Numerical Experiments

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The Problem

Strongly convex

clients/devices

2nd order smooth

$$\min_{x \in \mathbb{R}^d} \left\{ f(x) \stackrel{\text{def}}{=} \frac{1}{n} \sum_{i=1}^{n} f_i(x) \right\}$$

model parameters

Empirical risk/loss

Local training data

Local loss function $f_i(x) = \mathbb{E}_{\xi \sim \mathcal{D}_i} [f_\xi(x)]$

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Existing Approaches and their Disadvantages

First order methods

- ✖ Rates depend on the condition number
- ✖ Hard to find optimal stepsizes

Second order methods

- ✖ Rates depend on the condition number
- ✖ Communication cost is high

Recently, [Islamov et al, 2021], [Qian et al., 2022], [Safaryan et al., 2022] develop new theory of second order methods for Federated and Distributed Learning solving most of the existing issues

GOAL

1. **Unify theory of these works into one creating more general class of compression operators**
2. **Improve the computational cost of the methods**

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1. The Problem
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3. Ideal Newton-type Method
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Newton Star Method

$$x^{k+1} = x^k - \left(\frac{1}{n}\sum_{i=1}^{n} \boxed{\nabla^2 f_i(x^*)}\right)^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} \boxed{\nabla f_i(x^k)}\right)$$

Can NOT be computed locally

Single communication of $\mathcal{O}(d^2)$

Can be computed locally

Easy to communicate: $\mathcal{O}(d)$

Rustem Islamov, Xun Qian and Peter Richtárik
Distributed second order methods with fast rates and compressed communication,
ICML 2021.

- ✔ $\mathcal{O}(d)$ communication cost per round
- ✘ Implementability in practice
- ✔ Local quadratic convergence rate independent of the condition number

Hessian Lipschitz constant

$$\|x^{k+1} - x^*\| \leq \frac{L}{2\mu}\|x^k - x^*\|^{\boxed{2}}$$

Strong convexity constant

Local quadratic rate

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Learning the Optimal Hessian Matrices

Newton Star

$$x^{k+1} = x^k - \left(\frac{1}{n}\sum_{i=1}^{n} \nabla^2 f_i(x^*)\right)^{-1} \nabla f(x^k)$$

Idea! Learn the optimal Hessians $\nabla^2 f_i(x^*)$ in communication efficient manner:

(i) $\mathbf{H}_i^k \to \nabla^2 f_i(x^*)$ as $k \to \infty$ $\quad$ (ii) $\mathbf{H}_i^{k+1} - \mathbf{H}_i^k$ is compressed

$$x^{k+1} = x^k - \left(\frac{1}{n}\sum_{i=1}^{n} \mathbf{H}_i^k\right)^{\boxed{-1}} \nabla f(x^k)$$

$$= x^k - \left(\mathbf{H}^k\right)^{\boxed{-1}} \nabla f(x^k)$$

Rustem Islamov, Xun Qian and Peter Richtárik
Distributed second order methods with fast rates and compressed communication,
ICML 2021.

Newton-3PC: Two Options for Updating the Global Model

Option 1

$$x^{k+1} = x^k - \left(\left[\mathbf{H}^k\right]_{\boxed{\mu}}\right)^{-1} \nabla f(x^k)$$

Projection onto the cone of positive definite matrices

Option 2

$$x^{k+1} = x^k - \left(\mathbf{H}^k + \boxed{l^k}\mathbf{I}\right)^{-1} \nabla f(x^k)$$

$$l^k = \frac{1}{n}\sum_{i=1}^{n} \|\mathbf{H}_i^k - \nabla^2 f_i(x^k)\|_{\mathrm{F}}$$

Newton-3PC: New Hessian Learning Technique

$$\mathbf{H}_i^{k+1} = \boxed{\mathcal{C}_{\mathbf{H}_i^k, \nabla f_i(x^k)}}(\nabla f_i(x^{k+1}))$$

3PC compressor

Definition: 3PC compressor

$$\mathcal{C}_{\mathbf{H},\mathbf{Y}}(\mathbf{X}) : \underbrace{\mathbb{R}^{d\times d}}_{\mathbf{H}\in} \times \underbrace{\mathbb{R}^{d\times d}}_{\mathbf{Y}\in} \times \underbrace{\mathbb{R}^{d\times d}}_{\mathbf{X}\in} \to \mathbb{R}^{d\times d}$$

$$\mathbb{E}\left[\|\mathcal{C}_{\mathbf{H},\mathbf{Y}}(\mathbf{X}) - \mathbf{X}\|^2\right] \le (1-A)\|\mathbf{H} - \mathbf{Y}\|^2 + B\|\mathbf{X} - \mathbf{Y}\|^2$$

3PC compressor: Examples

Contractive compressor

$$\mathbb{E}\left[\|\mathcal{C}(\mathbf{X})-\mathbf{X}\|^2\right] \leq (1-\alpha)\|\mathbf{X}\|^2$$

$$\alpha \in (0,1]$$

Top-K satisfies this with $\alpha = \frac{K}{d^2}$

EF21 compressor

$$\mathcal{C}_{\mathbf{H}}(\mathbf{X}) = \mathbf{H} + \mathcal{C}(\mathbf{X}-\mathbf{H})$$

Contractive comp.

CBAG compressor

$$\mathcal{C}_{\mathbf{H}}(\mathbf{X}) = \begin{cases} \mathbf{H} + \mathcal{C}(\mathbf{X}-\mathbf{H}) & \text{with prob.} p \\ \mathbf{H} & \text{with prob.} 1-p \end{cases}$$

CLAG compressor

$$\mathcal{C}_{\mathbf{H},\mathbf{Y}}(\mathbf{X}) = \begin{cases} \mathbf{H} + \mathcal{C}(\mathbf{X}-\mathbf{H}) & \text{if } \|\mathbf{X}-\mathbf{H}\|^2 > \zeta\|\mathbf{X}-\mathbf{Y}\|^2 \\ \mathbf{H} & \text{otherwise} \end{cases}$$

Newton-3PC: Pseudocode

Algorithm 1 Newton-3PC (Newton's method with three point compressor)

Input: $x^0 \in \mathbb{R}^d$, $\mathbf{H}_1^0, \dots, \mathbf{H}_n^0 \in \mathbb{R}^{d\times d}$, $\mathbf{H}^0 := \frac{1}{n}\sum_{i=1}^n \mathbf{H}_i^0$, $l^0 = \frac{1}{n}\sum_{i=1}^n \|\mathbf{H}_i^0 - \nabla^2 f_i(x^0)\|_{\mathrm{F}}$.
on server
    Option 1: $x^{k+1} = x^k - [\mathbf{H}^k]_\mu^{-1}\nabla f(x^k)$
    Option 2: $x^{k+1} = x^k - [\mathbf{H}^k + l^k\mathbf{I}]^{-1}\nabla f(x^k)$
    Broadcast x^{k+1} to all nodes
for each device $i = 1, \dots, n$ in parallel **do**
    Get x^{k+1} and compute local gradient $\nabla f_i(x^{k+1})$ and local Hessian $\nabla^2 f_i(x^{k+1})$
    Apply 3PC and update local Hessian estimator to $\mathbf{H}_i^{k+1} = \mathcal{C}_{\mathbf{H}_i^k, \nabla^2 f_i(x^k)}\left(\nabla^2 f_i(x^{k+1})\right)$
    Send $\nabla f_i(x^{k+1})$, $\mathbf{H}_i^{k+1}$ and $l_i^{k+1} := \|\mathbf{H}_i^{k+1} - \nabla^2 f_i(x^{k+1})\|_{\mathrm{F}}$ to the server
end for
on server
    Aggregate $\nabla f(x^{k+1}) = \frac{1}{n}\sum_{i=1}^n \nabla f_i(x^{k+1}), \mathbf{H}^{k+1} = \frac{1}{n}\sum_{i=1}^n \mathbf{H}_i^{k+1}, l^{k+1} = \frac{1}{n}\sum_{i=1}^n l_i^{k+1}$

Newton-3PC: Assumptions

1. For a given input $x \in \mathbb{R}^d$, clients can compute gradient $\nabla f_i(x)$ and Hessian $\nabla^2 f_i(x)$.
2. The average loss function $f(x)$ is μ-strongly convex for some $\mu \geq 0$.
3. Local loss functions $f_i(x)$ have Lipschitz continuous Hessians with constant $L \geq 0$, i.e.,

$$\|\nabla^2 f_i(x) - \nabla^2 f_i(y)\| \leq L\|x - y\|$$

   holds for any $x, y \in \mathbb{R}^d$.

Newton-3PC: Local Convergence Theory

Local rate

$$\|x^0 - x^*\| \leq \frac{\mu}{\sqrt{D}}$$

$$\|x^k - x^*\|^2 \leq \frac{1}{2^k}\|x^0 - x^*\|^2$$

Local (fixed) linear rate

Lyapunov function

$$\Phi^k := \mathcal{H}^k + 6\left(\frac{1}{A} + 3AB\right) L_{\mathrm{F}}^2 \left\|x^k - x^*\right\|^2$$

where $\mathcal{H}^k := \frac{1}{n}\sum_{i=1}^{n} \|\mathbf{H}_i^k - \nabla^2 f_i(x^*)\|_{\mathrm{F}}^2$

$$\mathbb{E}\left[\boxed{\Phi^k}\right] \leq \left(1 - \min\left\{\frac{\boxed{A}}{2}, \frac{1}{3}\right\}\right)^k \boxed{\Phi^0}$$

Local linear rates

$x^k \to x^*$

$\mathbf{H}_i^k \to \nabla^2 f_i(x^*)$ for all $i \in [n]$

Constant depending on the choice of the compressor and stepsize

$$\mathbb{E}\left[\frac{\left\|x^{k+1} - x^*\right\|^2}{\left\|x^k - x^*\right\|^2}\right] \leq \left(1 - \min\left\{\frac{\boxed{A}}{2}, \frac{1}{3}\right\}\right)^k \left(C + \frac{AD}{12(1+3AB)L_{\mathrm{F}}^2}\right)\frac{\Phi^0}{\mu^2}$$

Local superlinear rate

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Special Compressors to Save Time

CBAG compressor

$$\mathcal{C}_{\mathbf{H}}(\mathbf{X}) = \begin{cases} \mathbf{H} + \mathcal{C}(\mathbf{X} - \mathbf{H}) & \text{with prob.} p \\ \mathbf{H} & \text{with prob.} 1 - p \end{cases}$$

We need to compute X only with probability p

Sketch&Project

$$\mathcal{C}(\mathbf{X}) = \mathbf{S}(\mathbf{S}^\top \mathbf{S})^\dagger \mathbf{S}^\top \mathbf{X} \quad \text{where } \mathbf{S} \in \mathbb{R}^{d \times \tau} \sim \mathcal{D} \text{ and } \tau \ll d$$

We need to compute Hessian-vector products only

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Experiments: Regularized Logistic Regression

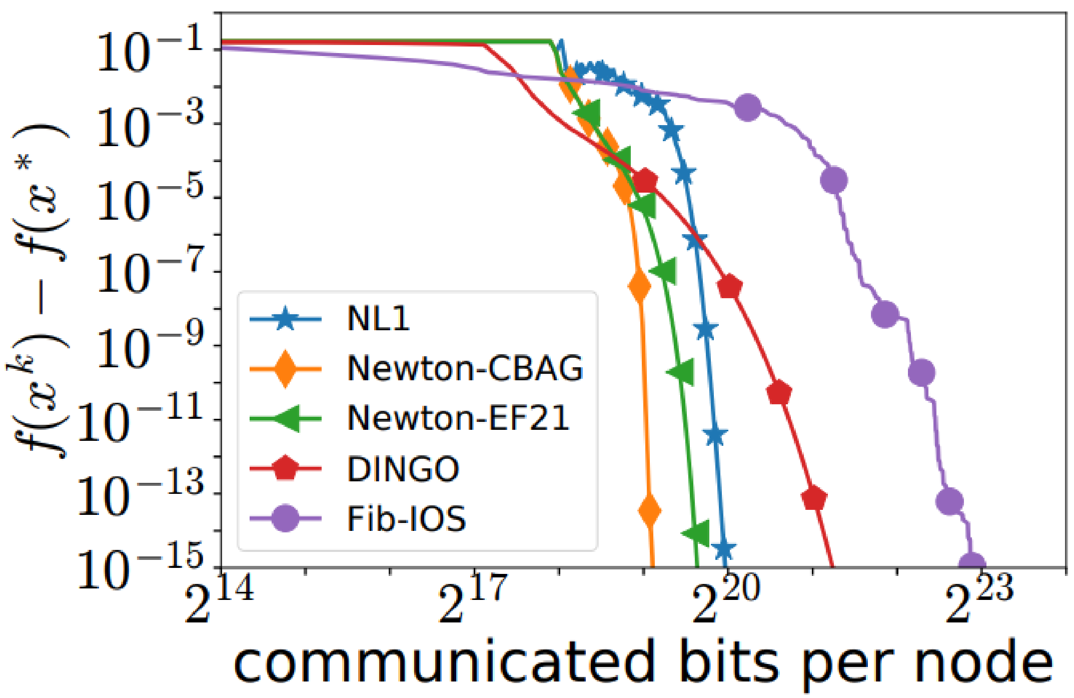
Regularization parameter

Training data points

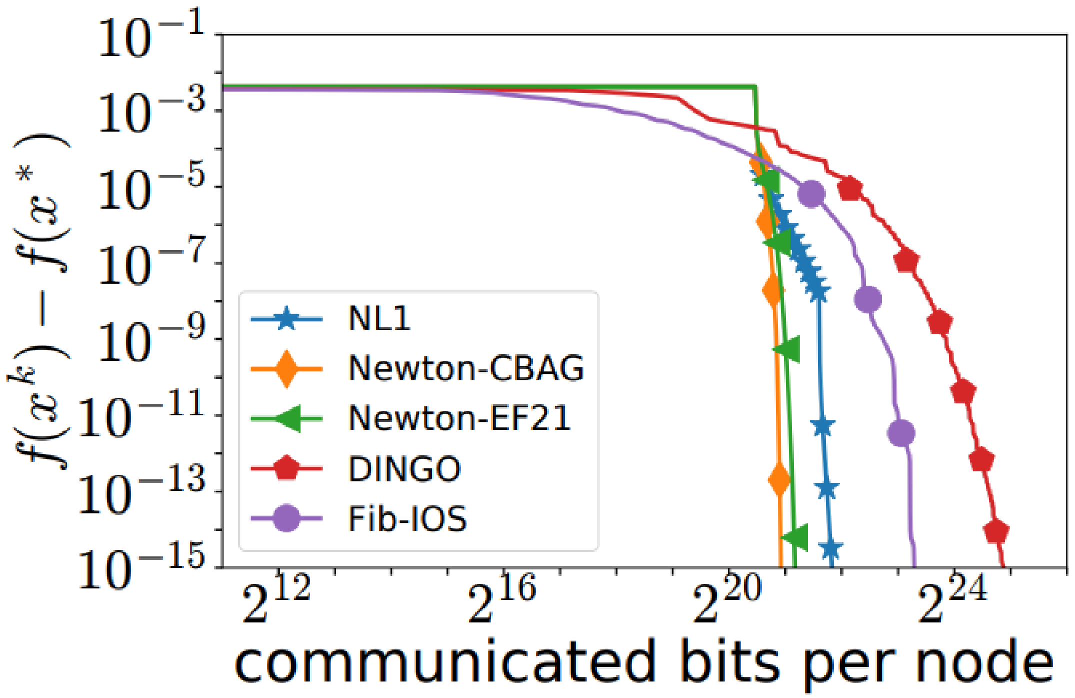
$$\min_{x\in\mathbb{R}^d}\left\{f(x) := \frac{1}{n}\sum_{i=1}^{n} f_i(x) + \frac{\lambda}{2}\|x\|^2\right\}, \qquad f_i(x) = \frac{1}{m}\sum_{j=1}^{m}\log\left(1+\exp(-b_{ij}a_{ij}^\top x)\right),$$

where $\{a_{ij}, b_{ij}\}_{j\in[m]}$ are data points at the i-th device. The datasets were taken from LibSVM library

Experiments: Local Comparison against Second Order Methods

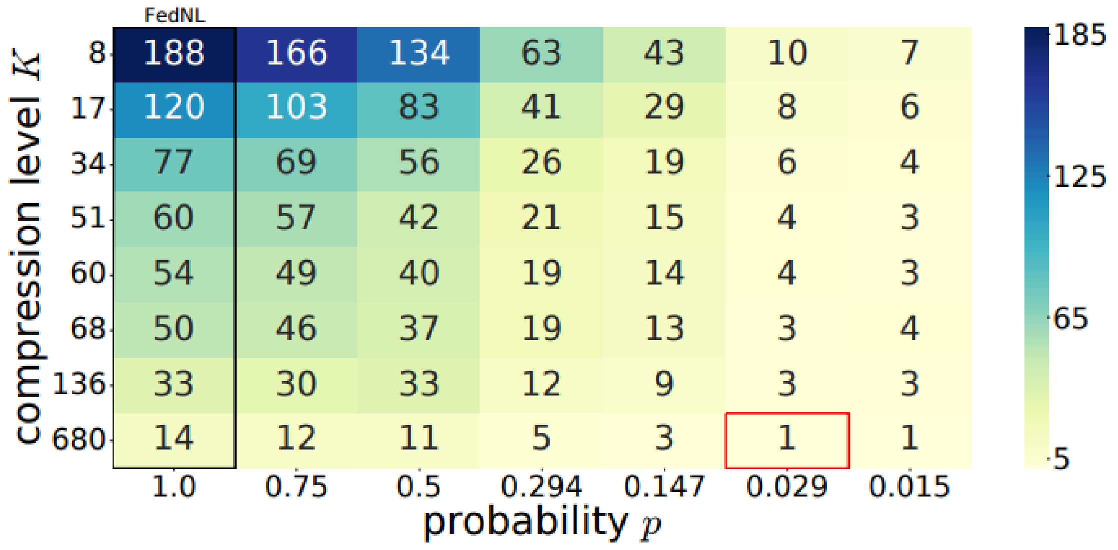


(a) a1a, $\lambda = 10^{-3}$

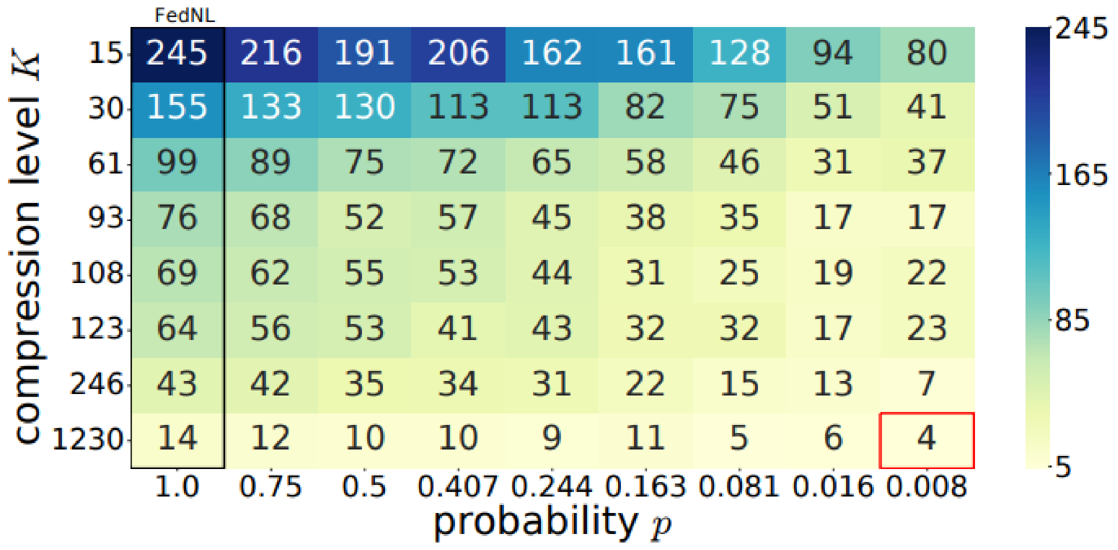


(b) w2a, $\lambda = 10^{-4}$

Experiments: CBAG indeed saves time



(i) phishing, $\lambda = 10^{-3}$



(j) a1a, $\lambda = 10^{-4}$

We use CBAG compressor based on Top-K contractive compressor and vary compression level K and probability p

Thank you