Minimum Spanning Tree – *DPHPC*

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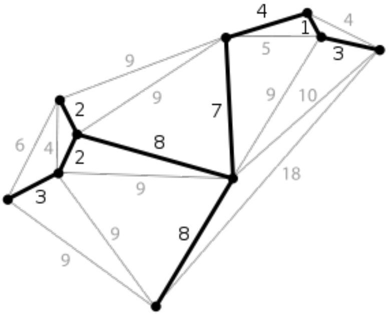
Problem definition - reminder

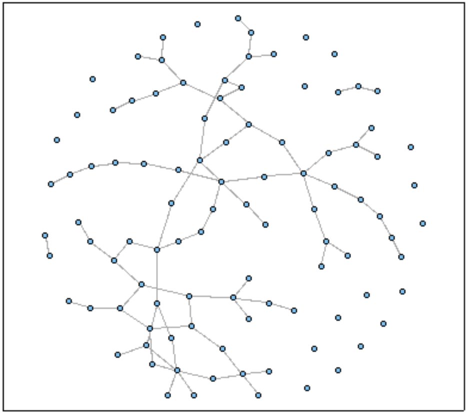
The MST problem

A minimum spanning tree (MST) or minimum weight spanning tree is a subset of the edges of a connected, edge-weighted (un)directed graph that connects all the vertices together, without any cycles and with the minimum possible total edge weight.



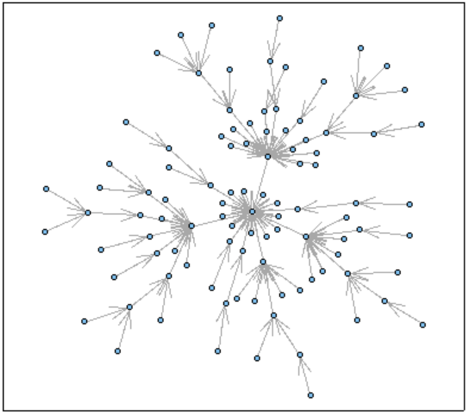
Input sets: $G(n, p)$

$G(100, 0.02)$



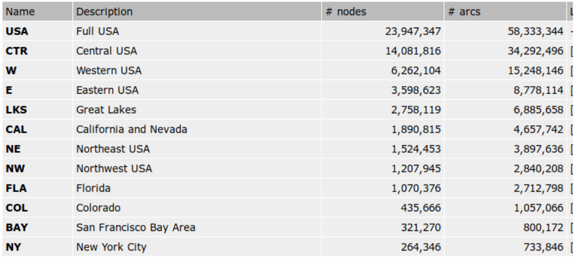
Input sets: $PA(n)$

$PA(100)$



Input sets: 9th DIMACS challenge dataset

USA Roads



Name	Description	# nodes	# arcs	l
USA	Full USA	23,947,347	58,333,344	-
CTR	Central USA	14,081,816	34,292,496	[
W	Western USA	6,262,104	15,248,146	[
E	Eastern USA	3,598,623	8,778,114	[
LKS	Great Lakes	2,758,119	6,885,658	[
CAL	California and Nevada	1,890,815	4,657,742	[
NE	Northeast USA	1,524,453	3,897,636	[
NW	Northwest USA	1,207,945	2,840,208	[
FLA	Florida	1,070,376	2,712,798	[
COL	Colorado	435,666	1,057,066	[
BAY	San Francisco Bay Area	321,270	800,172	[
NY	New York City	264,346	733,846	[

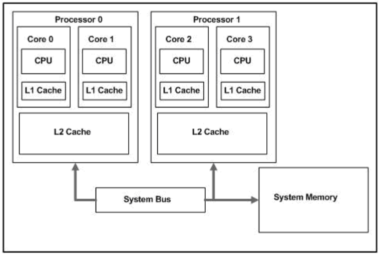
Setup

Software

- OMP
- Intel Threading Building Blocks (TBB)
- Parallel Streaming Transformation Loader Service (PSTL)

EULER

- 1 node limitation (OMP)
- 2 sockets filled with 18 cores - up to 3.7Ghz
- Inter-sockets bus speed: 10.4 GT/s



Algorithms and parallel implementations

Sollin

1. For each connected component, find adjacent edge with minimum weight
2. Add edge to mst (each edge add at most twice)
3. Merge connected components

Kruskal

```
1:  $A = \emptyset$
2:  for all $v \in G.V$ do
3:      MAKE-SET($v$)
4:  end for
5:  Sort (asc.) $(weight(u, v))_{(u,v) \in G.E}$
6:  for all $(u, v)$ in $G.E$ ordered by weight do
7:      if FIND-SET($u$) $\neq$ FIND-SET($v$) then
8:          $A = A \cup (u, v)$
9:          UNION($u, v$)
10:     end if
11: end for
12: return A
```

Boost implementations

- Boost-Kruskal used as a reference

Parallel sorting on Kruskal

Filter Kruskal

Amdahl's law

S_p is the speedup with p cores, f is the part of the program that is sequential.

$$S_p = \frac{1}{\frac{1-f}{p} + f}$$

$$f = \frac{\frac{p}{S_p} - 1}{p - 1}$$

Amdahl's law: Kruskal

Graph: Erdos-Renyi (100,000 nodes, $p = 0.0005$)

Cores	Median speed-up	Standard deviation	f
1	1	0.0129805395	-
2	1.1881513396	0.0400305172	0.6832872491
4	1.3340592415	0.0130798641	0.6661225318
8	1.3134515984	0.01048841	0.7272602975
16	1.4399618642	0.0088220116	0.6740936918
32	1.3877640643	0.0442081933	0.7115701488

Amdahl's law: Filter Kruskal

Graph: Preferential attachment (10,000 nodes, 1,000 edges per vertex)

Cores	Median speed-up	Standard deviation	f
1	1	0.174182691	-
2	1.6268639022	0.2472258016	0.2293591353
4	2.669168898	0.5213238241	0.1661979381
8	5.3584384444	0.9757941767	0.0704246098
16	5.7447285937	1.108109986	0.1190108026
32	5.6322979489	1.497976882	0.1510166972

Kruskal vs Sollin

Same complexity, but Sollin computing steps are more fine-grained

Sollin is entirely paralelizable

Filter Kruskal can be adapted to Filter Sollin

Results overview

EULER Cluster

Scalability

Speedups