Note

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September 16, 2017

1 Basic Construction

The Hamiltonian for t-J model is $H_{t\text{-}J} = H_t + H_J$ where

$$\begin{aligned} H_t &= -t \sum_{\langle ij\rangle,\sigma} (c^\dagger_{i\sigma} c_{j\sigma} + h.c.), \\ H_J &= J \sum_{\langle ij\rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j \right). \end{aligned} \tag{1}$$

Suppose the square lattice is formed with $N_x \cdot N_y = N$ sites and they have been numbered as $0, \cdots, N-1$ in a certain way, for instance, a *snake*. With consideration of one hole doped case, a generic basis can be defined in such a one-dimensional way

$$c^\dagger_{0\sigma_0} \cdots c^\dagger_{h-1\sigma_{h-1}} c^\dagger_{h+1\sigma_{h+1}} \cdots c^\dagger_{N-1\sigma_{N-1}} |0\rangle = (-)^h c_{h\sigma_h} |s\rangle \equiv |h; s\rangle, \tag{2}$$

where $|s\rangle \equiv c^\dagger_{0\sigma_0} \cdots c^\dagger_{N-1\sigma_{N-1}} |0\rangle$ is the half-filled spin background created by *ordered* fermionic operators. $|h; s\rangle$ thus can be represented as a bosonic configuration in computational program. Here our major task is to compute the vector multiplication required by the package ARPACKPP[Reuter *et al.*()Reuter, Gomes, and Sorensen]. H_J can be evaluated as same as the bosonic Heisenberg spin model as one diagonal block of the $H_{t\text{-}J}$ matrix in our representation. For H_t, we would like to compute the hole's hopping term from site h to site h' (electron's hopping frome h' to h)

$$\begin{aligned} &\sum_\sigma (c^\dagger_{h\sigma} c_{h'\sigma}) |h; s\rangle \\ &= c^\dagger_{h\sigma_{h'}} c_{h'\sigma_{h'}} (-)^h c_{h\sigma_h} |s\rangle = c_{h'\sigma_{h'}} (-)^{h+1} (c^\dagger_{h\sigma_{h'}} c_{h\sigma_h}) |s\rangle \\ &= (-)^{h-h'+1} (-)^{h'} c_{h'\sigma_{h'}} |s'\rangle, \end{aligned} \tag{3}$$

where spin summation $\sum_\sigma c_{h'\sigma}$ should match $\sigma_{h'}$ of which $c^\dagger_{h'\sigma_{h'}}$ in $|s\rangle$ otherwise leads to zero. Note that what $|s'\rangle$ differs from $|s\rangle$ is that the fermionic creation

operator $c^{\dagger}_{h\sigma_h}$ in $|s\rangle$ is replaced by $c^{\dagger}_{h\sigma_{h'}}$ at site h. Its Hermatian conjugate part is similar. That is to say, in order to evaluate the non-zero matrix elements in terms of H_t which connects different bosonic Heisenberg sub-blocks of the total Hilbert space, despite considering the change of bosonic configuration in $|h';s'\rangle$, an extra fermionic sign $(-)^{h-h'+1}$ should be taken in to consideration.

References

[Reuter *et al.*()Reuter, Gomes, and Sorensen] M. Reuter, F. M. Gomes, and D. Sorensen, "BSD arpack++ package," .