

Note

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1 Basic Construction

The Hamiltonian for t - J model is $H_{t-J} = H_t + H_J$ where

$$\begin{aligned} H_t &= -t \sum_{\langle ij \rangle, \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.), \\ H_J &= J \sum_{\langle ij \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j \right). \end{aligned} \quad (1)$$

Suppose the square lattice is formed with $N_x \cdot N_y = N$ sites and they have been numbered as $0, \dots, N-1$ in a certain way, for instance, a *snake*. With consideration of one hole doped case, a generic basis can be defined in such a one-dimensional way

$$c_{0\sigma_0}^\dagger \cdots c_{h-1\sigma_{h-1}}^\dagger c_{h+1\sigma_{h+1}}^\dagger \cdots c_{N-1\sigma_{N-1}}^\dagger |0\rangle = (-)^h c_{h\sigma_h} |s\rangle \equiv |h; s\rangle, \quad (2)$$

where $|s\rangle \equiv c_{0\sigma_0}^\dagger \cdots c_{N-1\sigma_{N-1}}^\dagger |0\rangle$ is the half-filled spin background created by *ordered* fermionic operators. $|h; s\rangle$ thus can be represented as a bosonic configuration in computational program. Here our major task is to compute the vector multiplication required by the package ARPACKPP[[Reuter et al.\(\)](#)Reuter, Gomes, and Sorensen]. H_J can be evaluated as same as the bosonic Heisenberg spin model as one diagonal block of the H_{t-J} matrix in our representation. For H_t , we would like to compute the electron's hopping term from site h to site h'

$$\begin{aligned} & \sum_{\sigma} (c_{h\sigma}^\dagger c_{h'\sigma} + h.c.) |h; s\rangle \\ &= c_{h\sigma_{h'}}^\dagger c_{h'\sigma_{h'}} (-)^h c_{h\sigma_h} |s\rangle = c_{h'\sigma_{h'}} (-)^{h+1} (c_{h\sigma_h}^\dagger c_{h\sigma_h}) |s\rangle \\ &= (-)^{h-h'+1} (-)^{h'} c_{h'\sigma_{h'}} |s'\rangle. \end{aligned} \quad (3)$$

Note that what $|s'\rangle$ differs from $|s\rangle$ is that the fermionic creation operator $c_{h\sigma_h}^\dagger$ in $|s\rangle$ is replaced by $c_{h\sigma_{h'}}^\dagger$ at site h . That is to say, in order to evaluate the non-zero matrix elements in terms of H_t connecting different bosonic Heisenberg sub-blocks, despite considering the bosonic $|h'; s'\rangle$ namely result hole's position and spin configuration, an extra fermionic sign $(-)^{h-h'+1}$ should be taken in to consideration.

References

- [Reuter *et al.*()Reuter, Gomes, and Sorensen] M. Reuter, F. M. Gomes, and D. Sorensen, “[BSD arpack++ package](#),” .