Undersampled Magnetic Resonance Image Reconstruction

Bair Mikhailov
MIPT, Skoltech
Scientific adviser: Ph.D. Dmitry Dylov

8 февраля 2024 г.

Undersampled MRI Reconstruction Task

Problem

MRI reconstruction with incomplete information.

Goal

Develop a method which preserves the quality of the Undersampled MRI.

Formal problem statement

1. $(\mathbf{M}, \mathbf{Y}) \in \mathcal{D}$ – Dataset
2. $\mathbf{M}, \mathbf{Y} \in \mathbb{R}^{k \times k}$, $\mathbf{Y} = \mathcal{F}(\mathbf{M})$ – MRI image and its Fourier transformation
3. $I : \mathbb{R}^{k \times k} \longrightarrow \mathbb{R}^{k \times k}$ – Filter function, which preserves other elements and zeroes other

The goal is to find function $B^* : \mathbb{R}^{k \times k} \longrightarrow \mathbb{R}^{k \times k}$ which minimizes the risk over the image distribution:

$$B^* = argmin_B R(B)$$

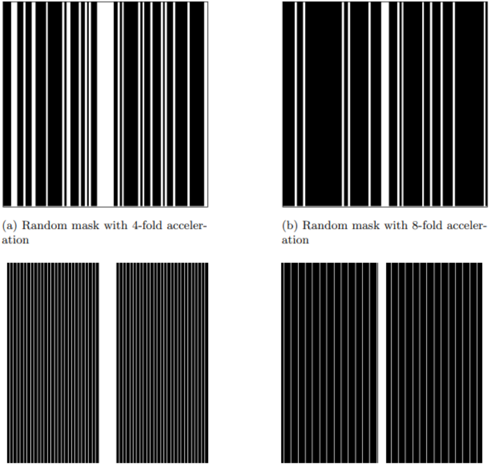
where

$$R(B) = \mathbb{E}_{\mathbf{Y},\mathbf{M}}[L(B(I(\mathbf{Y})), \mathbf{M})]$$

Evaluation functions

1. Normalized Mean Square Error: $NMSE(\hat{v}, v) = \frac{||v - \hat{v}||_2^2}{||v||_2^2}$
2. Peak Signal-to-Noise Ratio: $PSNR(\hat{v}, v) = 10log_{10}\frac{max(v)^2}{MSE(\hat{v}, v)}$
3. Structural Similarity Index Metric:
   $SSIM(\hat{\mathbf{M}}, \mathbf{M}) = l(\hat{\mathbf{M}}, \mathbf{M})^{\alpha} \cdot c(\hat{\mathbf{M}}, \mathbf{M})^{\beta} \cdot s(\hat{\mathbf{M}}, \mathbf{M})^{\gamma}$

Examples of filters



(a) Random mask with 4-fold acceleration

(b) Random mask with 8-fold acceleration

(c) Equispaced mask with 4-fold acceleration

(d) Equispaced mask with 8-fold acceleration

Figure 5: Examples of undersampled k-space trajectories

Taken from Zbonta[1].

Methods

Iterative shrinkage threshold algorithm

$\mathbf{R}_{i+1} = \mathbf{M}_i - \rho\mathcal{F}^{-1}[I(\mathcal{F}(\mathbf{M}_i)) - I(\mathbf{Y})]$

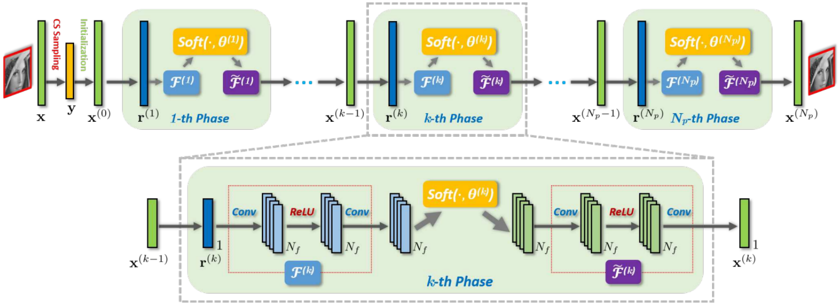
$\mathbf{M}_{i+1} = argmin_{\hat{\mathbf{M}}} \frac{1}{2}||\hat{\mathbf{M}} - \mathbf{R}_{i+1}||_2^2 + \lambda Reg(\hat{\mathbf{M}})$

Stops when: $||\mathbf{R}_i|| \leq \epsilon$

1. $Reg_{L1}(\mathbf{M}) = ||\mathbf{M}||_1$
2. $Reg_W(\mathbf{M}) = ||\Psi(\mathbf{M})||_1$

   $\mathbf{M}_{i+1} = \mathbf{W}^{-1} Soft(\mathbf{W}\mathbf{R}_{i+1}, \lambda)$, $Soft(\mathbf{U}, \lambda) = max(|\mathbf{U}| - \lambda, 0)\frac{\mathbf{U}}{|\mathbf{U}|}$
3. $Reg_{TV}(\mathbf{M}) = \sum_{i,j} \sqrt{|m_{i+1,j} - m_{i,j}|^2 + |m_{i,j+1} - m_{i,j}|^2}$

Methods



CS Sampling
Initialization
x
y
$\mathbf{x}^{(0)}$
$\mathbf{r}^{(1)}$
$Soft(\cdot, \theta^{(1)})$
$\mathcal{F}^{(1)}$
$\tilde{\mathcal{F}}^{(1)}$
1-th Phase
$\mathbf{x}^{(k-1)}$
$\mathbf{r}^{(k)}$
$Soft(\cdot, \theta^{(k)})$
$\mathcal{F}^{(k)}$
$\tilde{\mathcal{F}}^{(k)}$
k-th Phase
$\mathbf{x}^{(N_p-1)}$
$\mathbf{r}^{(N_p)}$
$Soft(\cdot, \theta^{(N_p)})$
$\mathcal{F}^{(N_p)}$
$\tilde{\mathcal{F}}^{(N_p)}$
N_p-th Phase
$\mathbf{x}^{(N_p)}$
Conv
ReLU
Conv
N_f
1
Conv
ReLU
Conv
k-th Phase
$\mathbf{x}^{(k)}$

Future work

1. Development a new method without multisteps
2. Proof of its quality
3. Tryout different filters I

Bibliography

Jure Zbonta, Florian Knoll et al.
fastMRI: A Publicly Available Raw k-Space and DICOM Dataset of Knee Images for Accelerated MR Image Reconstruction Using Machine Learning
Radiology: Artificial Intelligence 2020 2:1.

N. Pezzotti et al.
An Adaptive Intelligence Algorithm for Undersampled Knee MRI Reconstruction
IEEE Access, vol. 8, pp. 204825-204838, 2020.

Xin B. et al.
Fill the K-Space and Refine the Image: Prompting for Dynamic and Multi-Contrast MRI Reconstruction
arXiv e-prints (2023): arXiv-2309.