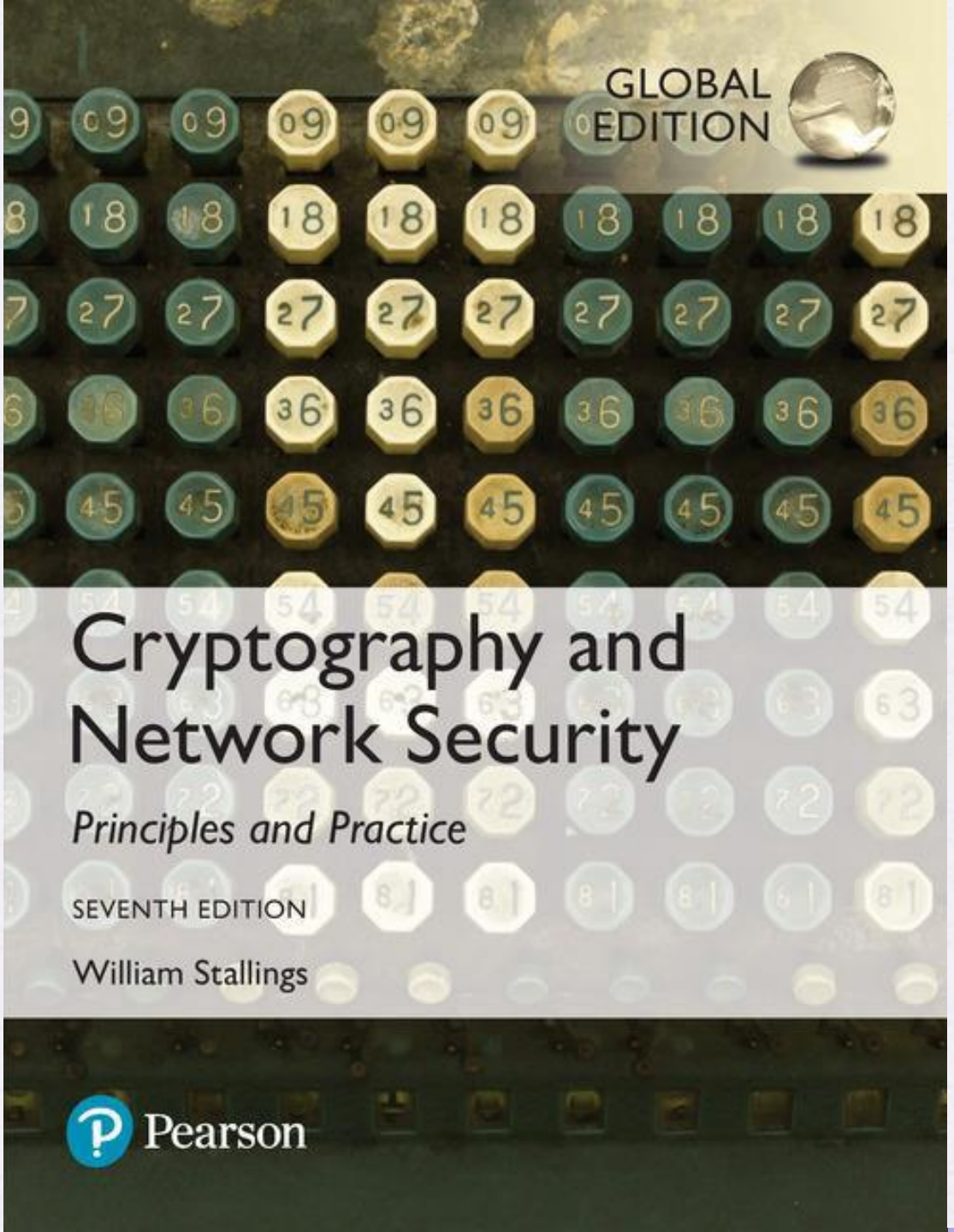
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Cryptography and Network Security
Principles and Practice
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Chapter 1

Computer and Network Security Concepts

Cryptographic algorithms and protocols can be grouped into four main areas:

Symmetric encryption

- Used to conceal the contents of blocks or streams of data of any size, including messages, files, encryption keys, and passwords

Asymmetric encryption

- Used to conceal small blocks of data, such as encryption keys and hash function values, which are used in digital signatures

Data integrity algorithms

- Used to protect blocks of data, such as messages, from alteration

Authentication protocols

- Schemes based on the use of cryptographic algorithms designed to authenticate the identity of entities

The field of network and Internet security consists of:

measures to deter, prevent, detect, and correct security violations that involve the transmission of information

Computer Security

The NIST *Computer Security Handbook* defines the term computer security as:

> "the protection afforded to an automated information system in order to attain the applicable objectives of preserving the integrity, availability and confidentiality of information system resources" (includes hardware, software, firmware, information/ data, and telecommunications)

Computer Security Objectives

Confidentiality

- Data confidentiality
  - Assures that private or confidential information is not made available or disclosed to unauthorized individuals
- Privacy
  - Assures that individuals control or influence what information related to them may be collected and stored and by whom and to whom that information may be disclosed

Integrity

- Data integrity
  - Assures that information and programs are changed only in a specified and authorized manner
- System integrity
  - Assures that a system performs its intended function in an unimpaired manner, free from deliberate or inadvertent unauthorized manipulation of the system

Availability

- Assures that systems work promptly and service is not denied to authorized users

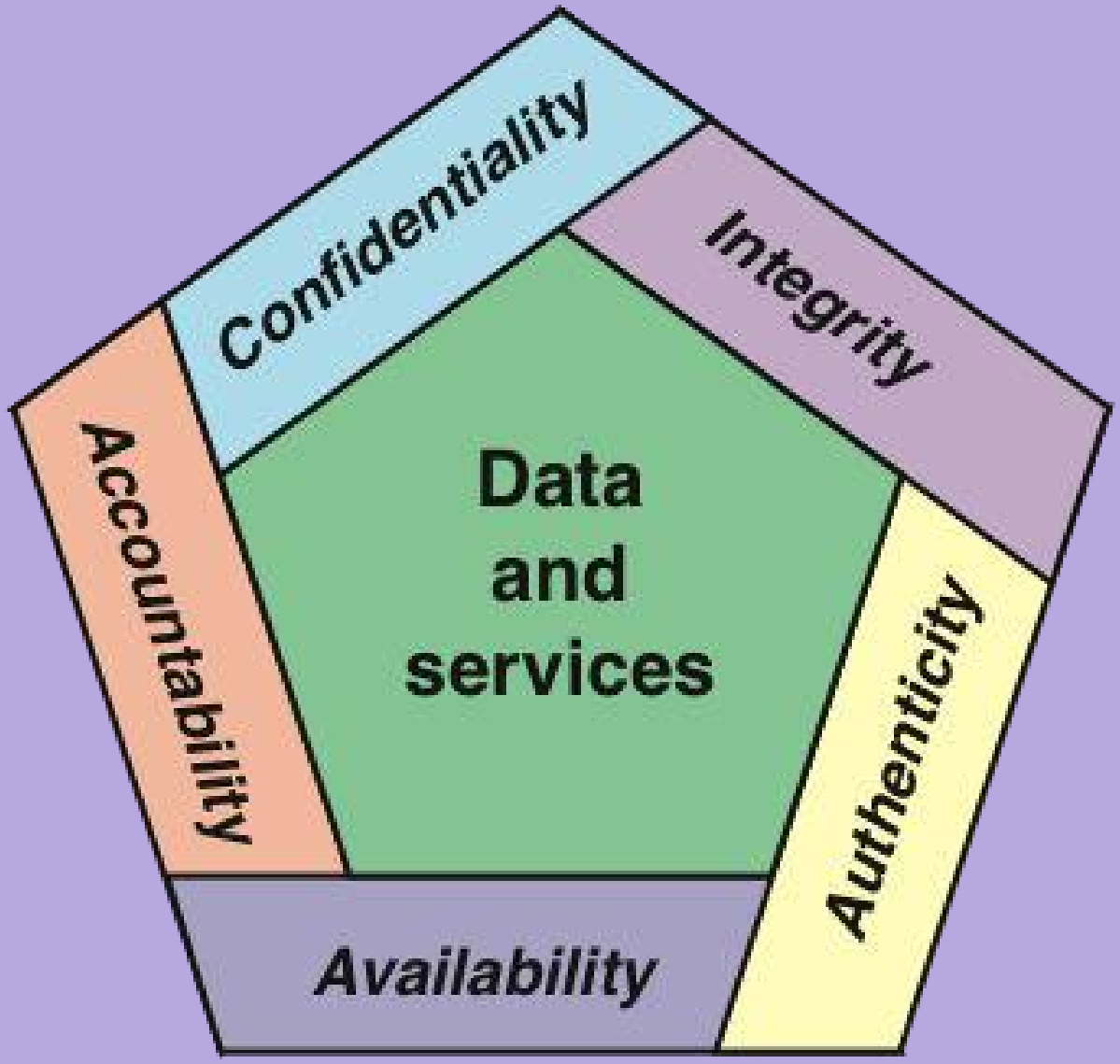
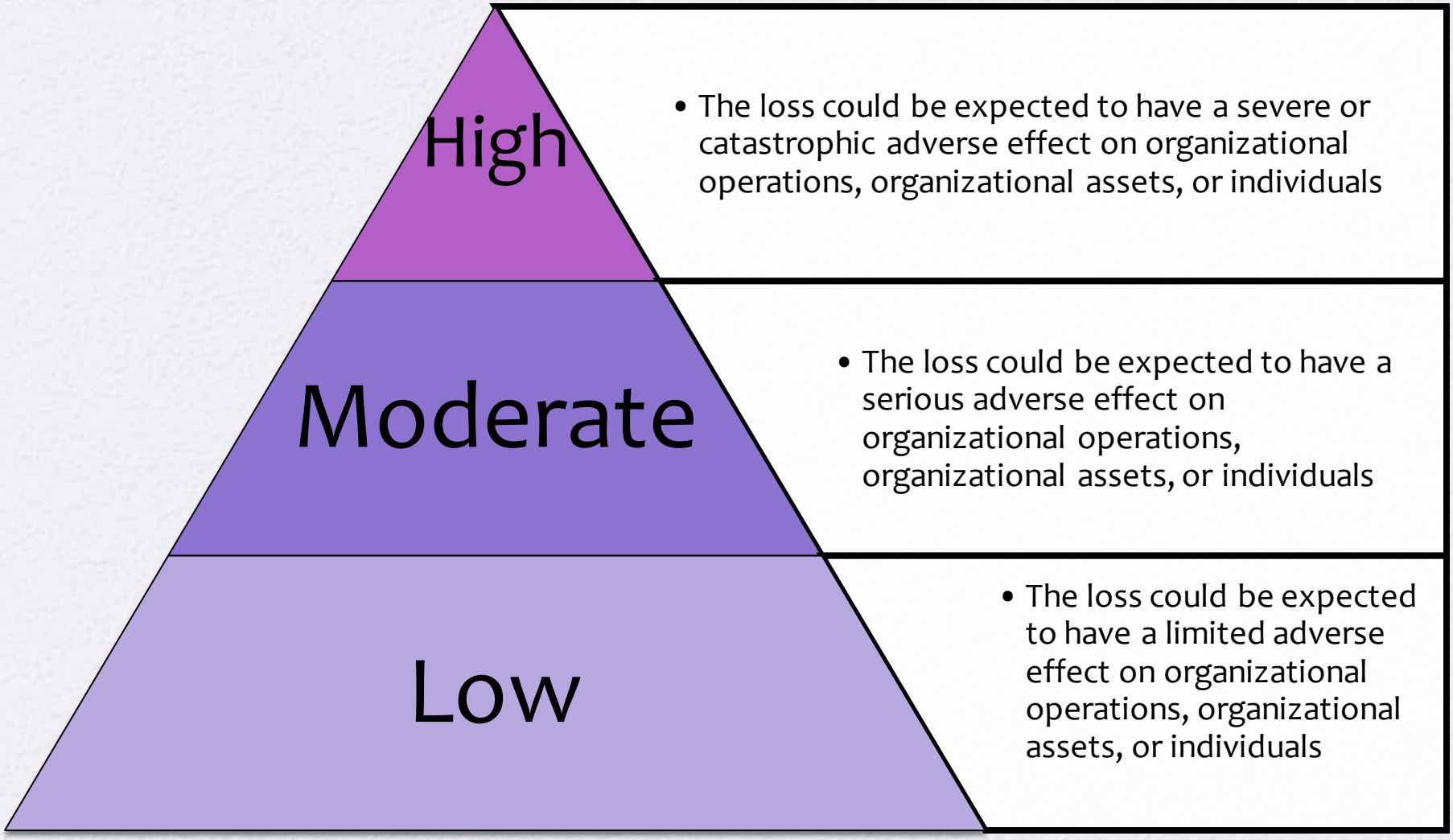


Figure 1.1 Essential Network and Computer Security Requirements

Breach of Security Levels of Impact



Computer Security Challenges

- Security is not simple
- Potential attacks on the security features need to be considered
- Procedures used to provide particular services are often counter-intuitive
- It is necessary to decide where to use the various security mechanisms
- Requires constant monitoring
- Is too often an afterthought
- Security mechanisms typically involve more than a particular algorithm or protocol
- Security is essentially a battle of wits between a perpetrator and the designer
- Little benefit from security investment is perceived until a security failure occurs
- Strong security is often viewed as an impediment to efficient and user-friendly operation

OSI Security Architecture

- Security attack
  - Any action that compromises the security of information owned by an organization
- Security mechanism
  - A process (or a device incorporating such a process) that is designed to detect, prevent, or recover from a security attack
- Security service
  - A processing or communication service that enhances the security of the data processing systems and the information transfers of an organization
  - Intended to counter security attacks, and they make use of one or more security mechanisms to provide the service

Table 1.1
Threats and Attacks (RFC 4949)

Threat

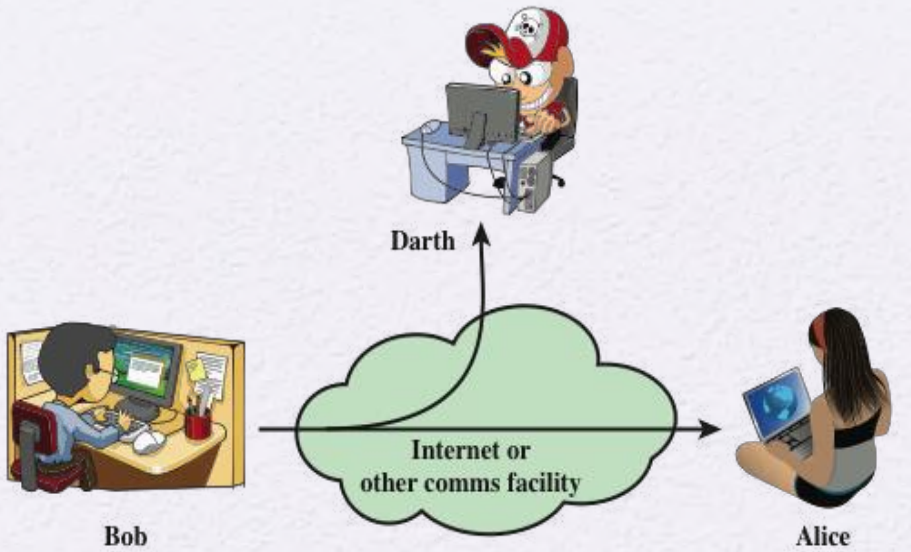
A potential for violation of security, which exists when there is a circumstance, capability, action, or event that could breach security and cause harm. That is, a threat is a possible danger that might exploit a vulnerability.

Attack

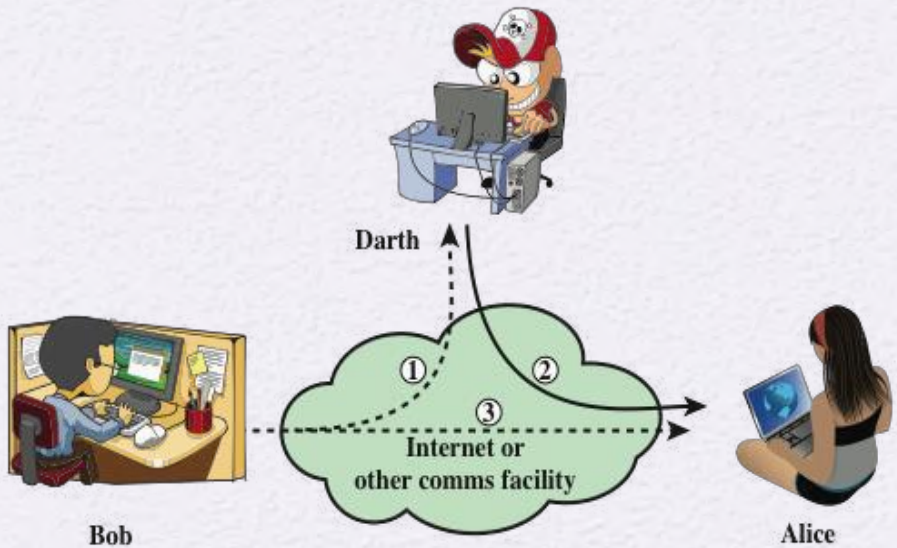
An assault on system security that derives from an intelligent threat; that is, an intelligent act that is a deliberate attempt (especially in the sense of a method or technique) to evade security services and violate the security policy of a system.

Security Attacks

- A means of classifying security attacks, used both in X.800 and RFC 4949, is in terms of *passive attacks* and *active attacks*
- A *passive attack* attempts to learn or make use of information from the system but does not affect system resources
- An *active attack* attempts to alter system resources or affect their operation



(a) Passive attacks



(b) Active attacks

Figure 1.2 Security Attacks

Passive Attacks

- Are in the nature of eavesdropping on, or monitoring of, transmissions
- Goal of the opponent is to obtain information that is being transmitted

- Two types of passive attacks are:
  - The release of message contents
  - Traffic analysis

Active Attacks

- Involve some modification of the data stream or the creation of a false stream
- Difficult to prevent because of the wide variety of potential physical, software, and network vulnerabilities
- Goal is to detect attacks and to recover from any disruption or delays caused by them

Masquerade

- Takes place when one entity pretends to be a different entity
- Usually includes one of the other forms of active attack

Replay

- Involves the passive capture of a data unit and its subsequent retransmission to produce an unauthorized effect

Modification of messages

- Some portion of a legitimate message is altered, or messages are delayed or reordered to produce an unauthorized effect

Denial of service

- Prevents or inhibits the normal use or management of communications facilities

Security Services

- Defined by X.800 as:
  - A service provided by a protocol layer of communicating open systems and that ensures adequate security of the systems or of data transfers
- Defined by RFC 4949 as:
  - A processing or communication service provided by a system to give a specific kind of protection to system resources

AUTHENTICATION

The assurance that the communicating entity is the one that it claims to be.

Peer Entity Authentication
Used in association with a logical connection to provide confidence in the identity of the entities connected.

Data-Origin Authentication
In a connectionless transfer, provides assurance that the source of received data is as claimed.

ACCESS CONTROL

The prevention of unauthorized use of a resource (i.e., this service controls who can have access to a resource, under what conditions access can occur, and what those accessing the resource are allowed to do).

DATA CONFIDENTIALITY

The protection of data from unauthorized disclosure.

Connection Confidentiality
The protection of all user data on a connection.

Connectionless Confidentiality
The protection of all user data in a single data block

Selective-Field Confidentiality
The confidentiality of selected fields within the user data on a connection or in a single data block.

Traffic-Flow Confidentiality
The protection of the information that might be derived from observation of traffic flows.

DATA INTEGRITY

The assurance that data received are exactly as sent by an authorized entity (i.e., contain no modification, insertion, deletion, or replay).

Connection Integrity with Recovery
Provides for the integrity of all user data on a connection and detects any modification, insertion, deletion, or replay of any data within an entire data sequence, with recovery attempted.

Connection Integrity without Recovery
As above, but provides only detection without recovery.

Selective-Field Connection Integrity
Provides for the integrity of selected fields within the user data of a data block transferred over a connection and takes the form of determination of whether the selected fields have been modified, inserted, deleted, or replayed.

Connectionless Integrity
Provides for the integrity of a single connectionless data block and may take the form of detection of data modification. Additionally, a limited form of replay detection may be provided.

Selective-Field Connectionless Integrity
Provides for the integrity of selected fields within a single connectionless data block; takes the form of determination of whether the selected fields have been modified.

NONREPUDIATION

Provides protection against denial by one of the entities involved in a communication of having participated in all or part of the communication.

Nonrepudiation, Origin
Proof that the message was sent by the specified party.

Nonrepudiation, Destination
Proof that the message was received by the specified party.

Table 1.2

Security Services (X.800)

(This table is found on page 12 in textbook)

Authentication

- Concerned with assuring that a communication is authentic
  - In the case of a single message, assures the recipient that the message is from the source that it claims to be from
  - In the case of ongoing interaction, assures the two entities are authentic and that the connection is not interfered with in such a way that a third party can masquerade as one of the two legitimate parties

Two specific authentication services are defined in X.800:

- Peer entity authentication
- Data origin authentication

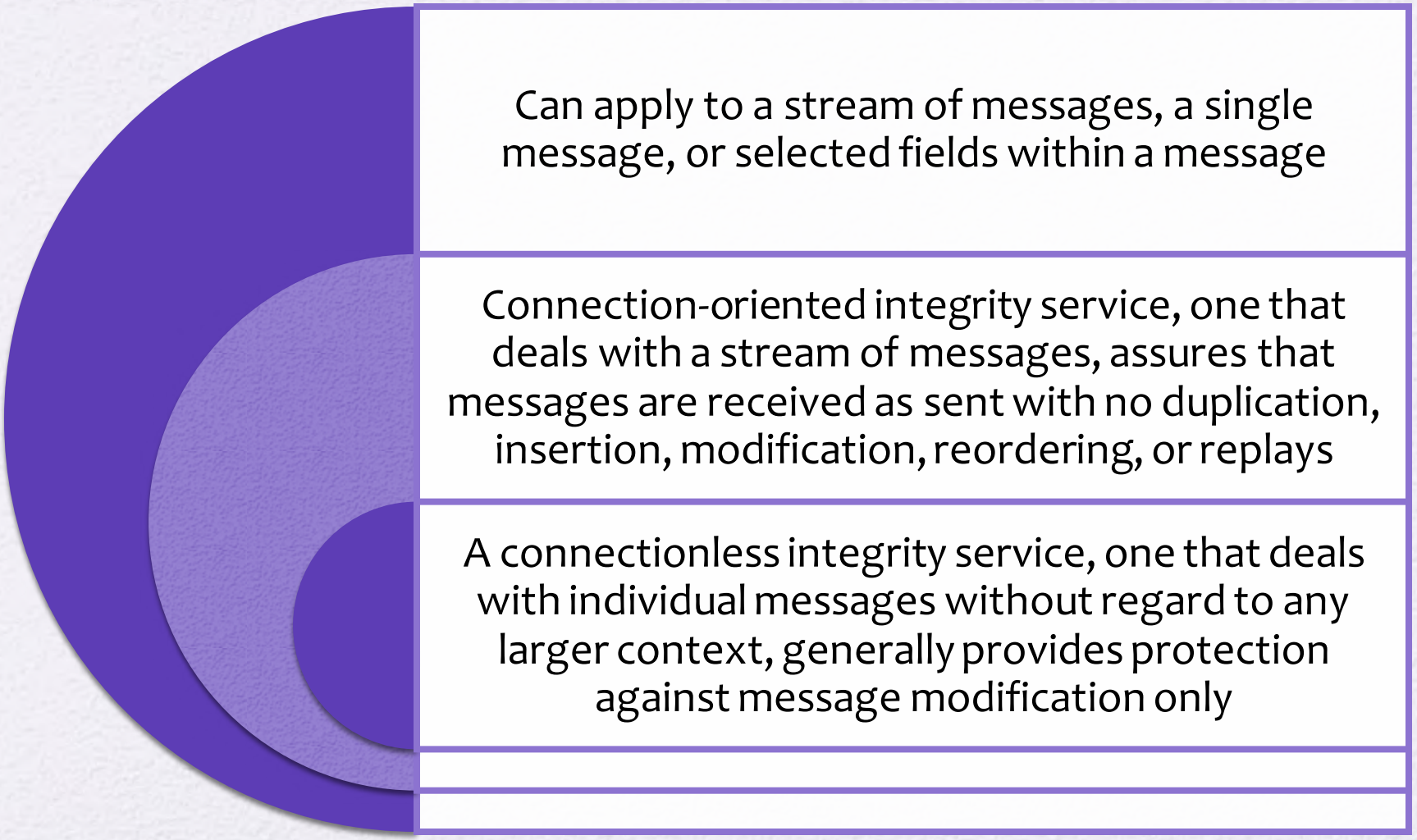
Access Control

- The ability to limit and control the access to host systems and applications via communications links
- To achieve this, each entity trying to gain access must first be indentified, or authenticated, so that access rights can be tailored to the individual

Data Confidentiality

- The protection of transmitted data from passive attacks
  - Broadest service protects all user data transmitted between two users over a period of time
  - Narrower forms of service includes the protection of a single message or even specific fields within a message
- The protection of traffic flow from analysis
  - This requires that an attacker not be able to observe the source and destination, frequency, length, or other characteristics of the traffic on a communications facility

Data Integrity



Nonrepudiation

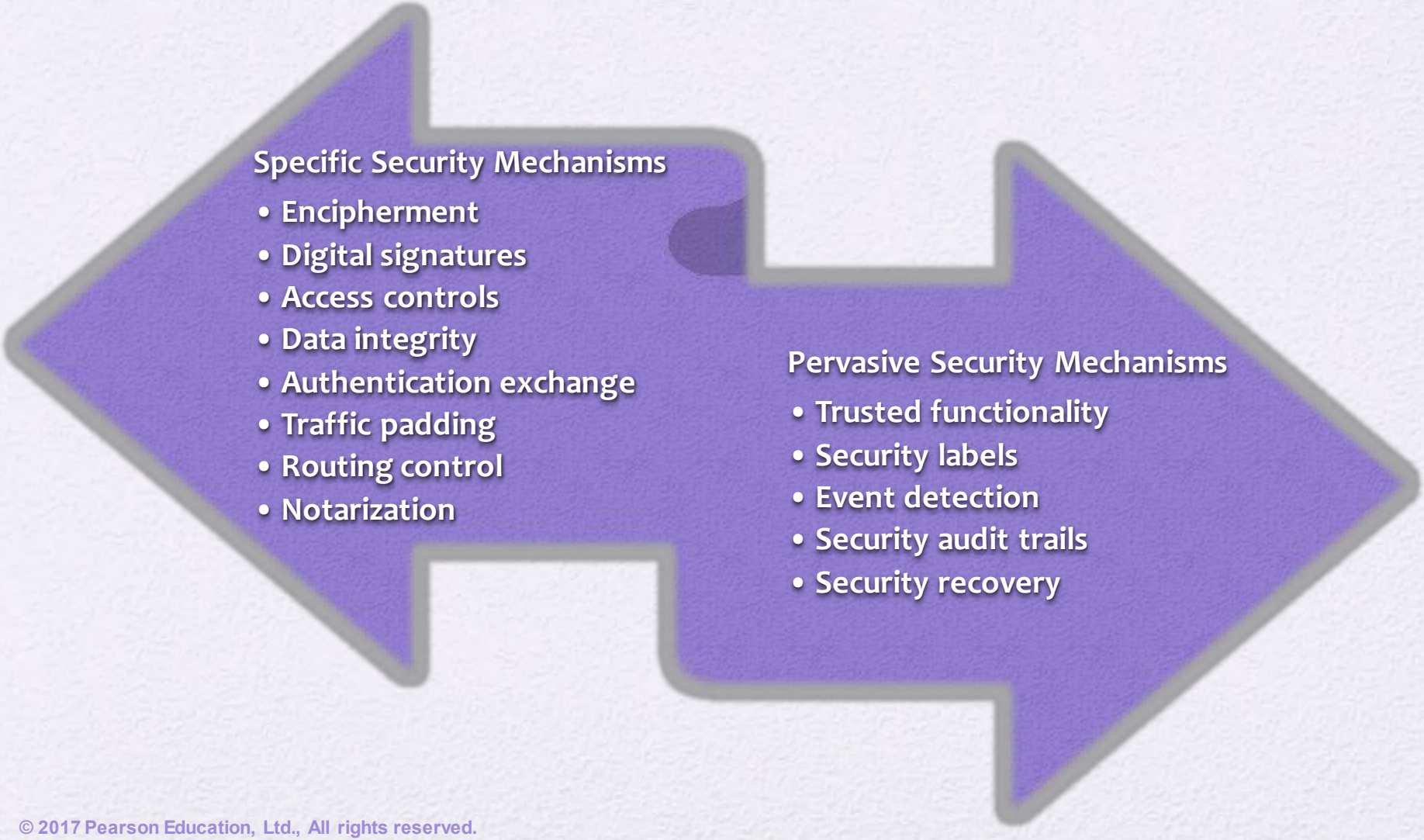
- Prevents either sender or receiver from denying a transmitted message

- When a message is sent, the receiver can prove that the alleged sender in fact sent the message
- When a message is received, the sender can prove that the alleged receiver in fact received the message

Availability Service

- Protects a system to ensure its availability
- This service addresses the security concerns raised by denial-of-service attacks
- It depends on proper management and control of system resources and thus depends on access control service and other security services

Security Mechanisms (X.800)



SPECIFIC SECURITY MECHANISMS

May be incorporated into the appropriate protocol layer in order to provide some of the OSI security services.

Encipherment
The use of mathematical algorithms to transform data into a form that is not readily intelligible. The transformation and subsequent recovery of the data depend on an algorithm and zero or more encryption keys.

Digital Signature
Data appended to, or a cryptographic transformation of, a data unit that allows a recipient of the data unit to prove the source and integrity of the data unit and protect against forgery (e.g., by the recipient).

Access Control
A variety of mechanisms that enforce access rights to resources.

Data Integrity
A variety of mechanisms used to assure the integrity of a data unit or stream of data units.

Authentication Exchange
A mechanism intended to ensure the identity of an entity by means of information exchange.

Traffic Padding
The insertion of bits into gaps in a data stream to frustrate traffic analysis attempts.

Routing Control
Enables selection of particular physically secure routes for certain data and allows routing changes, especially when a breach of security is suspected.

Notarization
The use of a trusted third party to assure certain properties of a data exchange.

PERVASIVE SECURITY MECHANISMS

Mechanisms that are not specific to any particular OSI security service or protocol layer.

Trusted Functionality
That which is perceived to be correct with respect to some criteria (e.g., as established by a security policy).

Security Label
The marking bound to a resource (which may be a data unit) that names or designates the security attributes of that resource.

Event Detection
Detection of security-relevant events.

Security Audit Trail
Data collected and potentially used to facilitate a security audit, which is an independent review and examination of system records and activities.

Security Recovery
Deals with requests from mechanisms, such as event handling and management functions, and takes recovery actions.

Table 1.3

Security Mechanisms (X.800)

(This table is found on pages 14-15 in textbook)

Fundamental Security Design Principles

- Economy of mechanism
- Fail-safe defaults
- Complete meditation
- Open design
- Separation of privilege
- Least privilege
- Least common mechanism
- Psychological acceptability
- Isolation
- Encapsulation
- Modularity
- Layering
- Least astonishment

Fundamental Security Design Principles

Economy of mechanism

- Means that the design of security measures embodied in both hardware and software should be as simple and small as possible
- Relatively simple, small design is easier to test and verify thoroughly
- With a complex design, there are many more opportunities for an adversary to discover subtle weaknesses to exploit that may be difficult to spot ahead of time

Fail-safe defaults

- Means that access decisions should be based on permission rather than exclusion
- The default situation is lack of access, and the protection scheme identifies conditions under which access is permitted
- Most file access systems and virtually all protected services on client/server use fail-safe defaults

Fundamental Security Design Principles

Complete mediation

- Means that every access must be checked against the access control mechanism
- Systems should not rely on access decisions retrieved from a cache
- To fully implement this, every time a user reads a field or record in a file, or a data item in a database, the system must exercise access control
- This resource-intensive approach is rarely used

Open design

- Means that the design of a security mechanism should be open rather than secret
- Although encryption keys must be secret, encryption algorithms should be open to public scrutiny
- Is the philosophy behind the NIST program of standardizing encryption and hash algorithms

Fundamental Security Design Principles

Separation of privilege

- Defined as a practice in which multiple privilege attributes are required to achieve access to a restricted resource
- Multifactor user authentication is an example which requires the use of multiple techniques, such as a password and a smart card, to authorize a user

Least privilege

- Means that every process and every user of the system should operate using the least set of privileges necessary to perform the task
- An example of the use of this principle is role-based access control; the system security policy can identify and define the various roles of users or processes and each role is assigned only those permissions needed to perform its functions

Fundamental Security Design Principles

Least common mechanism

- Means that the design should minimize the functions shared by different users, providing mutual security
- This principle helps reduce the number of unintended communication paths and reduces the amount of hardware and software on which all users depend, thus making it easier to verify if there are any undesirable security implications

Psychological acceptability

- Implies that the security mechanisms should not interfere unduly with the work of users, while at the same time meeting the needs of those who authorize access
- Where possible, security mechanisms should be transparent to the users of the system or, at most, introduce minimal obstruction
- In addition to not being intrusive or burdensome, security procedures must reflect the user's mental model of protection

Fundamental Security Design Principles

Isolation

- Applies in three contexts:
  - Public access systems should be isolated from critical resources to prevent disclosure or tampering
  - Processes and files of individual users should be isolated from one another except where it is explicitly desired
  - Security mechanisms should be isolated in the sense of preventing access to those mechanisms

Encapsulation

- Can be viewed as a specific form of isolation based on object-oriented functionality
- Protection is provided by encapsulating a collection of procedures and data objects in a domain of its own so that the internal structure of a data object is accessible only to the procedures of the protected subsystem, and the procedures may be called only at designated domain entry points

Fundamental Security Design Principles

Modularity

- Refers both to the development of security functions as separate, protected modules and to the use of a modular architecture for mechanism design and implementation

Layering

- Refers to the use of multiple, overlapping protection approaches addressing the people, technology, and operational aspects of information systems
- The failure or circumvention of any individual protection approach will not leave the system unprotected

Fundamental Security Design Principles

Least astonishment

- Means that a program or user interface should always respond in the way that is least likely to astonish the user
- The mechanism for authorization should be transparent enough to a user that the user has a good intuitive understanding of how the security goals map to the provided security mechanism

Attack Surfaces

- An attack surface consists of the reachable and exploitable vulnerabilities in a system
- Examples:
  - Open ports on outward facing Web and other servers, and code listening on those ports
  - Services available on the inside of a firewall
  - Code that processes incoming data, email, XML, office documents, and industry-specific custom data exchange formats
  - Interfaces, SQL, and Web forms
  - An employee with access to sensitive information vulnerable to a social engineering attack

Attack Surface Categories

- Network attack surface
  - Refers to vulnerabilities over an enterprise network, wide-area network, or the Internet
- Software attack surface
  - Refers to vulnerabilities in application, utility, or operating system code
- Human attack surface
  - Refers to vulnerabilities created by personnel or outsiders

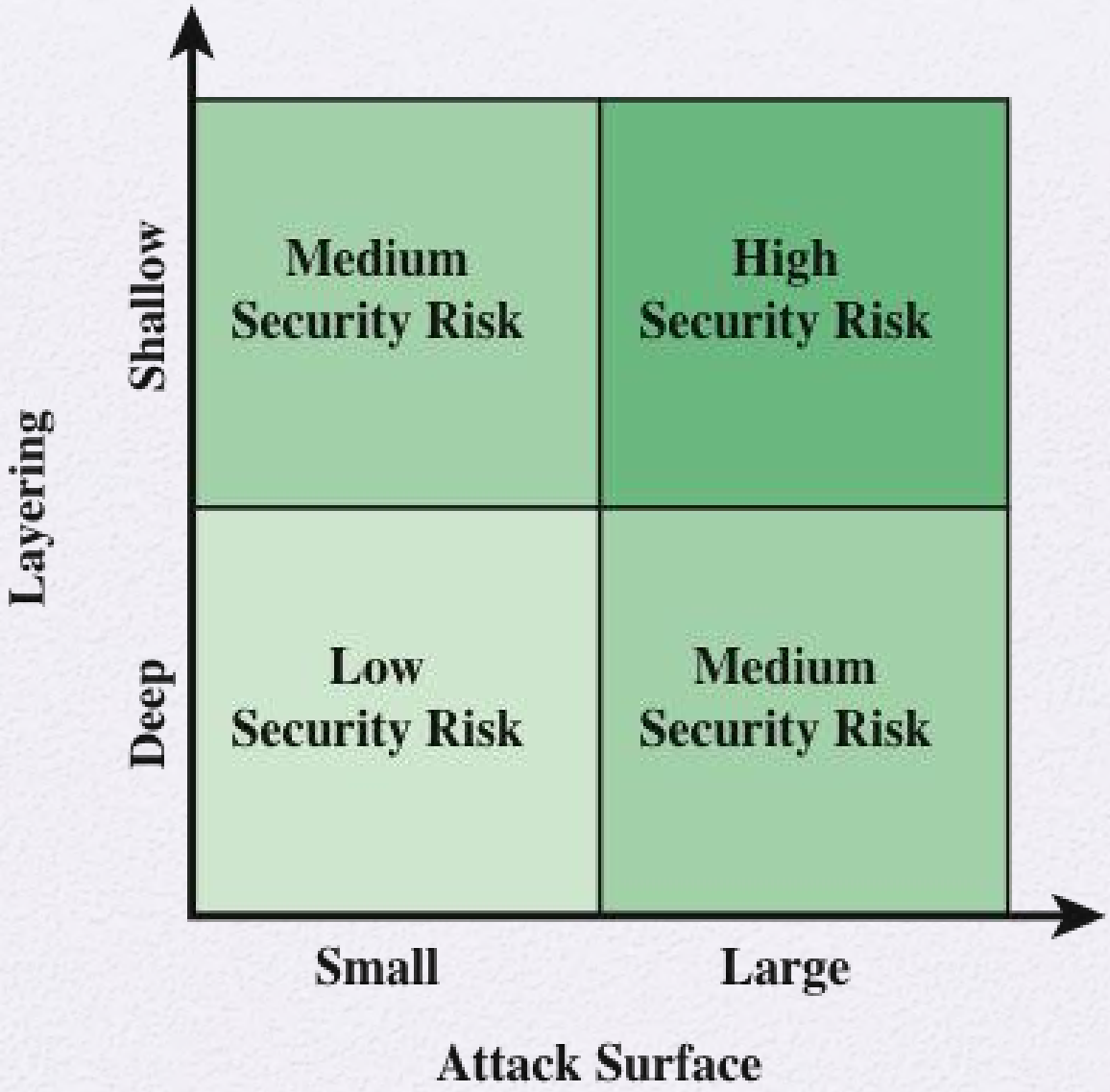


Figure 1.3 Defense in Depth and Attack Surface

Attack Tree

- A branching, hierarchical data structure that represents a set of potential techniques for exploiting security vulnerabilities
- The security incident that is the goal of the attack is represented as the root node of the tree, and the ways that an attacker could reach that goal are represented as branches and subnodes of the tree
- The final nodes on the paths outward from the root, (leaf nodes), represent different ways to initiate an attack
- The motivation for the use of attack trees is to effectively exploit the information available on attack patterns

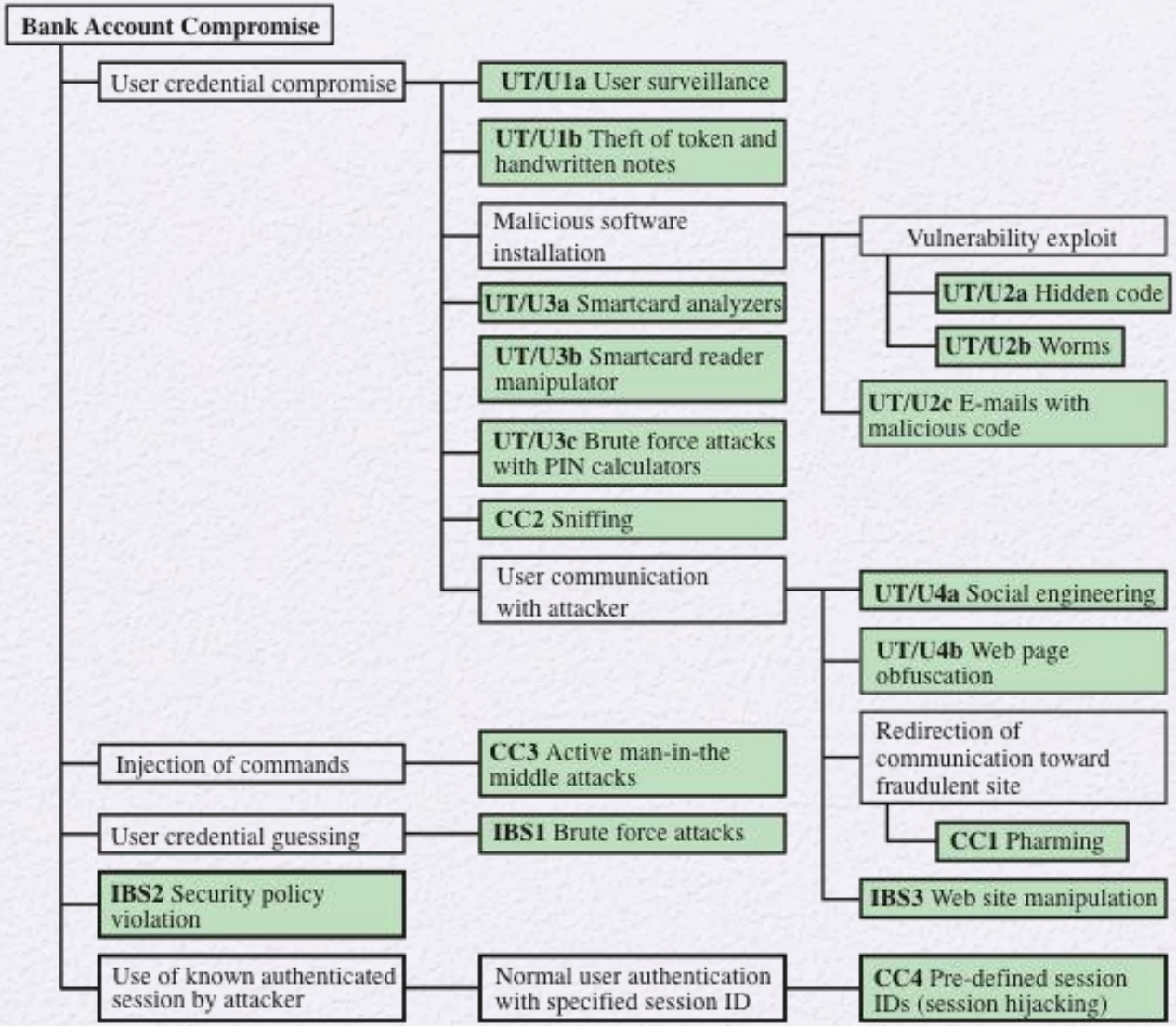


Figure 1.4 An Attack Tree for Internet Banking Authentication

Model for Network Security

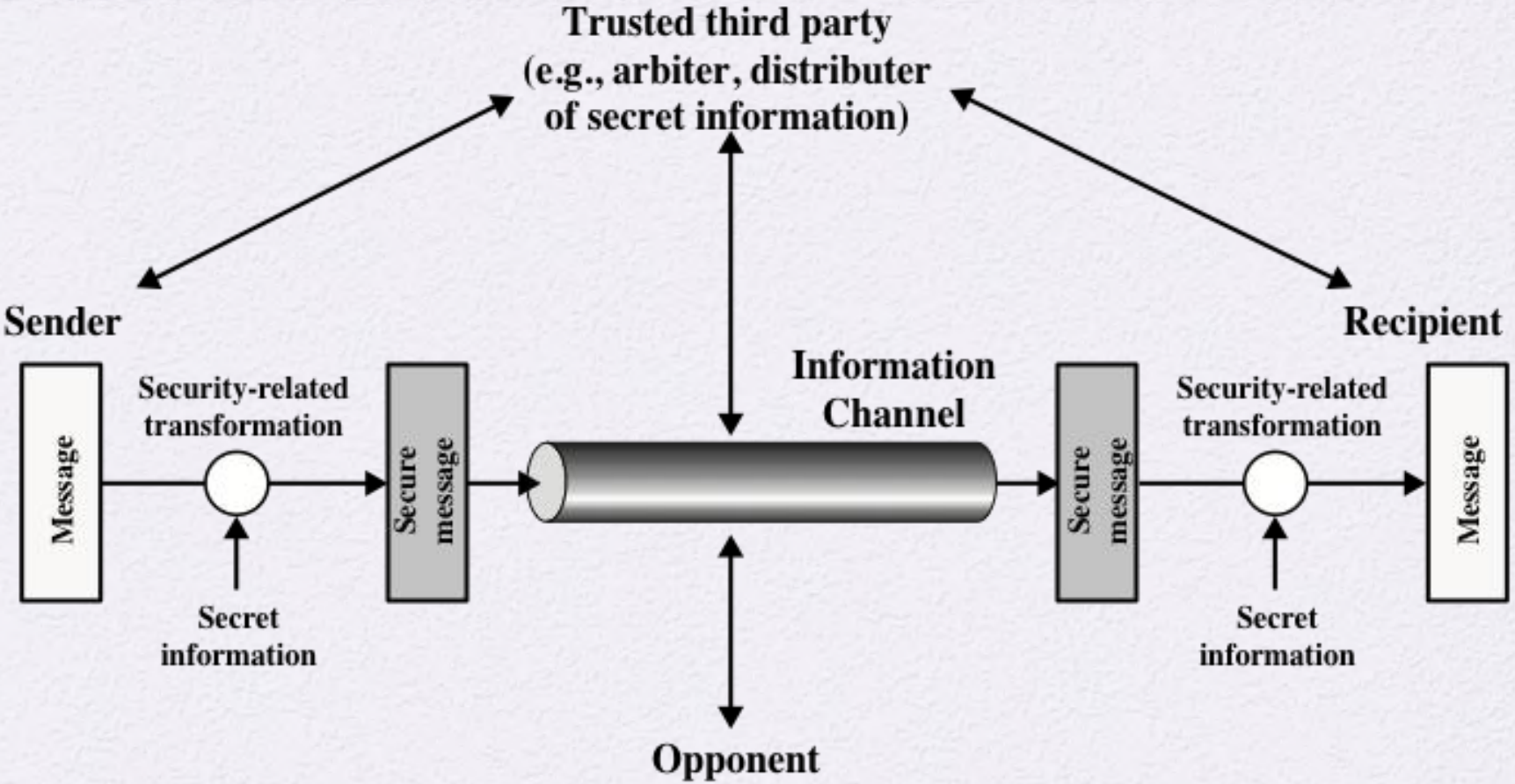


Figure 1.5 Model for Network Security

Network Access Security Model

Figure 1.6 Network Access Security Model

Unwanted Access

- Placement in a computer system of logic that exploits vulnerabilities in the system and that can affect application programs as well as utility programs such as editors and compilers
- Programs can present two kinds of threats:
  - Information access threats
    - Intercept or modify data on behalf of users who should not have access to that data
  - Service threats
    - Exploit service flaws in computers to inhibit use by legitimate users

Standards

National Institute of Standards and Technology

- NIST is a U.S. federal agency that deals with measurement science, standards, and technology related to U.S. government use and to the promotion of U.S. private-sector innovation
- Despite its national scope, NIST Federal Information Processing Standards (FIPS) and Special Publications (SP) have a worldwide impact

Internet Society

- ISOC is a professional membership society with world-wide organizational and individual membership
- Provides leadership in addressing issues that confront the future of the Internet and is the organization home for the groups responsible for Internet infrastructure standards

ITU-T

- The International Telecommunication Union (ITU) is an international organization within the United Nations System in which governments and the private sector coordinate global telecom networks and services
- The ITU Telecommunication Standardization Sector (ITU-T) is one of the three sectors of the ITU and whose mission is the development of technical standards covering all fields of telecommunications

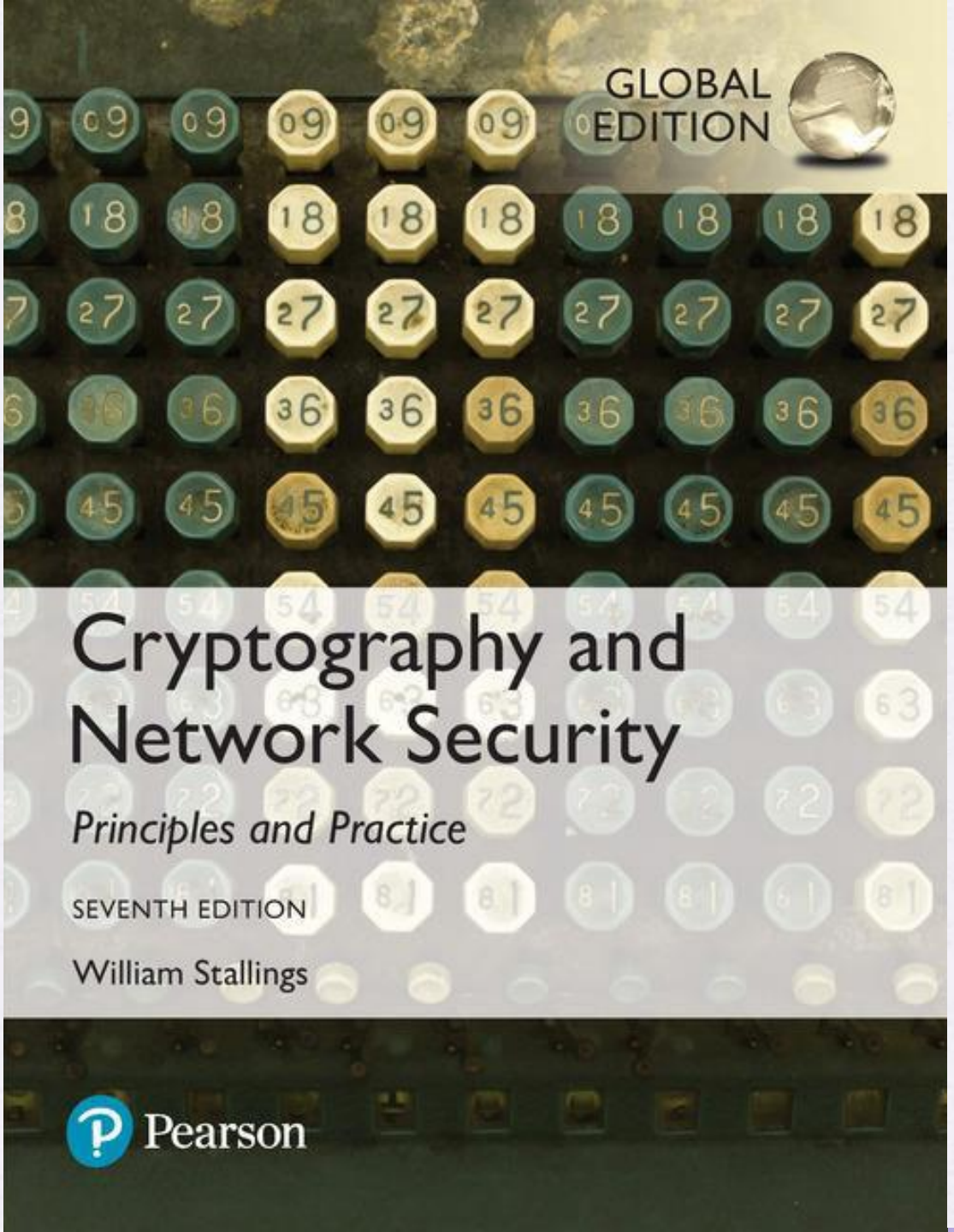
ISO

- The International Organization for Standardization is a world-wide federation of national standards bodies from more than 140 countries
- ISO is a nongovernmental organization that promotes the development of standardization and related activities with a view to facilitating the international exchange of goods and services and to developing cooperation in the spheres of intellectual, scientific, technological, and economic activity

Summary

- Computer security concepts
  - Definition
  - Examples
  - Challenges
- The OSI security architecture
- Security attacks
  - Passive attacks
  - Active attacks
- Attack surfaces and attack trees

- Security services
  - Authentication
  - Access control
  - Data confidentiality
  - Data integrity
  - Nonrepudiation
  - Availability service
- Security mechanisms
- Fundamental security design principles
- Network security model
- Standards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Chapter 2

Introduction to Number Theory

Divisibility

- We say that a nonzero b **divides** a if $a = mb$ for some m, where a, b, and m are integers
- b divides a if there is no remainder on division
- The notation $b \mid a$ is commonly used to mean b divides a
- If $b \mid a$ we say that b is a **divisor** of a

The positive divisors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24
13 | 182; - 5 | 30; 17 | 289; - 3 | 33; 17 | 0

Properties of Divisibility

- If $a \mid 1$, then $a = \pm 1$
- If $a \mid b$ and $b \mid a$, then $a = \pm b$
- Any $b \neq 0$ divides 0
- If $a \mid b$ and $b \mid c$, then $a \mid c$

$11 \mid 66$ and $66 \mid 198 = 11 \mid 198$

- If $b \mid g$ and $b \mid h$, then $b \mid (mg + nh)$ for arbitrary integers m and n

Properties of Divisibility

- To see this last point, note that:
  - If $b \mid g$, then g is of the form $g = b * g_1$ for some integer g_1
  - If $b \mid h$, then h is of the form $h = b * h_1$ for some integer h_1
- So:
  - $mg + nh = mbg_1 + nbh_1 = b * (mg_1 + nh_1)$

    and therefore b divides $mg + nh$

$b = 7$; $g = 14$; $h = 63$; $m = 3$; $n = 2$
$7 \mid 14$ and $7 \mid 63$.
To show $7 (3 * 14 + 2 * 63)$,
we have $(3 * 14 + 2 * 63) = 7(3 * 2 + 2 * 9)$,
and it is obvious that $7 \mid (7(3 * 2 + 2 * 9))$.

Division Algorithm

- Given any positive integer n and any nonnegative integer a, if we divide a by n we get an integer quotient q and an integer remainder r that obey the following relationship:

$$a = qn + r \qquad 0 \leq r < n;\ q = [a/n]$$

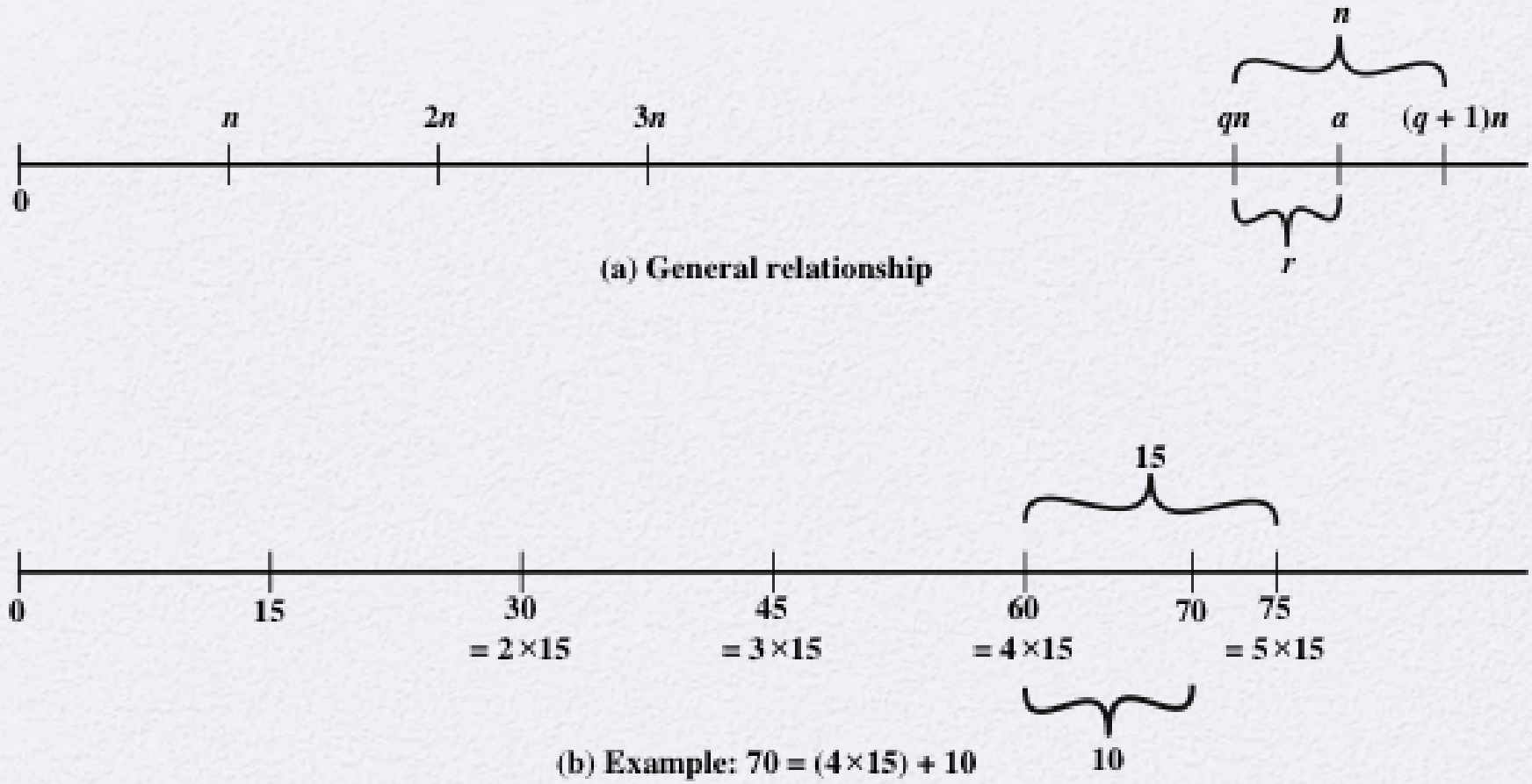


Figure 2.1 The Relationship $a = qn + r;\ 0 \leq r < n$

Euclidean Algorithm

- One of the basic techniques of number theory
- Procedure for determining the greatest common divisor of two positive integers
- Two integers are **relatively prime** if their only common positive integer factor is 1

Greatest Common Divisor (GCD)

- The greatest common divisor of a and b is the largest integer that divides both a and b
- We can use the notation $\gcd(a,b)$ to mean the **greatest common divisor** of a and b
- We also define $\gcd(0,0) = 0$
- Positive integer c is said to be the gcd of a and b if:
  - c is a divisor of a and b
  - Any divisor of a and b is a divisor of c
- An equivalent definition is:

$$\gcd(a,b) = \max[k, \text{ such that } k \mid a \text{ and } k \mid b]$$

GCD

- Because we require that the greatest common divisor be positive, $\gcd(a,b) = \gcd(a,-b) = \gcd(-a,b) = \gcd(-a,-b)$
- In general, $\gcd(a,b) = \gcd(|a|, |b|)$

$$\gcd(60, 24) = \gcd(60, -24) = 12$$

- Also, because all nonzero integers divide 0, we have $\gcd(a,0) = |a|$
- We stated that two integers a and b are relatively prime if their only common positive integer factor is 1; this is equivalent to saying that a and b are relatively prime if $\gcd(a,b) = 1$

8 and 15 are relatively prime because the positive divisors of 8 are 1, 2, 4, and 8, and the positive divisors of 15 are 1, 3, 5, and 15. So 1 is the only integer on both lists.

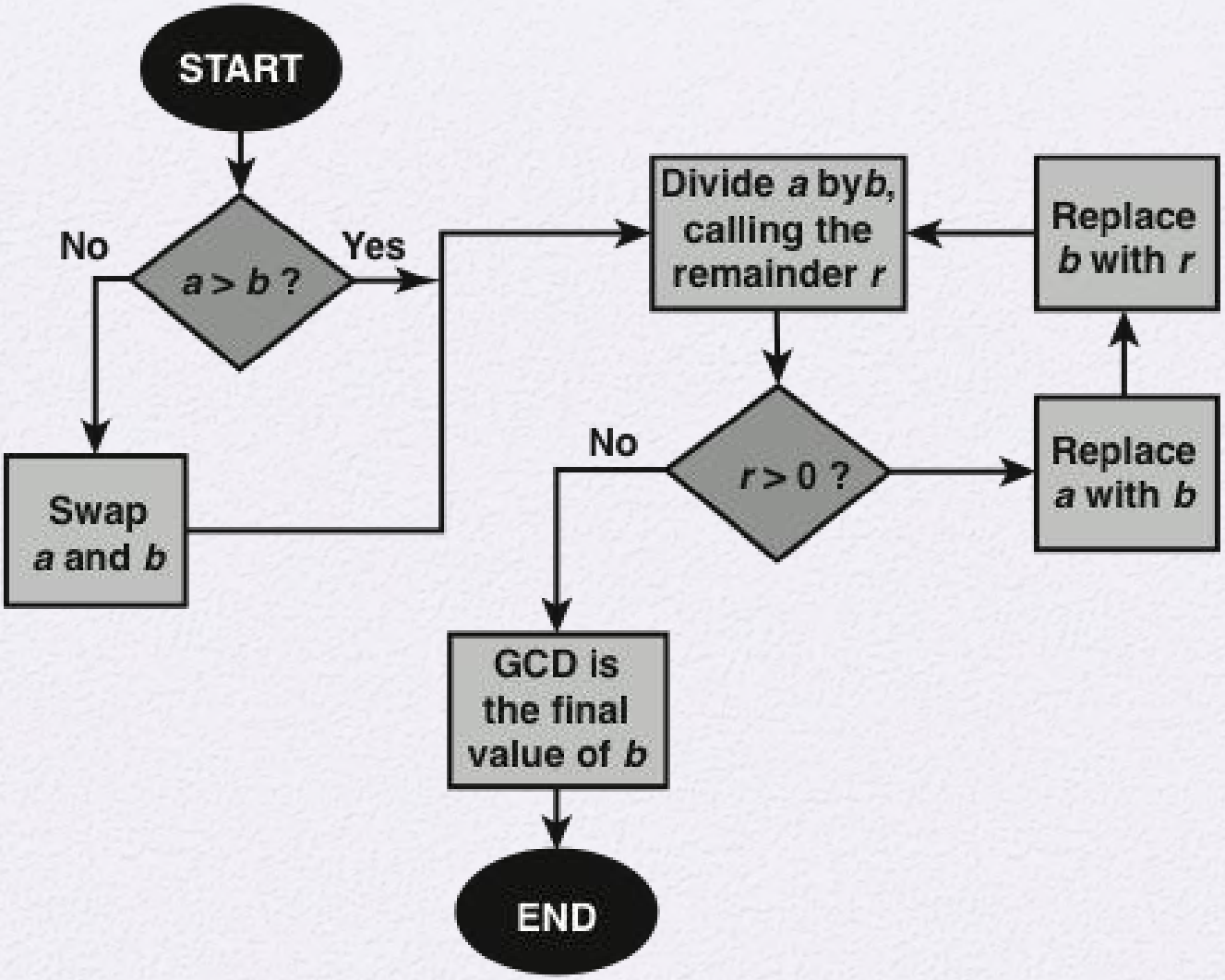


Figure 2.2 Euclidean Algorithm

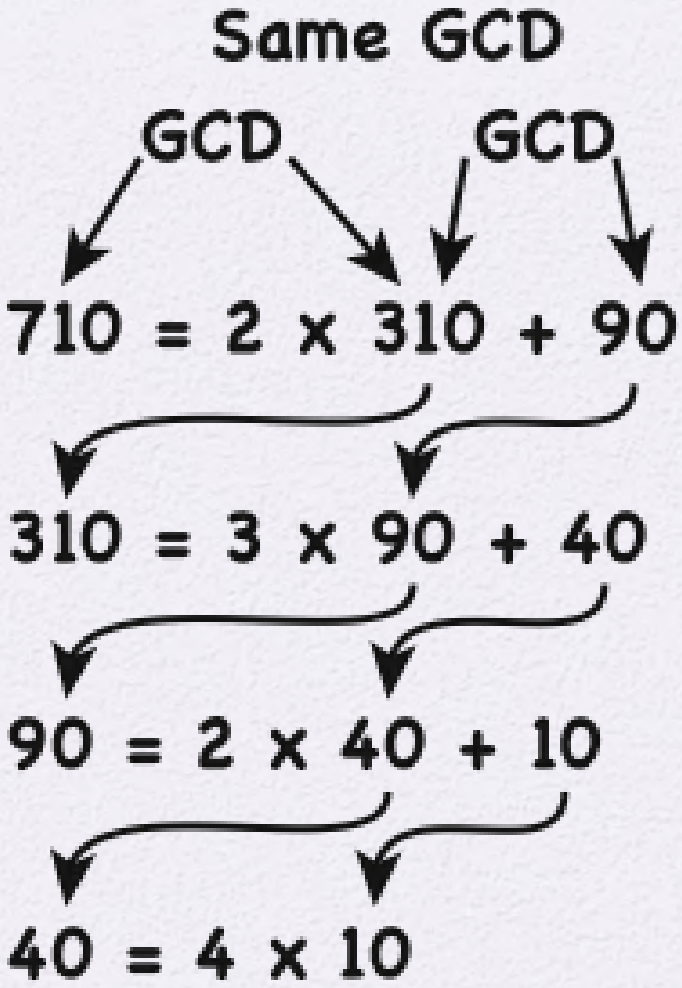


Figure 2.3 Euclidean Algorithm Example: gcd(710, 310)

Table 2.1
Euclidean Algorithm Example

Dividend	Divisor	Quotient	Remainder
$a = 1160718174$	$b = 316258250$	$q_1 = 3$	$r_1 = 211943424$
$b = 316258250$	$r_1 = 211943424$	$q_2 = 1$	$r_2 = 104314826$
$r_1 = 211943424$	$r_2 = 104314826$	$q_3 = 2$	$r_3 = 3313772$
$r_2 = 104314826$	$r_3 = 3313772$	$q_4 = 31$	$r_4 = 1587894$
$r_3 = 3313772$	$r_4 = 1587894$	$q_5 = 2$	$r_5 = 137984$
$r_4 = 1587894$	$r_5 = 137984$	$q_6 = 11$	$r_6 = 70070$
$r_5 = 137984$	$r_6 = 70070$	$q_7 = 1$	$r_7 = 67914$
$r_6 = 70070$	$r_7 = 67914$	$q_8 = 1$	$r_8 = 2156$
$r_7 = 67914$	$r_8 = 2156$	$q_9 = 31$	$r_9 = 1078$
$r_8 = 2156$	$r_9 = 1078$	$q_{10} = 2$	$r_{10} = 0$

(This table can be found on page 34 in the textbook)

Modular Arithmetic

- The modulus
  - If a is an integer and n is a positive integer, we define a mod n to be the remainder when a is divided by n; the integer n is called the **modulus**
  - Thus, for any integer a:

$$a = qn + r \quad 0 \leq r < n; \; q = [a/n]$$

$$a = [a/n] * n + (a \bmod n)$$

11 mod 7 = 4; - 11 mod 7 = 3

Modular Arithmetic

- Congruent modulo n
  - Two integers a and b are said to be **congruent modulo n** if $(a \bmod n) = (b \bmod n)$
  - This is written as $a = b \pmod{n}$[2]
  - Note that if $a = 0 \pmod{n}$, then $n \mid a$

$73 = 4 \pmod{23}$; $21 = -9 \pmod{10}$

Properties of Congruences

- Congruences have the following properties:

  1. $a = b \pmod{n}$ if $n \mid (a - b)$

  2. $a = b \pmod{n}$ implies $b = a \pmod{n}$

  3. $a = b \pmod{n}$ and $b = c \pmod{n}$ imply $a = c \pmod{n}$

- To demonstrate the first point, if $n \mid (a - b)$, then $(a - b) = kn$ for some k
  - So we can write $a = b + kn$
  - Therefore, (a mod n) = (remainder when $b + kn$ is divided by n) = (remainder when b is divided by n) = (b mod n)

$23 = 8 \pmod{5}$ because $23 - 8 = 15 = 5 * 3$
$-11 = 5 \pmod{8}$ because $-11 - 5 = -16 = 8 * (-2)$
$81 = 0 \pmod{27}$ because $81 - 0 = 81 = 27 * 3$

Modular Arithmetic

- Modular arithmetic exhibits the following properties:

  1. $[(a \bmod n) + (b \bmod n)] \bmod n = (a + b) \bmod n$
  2. $[(a \bmod n) - (b \bmod n)] \bmod n = (a - b) \bmod n$
  3. $[(a \bmod n) * (b \bmod n)] \bmod n = (a * b) \bmod n$

- We demonstrate the first property:
  - Define $(a \bmod n) = r_a$ and $(b \bmod n) = r_b$. Then we can write $a = r_a + jn$ for some integer j and $b = r_b + kn$ for some integer k
  - Then:

$$
\begin{aligned}
(a + b) \bmod n &= (ra + jn + rb + kn) \bmod n \\
&= (ra + rb + (k + j)n) \bmod n \\
&= (ra + rb) \bmod n \\
&= [(a \bmod n) + (b \bmod n)] \bmod n
\end{aligned}
$$

Remaining Properties:

- Examples of the three remaining properties:

11 mod 8 = 3; 15 mod 8 = 7

[(11 mod 8) + (15 mod 8)] mod 8 = 10 mod 8 = 2

(11 + 15) mod 8 = 26 mod 8 = 2

[(11 mod 8) - (15 mod 8)] mod 8 = - 4 mod 8 = 4

(11 - 15) mod 8 = - 4 mod 8 = 4

[(11 mod 8) * (15 mod 8)] mod 8 = 21 mod 8 = 5

(11 * 15) mod 8 = 165 mod 8 = 5

Table 2.2(a) Arithmetic Modulo 8

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6

(This table can be found on page 37 in the textbook)

Table 2.2(b)
Multiplication Modulo 8

×	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	0	2	4	6
3	0	3	6	1	4	7	2	5
4	0	4	0	4	0	4	0	4
5	0	5	2	7	4	1	6	3
6	0	6	4	2	0	6	4	2
7	0	7	6	5	4	3	2	1

(This table can be found on page 37 in the textbook)

Table 2.2(c)

Additive and Multiplicative Inverse Modulo 8

w	$-w$	w^{-1}
0	0	—
1	7	1
2	6	—
3	5	3
4	4	—
5	3	5
6	2	—
7	1	7

(This table can be found on page 37 in the textbook)

Table 2.3

Properties of Modular Arithmetic for Integers in Z_n

Property	Expression
Commutative Laws	$(w + x) \bmod n = (x + w) \bmod n$ $(w \times x) \bmod n = (x \times w) \bmod n$
Associative Laws	$[(w + x) + y] \bmod n = [w + (x + y)] \bmod n$ $[(w \times x) \times y] \bmod n = [w \times (x \times y)] \bmod n$
Distributive Law	$[w \times (x + y)] \bmod n = [(w \times x) + (w \times y)] \bmod n$
Identities	$(0 + w) \bmod n = w \bmod n$ $(1 \times w) \bmod n = w \bmod n$
Additive Inverse ($-w$)	For each $w \in Z_n$, there exists a z such that $w + z \equiv 0 \bmod n$

(This table can be found on page 38 in the textbook)

Table 2.4

Extended Euclidean Algorithm Example

i	r_i	q_i	x_i	Y_i
–1	1759		1	0
0	550		0	1
1	109	3	1	–3
2	5	5	–5	16
3	4	21	106	–339
4	1	1	–111	355
5	0	4		

Result: $d = 1$; $x = -111$; $y = 355$

(This table can be found on page 43 in the textbook)

Prime Numbers

- Prime numbers only have divisors of 1 and itself
  - They cannot be written as a product of other numbers
- Prime numbers are central to number theory
- Any integer $a > 1$ can be factored in a unique way as

$$a = p_1^{a1} * p_2^{a2} * \ldots * p_{p1}^{a1}$$

where $p_1 < p_2 < \ldots < p_t$ are prime numbers and where each a_i is a positive integer

- This is known as the fundamental theorem of arithmetic

Table 2.5
Primes Under 2000

2	101	211	307	401	503	601	701	809	907	1009	1103	1201	1301	1409	1511	1601	1709	1801	1901
3	103	223	311	409	509	607	709	811	911	1013	1109	1213	1303	1423	1523	1607	1721	1811	1907
5	107	227	313	419	521	613	719	821	919	1019	1117	1217	1307	1427	1531	1609	1723	1823	1913
7	109	229	317	421	523	617	727	823	929	1021	1123	1223	1319	1429	1543	1613	1733	1831	1931
11	113	233	331	431	541	619	733	827	937	1031	1129	1229	1321	1433	1549	1619	1741	1847	1933
13	127	239	337	433	547	631	739	829	941	1033	1151	1231	1327	1439	1553	1621	1747	1861	1949
17	131	241	347	439	557	641	743	839	947	1039	1153	1237	1361	1447	1559	1627	1753	1867	1951
19	137	251	349	443	563	643	751	853	953	1049	1163	1249	1367	1451	1567	1637	1759	1871	1973
23	139	257	353	449	569	647	757	857	967	1051	1171	1259	1373	1453	1571	1657	1777	1873	1979
29	149	263	359	457	571	653	761	859	971	1061	1181	1277	1381	1459	1579	1663	1783	1877	1987
31	151	269	367	461	577	659	769	863	977	1063	1187	1279	1399	1471	1583	1667	1787	1879	1993
37	157	271	373	463	587	661	773	877	983	1069	1193	1283		1481	1597	1669	1789	1889	1997
41	163	277	379	467	593	673	787	881	991	1087		1289		1483		1693			1999
43	167	281	383	479	599	677	797	883	997	1091		1291		1487		1697			
47	173	283	389	487		683		887		1093		1297		1489		1699			
53	179	293	397	491		691				1097				1493					
59	181			499										1499					
61	191																		
67	193																		
71	197																		
73	199																		
79																			
83																			
89																			
97																			

(This table can be found on page 44 in the textbook)

Fermat's Theorem

- States the following:
  - If p is prime and a is a positive integer not divisible by p then

$$a^{p-1} = 1 \pmod{p}$$

- An alternate form is:
  - If p is prime and a is a positive integer then

$$a^{p} = a \pmod{p}$$

Table 2.6

Some Values of Euler's Totient Function ø(*n*)

n	$\phi(n)$
1	1
2	1
3	2
4	2
5	4
6	2
7	6
8	4
9	6
10	4

n	$\phi(n)$
11	10
12	4
13	12
14	6
15	8
16	8
17	16
18	6
19	18
20	8

n	$\phi(n)$
21	12
22	10
23	22
24	8
25	20
26	12
27	18
28	12
29	28
30	8

(This table can be found on page 48 in the textbook)

Euler's Theorem

- States that for every *a* and *n* that are relatively prime:

$$a^{\phi(n)} = 1 \pmod{n}$$

- An alternative form is:

$$a^{\phi(n)+1} = a \pmod{n}$$

Miller-Rabin Algorithm

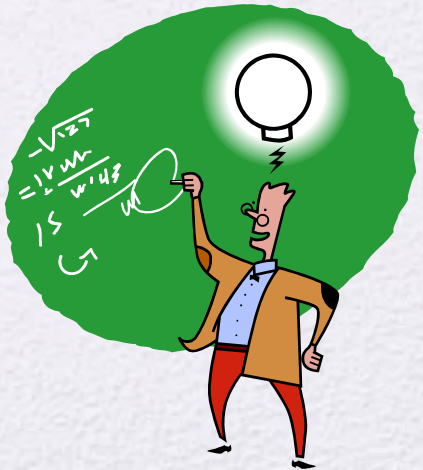
- Typically used to test a large number for primality
- Algorithm is:

TEST (n)

1. Find integers k, q, with $k > 0$, q odd, so that $(n-1)=2^k q$;
2. Select a random integer a, $1 < a < n-1$;
3. **if** $a^q \bmod n = 1$ **then** return ("inconclusive") ;
4. **for** $j = 0$ **to** $k-1$ **do**
5. **if** $(a^{2jq} \bmod n = n-1)$ **then** return ("inconclusive") ;
6. return ("composite");

Deterministic Primality Algorithm

- Prior to 2002 there was no known method of efficiently proving the primality of very large numbers
- All of the algorithms in use produced a probabilistic result
- In 2002 Agrawal, Kayal, and Saxena developed an algorithm that efficiently determines whether a given large number is prime
  - Known as the AKS algorithm
  - Does not appear to be as efficient as the Miller-Rabin algorithm



Chinese Remainder Theorem (CRT)

- Believed to have been discovered by the Chinese mathematician Sun-Tsu in around 100 A.D.
- One of the most useful results of number theory
- Says it is possible to reconstruct integers in a certain range from their residues modulo a set of pairwise relatively prime moduli
- Can be stated in several ways

Provides a way to manipulate (potentially very large) numbers mod M in terms of tuples of smaller numbers

- This can be useful when M is 150 digits or more
- However, it is necessary to know beforehand the factorization of M

Table 2.7

Powers of Integers, Modulo 19

a	a^2	a^3	a^4	a^5	a^6	a^7	a^8	a^9	a^{10}	a^{11}	a^{12}	a^{13}	a^{14}	a^{15}	a^{16}	a^{17}	a^{18}
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	4	8	16	13	7	14	9	18	17	15	11	3	6	12	5	10	1
3	9	8	5	15	7	2	6	18	16	10	11	14	4	12	17	13	1
4	16	7	9	17	11	6	5	1	4	16	7	9	17	11	6	5	1
5	6	11	17	9	7	16	4	1	5	6	11	17	9	7	16	4	1
6	17	7	4	5	11	9	16	1	6	17	7	4	5	11	9	16	1
7	11	1	7	11	1	7	11	1	7	11	1	7	11	1	7	11	1
8	7	18	11	12	1	8	7	18	11	12	1	8	7	18	11	12	1
9	5	7	6	16	11	4	17	1	9	5	7	6	16	11	4	17	1
10	5	12	6	3	11	15	17	18	9	14	7	13	16	8	4	2	1
11	7	1	11	7	1	11	7	1	11	7	1	11	7	1	11	7	1
12	11	18	7	8	1	12	11	18	7	8	1	12	11	18	7	8	1
13	17	12	4	14	11	10	16	18	6	2	7	15	5	8	9	3	1
14	6	8	17	10	7	3	4	18	5	13	11	2	9	12	16	15	1
15	16	12	9	2	11	13	5	18	4	3	7	10	17	8	6	14	1
16	9	11	5	4	7	17	6	1	16	9	11	5	4	7	17	6	1
17	4	11	16	6	7	5	9	1	17	4	11	16	6	7	5	9	1
18	1	18	1	18	1	18	1	18	1	18	1	18	1	18	1	18	1

(This table can be found on page 57 in the textbook)

Table 2.8

Tables of Discrete Logarithms, Modulo 19

(a) Discrete logarithms to the base 2, modulo 19

a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$\log_{2,19}(a)$	18	1	13	2	16	14	6	3	8	17	12	15	5	7	11	4	10	9

(b) Discrete logarithms to the base 3, modulo 19

a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$\log_{3,19}(a)$	18	7	1	14	4	8	6	3	2	11	12	15	17	13	5	10	16	9

(c) Discrete logarithms to the base 10, modulo 19

a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$\log_{10,19}(a)$	18	17	5	16	2	4	12	15	10	1	6	3	13	11	7	14	8	9

(d) Discrete logarithms to the base 13, modulo 19

a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$\log_{13,19}(a)$	18	11	17	4	14	10	12	15	16	7	6	3	1	5	13	8	2	9

(e) Discrete logarithms to the base 14, modulo 19

a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$\log_{14,19}(a)$	18	13	7	8	10	2	6	3	14	5	12	15	11	1	17	16	4	9

(f) Discrete logarithms to the base 15, modulo 19

a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$\log_{15,19}(a)$	18	5	11	10	8	16	12	15	4	13	6	3	7	17	1	2	14	9

(This table can be found on page 60 in the textbook)

Summary

- Divisibility and the division algorithm
- The Euclidean algorithm
  - Greatest Common Divisor
  - Finding the Greatest Common Divisor
- Modular arithmetic
  - The modulus
  - Properties of congruences
  - Modular arithmetic operations
  - Properties of modular arithmetic
  - Euclidean algorithm revisited
  - The extended Euclidean algorithm
- Prime numbers

- Fermat's Theorem
- Euler's totient function
- Euler's Theorem
- Testing for primality
  - Miller-Rabin algorithm
  - A deterministic primality algorithm
  - Distribution of primes
- The Chinese Remainder Theorem
- Discrete logarithms
  - Powers of an integer, modulo *n*
  - Logarithms for modular arithmetic
  - Calculation of discrete logarithms

Chapter 3

Block Ciphers and The Data Encryption Standard

Motivation for the Feistel Cipher Structure

- What we need is a reversible mapping
- Ex

PLAINTEXT	CIPHERTEXT
00	11
01	10
10	00
11	01

- Ex. of an irreversible mapping
- Note that decryption is not possible

PLAINTEXT	CIPHERTEXT
00	11
01	00
10	00
11	01

How many reversible mappings do we have

- Let n be the block size
  - The number of plaintext blocks is 2^n
  - This is also the number of cipher text blocks
  - $2^n!$ is the number of reversible mapping
- N should be sufficiently large (remember the lesson from the Hill cipher)

A strongly ideal cipher

- Would choose an arbitrary reversible mapping for a large block size
- The problem with this is the large key size
  - The key would be the table (the second column only)
  - Therefore the key size is n 2^n bits
  - Imagine 64 * 2^{64}

Shannon's proposal

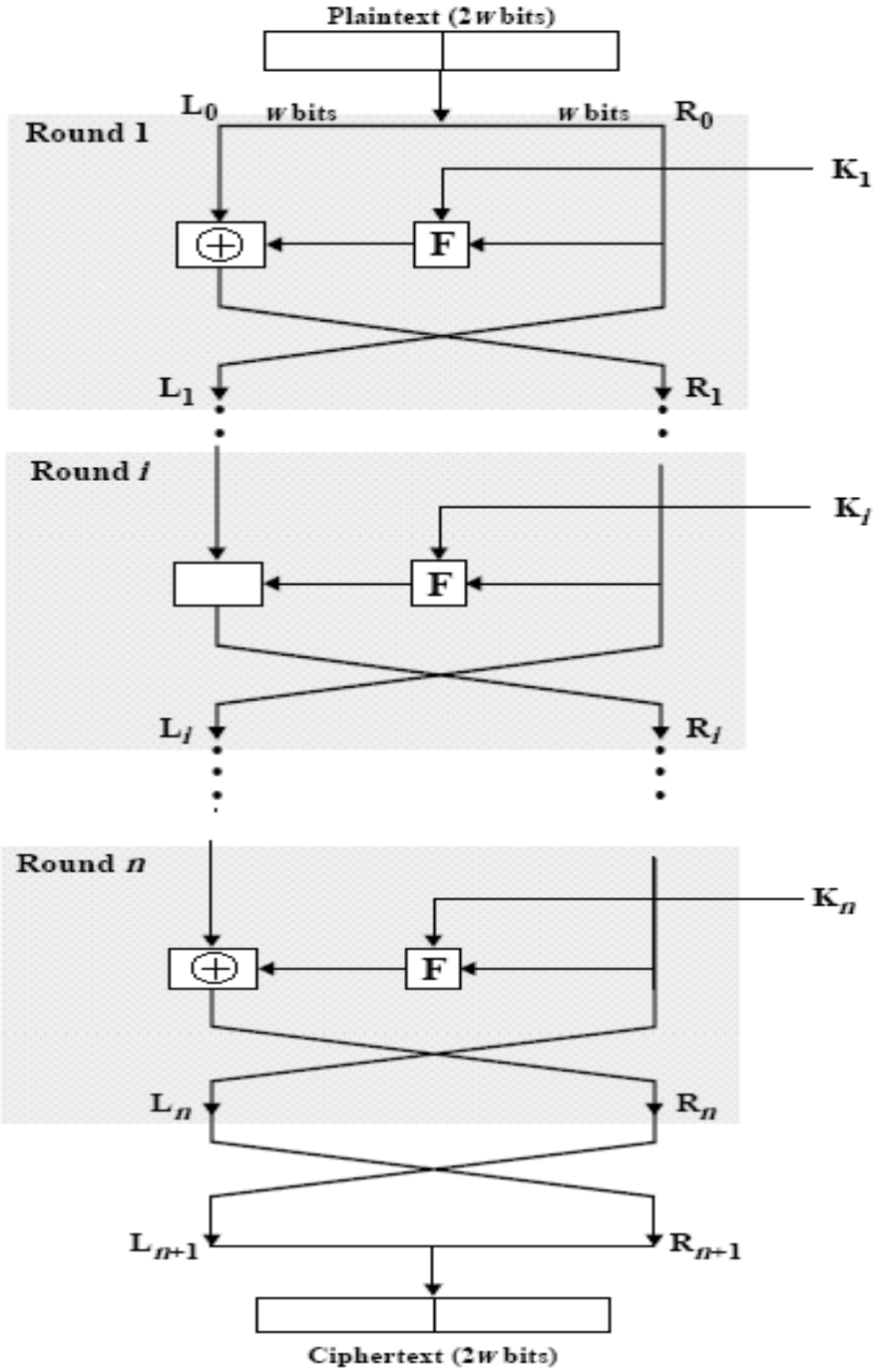
- Shannon suggests two methods for frustrating statistical cryptanalysis: Diffusion and Confusion
- Diffusion
  - The statistical structure of the plaintext is dissipated into long range statistics of the cipher text.
  - This is achieved by having each plaintext digit affect many ciphertext digits.
  - Remember the first lesson from the Hill cipher

- Confusion
  - Seeks to make the relationship between the statistics of the ciphertext and the value of the key used as complex as possible
  - This is to thwart attempts to discover the key.
  - This is achieved by the use of a complex substitution algorithm
  - Recall the second lesson of the Hill cipher.
  -

The Feistel Cipher

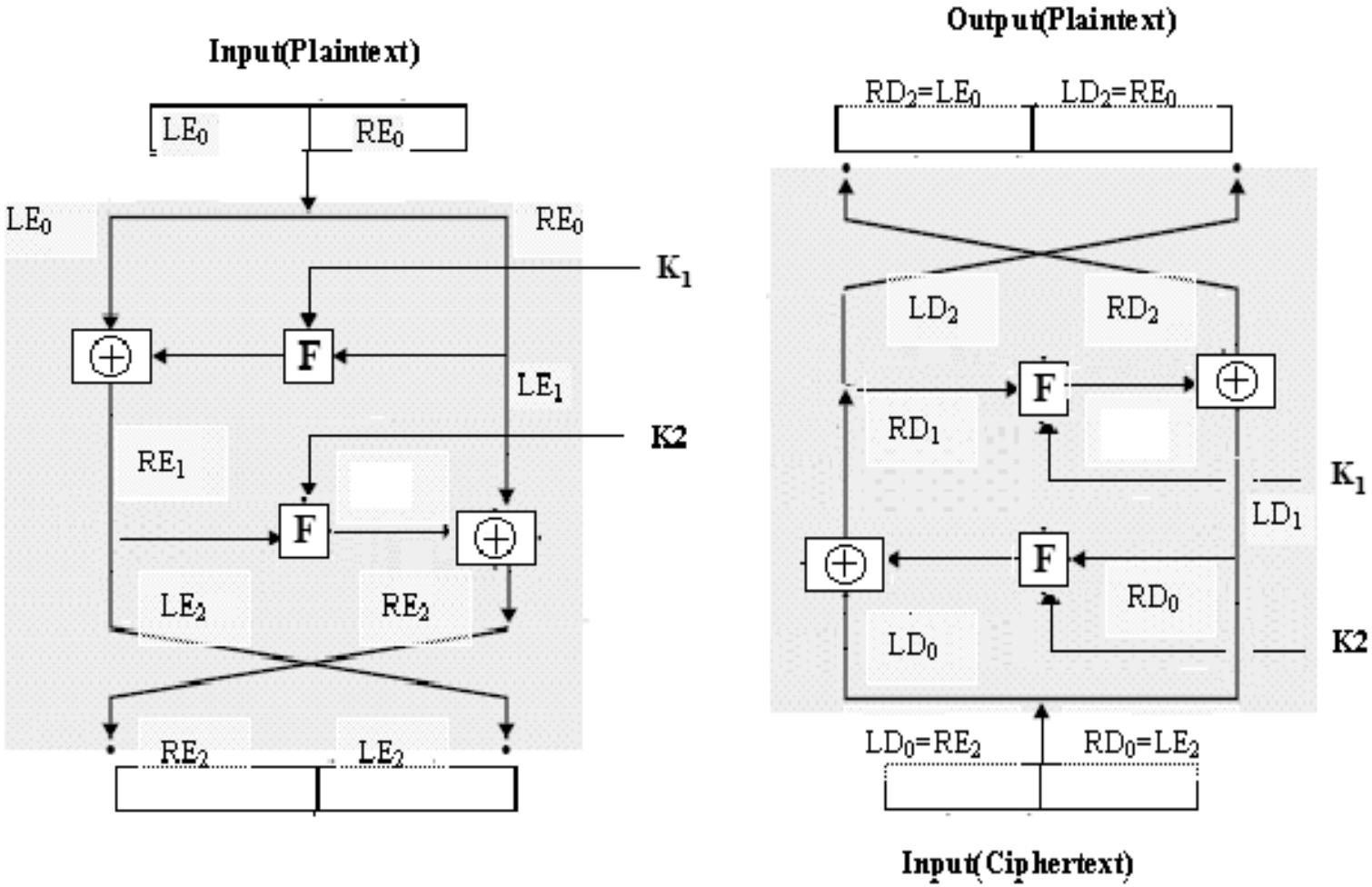
- A structure that is used by most significant symmetric block ciphers currently in use.
- A practical application of Shannon's proposal
- Alternates confusion and diffusion functions.
- Uses substitution and transposition techniques.

Feistel Cipher Structure


Plaintext (2w bits)
L0
w bits
w bits
R0
Round 1
K1
F
L1
R1
Round i
Ki
F
Li
Ri
Round n
Kn
F
Ln
Rn
Ln+1
Rn+1
Ciphertext (2w bits)

Feistel Encryption and Decryption

- Decryption is the same as encryption but uses the subkeys in reversed order.
  - This is good because it reduces the implementation cost.


Input(Plaintext)
LE0
RE0
LE0
RE0
K1
F
LE1
K2
RE1
F
LE2
RE2
RE2
LE2
Output(Plaintext)
RD2=LE0
LD2=RE0
LD2
RD2
F
RD1
K1
LD1
F
RD0
LD0
K2
LD0=RE2
RD0=LE2
Input(Ciphertext)

- To see that the decryption of the cipher really produces the original plaintext, we need to show that the output of each decryption round is the same as the input of the corresponding encryption round.

- Note that
  - LD0=RE2 and
  - RD0=LE2
- So in order to show that the decryption phase really works we need to show that for the first decryption round
  - LD1=RE1 and
  - RD1=LE1
- and for the second decryption round we need to show that
  - LD2=RE0 and
  - RD2=LE0

The First decryption round

- Note that (from the encryption side)
  - LE2=RE1
  - RE2=LE1⊕ F(RE1, K2)
- and on the decryption side
  - LD1 =RD0= LE2= RE1
  - RD1=LD0⊕ F(RD0, K2)
  - =RE2⊕ F(RE1, K2)
  - = (LE1⊕ F(RE1, K2)) ⊕ F(RE1, K2)
  - = LE1

- This is true because the XOR operation has the following properties:
  1. $(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z)$
  2. $(X \oplus X) = 0$
  3. $(X \oplus 0) = Y$

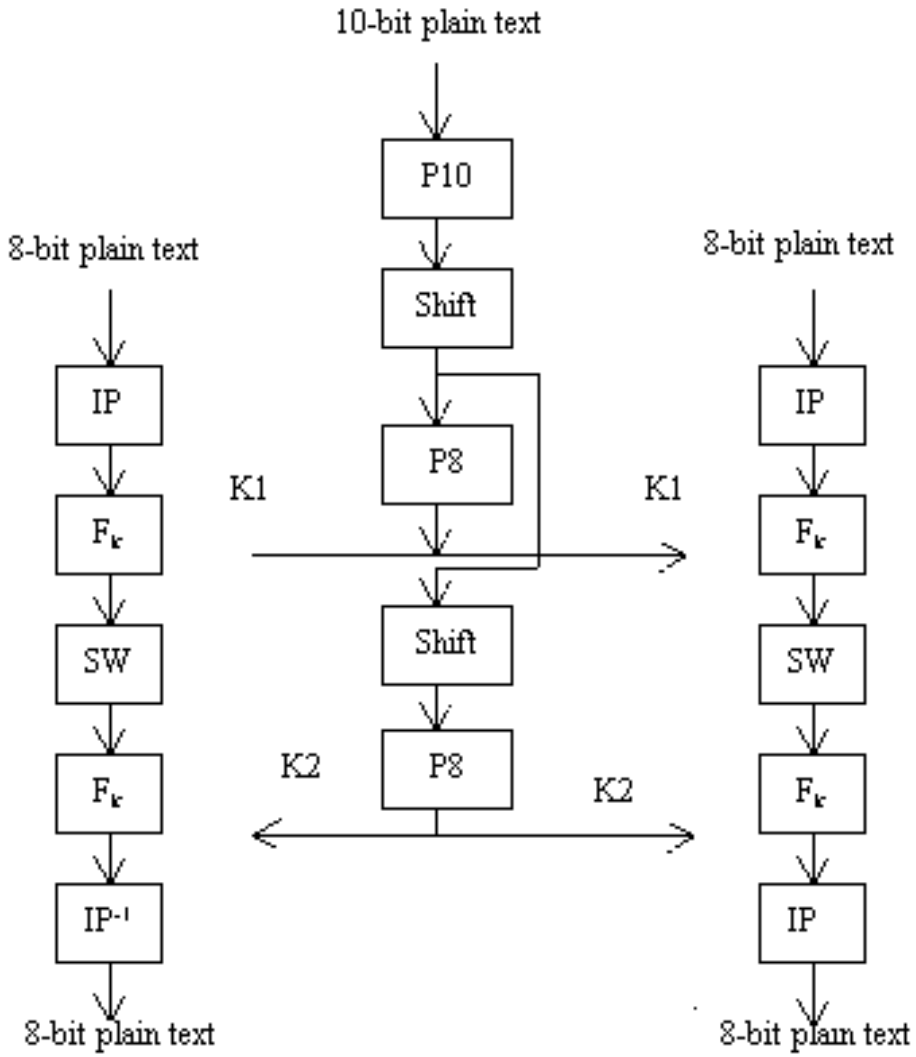
the second decryption round

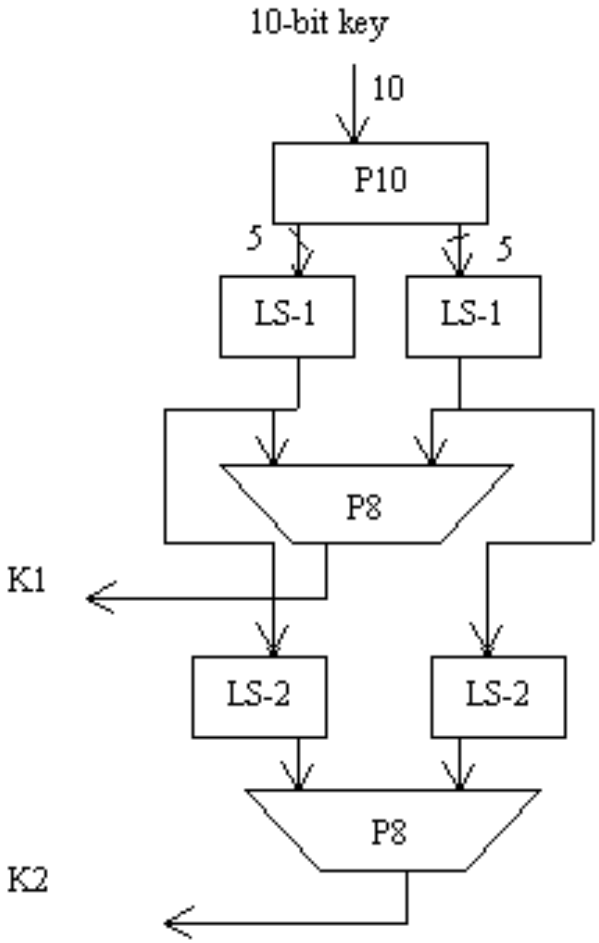
- Now for the second decryption round, we need to show that:
  - LD2=RE0
  - RD2=LE0
- This is true because:
  - LD2 =RD1= LE1= RE0
- and
  - RD2=LD1$\oplus$ F(RD1, K1)
  - =RE1$\oplus$ F(LE1, K1)
  - = (LE0$\oplus$ F(LE1, K1)) $\oplus$ F(LE1, K1)
  - = LE0

Simplified DES

- Simplified DES was developed for educational purposes and it was not intended as a secure algorithm (Schaefer 96).
- It uses a small key size of 10 bits only.
- The block size is also small, 8 bits only. In other words, it encrypts 8 plaintext bits at a time.
- Therefore, the degree of diffusion achieved is quite limited.
- More importantly, the small key size makes it very vulnerable to brute-force attack.
- Minimal number of rounds (2)

- S-DES encryption performs 5 main steps.
- It starts by performing an initial permutation (IP),
- followed by a complex function fk.
- Next, the left and right halves are switched before applying the function fk once again.
- Finally, another permutation step (IP-1) which is the inverse of the initial permutation is applied.


10-bit plain text
P10
Shift
P8
K1
Shift
P8
K2
8-bit plain text
IP
F_k
SW
F_k
IP^{-1}
8-bit plain text
K1
K2
8-bit plain text
IP
F_k
SW
F_k
IP
8-bit plain text


10-bit key
10
P10
5
5
LS-1
LS-1
P8
K1
LS-2
LS-2
P8
K2

Feistel Cipher Design Elements

- block size
  - DES 64 bits SDES=8 bits
  - diffusion
- key size
  - DES 56 bits SDES= 10 bits
  - Brute force attack
  - 16 subkeys each with 48 bits
  - Better confusion: more bits in the subkeys which means we have more bits to make the relationship between the values taken from the sboxes and the key complex

- number of rounds
  - DES 16 rounds ➔better diffusion and confusion
- subkey generation algorithm
  - We need substantially different subkeys
- round function
  - An important part of which is the s-boexes
- Other considerations
  - fast software en/decryption
  - ease of analysis
    - Of the algorithm
    - To earn user confidence
    - We discover any weakness point

Data Encryption Standard (DES)

- most widely used block cipher in world
- adopted in 1977 by NBS (now NIST)
  - as FIPS PUB 46
- encrypts 64-bit data using 56-bit key
- has widespread use
- has been considerable controversy over its security

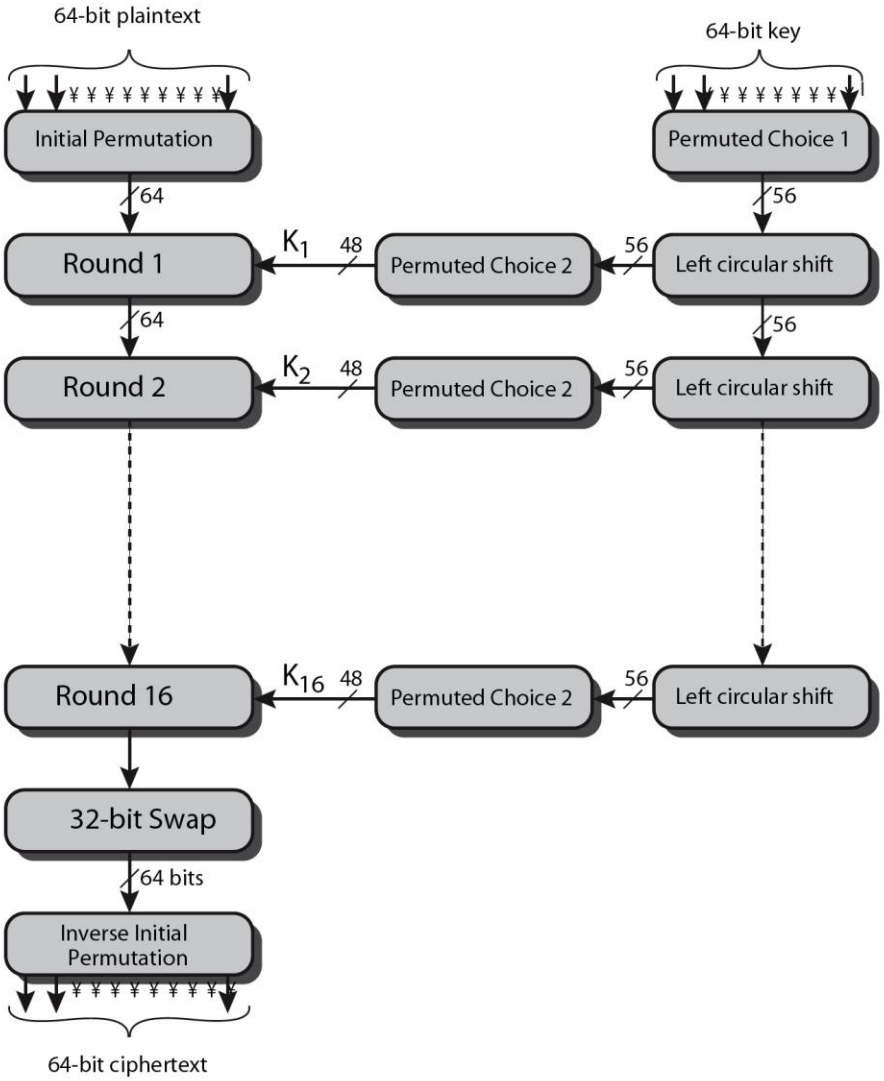
DES History

- IBM developed Lucifer cipher
  - by team led by Feistel in late 60's
  - used 64-bit data blocks with 128-bit key
- then redeveloped as a commercial cipher with input from NSA and others
- in 1973 NBS issued request for proposals for a national cipher standard
- IBM submitted their revised Lucifer which was eventually accepted as the DES

DES Design Controversy

- although DES standard is public
- was considerable controversy over design
  - in choice of 56-bit key (vs Lucifer 128-bit)
  - and because design criteria were classified
- subsequent events and public analysis show in fact design was appropriate
- use of DES has flourished
  - especially in financial applications
  - still standardised for legacy application use

DES Encryption Overview



Initial Permutation IP

- first step of the data computation
- IP reorders the input data bits
- even bits to LH half, odd bits to RH half
- quite regular in structure (easy in h/w)
- example:

```
IP(675a6967 5e5a6b5a) = (ffb2194d
004df6fb)
```

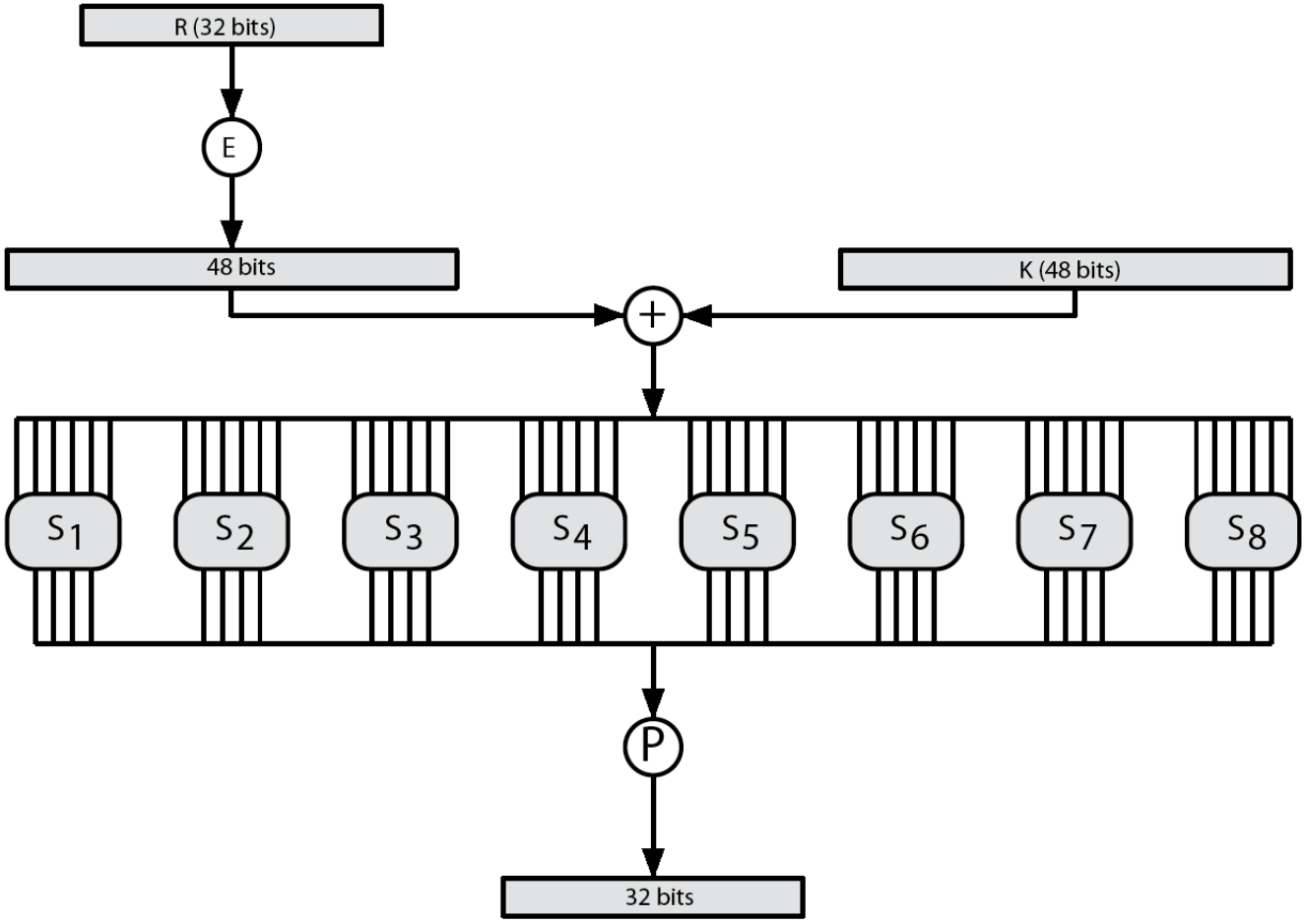
DES Round Structure

- uses two 32-bit L & R halves
- as for any Feistel cipher can describe as:

  $L_i = R_{i-1}$

  $R_i = L_{i-1} \oplus \mathrm{F}(R_{i-1}, K_i)$
- F takes 32-bit R half and 48-bit subkey:
  - expands R to 48-bits using perm E
  - adds to subkey using XOR
  - 8 S-boxes, each is a 4*16 matrix
  - Each S-box produces a 4-bit binary value
  - passes through 8 S-boxes to get 32-bit result
  - finally permutes using 32-bit perm P

DES Round Structure



Substitution Boxes S

- have eight S-boxes which map 6 to 4 bits
- each S-box is actually 4 little 4 bit boxes
  - outer bits 1 & 6 (**row** bits) select one row of 4
  - inner bits 2-5 (**col** bits) are substituted
  - result is 8 lots of 4 bits, or 32 bits
- row selection depends on both data & key
  - feature known as autoclaving (autokeying)
- example:
  - `S(18 09 12 3d 11 17 38 39) = 5fd25e03`

DES Key Schedule

- forms subkeys used in each round
  - initial permutation of the key (PC1) which selects 56-bits in two 28-bit halves
  - 16 stages consisting of:
    - rotating **each half** separately either 1 or 2 places depending on the **key rotation schedule** K
    - selecting 24-bits from each half & permuting them by PC2 for use in round function F
- note practical use issues in h/w vs s/w

DES Decryption

- decrypt must unwind steps of data computation
- with Feistel design, do encryption steps again using subkeys in reverse order (SK16 … SK1)
  - IP undoes final FP step of encryption
  - 1st round with SK16 undoes 16th encrypt round
  - ….
  - 16th round with SK1 undoes 1st encrypt round
  - then final FP undoes initial encryption IP
  - thus recovering original data value

Avalanche Effect

- key desirable property of encryption alg
- where a change of **one** input or key bit results in changing approx **half** output bits
- making attempts to “home-in” by guessing keys impossible
- DES exhibits strong avalanche

The Strength of DES

- The Use of 56-bit keys
  - The 1977 proposal to build a machine that could cryptanalyse DES in less than 10 hours
    - The cost $20 million
  - In 1998 EFF announced that it built a machine that broke DES in less than 3 days
  - The cost $25,000
  - There is more to brute force attack than just a powerful machine

- Timing Attacks
  - Given how long it takes a particular to perform decryption of various ciphertexts, some information about the key or the plaintext is obtained.
  - Could determine the number of bits equal to 1 of the secret key
  - Along way from knowing the key

- Differential Cryptanalysis
  - Requires 2^{47} chosen plaintexts
- Linear Cryptanalysis
  - Requires 2^{47} known plaintexts
  - Slightly easier than Diff cryptanalysis

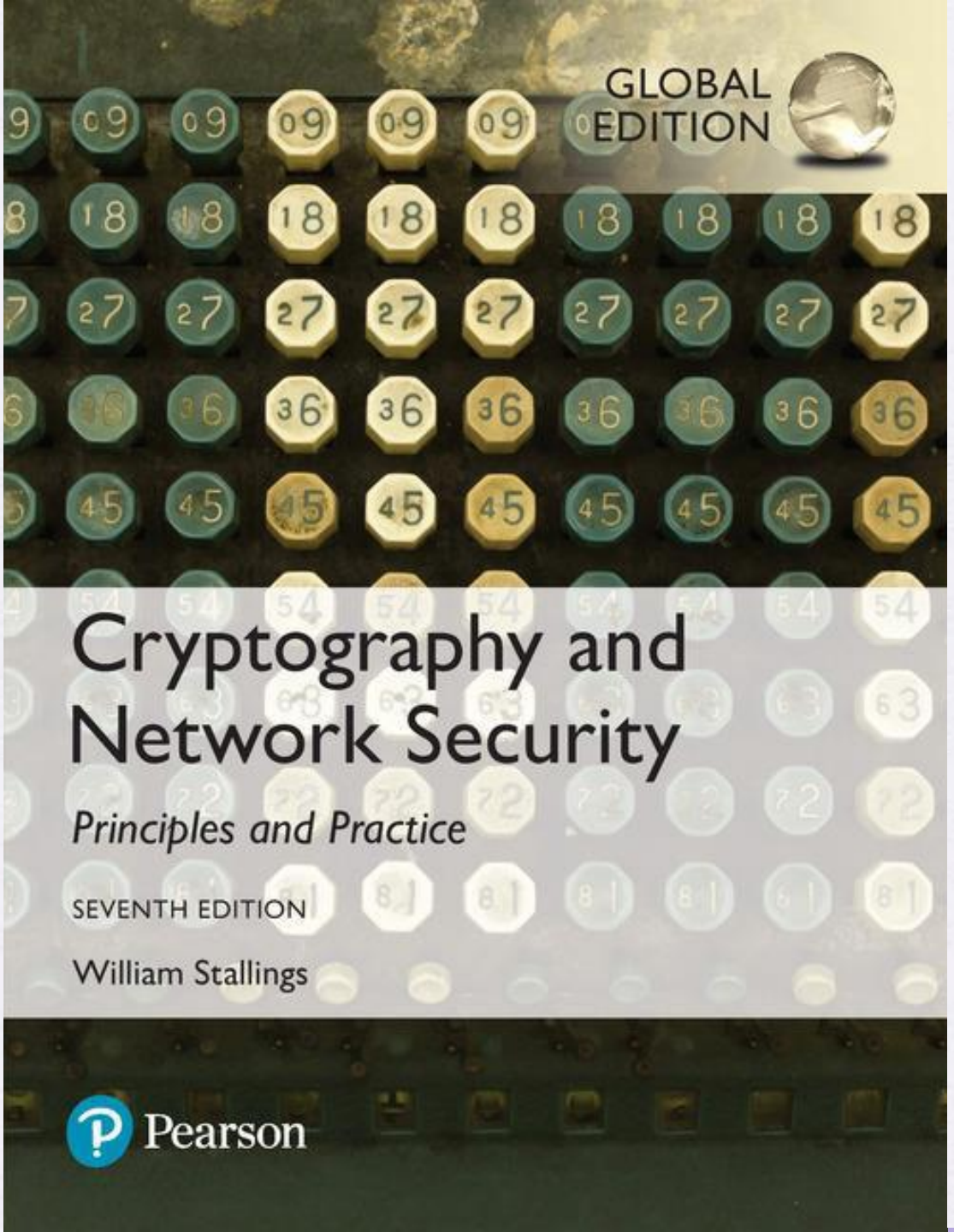
Block Cipher Design Principles

- as reported by Coppersmith in [COPP94]
- 7 criteria for S-boxes provide for
  - non-linearity
  - resistance to differential cryptanalysis
  - good confusion
- 3 criteria for permutation P provide for
  - increased diffusion
- Number of rounds
  - Known cryptanalysis takes more effort than brute force attack
  - Diff cryptanalysis requires $2^{55.1}$ operations

Block Cipher Modes of Operations

- Electornic code book mode
- Cipher Block Chaining Mode
- Cipher Feedback Mode
- Output Feedback Mode
- Counter Mode

  +ve
  - Hardware efficiency (multiprocessor machines)
  - Software efficiency
  - Random Access
  - Provable security
  - Simplicity
  - Preprocessing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William Stallings
Pearson

Chapter 3

Classical Encryption Techniques

Definitions

Plaintext

- An original message

Ciphertext

- The coded message

Enciphering/encryption

- The process of converting from plaintext to ciphertext

Deciphering/decryption

- Restoring the plaintext from the ciphertext

Cryptography

- The area of study of the many schemes used for encryption

Cryptographic system/cipher

- A scheme

Cryptanalysis

- Techniques used for deciphering a message without any knowledge of the enciphering details

Cryptology

- The areas of cryptography and cryptanalysis

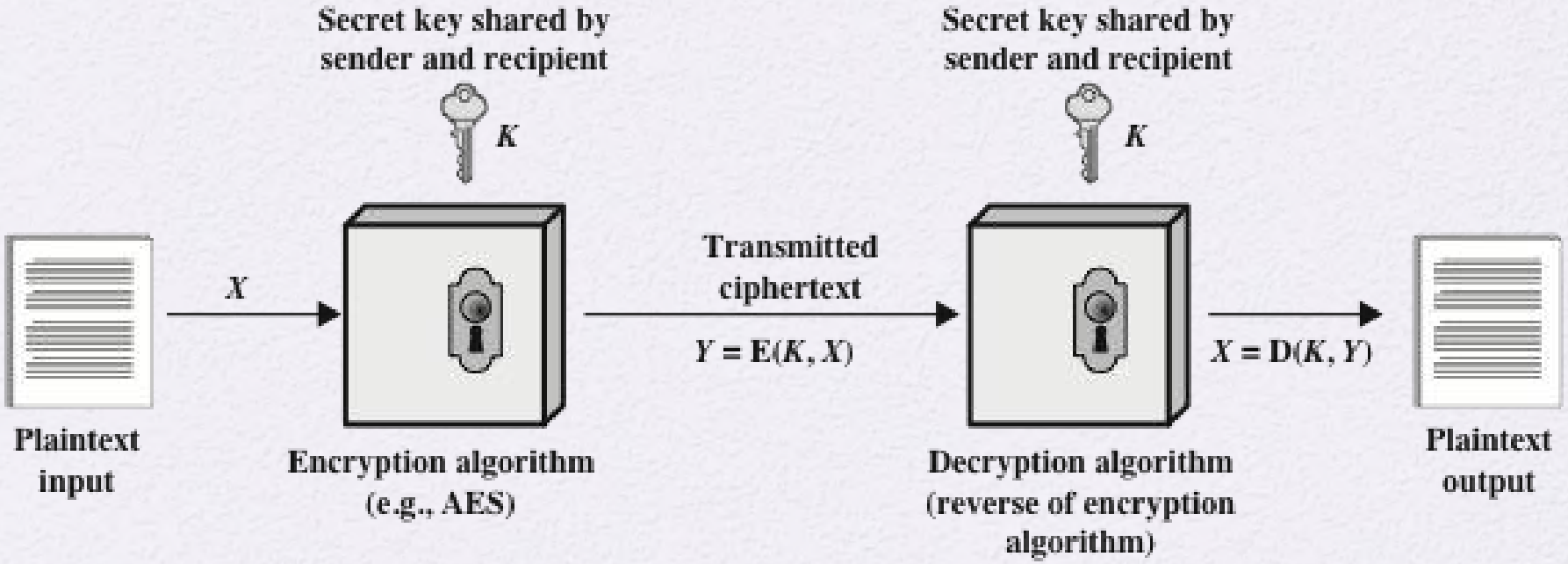
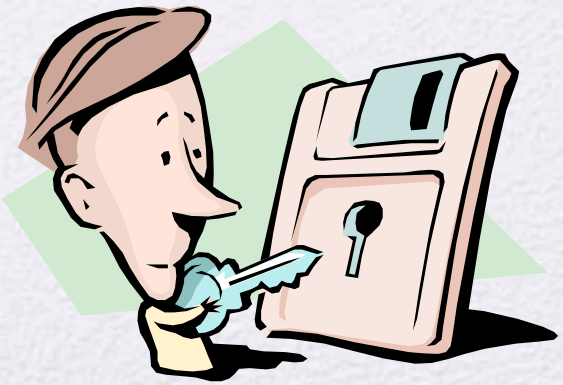


Figure 3.1 Simplified Model of Symmetric Encryption

Symmetric Cipher Model

- There are two requirements for secure use of conventional encryption:
  - A strong encryption algorithm
  - Sender and receiver must have obtained copies of the secret key in a secure fashion and must keep the key secure



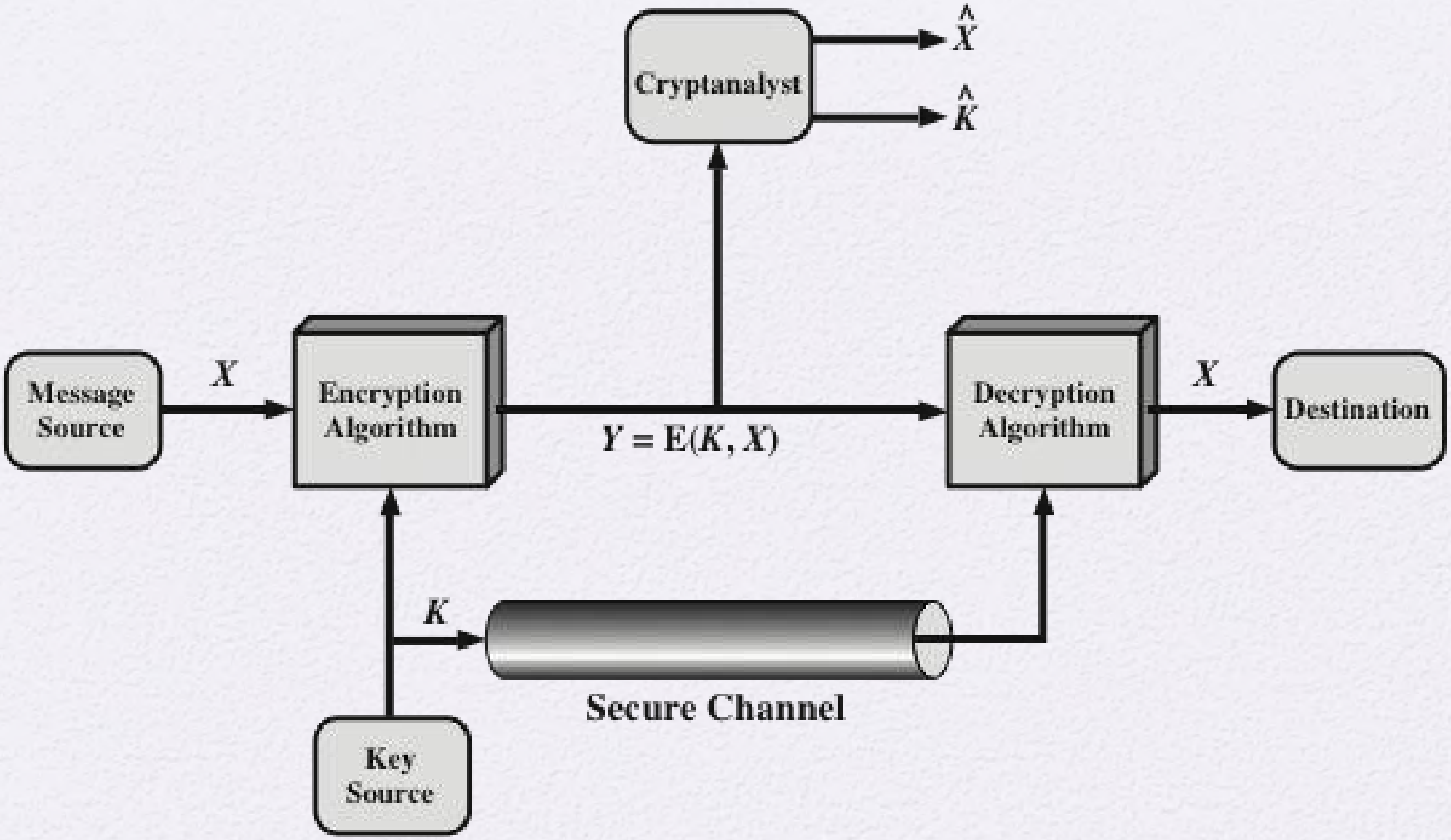
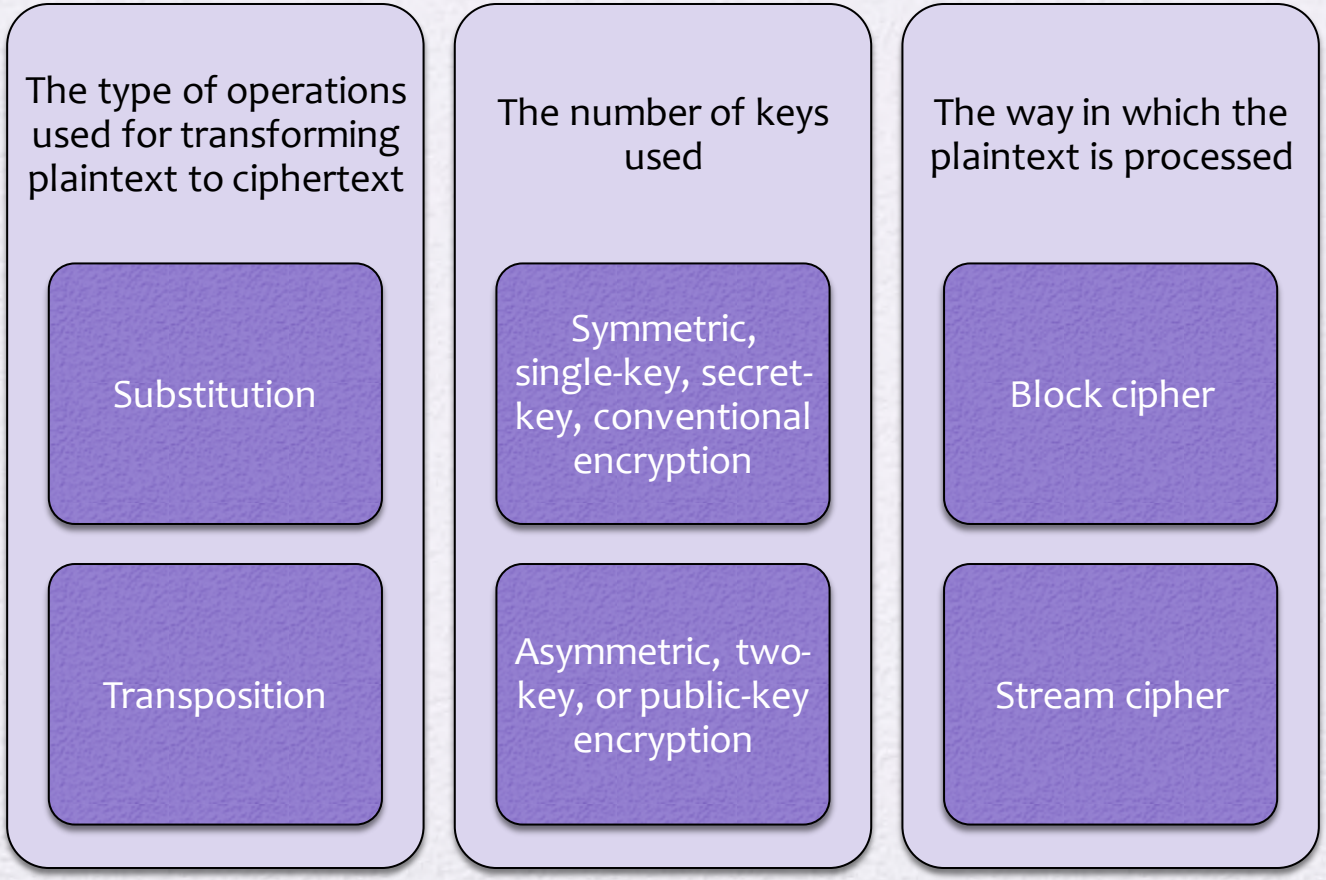


Figure 3.2 Model of Symmetric Cryptosystem

Cryptographic Systems

- Characterized along three independent dimensions:



Cryptanalysis and Brute-Force Attack

Cryptanalysis

- Attack relies on the nature of the algorithm plus some knowledge of the general characteristics of the plaintext
- Attack exploits the characteristics of the algorithm to attempt to deduce a specific plaintext or to deduce the key being used

Brute-force attack

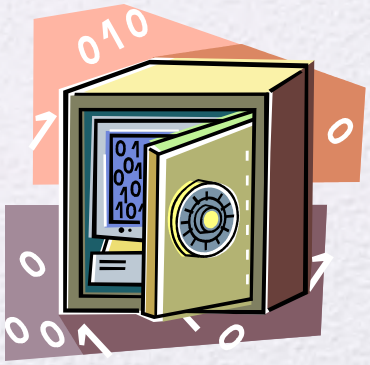
- Attacker tries every possible key on a piece of ciphertext until an intelligible translation into plaintext is obtained
- On average, half of all possible keys must be tried to achieve success

Type of Attack	Known to Cryptanalyst
Ciphertext Only	• Encryption algorithm • Ciphertext
Known Plaintext	• Encryption algorithm • Ciphertext • One or more plaintext-ciphertext pairs formed with the secret key
Chosen Plaintext	• Encryption algorithm • Ciphertext • Plaintext message chosen by cryptanalyst, together with its corresponding ciphertext generated with the secret key
Chosen Ciphertext	• Encryption algorithm • Ciphertext • Ciphertext chosen by cryptanalyst, together with its corresponding decrypted plaintext generated with the secret key
Chosen Text	• Encryption algorithm • Ciphertext • Plaintext message chosen by cryptanalyst, together with its corresponding ciphertext generated with the secret key • Ciphertext chosen by cryptanalyst, together with its corresponding decrypted plaintext generated with the secret key

Table 3.1
Types of Attacks on Encrypted Messages

Encryption Scheme Security

- Unconditionally secure
  - No matter how much time an opponent has, it is impossible for him or her to decrypt the ciphertext simply because the required information is not there
- Computationally secure
  - The cost of breaking the cipher exceeds the value of the encrypted information
  - The time required to break the cipher exceeds the useful lifetime of the information



Brute-Force Attack

Involves trying every possible key until an intelligible translation of the ciphertext into plaintext is obtained

On average, half of all possible keys must be tried to achieve success

To supplement the brute-force approach, some degree of knowledge about the expected plaintext is needed, and some means of automatically distinguishing plaintext from garble is also needed

Substitution Technique

- Is one in which the letters of plaintext are replaced by other letters or by numbers or symbols
- If the plaintext is viewed as a sequence of bits, then substitution involves replacing plaintext bit patterns with ciphertext bit patterns

Caesar Cipher

- Simplest and earliest known use of a substitution cipher
- Used by Julius Caesar
- Involves replacing each letter of the alphabet with the letter standing three places further down the alphabet
- Alphabet is wrapped around so that the letter following Z is A

plain: meet me after the toga party

cipher: PHHW PH DIWHU WKH WRJD SDUWB

Caesar Cipher Algorithm

- Can define transformation as:

  a b c d e f g h i j k l m n o p q r s t u v w x y z

  D E F G H I J K L M N O P Q R S T U V W X Y Z A B C

- Mathematically give each letter a number

  a b c d e f g h i j k l m n o p q r s t u v w x y z

  0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

- Algorithm can be expressed as:

$$c = E(3, p) = (p + 3) \bmod (26)$$

- A shift may be of any amount, so that the general Caesar algorithm is:

$$C = E(k, p) = (p + k) \bmod 26$$

- Where k takes on a value in the range 1 to 25; the decryption algorithm is simply:

$$p = D(k, C) = (C - k) \bmod 26$$

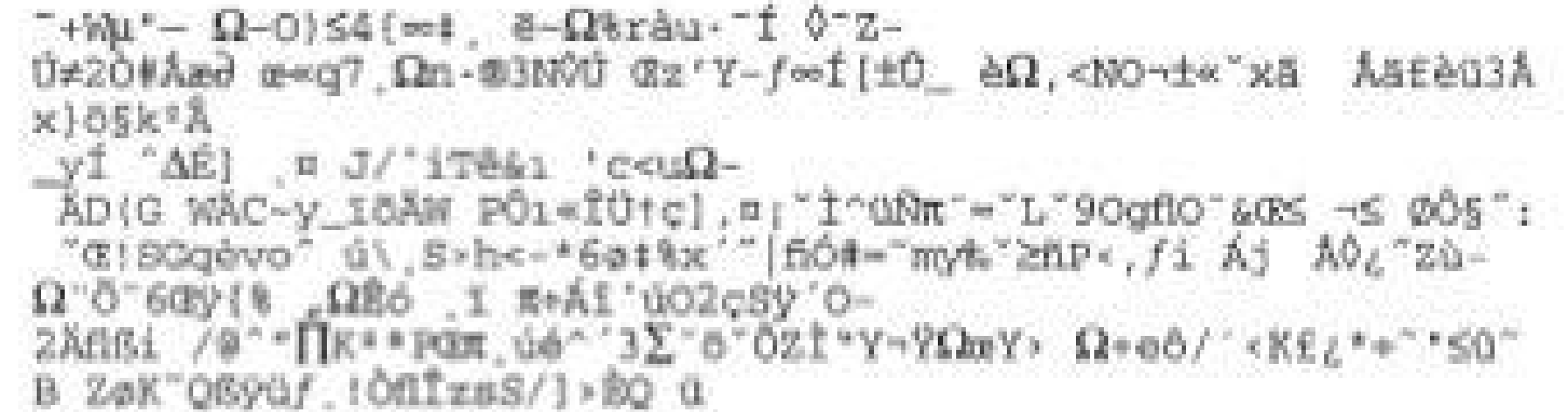
Figure 3.3

Brute-Force Cryptanalysis of Caesar Cipher

(This chart can be found on page 75 in the textbook)

KEY	PHHW PH DIWHU WKH WRJD SDUWB
1	oggv og chvgt vjg vqic rctva
2	nffu nf bgufs uif uphb qbsuz
3	meet me after the toga party
4	ldds ld zesdq sgd snfz ozqsx
5	kccr kc ydrcp rfc rmey nyprw
6	jbbq jb xcqbo qeb qldx mxoqv
7	iaap ia wbpan pda pkcw lwnpu
8	hzzo hz vaozm ocz ojbv kvmot
9	gyyn gy uznyl nby niau julns
10	fxxm fx tymxk max mhzt itkmr
11	ewwl ew sxlwj lzw lgys hsjlq
12	dvvk dv rwkvi kyv kfxr grikp
13	cuuj cu qvjuh jxu jewq fqhjo
14	btti bt puitg iwt idvp epgin
15	assh as othsf hvs hcuo dofhm
16	zrrg zr nsgre gur gbtn cnegl
17	yqqf yq mrfqd ftq fasm bmdfk
18	xppe xp lqepc esp ezrl alcej
19	wood wo kpdob dro dyqk zkbdi
20	vnnc vn jocna cqn cxpj yjach
21	ummb um inbmz bpm bwoi xizbg
22	tlla tl hmaly aol avnh whyaf
23	skkz sk glzkx znk zumg vgxze
24	rjjy rj fkyjw ymj ytlf ufwyd
25	qiix qi ejxiv xli xske tevxc

Sample of Compressed Text



Monoalphabetic Cipher

- Permutation
  - Of a finite set of elements *S* is an ordered sequence of all the elements of *S* , with each element appearing exactly once
- If the "cipher" line can be any permutation of the 26 alphabetic characters, then there are 26! or greater than 4×10^{26} possible keys
  - This is 10 orders of magnitude greater than the key space for DES
  - Approach is referred to as a *monoalphabetic substitution cipher* because a single cipher alphabet is used per message

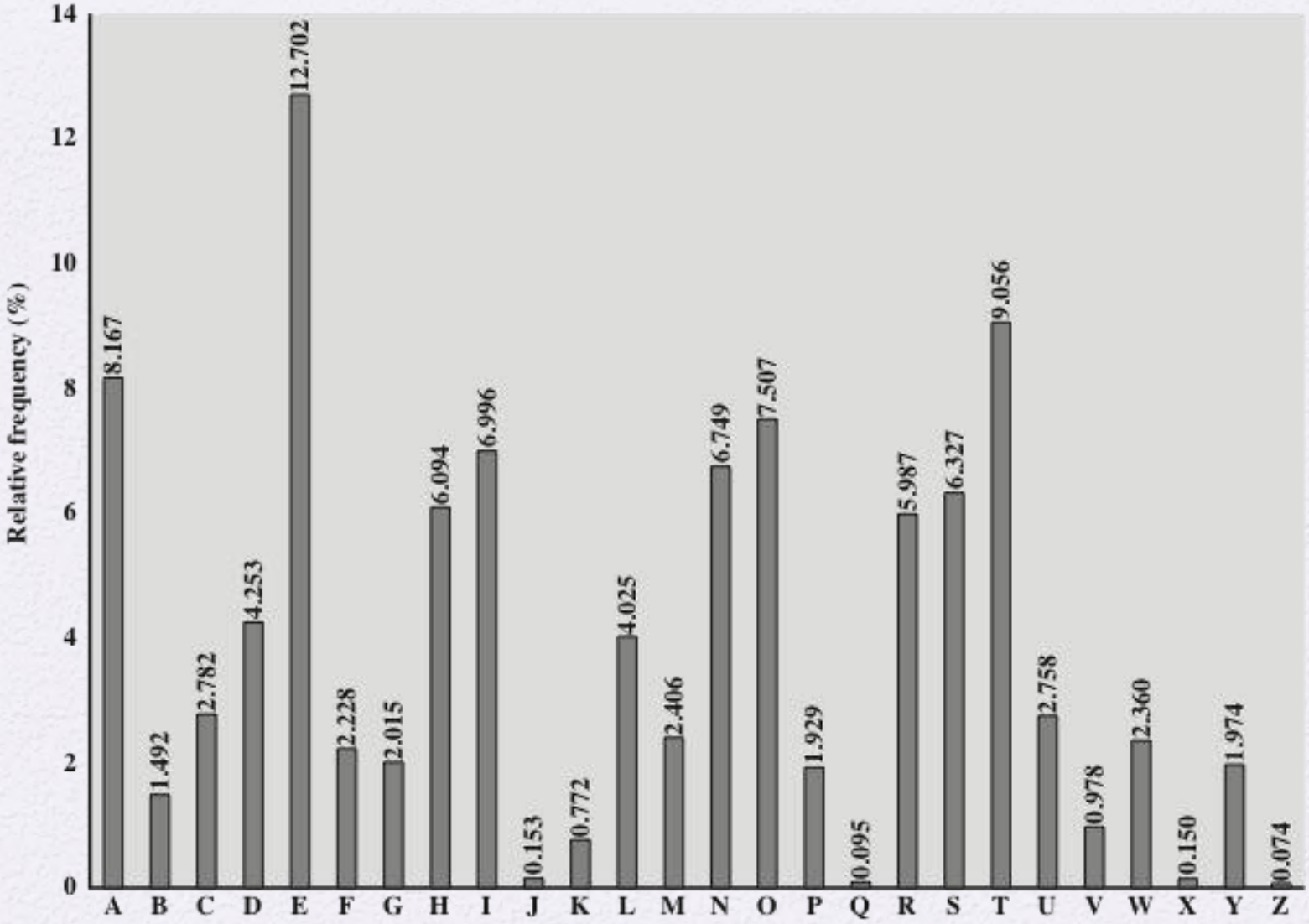
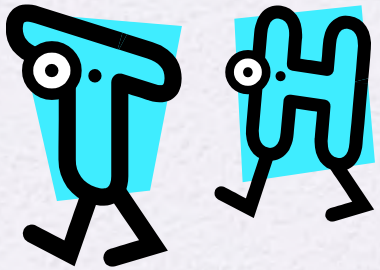


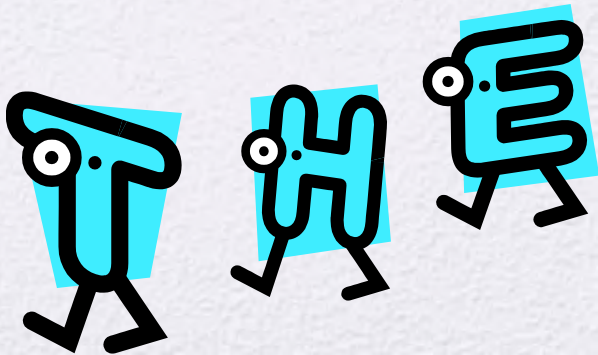
Figure 3.5 Relative Frequency of Letters in English Text

Monoalphabetic Ciphers

- Easy to break because they reflect the frequency data of the original alphabet
- Countermeasure is to provide multiple substitutes (homophones) for a single letter
- Digram
  - Two-letter combination
  - Most common is *th*



- Trigram
  - Three-letter combination
  - Most frequent is *the*



Playfair Cipher

- Best-known multiple-letter encryption cipher
- Treats digrams in the plaintext as single units and translates these units into ciphertext digrams
- Based on the use of a 5 x 5 matrix of letters constructed using a keyword
- Invented by British scientist Sir Charles Wheatstone in 1854
- Used as the standard field system by the British Army in World War I and the U.S. Army and other Allied forces during World War II

Playfair Key Matrix

- Fill in letters of keyword (minus duplicates) from left to right and from top to bottom, then fill in the remainder of the matrix with the remaining letters in alphabetic order
- Using the keyword MONARCHY:

M	O	N	A	R
C	H	Y	B	D
E	F	G	I/J	K
L	P	Q	S	T
U	V	W	X	Z

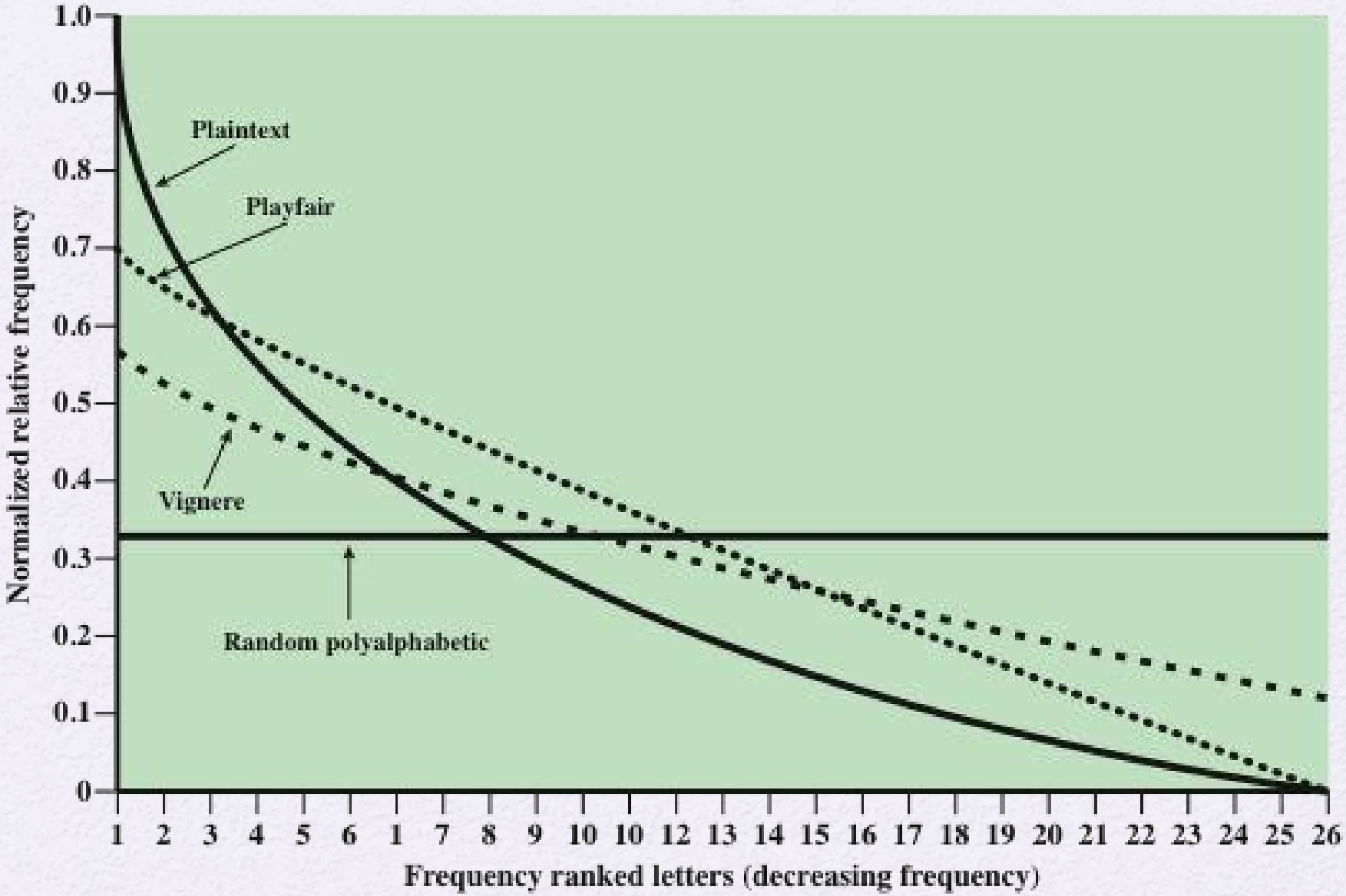


Figure 3.6 Relative Frequency of Occurrence of Letters

Hill Cipher

- Developed by the mathematician Lester Hill in 1929
- Strength is that it completely hides single-letter frequencies
  - The use of a larger matrix hides more frequency information
  - A 3 x 3 Hill cipher hides not only single-letter but also two-letter frequency information
- Strong against a ciphertext-only attack but easily broken with a known plaintext attack

Polyalphabetic Ciphers

- Polyalphabetic substitution cipher
  - Improves on the simple monoalphabetic technique by using different monoalphabetic substitutions as one proceeds through the plaintext message

All these techniques have the following features in common:

- A set of related monoalphabetic substitution rules is used
- A key determines which particular rule is chosen for a given transformation

Vigenère Cipher

- Best known and one of the simplest polyalphabetic substitution ciphers
- In this scheme the set of related monoalphabetic substitution rules consists of the 26 Caesar ciphers with shifts of 0 through 25
- Each cipher is denoted by a key letter which is the ciphertext letter that substitutes for the plaintext letter a

Example of Vigenère Cipher

- To encrypt a message, a key is needed that is as long as the message
- Usually, the key is a repeating keyword
- For example, if the keyword is *deceptive*, the message "we are discovered save yourself" is encrypted as:

key: deceptivedeceptivedeceptive
plaintext: wearediscoveredsaveyourself
ciphertext: ZICVTWQNGRZGVTWAVZHCQYGLMGJ

Vigenère Autokey System

- A keyword is concatenated with the plaintext itself to provide a running key
- Example:

  key: deceptivewearediscoveredsav

  plaintext: wearediscoveredsaveyourself

  ciphertext: ZICVTWQNGKZEIIGASXSTSLVVWLA
- Even this scheme is vulnerable to cryptanalysis
  - Because the key and the plaintext share the same frequency distribution of letters, a statistical technique can be applied

Vernam Cipher

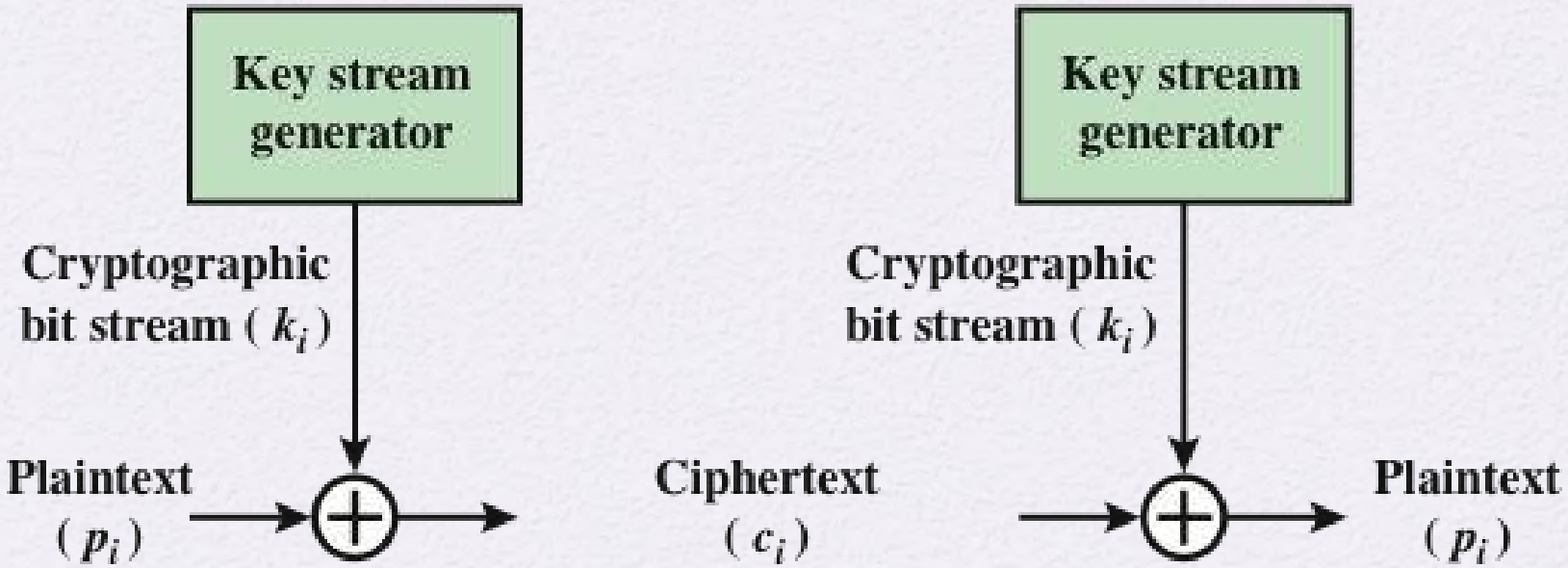


Figure 3.7 Vernam Cipher

One-Time Pad

- Improvement to Vernam cipher proposed by an Army Signal Corp officer, Joseph Mauborgne
- Use a random key that is as long as the message so that the key need not be repeated
- Key is used to encrypt and decrypt a single message and then is discarded
- Each new message requires a new key of the same length as the new message
- Scheme is unbreakable
  - Produces random output that bears no statistical relationship to the plaintext
  - Because the ciphertext contains no information whatsoever about the plaintext, there is simply no way to break the code

Difficulties

- The one-time pad offers complete security but, in practice, has two fundamental difficulties:
  - There is the practical problem of making large quantities of random keys
    - Any heavily used system might require millions of random characters on a regular basis
  - Mammoth key distribution problem
    - For every message to be sent, a key of equal length is needed by both sender and receiver
- Because of these difficulties, the one-time pad is of limited utility
  - Useful primarily for low-bandwidth channels requiring very high security
- The one-time pad is the only cryptosystem that exhibits *perfect secrecy* (see Appendix F)

Rail Fence Cipher

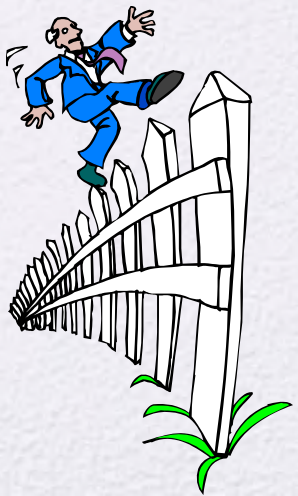
- Simplest transposition cipher
- Plaintext is written down as a sequence of diagonals and then read off as a sequence of rows
- To encipher the message "meet me after the toga party" with a rail fence of depth 2, we would write:

m e m a t r h t g p r y

e t e f e t e o a a t

Encrypted message is:

MEMATRHTGPRYETEFETEOAAT



Row Transposition Cipher

- Is a more complex transposition
- Write the message in a rectangle, row by row, and read the message off, column by column, but permute the order of the columns
  - The order of the columns then becomes the key to the algorithm

Key:	4 3 1 2 5 6 7
Plaintext:	a t t a c k p
	o s t p o n e
	d u n t i l t
	w o a m x y z
Ciphertext:	TTNAAPTMTSUOAODWCOIXKNLYPETZ

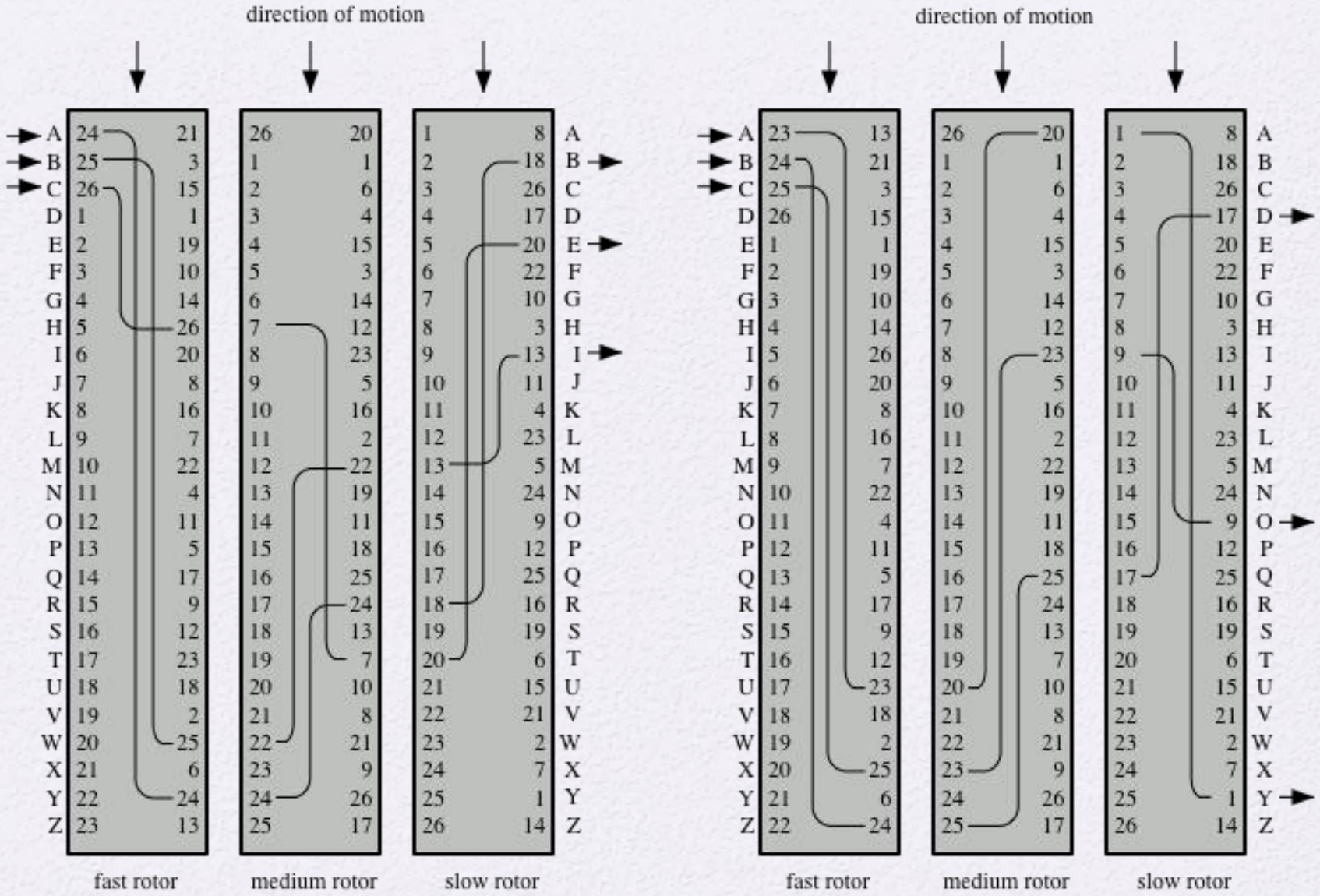


Figure 3.8 Three-Rotor Machine With Wiring Represented by Numbered Contacts

Steganography

3rd March

Dear George,

Greetings to all at Oxford. Many thanks for your letter and for the Summer examination package. All Entry Forms and Fees Forms should be ready for final despatch to the Syndicate by Friday 20th or at the very latest, I'm told, by the 21st. Admin has improved here, though there's room for improvement still; just give us all two or three more years and we'll really show you! Please don't let these wretched 16+ proposals destroy your basic O and A pattern. Certainly this sort of change, if implemented immediately, would bring chaos.

Sincerely yours,

A Puzzle for Inspector Morse
(from *The Silent World of Nicholas Quinn*, by Colin Dexter)

Other Steganography Techniques

- Character marking
  - Selected letters of printed or typewritten text are over-written in pencil
  - The marks are ordinarily not visible unless the paper is held at an angle to bright light
- Invisible ink
  - A number of substances can be used for writing but leave no visible trace until heat or some chemical is applied to the paper
- Pin punctures
  - Small pin punctures on selected letters are ordinarily not visible unless the paper is held up in front of a light
- Typewriter correction ribbon
  - Used between lines typed with a black ribbon, the results of typing with the correction tape are visible only under a strong light

Steganography vs. Encryption

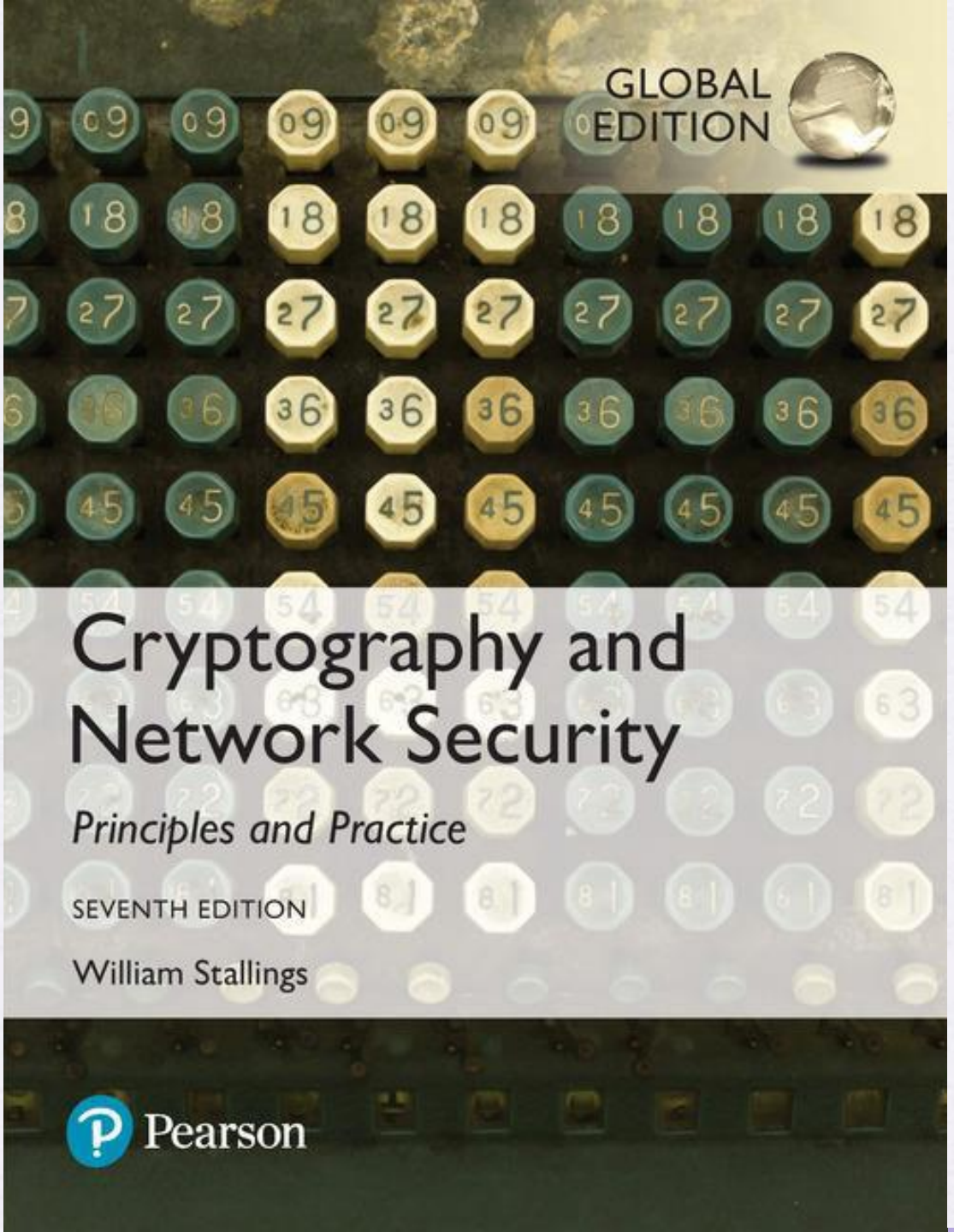
- Steganography has a number of drawbacks when compared to encryption
  - It requires a lot of overhead to hide a relatively few bits of information
  - Once the system is discovered, it becomes virtually worthless

- The advantage of steganography
  - It can be employed by parties who have something to lose should the fact of their secret communication (not necessarily the content) be discovered
- Encryption flags traffic as important or secret or may identify the sender or receiver as someone with something to hide

Summary

- Symmetric Cipher Model
  - Cryptography
  - Cryptanalysis and Brute-Force Attack
- Transposition techniques
- Rotor machines

- Substitution techniques
  - Caesar cipher
  - Monoalphabetic ciphers
  - Playfair cipher
  - Hill cipher
  - Polyalphabetic ciphers
  - One-time pad
- Steganography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Chapter 4

Block Ciphers and the Data Encryption Standard

Stream Cipher

Encrypts a digital data stream one bit or one byte at a time

Examples:

- Autokeyed Vigenère cipher
- Vernam cipher

In the ideal case, a one-time pad version of the Vernam cipher would be used, in which the keystream is as long as the plaintext bit stream

If the cryptographic keystream is random, then this cipher is unbreakable by any means other than acquiring the keystream

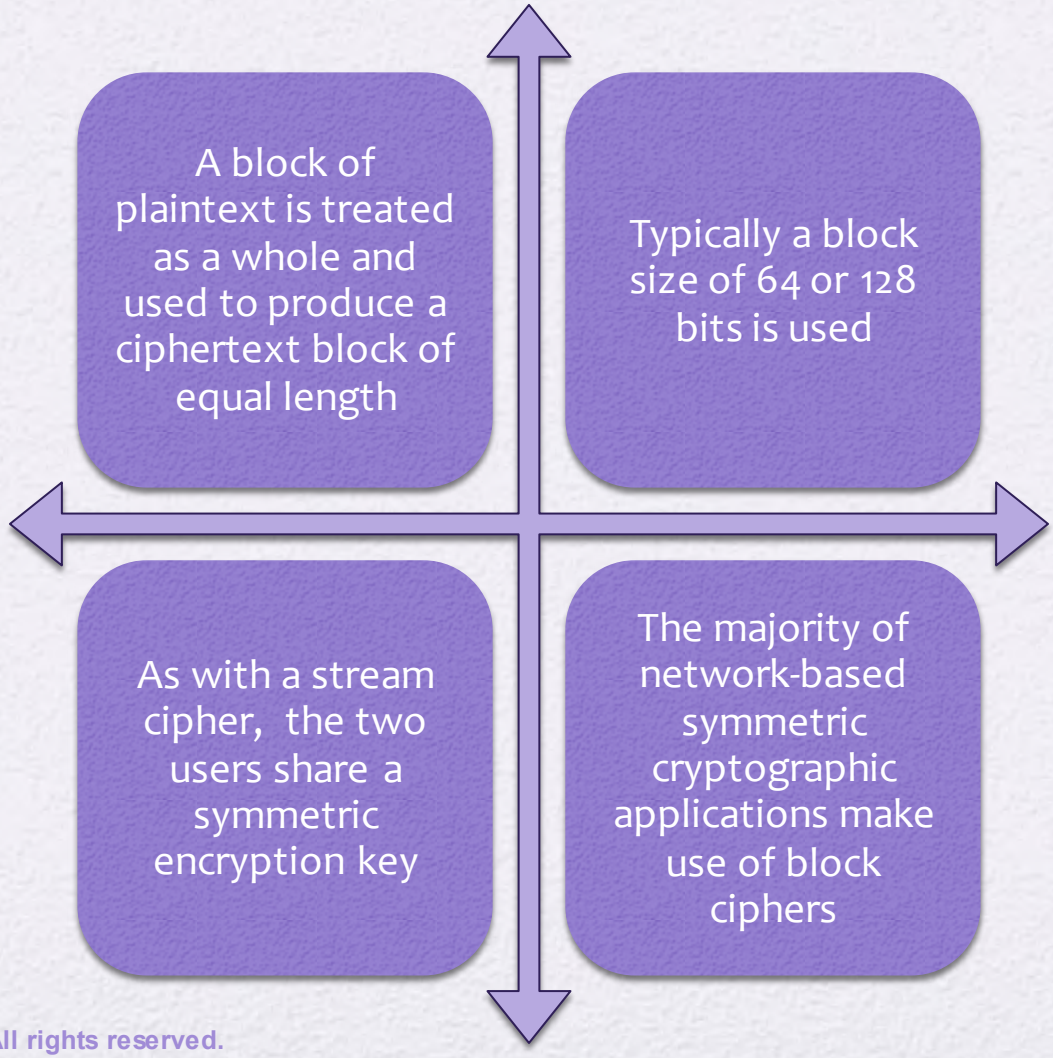
- Keystream must be provided to both users in advance via some independent and secure channel
- This introduces insurmountable logistical problems if the intended data traffic is very large

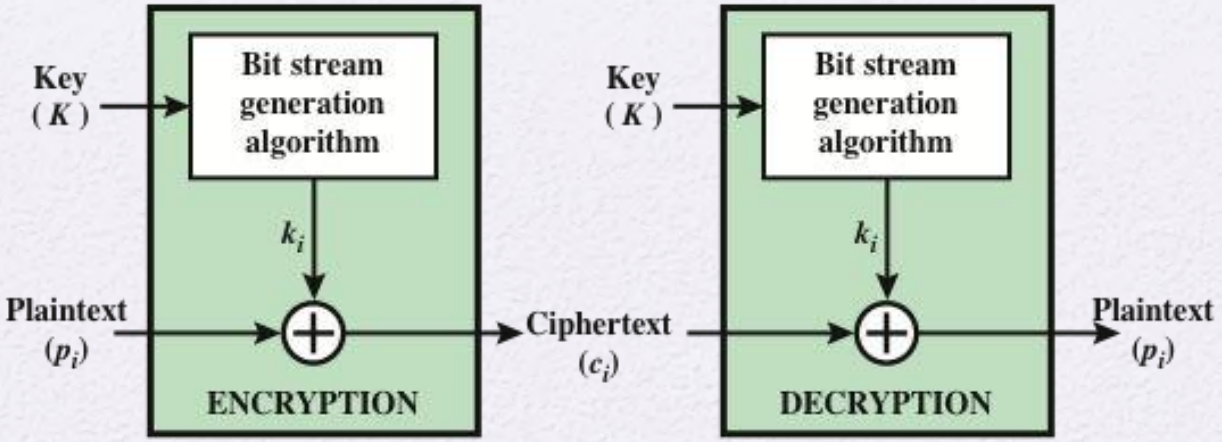
For practical reasons the bit-stream generator must be implemented as an algorithmic procedure so that the cryptographic bit stream can be produced by both users

It must be computationally impractical to predict future portions of the bit stream based on previous portions of the bit stream

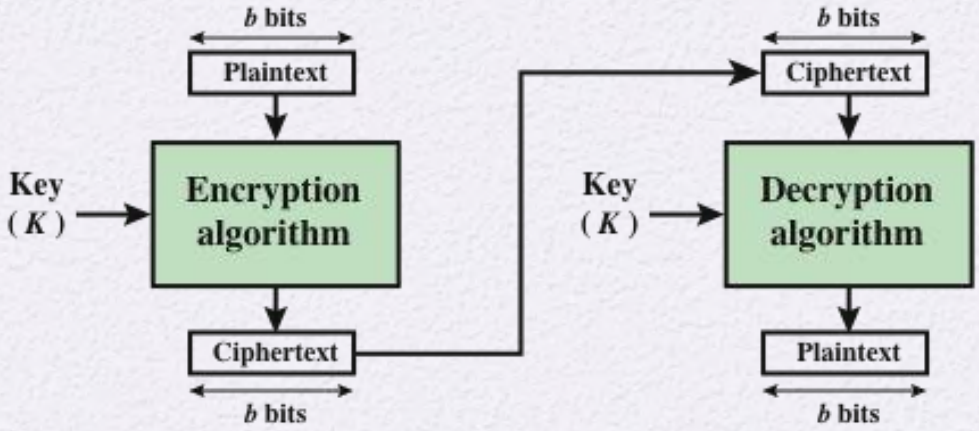
The two users need only share the generating key and each can produce the keystream

Block Cipher





(a) Stream Cipher Using Algorithmic Bit Stream Generator



(b) Block Cipher

Figure 4.1 Stream Cipher and Block Cipher

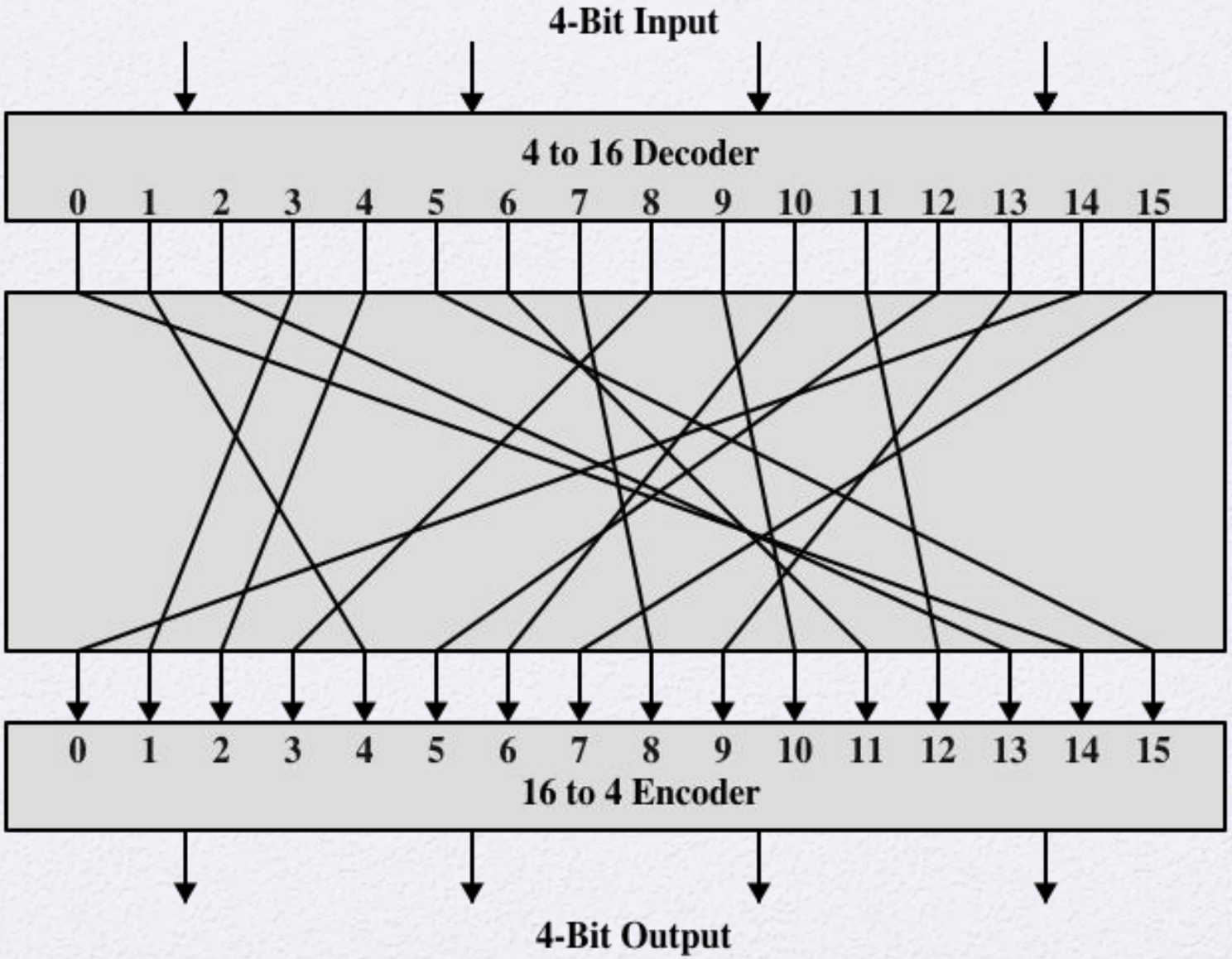


Figure 4.2 General *n*-bit-*n*-bit Block Substitution (shown with $n = 4$)

Table 4.1

Encryption and Decryption Tables for Substitution Cipher of Figure 4.2

Plaintext	Ciphertext
0000	1110
0001	0100
0010	1101
0011	0001
0100	0010
0101	1111
0110	1011
0111	1000
1000	0011
1001	1010
1010	0110
1011	1100
1100	0101
1101	1001
1110	0000
1111	0111

Ciphertext	Plaintext
0000	1110
0001	0011
0010	0100
0011	1000
0100	0001
0101	1100
0110	1010
0111	1111
1000	0111
1001	1101
1010	1001
1011	0110
1100	1011
1101	0010
1110	0000
1111	0101

Feistel Cipher

- Feistel proposed the use of a cipher that alternates substitutions and permutations

Substitutions

- Each plaintext element or group of elements is uniquely replaced by a corresponding ciphertext element or group of elements

Permutation

- No elements are added or deleted or replaced in the sequence, rather the order in which the elements appear in the sequence is changed

- Is a practical application of a proposal by Claude Shannon to develop a product cipher that alternates confusion and diffusion functions
- Is the structure used by many significant symmetric block ciphers currently in use

Diffusion and Confusion

- Terms introduced by Claude Shannon to capture the two basic building blocks for any cryptographic system
  - Shannon's concern was to thwart cryptanalysis based on statistical analysis

Diffusion

- The statistical structure of the plaintext is dissipated into long-range statistics of the ciphertext
- This is achieved by having each plaintext digit affect the value of many ciphertext digits

Confusion

- Seeks to make the relationship between the statistics of the ciphertext and the value of the encryption key as complex as possible
- Even if the attacker can get some handle on the statistics of the ciphertext, the way in which the key was used to produce that ciphertext is so complex as to make it difficult to deduce the key

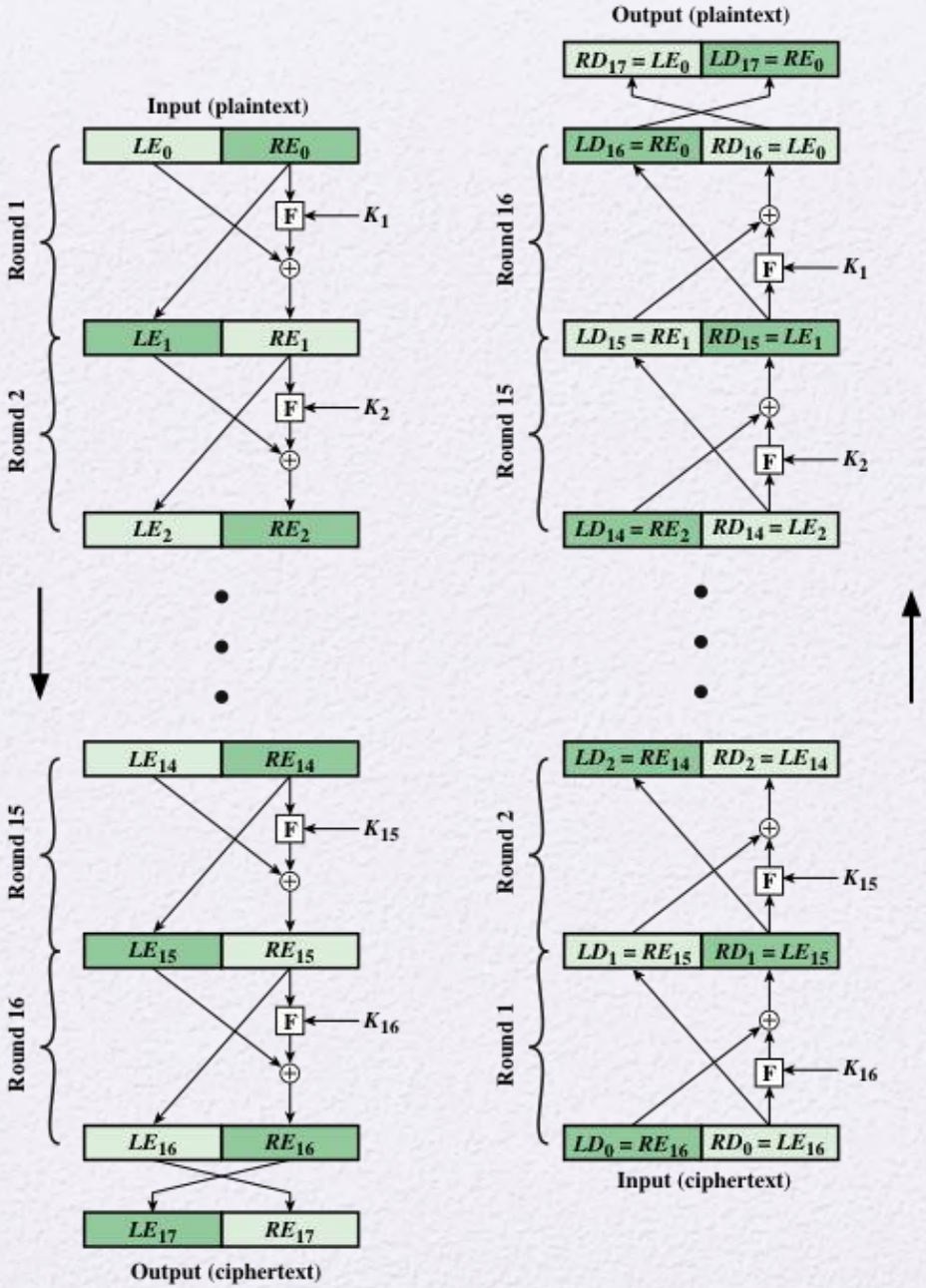


Figure 4.3 Feistel Encryption and Decryption (16 rounds)

Feistel Cipher Design Features

- Block size
  - Larger block sizes mean greater security but reduced encryption/decryption speed for a given algorithm
- Key size
  - Larger key size means greater security but may decrease encryption/decryption speeds
- Number of rounds
  - The essence of the Feistel cipher is that a single round offers inadequate security but that multiple rounds offer increasing security
- Subkey generation algorithm
  - Greater complexity in this algorithm should lead to greater difficulty of cryptanalysis
- Round function F
  - Greater complexity generally means greater resistance to cryptanalysis
- Fast software encryption/decryption
  - In many cases, encrypting is embedded in applications or utility functions in such a way as to preclude a hardware implementation; accordingly, the speed of execution of the algorithm becomes a concern
- Ease of analysis
  - If the algorithm can be concisely and clearly explained, it is easier to analyze that algorithm for cryptanalytic vulnerabilities and therefore develop a higher level of assurance as to its strength

Feistel Example

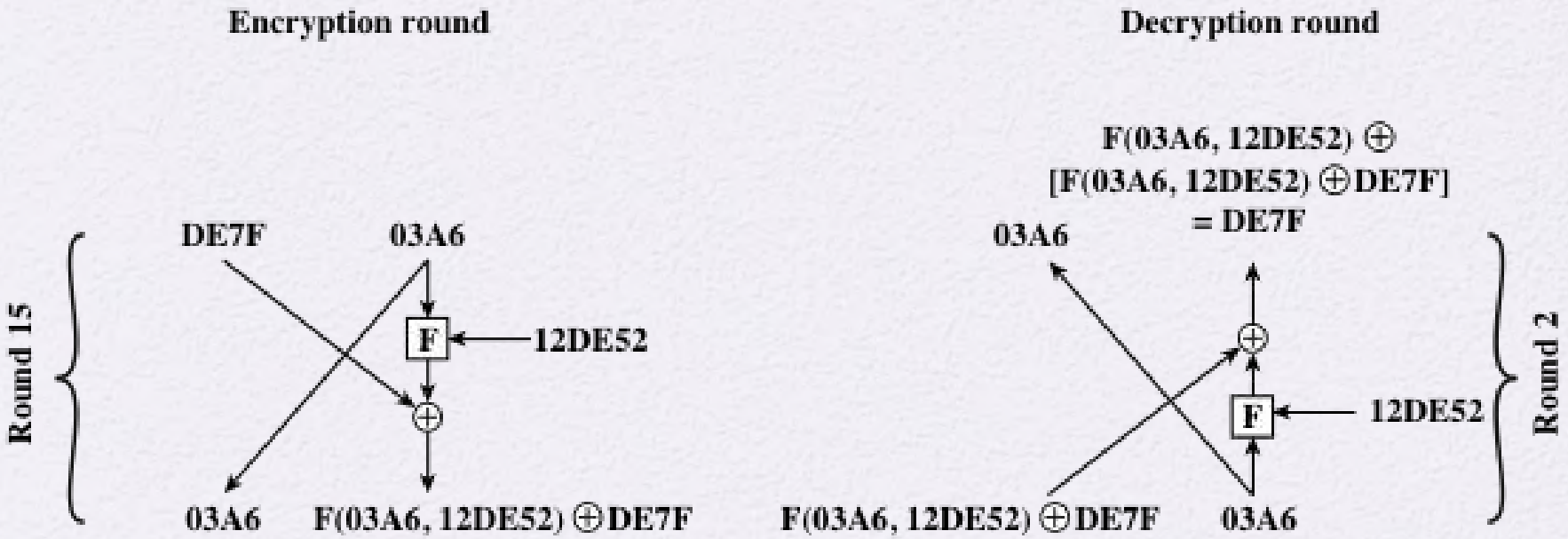


Figure 4.4 Feistel Example

Data Encryption Standard (DES)

- Issued in 1977 by the National Bureau of Standards (now NIST) as Federal Information Processing Standard 46
- Was the most widely used encryption scheme until the introduction of the Advanced Encryption Standard (AES) in 2001
- Algorithm itself is referred to as the Data Encryption Algorithm (DEA)
  - Data are encrypted in 64-bit blocks using a 56-bit key
  - The algorithm transforms 64-bit input in a series of steps into a 64-bit output
  - The same steps, with the same key, are used to reverse the encryption

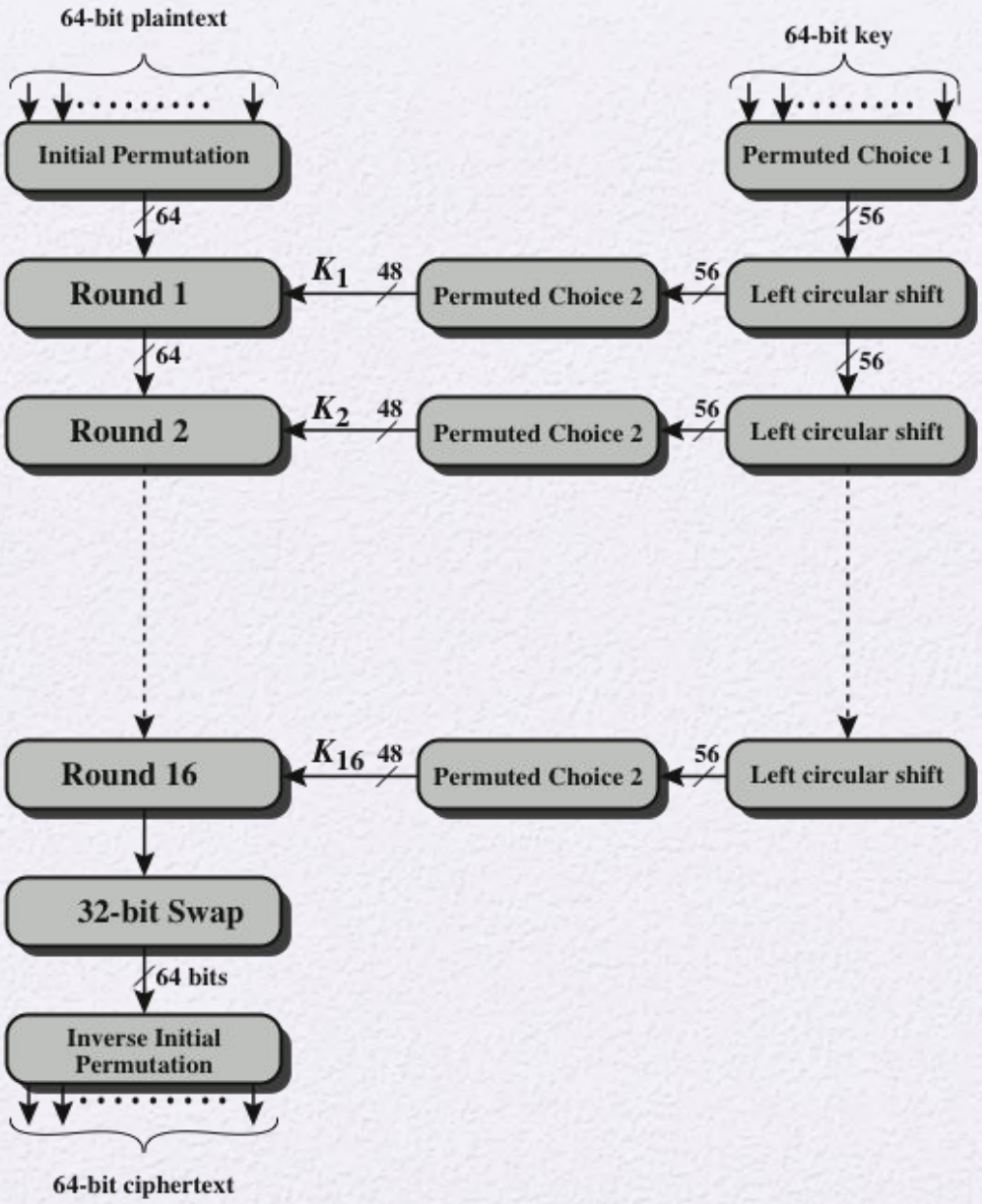


Figure 4.5 General Depiction of DES Encryption Algorithm

Table 4.2

DES Example

(Table can be found on page 114 in textbook)

Round	*Ki*	*Li*	*Ri*
IP		5a005a00	3cf03c0f
1	1e030f03080d2930	3cf03c0f	bad22845
2	0a31293432242318	bad22845	99e9b723
3	23072318201d0c1d	99e9b723	0bae3b9e
4	05261d3824311a20	0bae3b9e	42415649
5	3325340136002c25	42415649	18b3fa41
6	123a2d0d04262a1c	18b3fa41	9616fe23
7	021f120b1c130611	9616fe23	67117cf2
8	1c10372a2832002b	67117cf2	c11bfc09
9	04292a380c341f03	c11bfc09	887fbc6c
10	2703212607280403	887fbc6c	600f7e8b
11	2826390c31261504	600f7e8b	f596506e
12	12071c241a0a0f08	f596506e	738538b8
13	300935393c0d100b	738538b8	c6a62c4e
14	311e09231321182a	c6a62c4e	56b0bd75
15	283d3e0227072528	56b0bd75	75e8fd8f
16	2921080b13143025	75e8fd8f	25896490
IP–1		da02ce3a	89ecac3b

Note: DES subkeys are shown as eight 6-bit values in hex format

Round		δ
	02468aceeca86420 12468aceeca86420	1
1	3cf03c0fbad22845 3cf03c0fbad32845	1
2	bad2284599e9b723 bad3284539a9b7a3	5
3	99e9b7230bae3b9e 39a9b7a3171cb8b3	18
4	0bae3b9e42415649 171cb8b3ccaca55e	34
5	4241564918b3fa41 ccaca55ed16c3653	37
6	18b3fa419616fe23 d16c3653cf402c68	33
7	9616fe2367117cf2 cf402c682b2cefbc	32
8	67117cf2c11bfc09 2b2cefbc99f91153	33

Round		δ
9	c11bfc09887fbc6c 99f911532eed7d94	32
10	887fbc6c600f7e8b 2eed7d94d0f23094	34
11	600f7e8bf596506e d0f23094455da9c4	37
12	f596506e738538b8 455da9c47f6e3cf3	31
13	738538b8c6a62c4e 7f6e3cf34bc1a8d9	29
14	c6a62c4e56b0bd75 4bc1a8d91e07d409	33
15	56b0bd7575e8fd8f 1e07d4091ce2e6dc	31
16	75e8fd8f25896490 1ce2e6dc365e5f59	32
IP–1	da02ce3a89ecac3b 057cde97d7683f2a	32

Table 4.3 Avalanche Effect in DES: Change in Plaintext

Round		δ
	02468aceeca86420 02468aceeca86420	0
1	3cf03c0fbad22845 3cf03c0f9ad628c5	3
2	bad2284599e9b723 9ad628c59939136b	11
3	99e9b7230bae3b9e 9939136b768067b7	25
4	0bae3b9e42415649 768067b75a8807c5	29
5	4241564918b3fa41 5a8807c5488dbe94	26
6	18b3fa419616fe23 488dbe94aba7fe53	26
7	9616fe2367117cf2 aba7fe53177d21e4	27
8	67117cf2c11bfc09 177d21e4548f1de4	32

Round		δ
9	c11bfc09887fbc6c 548f1de471f64dfd	34
10	887fbc6c600f7e8b 71f64dfd4279876c	36
11	600f7e8bf596506e 4279876c399fdc0d	32
12	f596506e738538b8 399fdc0d6d208dbb	28
13	738538b8c6a62c4e 6d208dbbb9bdeeaa	33
14	c6a62c4e56b0bd75 b9bdeeaad2c3a56f	30
15	56b0bd7575e8fd8f d2c3a56f2765c1fb	33
16	75e8fd8f25896490 2765c1fb01263dc4	30
IP–1	da02ce3a89ecac3b ee92b50606b62b0b	30

Table 4.4 Avalanche Effect in DES: Change in Key

Table 4.5

Average Time Required for Exhaustive Key Search

Key size (bits)	Cipher	Number of Alternative Keys	Time Required at 10^9 decryptions/s	Time Required at 10^{13} decryptions/s
56	DES	$2^{56} \approx 7.2 \times 10^{16}$	2^{55} ns = 1.125 years	1 hour
128	AES	$2^{128} \approx 3.4 \times 10^{38}$	2^{127} ns = 5.3×10^{21} years	5.3×10^{17} years
168	Triple DES	$2^{168} \approx 3.7 \times 10^{50}$	2^{167} ns = 5.8×10^{33} years	5.8×10^{29} years
192	AES	$2^{192} \approx 6.3 \times 10^{57}$	2^{191} ns = 9.8×10^{40} years	9.8×10^{36} years
256	AES	$2^{256} \approx 1.2 \times 10^{77}$	2^{255} ns = 1.8×10^{60} years	1.8×10^{56} years
26 characters (permutation)	Monoalphabetic	$26! = 4 \times 10^{26}$	2×10^{26} ns = 6.3×10^{9} years	6.3×10^{6} years

Strength of DES

- Timing attacks
  - One in which information about the key or the plaintext is obtained by observing how long it takes a given implementation to perform decryptions on various ciphertexts
  - Exploits the fact that an encryption or decryption algorithm often takes slightly different amounts of time on different inputs
  - So far it appears unlikely that this technique will ever be successful against DES or more powerful symmetric ciphers such as triple DES and AES

Block Cipher Design Principles: Number of Rounds

The greater the number of rounds, the more difficult it is to perform cryptanalysis

In general, the criterion should be that the number of rounds is chosen so that known cryptanalytic efforts require greater effort than a simple brute-force key search attack

If DES had 15 or fewer rounds, differential cryptanalysis would require less effort than a brute-force key search

Block Cipher Design Principles: Design of Function F

- The heart of a Feistel block cipher is the function F
- The more nonlinear F, the more difficult any type of cryptanalysis will be
- The SAC and BIC criteria appear to strengthen the effectiveness of the confusion function

The algorithm should have good avalanche properties

Strict avalanche criterion (SAC)

States that any output bit j of an S-box should change with probability 1/2 when any single input bit i is inverted for all i , j

Bit independence criterion (BIC)

States that output bits j and k should change independently when any single input bit i is inverted for all i , j , and k

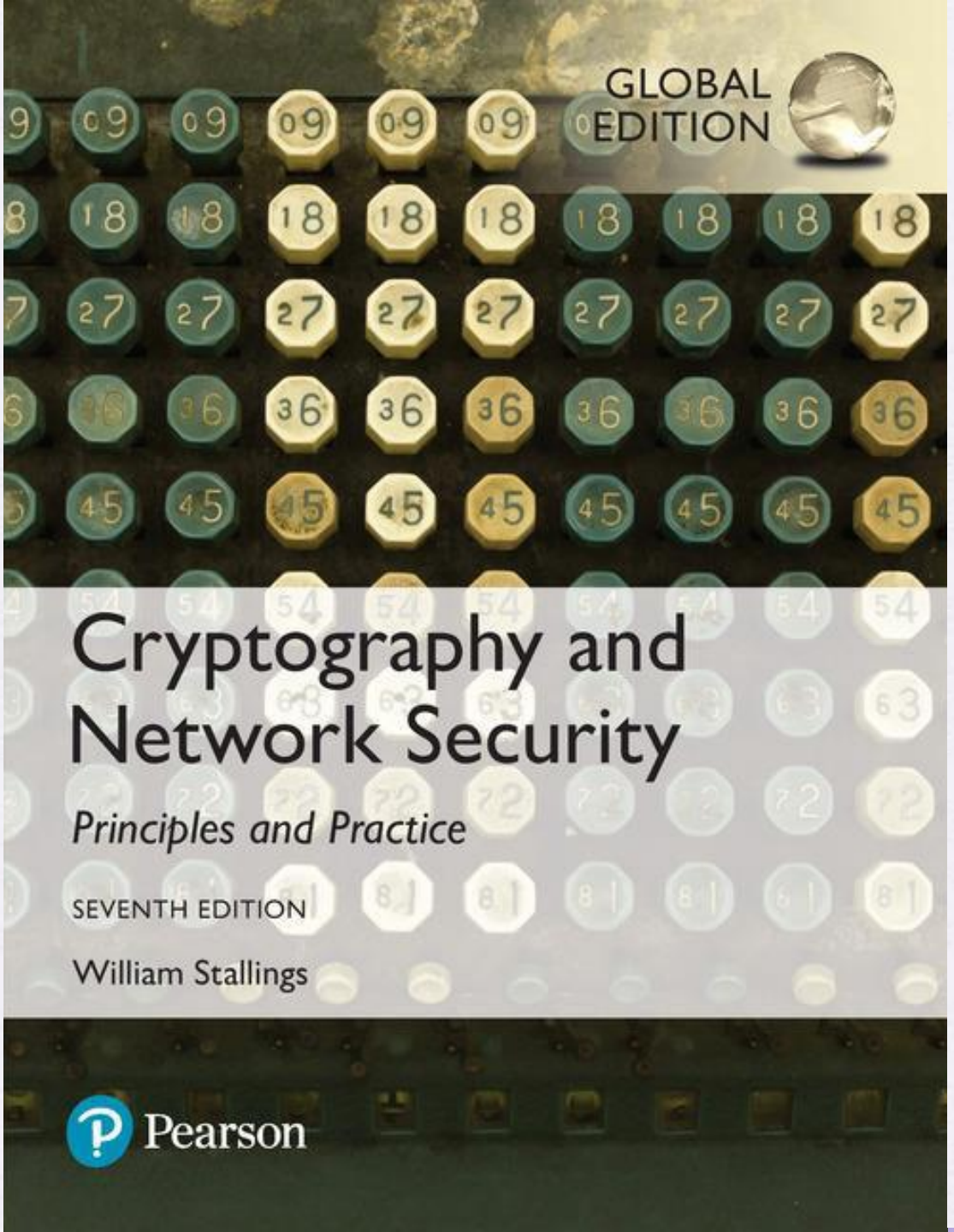
Block Cipher Design Principles: Key Schedule Algorithm

- With any Feistel block cipher, the key is used to generate one subkey for each round
- In general, we would like to select subkeys to maximize the difficulty of deducing individual subkeys and the difficulty of working back to the main key
- It is suggested that, at a minimum, the key schedule should guarantee key/ciphertext Strict Avalanche Criterion and Bit Independence Criterion

Summary

- Traditional Block Cipher Structure
  - Stream ciphers
  - Block ciphers
  - Motivation for the Feistel cipher structure
  - Feistel cipher
- The Data Encryption Standard (DES)
  - Encryption
  - Decryption
  - Avalanche effect

- The strength of DES
  - Use of 56-bit keys
  - Nature of the DES algorithm
  - Timing attacks
- Block cipher design principles
  - Number of rounds
  - Design of function F
  - Key schedule algorithm


GLOBAL EDITION
Cryptography and Network Security
Principles and Practice
SEVENTH EDITION
William Stallings
Pearson

Chapter 5

Finite Fields

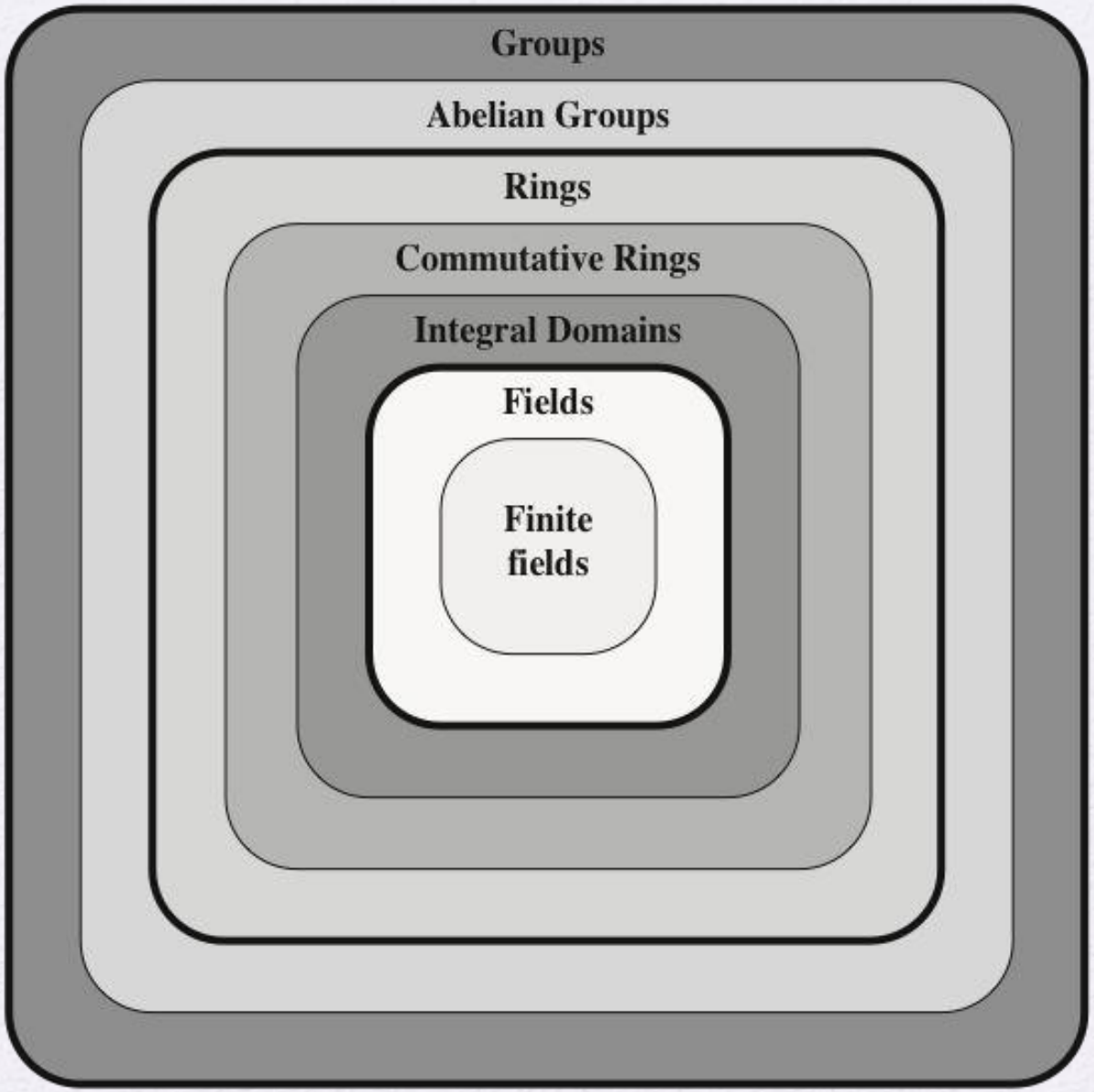


Figure 5.1 Groups, Rings, and Fields

Groups

- A set of elements with a binary operation denoted by • that associates to each ordered pair (a,b) of elements in G an element $(a \bullet b)$ in G , such that the following axioms are obeyed:
  - (A1) Closure:
    - If a and b belong to G, then $a \bullet b$ is also in G
  - (A2) Associative:
    - $a \bullet (b \bullet c) = (a \bullet b) \bullet c$ for all a, b, c in G
  - (A3) Identity element:
    - There is an element e in G such that $a \bullet e = e \bullet a = a$ for all a in G
  - (A4) Inverse element:
    - For each a in G, there is an element a^{1} in G such that $a \bullet a^{1} = a^{1} \bullet a = e$
  - (A5) Commutative:
    - $a \bullet b = b \bullet a$ for all a, b in G

Cyclic Group

- Exponentiation is defined within a group as a repeated application of the group operator, so that $a^3 = a \bullet a \bullet a$
- We define $a^0 = e$ as the identity element, and $a^{-n} = (a')^n$, where a' is the inverse element of a within the group
- A group G is **cyclic** if every element of G is a power a^k (k is an integer) of a fixed element $a \in G$
- The element a is said to **generate** the group G or to be a **generator** of G
- A cyclic group is always abelian and may be finite or infinite

Rings

- A **ring** R, sometimes denoted by $\{R, +, *\}$, is a set of elements with two binary operations, called *addition* and *multiplication*, such that for all a, b, c in R the following axioms are obeyed:

(A1–A5)

R is an abelian group with respect to addition; that is, R satisfies axioms A1 through A5. For the case of an additive group, we denote the identity element as 0 and the inverse of a as $-a$

(M1) Closure under multiplication:

If a and b belong to R, then ab is also in R

(M2) Associativity of multiplication:

$a(bc) = (ab)c$ for all a, b, c in R

(M3) Distributive laws:

$a(b + c) = ab + ac$ for all a, b, c in R

$(a + b)c = ac + bc$ for all a, b, c in R

- In essence, a ring is a set in which we can do addition, subtraction $[a - b = a + (-b)]$, and multiplication without leaving the set

Rings (cont.)

- A ring is said to be commutative if it satisfies the following additional condition:

  (M4) Commutativity of multiplication:

  $ab = ba$ for all a, b in R

- An *integral domain* is a commutative ring that obeys the following axioms.

  (M5) Multiplicative identity:

  There is an element 1 in R such that $a1 = 1a = a$ for all a in R

  (M6) No zero divisors:

  If a, b in R and $ab = 0$, then either $a = 0$ or $b = 0$

Fields

- A **field** F , sometimes denoted by {F, +,* }, is a set of elements with two binary operations, called *addition* and *multiplication*, such that for all a, b, c in F the following axioms are obeyed:

  (A1–M6)

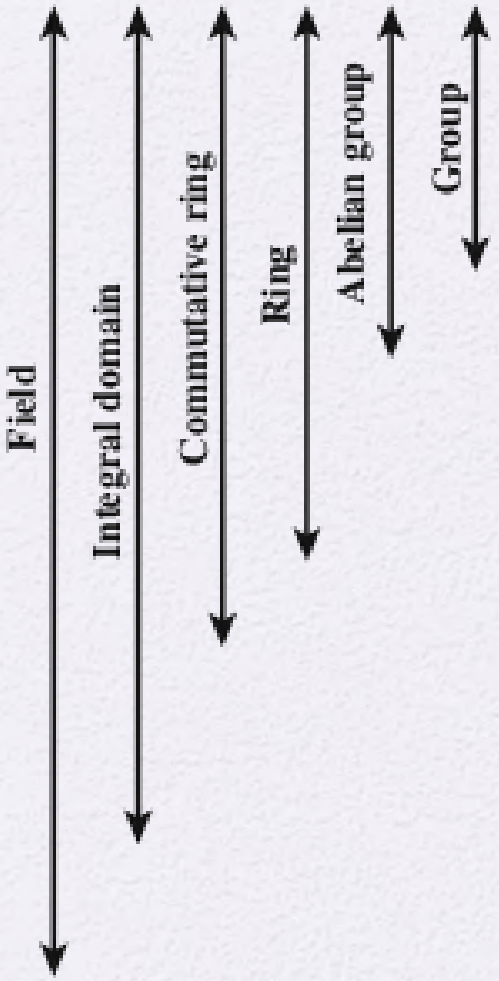
  F is an integral domain; that is, F satisfies axioms A1 through A5 and M1 through M6

  (M7) Multiplicative inverse:

  For each a in F, except 0, there is an element a^{-1} in F such that $aa^{-1} = (a^{-1})a = 1$

- In essence, a field is a set in which we can do addition, subtraction, multiplication, and division without leaving the set. Division is defined with the following rule: $a/b = a(b^{-1})$

Familiar examples of fields are the rational numbers, the real numbers, and the complex numbers. Note that the set of all integers is not a field, because not every element of the set has a multiplicative inverse.



(A1) Closure under addition:	If a and b belong to S, then $a + b$ is also in S
(A2) Associativity of addition:	$a + (b + c) = (a + b) + c$ for all a, b, c in S
(A3) Additive identity:	There is an element 0 in R such that $a + 0 = 0 + a = a$ for all a in S
(A4) Additive inverse:	For each a in S there is an element $-a$ in S such that $a + (-a) = (-a) + a = 0$
(A5) Commutativity of addition:	$a + b = b + a$ for all a, b in S
(M1) Closure under multiplication:	If a and b belong to S, then ab is also in S
(M2) Associativity of multiplication:	$a(bc) = (ab)c$ for all a, b, c in S
(M3) Distributive laws:	$a(b + c) = ab + ac$ for all a, b, c in S $(a + b)c = ac + bc$ for all a, b, c in S
(M4) Commutativity of multiplication:	$ab = ba$ for all a, b in S
(M5) Multiplicative identity:	There is an element 1 in S such that $a1 = 1a = a$ for all a in S
(M6) No zero divisors:	If a, b in S and $ab = 0$, then either $a = 0$ or $b = 0$
(M7) Multiplicative inverse:	If a belongs to S and $a \neq 0$, there is an element a^{-1} in S such that $aa^{-1} = a^{-1}a = 1$

Figure 5.2 Properties of Groups, Rings, and Fields

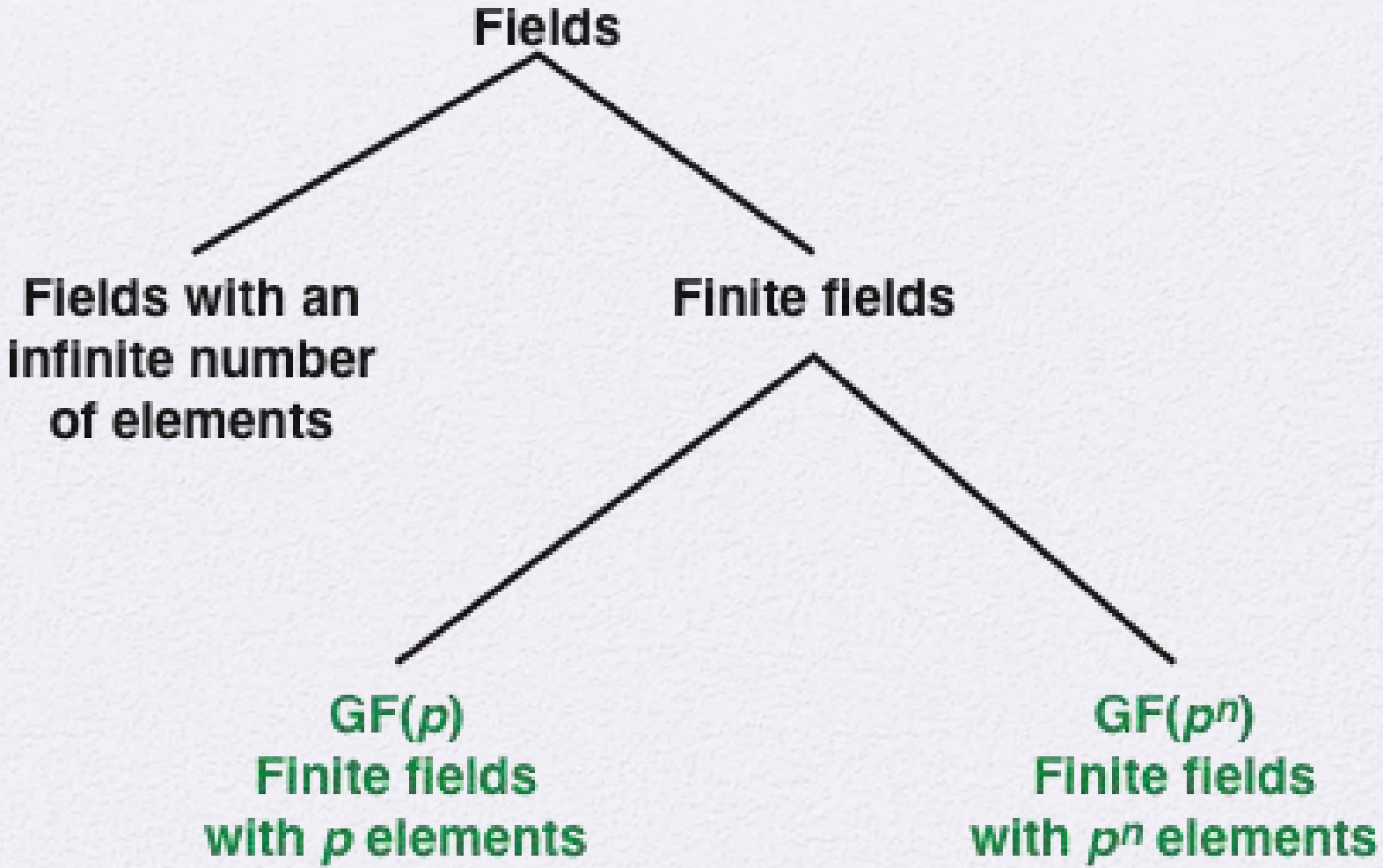


Figure 5.3 Types of Fields

Finite Fields of the Form GF(p)

- Finite fields play a crucial role in many cryptographic algorithms
- It can be shown that the order of a finite field must be a power of a prime p^n, where n is a positive integer
  - The finite field of order p^n is generally written GF(p^n)
  - GF stands for Galois field, in honor of the mathematician who first studied finite fields

Table 5.1(a)

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6

(a) Addition modulo 8

Table 5.1(b)

×	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	0	2	4	6
3	0	3	6	1	4	7	2	5
4	0	4	0	4	0	4	0	4
5	0	5	2	7	4	1	6	3
6	0	6	4	2	0	6	4	2
7	0	7	6	5	4	3	2	1

(b) Multiplication modulo 8

Table 5.1(c)

w	$-w$	w^{-1}
0	0	—
1	7	1
2	6	—
3	5	3
4	4	—
5	3	5
6	2	—
7	1	7

(c) Additive and multiplicative inverses modulo 8

Table 5.1(d)

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

(d) Addition modulo 7

Table 5.1(e)

×	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

(e) Multiplication modulo 7

Table 5.1(f)

w	$-w$	w^{-1}
0	0	—
1	6	1
2	5	4
3	4	5
4	3	2
5	2	3
6	1	6

(f) Additive and multiplicative inverses modulo 7

In this section, we have shown how to construct a finite field of order p, where p is prime.

GF(p) is defined with the following properties:

- 1. GF(p) consists of p elements
- 2. The binary operations + and * are defined over the set. The operations of addition, subtraction, multiplication, and division can be performed without leaving the set. Each element of the set other than 0 has a multiplicative inverse
- We have shown that the elements of GF(p) are the integers $\{0, 1, \ldots, p - 1\}$ and that the arithmetic operations are addition and multiplication mod p

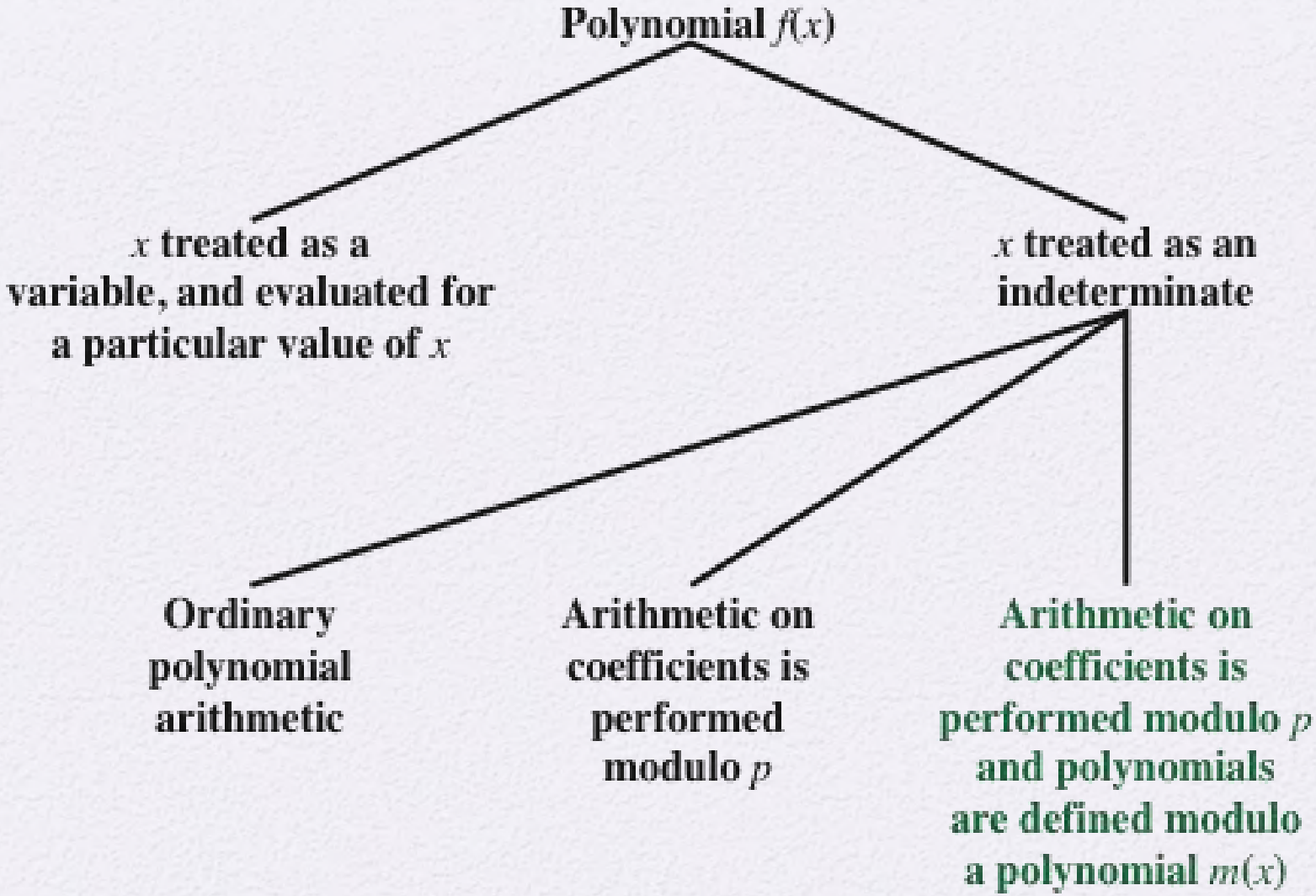
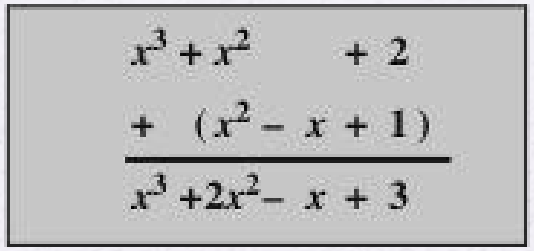
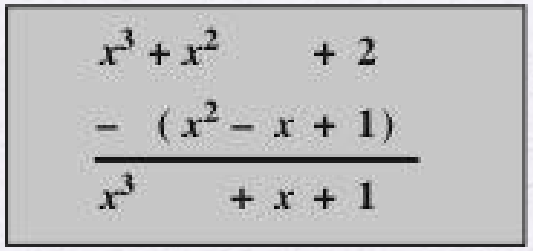


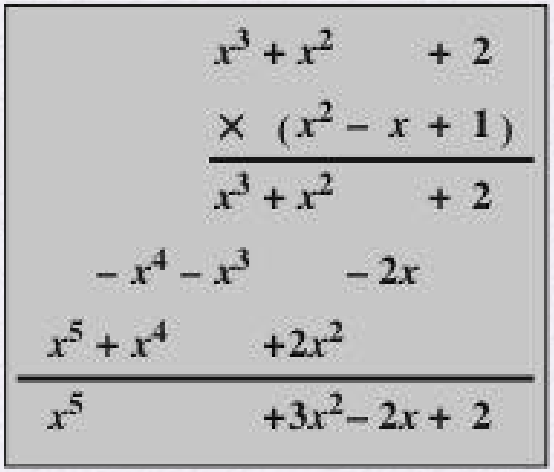
Figure 5.4 Treatment of Polynomials



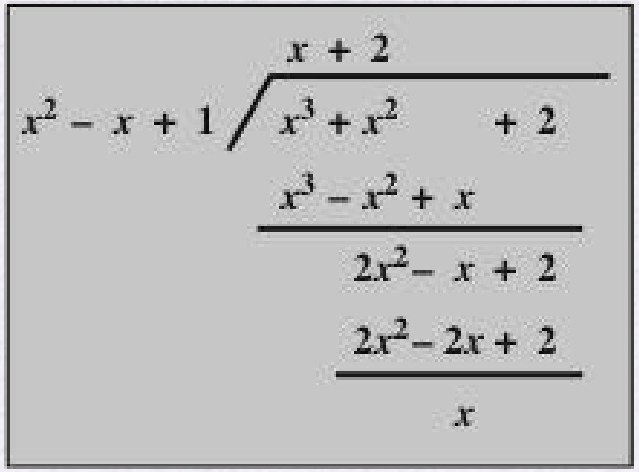
(a) Addition



(b) Subtraction



(c) Multiplication



(d) Division

Figure 5.5 Examples of Polynomial Arithmetic

Polynomial Arithmetic With Coefficients in Z_p

- If each distinct polynomial is considered to be an element of the set, then that set is a ring
- When polynomial arithmetic is performed on polynomials over a field, then division is possible
  - Note: this does not mean that *exact division* is possible
- If we attempt to perform polynomial division over a coefficient set that is not a field, we find that division is not always defined
  - Even if the coefficient set is a field, polynomial division is not necessarily exact
  - With the understanding that remainders are allowed, we can say that polynomial division is possible if the coefficient set is a field

Polynomial Division

- We can write any polynomial in the form:

$$f(x) = q(x)\, g(x) + r(x)$$

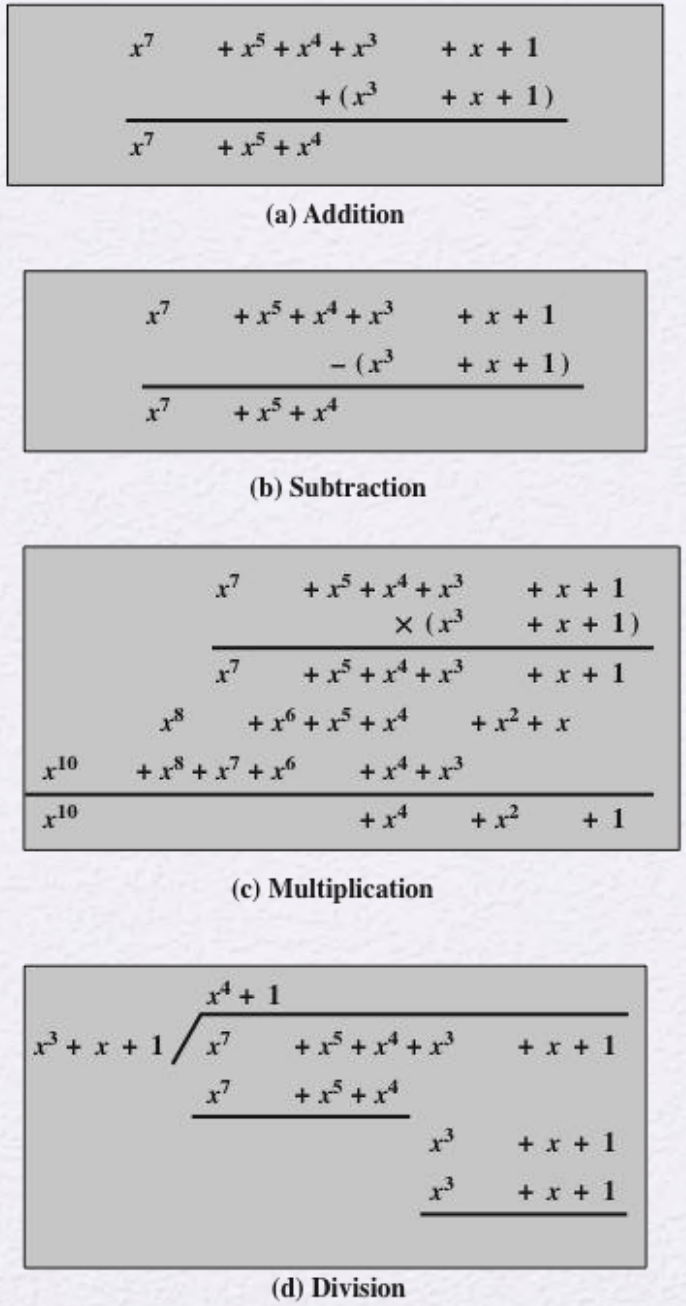
  - $r(x)$ can be interpreted as being a remainder
  - *So* $r(x) = f(x) \bmod g(x)$

- If there is no remainder we can say $g(x)$ **divides** $f(x)$
  - Written as $g(x) \mid f(x)$
  - We can say that $g(x)$ is a **factor** of $f(x)$
  - *Or* $g(x)$ is a **divisor** of $f(x)$

- A polynomial $f(x)$ over a field F is called **irreducible** if and only if $f(x)$ cannot be expressed as a product of two polynomials, both over F, and both of degree lower than that of $f(x)$
  - An irreducible polynomial is also called a **prime polynomial**

Example of Polynomial Arithmetic Over GF(2)

(Figure 5.6 can be found on page 137 in the textbook)



$$x^7 + x^5 + x^4 + x^3 + x + 1$$
$$+ (x^3 + x + 1)$$
$$x^7 + x^5 + x^4$$

(a) Addition

$$x^7 + x^5 + x^4 + x^3 + x + 1$$
$$- (x^3 + x + 1)$$
$$x^7 + x^5 + x^4$$

(b) Subtraction

$$x^7 + x^5 + x^4 + x^3 + x + 1$$
$$\times (x^3 + x + 1)$$
$$x^7 + x^5 + x^4 + x^3 + x + 1$$
$$x^8 + x^6 + x^5 + x^4 + x^2 + x$$
$$x^{10} + x^8 + x^7 + x^6 + x^4 + x^3$$
$$x^{10} + x^4 + x^2 + 1$$

(c) Multiplication

$$x^3 + x + 1 \;\big/\; x^7 + x^5 + x^4 + x^3 + x + 1 = x^4 + 1$$
$$x^7 + x^5 + x^4$$
$$x^3 + x + 1$$
$$x^3 + x + 1$$

(d) Division

Figure 5.6 Examples of Polynomial Arithmetic over GF(2)

Polynomial GCD

- The polynomial $c(x)$ is said to be the greatest common divisor of $a(x)$ and $b(x)$ if the following are true:
  - $c(x)$ divides both $a(x)$ and $b(x)$
  - Any divisor of $a(x)$ and $b(x)$ is a divisor of $c(x)$
- An equivalent definition is:
  - $\gcd[a(x), b(x)]$ is the polynomial of maximum degree that divides both $a(x)$ and $b(x)$
- The Euclidean algorithm can be extended to find the greatest common divisor of two polynomials whose coefficients are elements of a field

Table 5.2(a) Arithmetic in GF(2^3)

	+	000	001	010	011	100	101	110	111
		0	1	2	3	4	5	6	7
000	0	0	1	2	3	4	5	6	7
001	1	1	0	3	2	5	4	7	6
010	2	2	3	0	1	6	7	4	5
011	3	3	2	1	0	7	6	5	4
100	4	4	5	6	7	0	1	2	3
101	5	5	4	7	6	1	0	3	2
110	6	6	7	4	5	2	3	0	1
111	7	7	6	5	4	3	2	1	0

(a) Addition

Table 5.2(b) Arithmetic in GF(2^3)

	×	000	001	010	011	100	101	110	111
		0	1	2	3	4	5	6	7
000	0	0	0	0	0	0	0	0	0
001	1	0	1	2	3	4	5	6	7
010	2	0	2	4	6	3	1	7	5
011	3	0	3	6	5	7	4	1	2
100	4	0	4	3	7	6	2	5	1
101	5	0	5	1	4	2	7	3	6
110	6	0	6	7	1	5	3	2	4
111	7	0	7	5	2	1	6	4	3

(b) Multiplication

Table 5.2(c)

Arithmetic in GF(2^3)

w	$-w$	w^{-1}
0	0	—
1	1	1
2	2	5
3	3	6
4	4	7
5	5	2
6	6	3
7	7	4

(c) Additive and multiplicative inverses

Table 5.3 (page 144 in textbook)

Polynomial Arithmetic Modulo ($x^3 + x + 1$)

	+	000 0	001 1	010 x	011 $x+1$	100 x^2	101 x^2+1	110 x^2+x	111 x^2+x+1
000	0	0	1	x	$x+1$	x^2	x^2+1	x^2+x	x^2+x+1
001	1	1	0	$x+1$	x	x^2+1	x^2	x^2+x+1	x^2+x
010	x	x	$x+1$	0	1	x^2+x	x^2+x+1	x^2	x^2+1
011	$x+1$	$x+1$	x	1	0	x^2+x+1	x^2+x	x^2+1	x^2
100	x^2	x^2	x^2+1	x^2+x	x^2+x+1	0	1	x	$x+1$
101	x^2+1	x^2+1	x^2	x^2+x+1	x^2+x	1	0	$x+1$	x
110	x^2+x	x^2+x	x^2+x+1	x^2	x^2+1	x	$x+1$	0	1
111	x^2+x+1	x^2+x+1	x^2+x	x^2+1	x^2	$x+1$	x	1	0

(a) Addition

	×	000 0	001 1	010 x	011 $x+1$	100 x^2	101 x^2+1	110 x^2+x	111 x^2+x+1
000	0	0	0	0	0	0	0	0	0
001	1	0	1	x	$x+1$	x^2	x^2+1	x^2+x	x^2+x+1
010	x	0	x	x^2	x^2+x	$x+1$	1	x^2+x+1	x^2+1
011	$x+1$	0	$x+1$	x^2+x	x^2+1	x^2+x+1	x^2	1	x
100	x^2	0	x^2	$x+1$	x^2+x+1	x^2+x	x	x^2+1	1
101	x^2+1	0	x^2+1	1	x^2	x	x^2+x+1	$x+1$	x^2+x
110	x^2+x	0	x^2+x	x^2+x+1	1	x^2+1	$x+1$	x	x^2
111	x^2+x+1	0	x^2+x+1	x^2+1	x	1	x^2+x	x^2	$x+1$

(b) Multiplication

Table 5.4
Extended Euclid $[(x^8 + x^4 + x^3 + x + 1), (x^7 + x + 1)]$

Initialization	$a(x) = x^8 + x^4 + x^3 + x + 1$; $v_{-1}(x) = 1$; $w_{-1}(x) = 0$ $b(x) = x^7 + x + 1$; $v_0(x) = 0$; $w_0(x) = 1$
Iteration 1	$q_1(x) = x$; $r_1(x) = x^4 + x^3 + x^2 + 1$ $v_1(x) = 1$; $w_1(x) = x$
Iteration 2	$q_2(x) = x^3 + x^2 + 1$; $r_2(x) = x$ $v_2(x) = x^3 + x^2 + 1$; $w_2(x) = x^4 + x^3 + x + 1$
Iteration 3	$q_3(x) = x^3 + x^2 + x$; $r_3(x) = 1$ $v_3(x) = x^6 + x^2 + x + 1$; $w_3(x) = x^7$
Iteration 4	$q_4(x) = x$; $r_4(x) = 0$ $v_4(x) = x^7 + x + 1$; $w_4(x) = x^8 + x^4 + x^3 + x + 1$
Result	$d(x) = r_3(x) = \gcd(a(x), b(x)) = 1$ $w(x) = w_3(x) = (x^7 + x + 1)^{-1} \bmod (x^8 + x^4 + x^3 + x + 1) = x^7$

(Table 5.4 can be found on page 146 in textbook)

Computational Considerations

- Since coefficients are 0 or 1, they can represent any such polynomial as a bit string
- Addition becomes XOR of these bit strings
- Multiplication is shift and XOR
  - cf long-hand multiplication
- Modulo reduction is done by repeatedly substituting highest power with remainder of irreducible polynomial (also shift and XOR)

Using a Generator

- A **generator** *g* of a finite field F of order *q* (contains *q* elements) is an element whose first *q*-1 powers generate all the nonzero elements of F
  - The elements of F consist of 0, g^0, g^1, ..., g^{q-2}
- Consider a field F defined by a polynomial *fx*
  - An element b contained in F is called a **root** of the polynomial if *f(b) = 0*
- Finally, it can be shown that a root *g* of an irreducible polynomial is a generator of the finite field defined on that polynomial

Table 5.5

Generator for GF(2^3) using $x^3 + x + 1$

Power Representation	Polynomial Representation	Binary Representation	Decimal (Hex) Representation
0	0	000	0
g^0 (= g^7)	1	001	1
g^1	g	010	2
g^2	g^2	100	4
g^3	$g + 1$	011	3
g^4	$g^2 + g$	110	6
g^5	$g^2 + g + 1$	111	7
g^6	$g^2 + 1$	101	5

Table 5.6 (page 150 in textbook)

GF(2^3) Arithmetic Using Generator for the Polynomial ($x^3 + x + 1$)

	+	000 0	001 1	010 G	100 g^2	011 g^3	110 g^4	111 g^5	101 g^6
000	0	0	1	G	g^2	$g+1$	g^2+g	g^2+g+1	g^2+1
001	1	1	0	$g+1$	g^2+1	g	g^2+g+1	g^2+g	g^2
010	g	g	$g+1$	0	g^2+g	1	g^2	g^2+1	g^2+g+1
100	g^2	g^2	g^2+1	g^2+g	0	g^2+g+1	g	$g+1$	1
011	g^3	$g+1$	g	1	g^2+g+1	0	g^2+1	g^2	g^2+g
110	g^4	g^2+g	g^2+g+1	g^2	g	g^2+1	0	1	$g+1$
111	g^5	g^2+g+1	g^2+g	g^2+1	$g+1$	g^2	1	0	g
101	g^6	g^2+1	g^2	g^2+g+1	1	g^2+g	$g+1$	g	0

(a) Addition

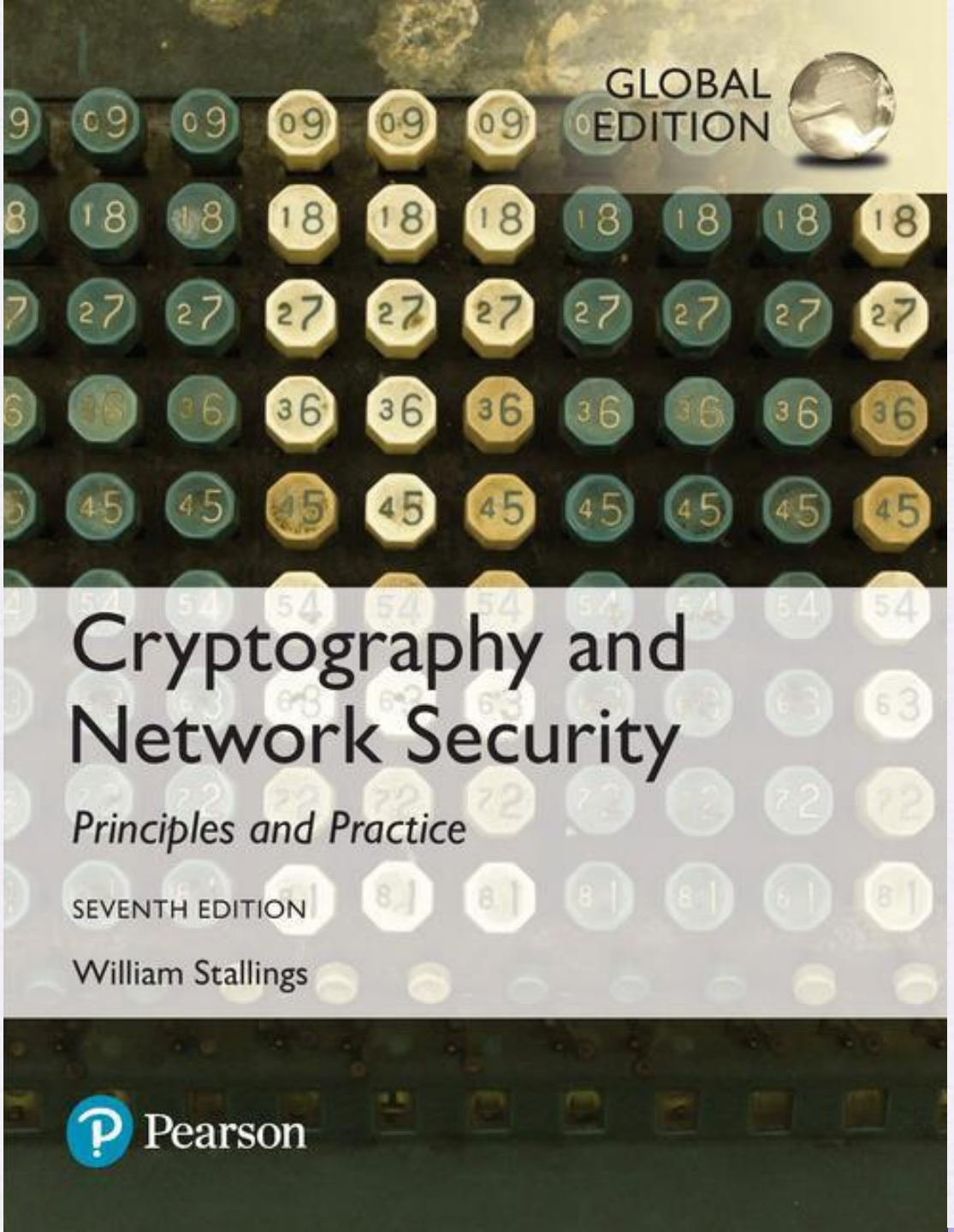
	×	000 0	001 1	010 G	100 g^2	011 g^3	110 g^4	111 g^5	101 g^6
000	0	0	0	0	0	0	0	0	0
001	1	0	1	G	g^2	$g+1$	g^2+g	g^2+g+1	g^2+1
010	g	0	g	g^2	$g+1$	g^2+g	g^2+g+1	g^2+1	1
100	g^2	0	g^2	$g+1$	g^2+g	g^2+g+1	g^2+1	1	g
011	g^3	0	$g+1$	g^2+g	g^2+g+1	g^2+1	1	g	g^2
110	g^4	0	g^2+g	g^2+g+1	g^2+1	1	g	g^2	$g+1$
111	g^5	0	g^2+g+1	g^2+1	1	g	g^2	$g+1$	g^2+g
101	g^6	0	g^2+1	1	g	g^2	$g+1$	g^2+g	g^2+g+1

(b) Multiplication

Summary

- Groups
  - Abelian group
  - Cyclic group
- Finite fields of the form GF(p)
  - Finite fields of Order p
  - Finding the multiplicative inverse in GF(p)
- Polynomial arithmetic
  - Ordinary polynomial arithmetic
  - Polynomial arithmetic with coefficients in Z_p
  - Finding the greatest common divisor

- Rings
- fields
- Finite fields of the form GF(2^n)
  - Motivation
  - Modular polynomial arithmetic
  - Finding the multiplicative inverse
  - Computational considerations
  - Using a generator


GLOBAL EDITION
Cryptography and Network Security
Principles and Practice
SEVENTH EDITION
William Stallings
Pearson

Chapter 6

Advanced Encryption Standard

Finite Field Arithmetic

- In the Advanced Encryption Standard (AES) all operations are performed on 8-bit bytes
- The arithmetic operations of addition, multiplication, and division are performed over the finite field $GF(2^8)$
- A field is a set in which we can do addition, subtraction, multiplication, and division without leaving the set
- Division is defined with the following rule:
  - $a/b = a(b^{-1})$
- An example of a finite field (one with a finite number of elements) is the set Z_p consisting of all the integers $\{0, 1, \ldots, p - 1\}$, where p is a prime number and in which arithmetic is carried out modulo p

Finite Field Arithmetic

If one of the operations used in the algorithm is division, then we need to work in arithmetic defined over a field

- Division requires that each nonzero element have a multiplicative inverse

For convenience and for implementation efficiency we would like to work with integers that fit exactly into a given number of bits with no wasted bit patterns

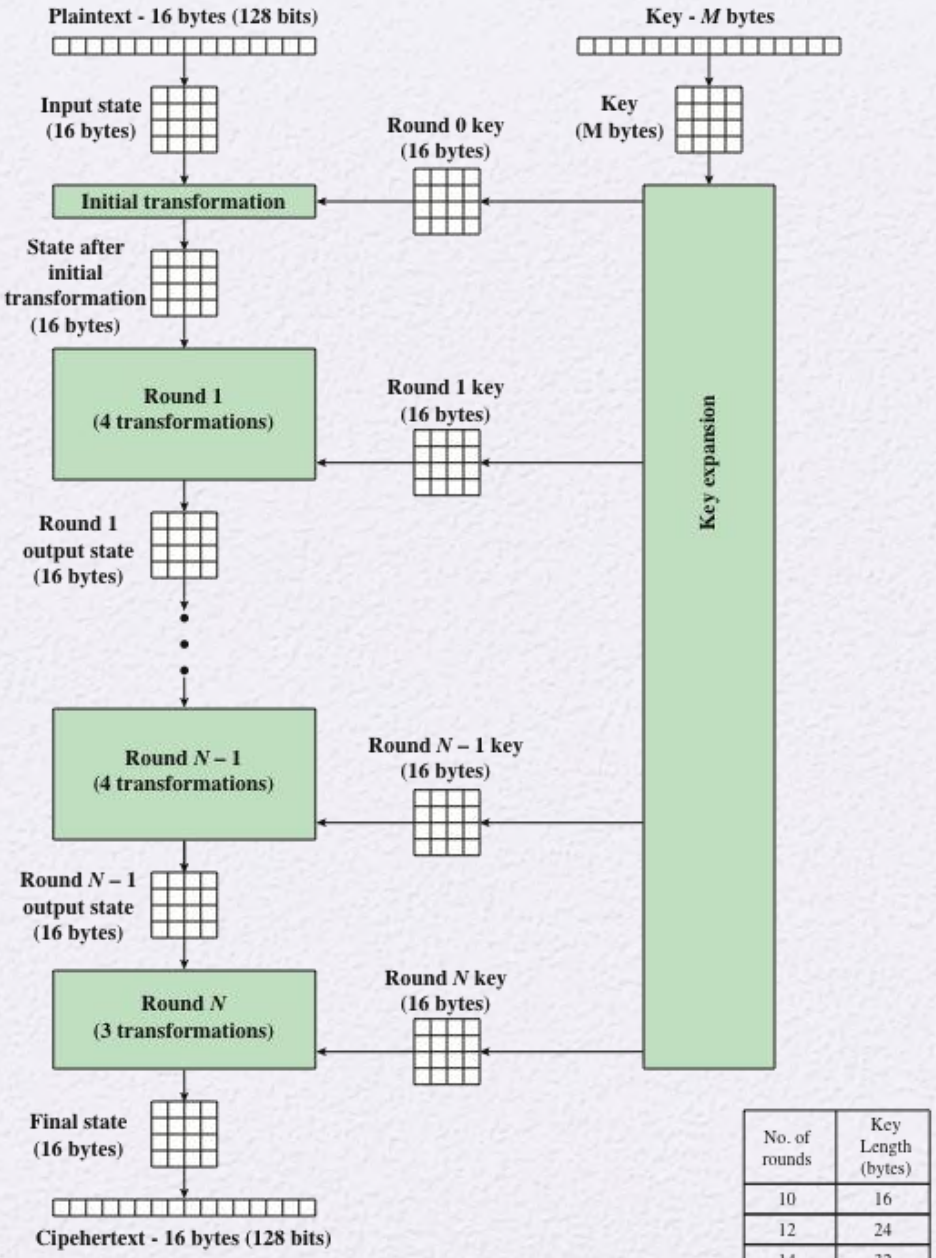
- Integers in the range 0 through $2^n - 1$, which fit into an *n*-bit word

The set of such integers, Z_{2^n}, using modular arithmetic, is not a field

- For example, the integer 2 has no multiplicative inverse in Z_{2^n}, that is, there is no integer *b*, such that $2b \bmod 2^n = 1$

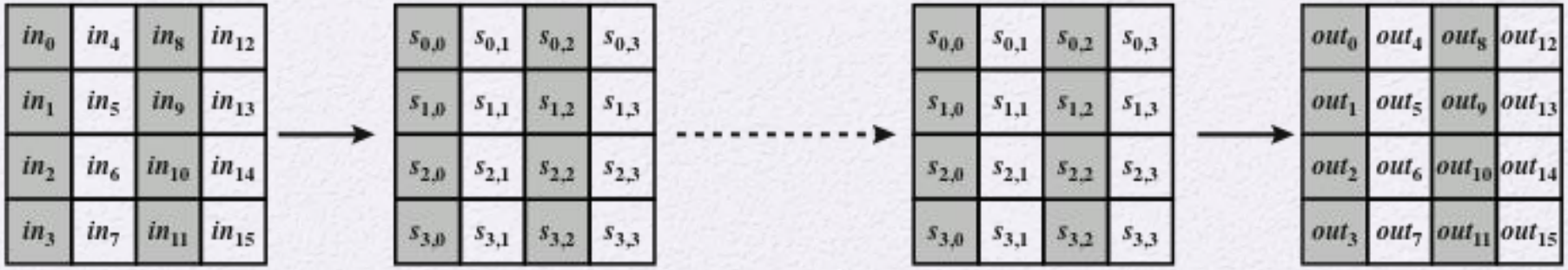
A finite field containing 2^n elements is referred to as $GF(2^n)$

- Every polynomial in $GF(2^n)$ can be represented by an n-bit number



No. of rounds	Key Length (bytes)
10	16
12	24
14	32

Figure 6.1 AES Encryption Process



(a) Input, state array, and output

(b) Key and expanded key

Figure 6.2 AES Data Structures

Table 6.1
AES Parameters

Key Size (words/bytes/bits)	4/16/128	6/24/192	8/32/256
Plaintext Block Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Number of Rounds	10	12	14
Round Key Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Expanded Key Size (words/bytes)	44/176	52/208	60/240

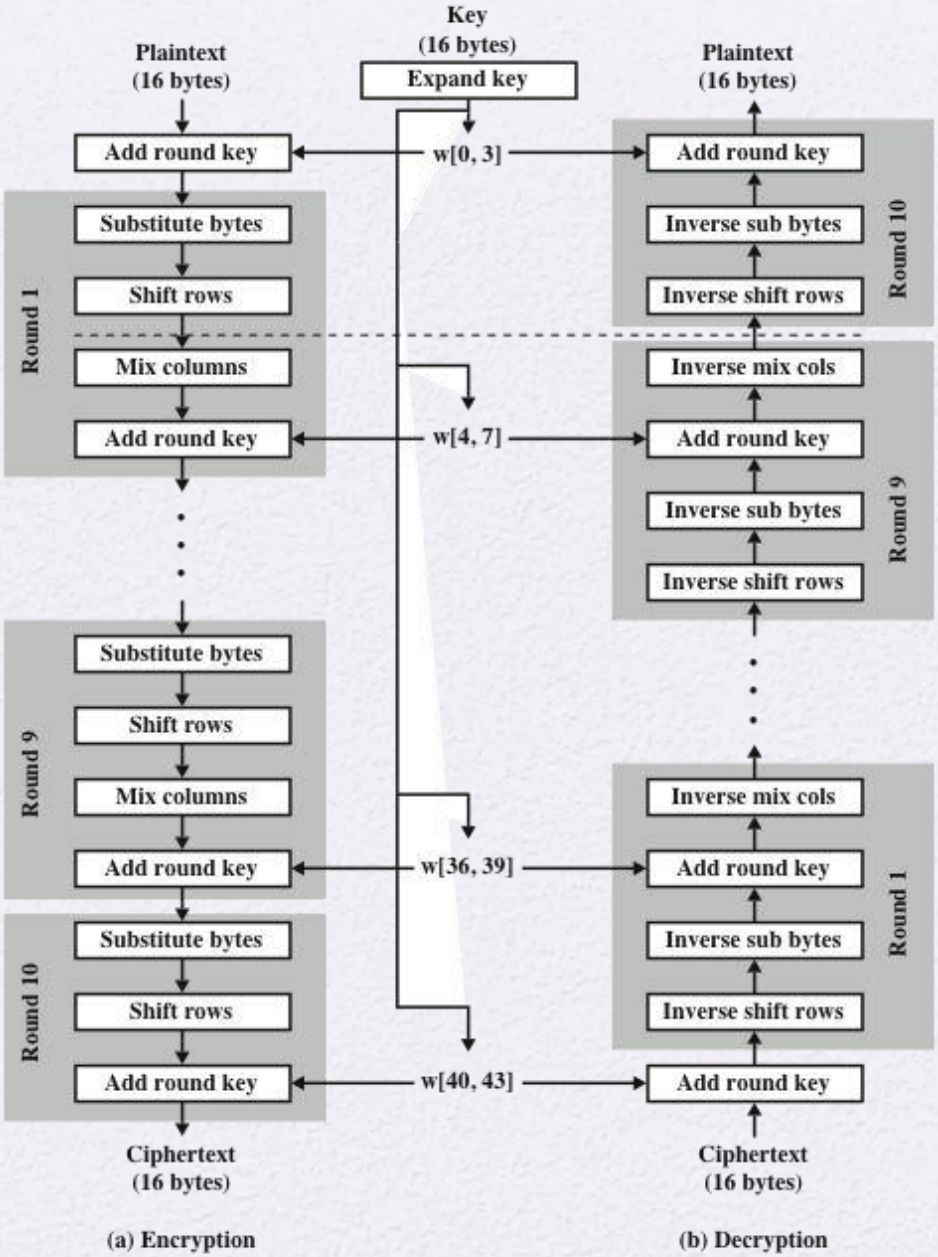


Figure 6.3 AES Encryption and Decryption

Detailed Structure

- Processes the entire data block as a single matrix during each round using substitutions and permutation
- The key that is provided as input is expanded into an array of forty-four 32-bit words, *w[i]*

Four different stages are used:

- Substitute bytes – uses an S-box to perform a byte-by-byte substitution of the block
- ShiftRows – a simple permutation
- MixColumns – a substitution that makes use of arithmetic over GF(2^8)
- AddRoundKey – a simple bitwise XOR of the current block with a portion of the expanded key

- The cipher begins and ends with an AddRoundKey stage
- Can view the cipher as alternating operations of XOR encryption (AddRoundKey) of a block, followed by scrambling of the block (the other three stages), followed by XOR encryption, and so on
- Each stage is easily reversible
- The decryption algorithm makes use of the expanded key in reverse order, however the decryption algorithm is not identical to the encryption algorithm
- State is the same for both encryption and decryption
- Final round of both encryption and decryption consists of only three stages

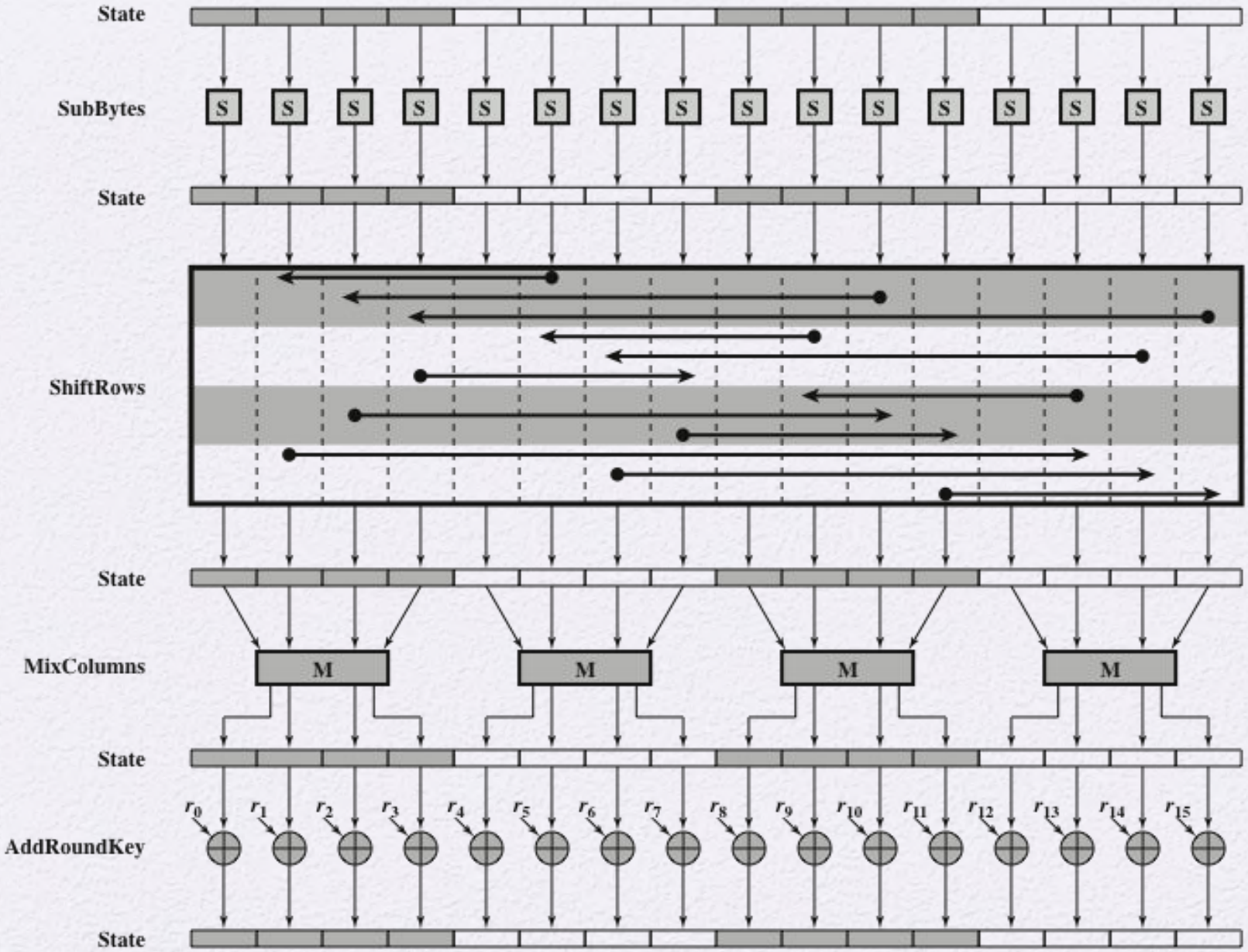
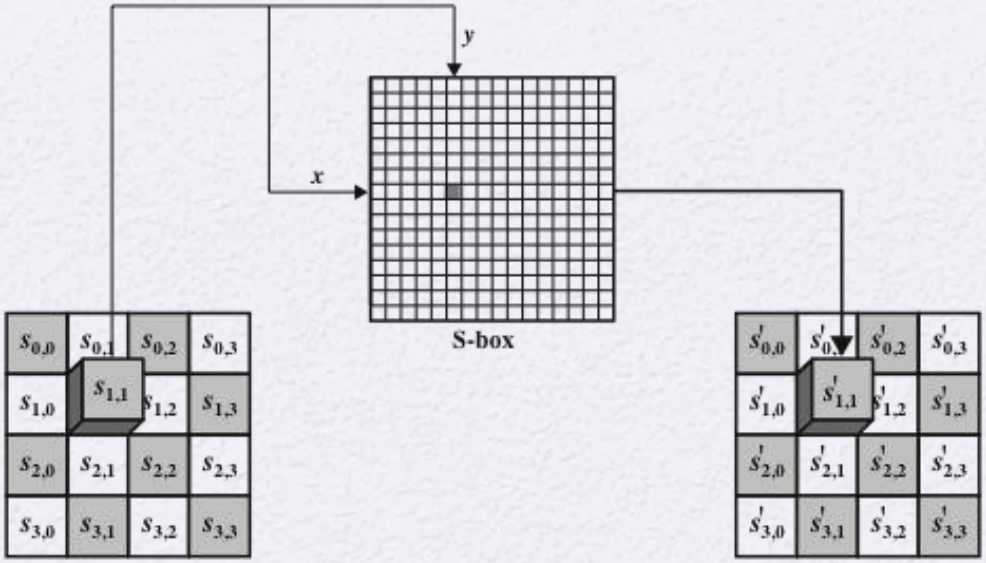
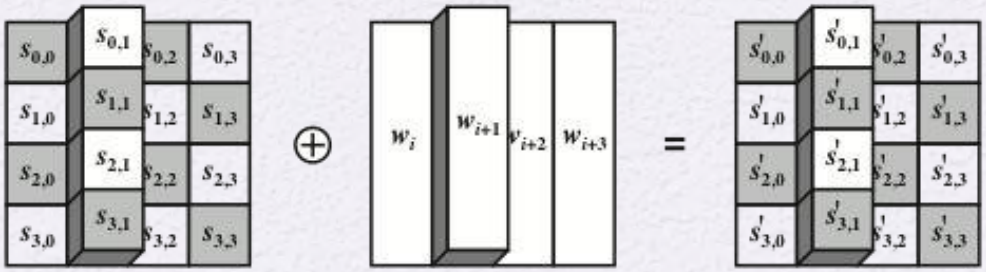


Figure 6.4 AES Encryption Round



(a) Substitute byte transformation



(b) Add round key Transformation

Figure 6.5 AES Byte-Level Operations

Table 6.2

		y														
		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
x	0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
	1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
	2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
	3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
	4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
	5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
	6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
	7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
	8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
	9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
	A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
	B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
	C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
	D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
	E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
	F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

(a) S-box

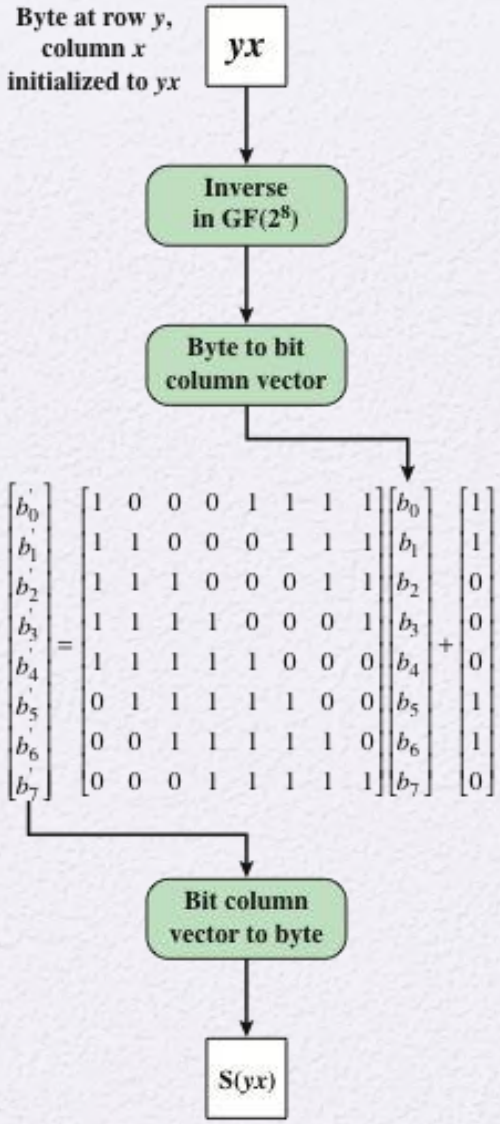
(Table can be found on page 163 in textbook)

Table 6.2

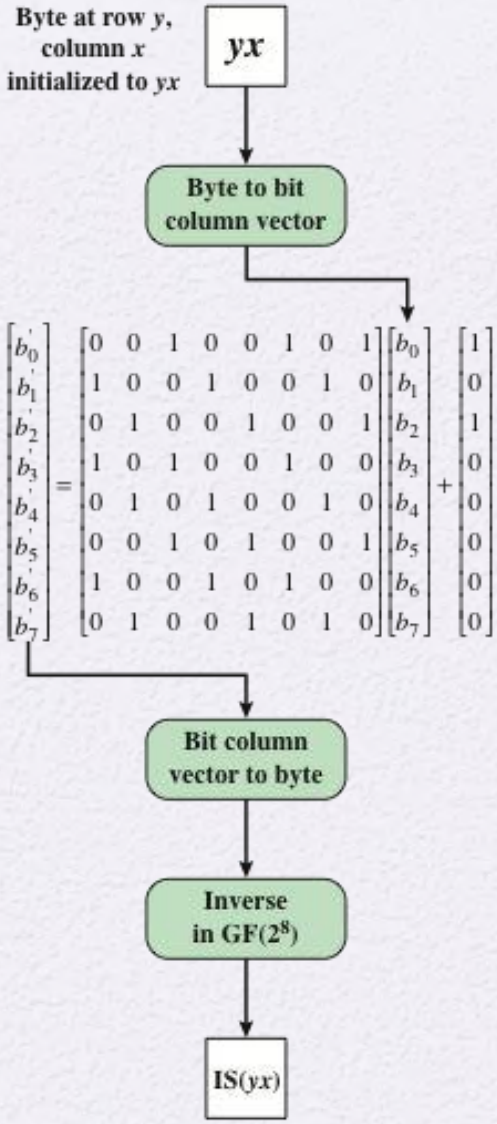
		y														
		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
x	0	52	09	6A	D5	30	36	A5	38	BF	40	A3	9E	81	F3	D7	FB
	1	7C	E3	39	82	9B	2F	FF	87	34	8E	43	44	C4	DE	E9	CB
	2	54	7B	94	32	A6	C2	23	3D	EE	4C	95	0B	42	FA	C3	4E
	3	08	2E	A1	66	28	D9	24	B2	76	5B	A2	49	6D	8B	D1	25
	4	72	F8	F6	64	86	68	98	16	D4	A4	5C	CC	5D	65	B6	92
	5	6C	70	48	50	FD	ED	B9	DA	5E	15	46	57	A7	8D	9D	84
	6	90	D8	AB	00	8C	BC	D3	0A	F7	E4	58	05	B8	B3	45	06
	7	D0	2C	1E	8F	CA	3F	0F	02	C1	AF	BD	03	01	13	8A	6B
	8	3A	91	11	41	4F	67	DC	EA	97	F2	CF	CE	F0	B4	E6	73
	9	96	AC	74	22	E7	AD	35	85	E2	F9	37	E8	1C	75	DF	6E
	A	47	F1	1A	71	1D	29	C5	89	6F	B7	62	0E	AA	18	BE	1B
	B	FC	56	3E	4B	C6	D2	79	20	9A	DB	C0	FE	78	CD	5A	F4
	C	1F	DD	A8	33	88	07	C7	31	B1	12	10	59	27	80	EC	5F
	D	60	51	7F	A9	19	B5	4A	0D	2D	E5	7A	9F	93	C9	9C	EF
	E	A0	E0	3B	4D	AE	2A	F5	B0	C8	EB	BB	3C	83	53	99	61
	F	17	2B	04	7E	BA	77	D6	26	E1	69	14	63	55	21	0C	7D

(b) Inverse S-box

(Table can be found on page 163 in textbook)



(a) Calculation of byte at row y, column x of S-box

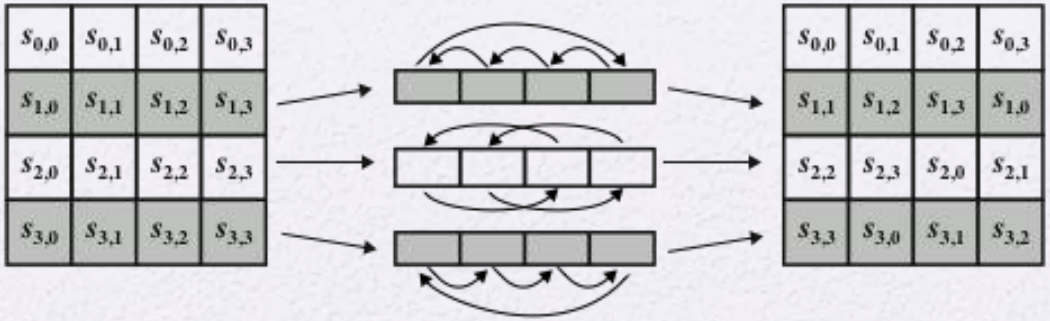


(a) Calculation of byte at row y, column x of IS-box

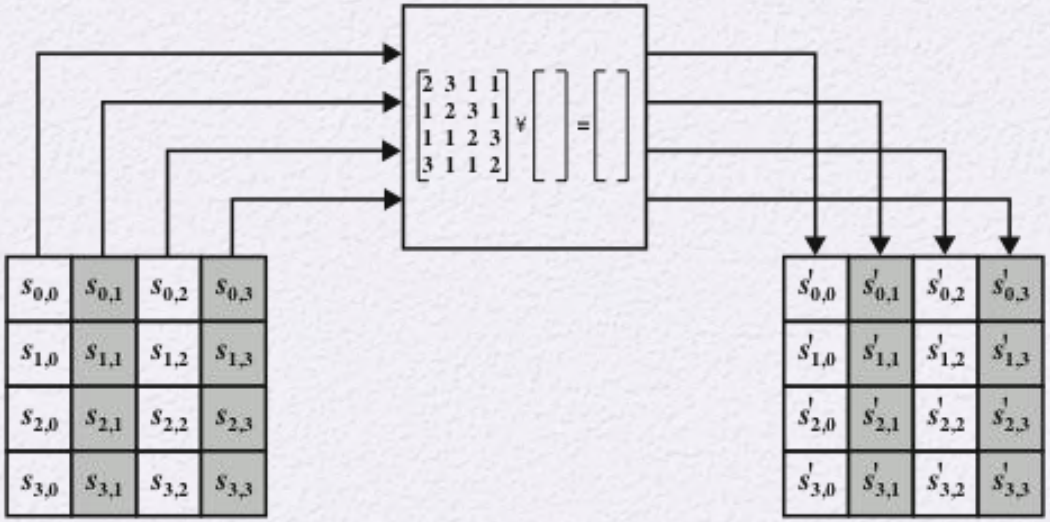
Figure 6.6 Construction of S-Box and IS-Box

S-Box Rationale

- The S-box is designed to be resistant to known cryptanalytic attacks
- The Rijndael developers sought a design that has a low correlation between input bits and output bits and the property that the output is not a linear mathematical function of the input
- The nonlinearity is due to the use of the multiplicative inverse



(a) Shift row transformation



(b) Mix column transformation

Figure 6.7 AES Row and Column Operations

Shift Row Rationale

- More substantial than it may first appear
- The State, as well as the cipher input and output, is treated as an array of four 4-byte columns
- On encryption, the first 4 bytes of the plaintext are copied to the first column of State, and so on
- The round key is applied to State column by column
  - Thus, a row shift moves an individual byte from one column to another, which is a linear distance of a multiple of 4 bytes
- Transformation ensures that the 4 bytes of one column are spread out to four different columns

Mix Columns Rationale

- Coefficients of a matrix based on a linear code with maximal distance between code words ensures a good mixing among the bytes of each column
- The mix column transformation combined with the shift row transformation ensures that after a few rounds all output bits depend on all input bits

AddRoundKey Transformation

- The 128 bits of State are bitwise XORed with the 128 bits of the round key
- Operation is viewed as a columnwise operation between the 4 bytes of a State column and one word of the round key
  - Can also be viewed as a byte-level operation

Rationale:

Is as simple as possible and affects every bit of State

The complexity of the round key expansion plus the complexity of the other stages of AES ensure security

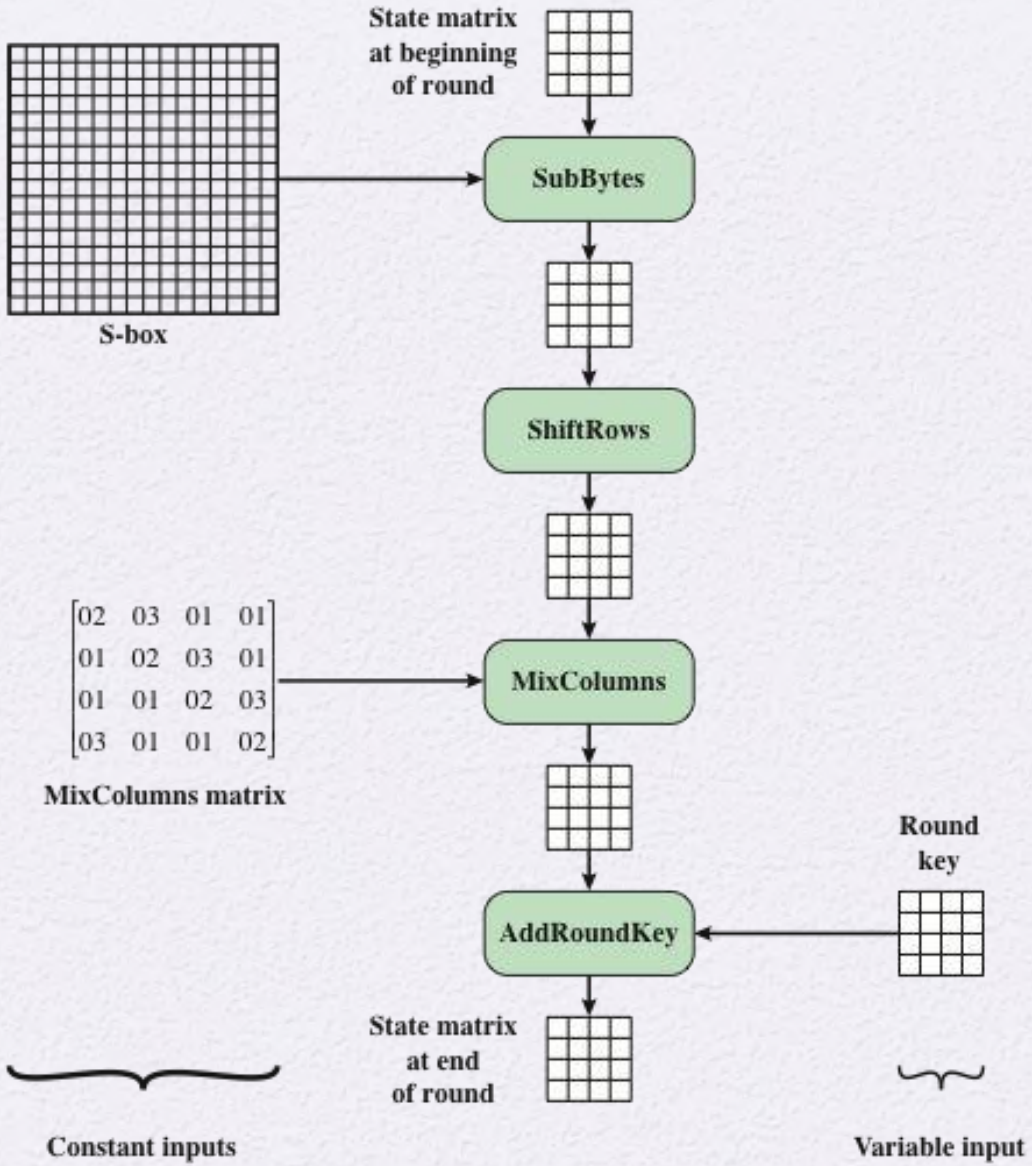


Figure 6.8 Inputs for Single AES Round

AES Key Expansion

- Takes as input a four-word (16 byte) key and produces a linear array of 44 words (176) bytes
  - This is sufficient to provide a four-word round key for the initial AddRoundKey stage and each of the 10 rounds of the cipher
- Key is copied into the first four words of the expanded key
  - The remainder of the expanded key is filled in four words at a time
- Each added word *w*[i] depends on the immediately preceding word, *w[i – 1]*, and the word four positions back, w[i – 4]
  - In three out of four cases a simple XOR is used
  - For a word whose position in the *w* array is a multiple of 4, a more complex function is used

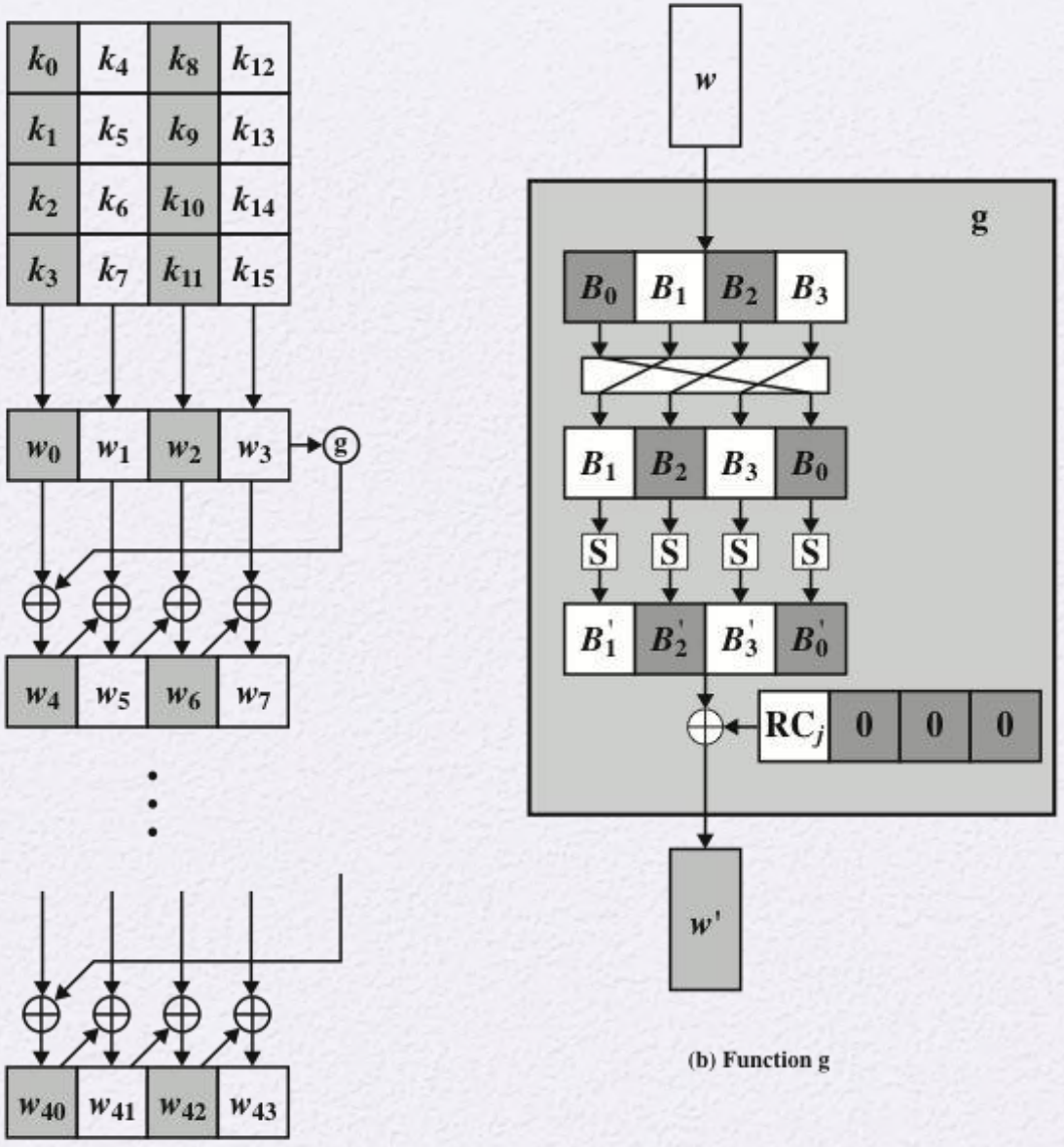


Figure 6.9 AES Key Expansion

Key Expansion Rationale

- The Rijndael developers designed the expansion key algorithm to be resistant to known cryptanalytic attacks
- Inclusion of a round-dependent round constant eliminates the symmetry between the ways in which round keys are generated in different rounds

The specific criteria that were used are:

- Knowledge of a part of the cipher key or round key does not enable calculation of many other round-key bits
- An invertible transformation
- Speed on a wide range of processors
- Usage of round constants to eliminate symmetries
- Diffusion of cipher key differences into the round keys
- Enough nonlinearity to prohibit the full determination of round key differences from cipher key differences only
- Simplicity of description

Table 6.3

AES Example Key Expansion

(Table is located on page 175 in textbook)

Key Words	Auxiliary Function
w0 = 0f 15 71 c9 w1 = 47 d9 e8 59 w2 = 0c b7 ad d6 w3 = af 7f 67 98	RotWord(w3)= 7f 67 98 af = x1 SubWord(x1)= d2 85 46 79 = y1 Rcon(1)= 01 00 00 00 y1 ⊕ Rcon(1)= d3 85 46 79 = z1
w4 = w0 ⊕ z1 = dc 90 37 b0 w5 = w4 ⊕ w1 = 9b 49 df e9 w6 = w5 ⊕ w2 = 97 fe 72 3f w7 = w6 ⊕ w3 = 38 81 15 a7	RotWord(w7)= 81 15 a7 38 = x2 SubWord(x4)= 0c 59 5c 07 = y2 Rcon(2)= 02 00 00 00 y2 ⊕ Rcon(2)= 0e 59 5c 07 = z2
w8 = w4 ⊕ z2 = d2 c9 6b b7 w9 = w8 ⊕ w5 = 49 80 b4 5e w10 = w9 ⊕ w6 = de 7e c6 61 w11 = w10 ⊕ w7 = e6 ff d3 c6	RotWord(w11)= ff d3 c6 e6 = x3 SubWord(x2)= 16 66 b4 8e = y3 Rcon(3)= 04 00 00 00 y3 ⊕ Rcon(3)= 12 66 b4 8e = z3
w12 = w8 ⊕ z3 = c0 af df 39 w13 = w12 ⊕ w9 = 89 2f 6b 67 w14 = w13 ⊕ w10 = 57 51 ad 06 w15 = w14 ⊕ w11 = b1 ae 7e c0	RotWord(w15)= ae 7e c0 b1 = x4 SubWord(x3)= e4 f3 ba c8 = y4 Rcon(4)= 08 00 00 00 y4 ⊕ Rcon(4)= ec f3 ba c8 = 4
w16 = w12 ⊕ z4 = 2c 5c 65 f1 w17 = w16 ⊕ w13 = a5 73 0e 96 w18 = w17 ⊕ w14 = f2 22 a3 90 w19 = w18 ⊕ w15 = 43 8c dd 50	RotWord(w19)= 8c dd 50 43 = x5 SubWord(x4)= 64 c1 53 1a = y5 Rcon(5)= 10 00 00 00 y5 ⊕ Rcon(5)= 74 c1 53 1a = z5
w20 = w16 ⊕ z5 = 58 9d 36 eb w21 = w20 ⊕ w17 = fd ee 38 7d w22 = w21 ⊕ w18 = 0f cc 9b ed w23 = w22 ⊕ w19 = 4c 40 46 bd	RotWord(w23)= 40 46 bd 4c = x6 SubWord(x5)= 09 5a 7a 29 = y6 Rcon(6)= 20 00 00 00 y6 ⊕ Rcon(6)= 29 5a 7a 29 = z6
w24 = w20 ⊕ z6 = 71 c7 4c c2 w25 = w24 ⊕ w21 = 8c 29 74 bf w26 = w25 ⊕ w22 = 83 e5 ef 52 w27 = w26 ⊕ w23 = cf a5 a9 ef	RotWord(w27)= a5 a9 ef cf = x7 SubWord(x6)= 06 d3 df 8a = y7 Rcon(7)= 40 00 00 00 y7 ⊕ Rcon(7)= 46 d3 df 8a = z7
w28 = w24 ⊕ z7 = 37 14 93 48 w29 = w28 ⊕ w25 = bb 3d e7 f7 w30 = w29 ⊕ w26 = 38 d8 08 a5 w31 = w30 ⊕ w27 = f7 7d a1 4a	RotWord(w31)= 7d a1 4a f7 = x8 SubWord(x7)= ff 32 d6 68 = y8 Rcon(8)= 80 00 00 00 y8 ⊕ Rcon(8)= 7f 32 d6 68 = z8
w32 = w28 ⊕ z8 = 48 26 45 20 w33 = w32 ⊕ w29 = f3 1b a2 d7 w34 = w33 ⊕ w30 = cb c3 aa 72 w35 = w34 ⊕ w32 = 3c be 0b 38	RotWord(w35)= be 0b 38 3c = x9 SubWord(x8)= ae 2b 07 eb = y9 Rcon(9)= 1B 00 00 00 y9 ⊕ Rcon(9)= b5 2b 07 eb = z9
w36 = w32 ⊕ z9 = fd 0d 42 cb w37 = w36 ⊕ w33 = 0e 16 e0 1c w38 = w37 ⊕ w34 = c5 d5 4a 6e w39 = w38 ⊕ w35 = f9 6b 41 56	RotWord(w39)= 6b 41 56 f9 = x10 SubWord(x9)= 7f 83 b1 99 = y10 Rcon(10)= 36 00 00 00 y10 ⊕ Rcon(10)= 49 83 b1 99 = z10
w40 = w36 ⊕ z10 = b4 8e f3 52 w41 = w40 ⊕ w37 = ba 98 13 4e w42 = w41 ⊕ w38 = 7f 4d 59 20 w43 = w42 ⊕ w39 = 86 26 18 76	

Table 6.4

AES Example

(Table is located on page 177 in textbook)

Start of round	After SubBytes	After ShiftRows	After MixColumns	Round Key
01 89 fe 76 23 ab dc 54 45 cd ba 32 67 ef 98 10				0f 47 0c af 15 d9 b7 7f 71 e8 ad 67 c9 59 d6 98
0e ce f2 d9 36 72 6b 2b 34 25 17 55 ae b6 4e 88	ab 8b 89 35 05 40 7f f1 18 3f f0 fc e4 4e 2f c4	ab 8b 89 35 40 7f f1 05 f0 fc 18 3f c4 e4 4e 2f	b9 94 57 75 e4 8e 16 51 47 20 9a 3f c5 d6 f5 3b	dc 9b 97 38 90 49 fe 81 37 df 72 15 b0 e9 3f a7
65 0f c0 4d 74 c7 e8 d0 70 ff e8 2a 75 3f ca 9c	4d 76 ba e3 92 c6 9b 70 51 16 9b e5 9d 75 74 de	4d 76 ba e3 c6 9b 70 92 9b e5 51 16 de 9d 75 74	8e 22 db 12 b2 f2 dc 92 df 80 f7 c1 2d c5 1e 52	d2 49 de e6 c9 80 7e ff 6b b4 c6 d3 b7 5e 61 c6
5c 6b 05 f4 7b 72 a2 6d b4 34 31 12 9a 9b 7f 94	4a 7f 6b bf 21 40 3a 3c 8d 18 c7 c9 b8 14 d2 22	4a 7f 6b bf 40 3a 3c 21 c7 c9 8d 18 22 b8 14 d2	b1 c1 0b cc ba f3 8b 07 f9 1f 6a c3 1d 19 24 5c	c0 89 57 b1 af 2f 51 ae df 6b ad 7e 39 67 06 c0
71 48 5c 7d 15 dc da a9 26 74 c7 bd 24 7e 22 9c	a3 52 4a ff 59 86 57 d3 f7 92 c6 7a 36 f3 93 de	a3 52 4a ff 86 57 d3 59 c6 7a f7 92 de 36 f3 93	d4 11 fe 0f 3b 44 06 73 cb ab 62 37 19 b7 07 ec	2c a5 f2 43 5c 73 22 8c 65 0e a3 dd f1 96 90 50
f8 b4 0c 4c 67 37 24 ff ae a5 c1 ea e8 21 97 bc	41 8d fe 29 85 9a 36 16 e4 06 78 87 9b fd 88 65	41 8d fe 29 9a 36 16 85 78 87 e4 06 65 9b fd 88	2a 47 c4 48 83 e8 18 ba 84 18 27 23 eb 10 0a f3	58 fd 0f 4c 9d ee cc 40 36 38 9b 46 eb 7d ed bd
72 ba cb 04 1e 06 d4 fa b2 20 bc 65 00 6d e7 4e	40 f4 1f f2 72 6f 48 2d 37 b7 65 4d 63 3c 94 2f	40 f4 1f f2 6f 48 2d 72 65 4d 37 b7 2f 63 3c 94	7b 05 42 4a 1e d0 20 40 94 83 18 52 94 c4 43 fb	71 8c 83 cf c7 29 e5 a5 4c 74 ef a9 c2 bf 52 ef
0a 89 c1 85 d9 f9 c5 e5 d8 f7 f7 fb 56 7b 11 14	67 a7 78 97 35 99 a6 d9 61 68 68 0f b1 21 82 fa	67 a7 78 97 99 a6 d9 35 68 0f 61 68 fa b1 21 82	ec 1a c0 80 0c 50 53 c7 3b d7 00 ef b7 22 72 e0	37 bb 38 f7 14 3d d8 7d 93 e7 08 a1 48 f7 a5 4a
db a1 f8 77 18 6d 8b ba a8 30 08 4e ff d5 d7 aa	b9 32 41 f5 ad 3c 3d f4 c2 04 30 2f 16 03 0e ac	b9 32 41 f5 3c 3d f4 ad 30 2f c2 04 ac 16 03 0e	b1 1a 44 17 3d 2f ec b6 0a 6b 2f 42 9f 68 f3 b1	48 f3 cb 3c 26 1b c3 be 45 a2 aa 0b 20 d7 72 38
f9 e9 8f 2b 1b 34 2f 08 4f c9 85 49 bf bf 81 89	99 1e 73 f1 af 18 15 30 84 dd 97 3b 08 08 0c a7	99 1e 73 f1 18 15 30 af 97 3b 84 dd a7 08 08 0c	31 30 3a c2 ac 71 8c c4 46 65 48 eb 6a 1c 31 62	fd 0e c5 f9 0d 16 d5 6b 42 e0 4a 41 cb 1c 6e 56
cc 3e ff 3b a1 67 59 af 04 85 02 aa a1 00 5f 34	4b b2 16 e2 32 85 cb 79 f2 97 77 ac 32 63 cf 18	4b b2 16 e2 85 cb 79 32 77 ac f2 97 18 32 63 cf	4b 86 8a 36 b1 cb 27 5a fb f2 f2 af cc 5a 5b cf	b4 ba 7f 86 8e 98 4d 26 f3 13 59 18 52 4e 20 76
ff 08 69 64 0b 53 34 14 84 bf ab 8f 4a 7c 43 b9				

Table 6.5

Avalanche Effect in AES: Change in Plaintext

(Table is located on page 178 in textbook)

Round		Number of Bits that Differ
	0123456789abcdeffedcba9876543210 0023456789abcdeffedcba9876543210	1
0	0e3634aece7225b6f26b174ed92b5588 0f3634aece7225b6f26b174ed92b5588	1
1	657470750fc7ff3fc0e8e8ca4dd02a9c c4a9ad090fc7ff3fc0e8e8ca4dd02a9c	20
2	5c7bb49a6b72349b05a2317ff46d1294 fe2ae569f7ee8bb8c1f5a2bb37ef53d5	58
3	7115262448dc747e5cdac7227da9bd9c ec093dfb7c45343d689017507d485e62	59
4	f867aee8b437a5210c24c1974cffeabc 43efdb697244df808e8d9364ee0ae6f5	61
5	721eb200ba06206dcbd4bce704fa654e 7b28a5d5ed643287e006c099bb375302	68
6	0ad9d85689f9f77bc1c5f71185e5fb14 3bc2d8b6798d8ac4fe36a1d891ac181a	64
7	db18a8ffa16d30d5f88b08d777ba4eaa 9fb8b5452023c70280e5c4bb9e555a4b	67
8	f91b4fbfe934c9bf8f2f85812b084989 20264e1126b219aef7feb3f9b2d6de40	65
9	cca104a13e678500ff59025f3bafaa34 b56a0341b2290ba7dfdfbddcd8578205	61
10	ff0b844a0853bf7c6934ab4364148fb9 612b89398d0600cde116227ce72433f0	58

Table 6.6

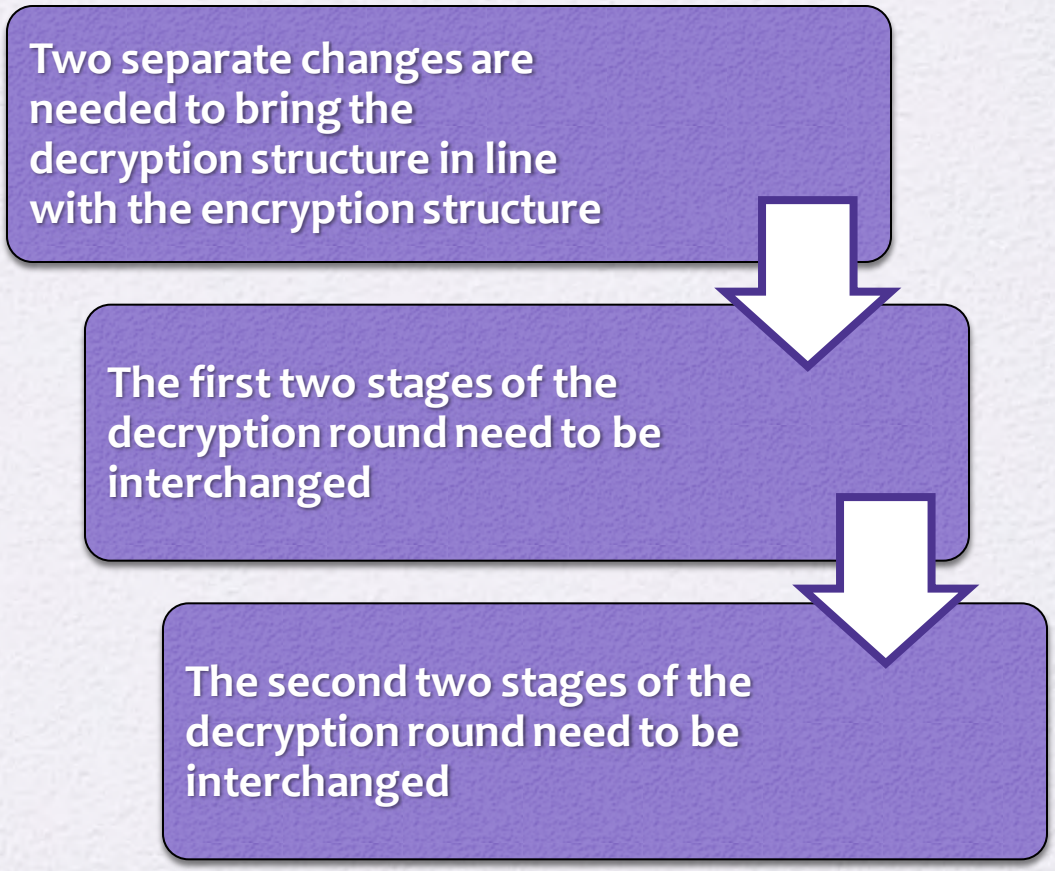
Avalanche Effect in AES: Change in Key

(Table is located on page 179 in textbook)

Round		Number of Bits that Differ
	0123456789abcdeffedcba9876543210 0123456789abcdeffedcba9876543210	0
0	0e3634aece7225b6f26b174ed92b5588 0f3634aece7225b6f26b174ed92b5588	1
1	657470750fc7ff3fc0e8e8ca4dd02a9c c5a9ad090ec7ff3fc1e8e8ca4cd02a9c	22
2	5c7bb49a6b72349b05a2317ff46d1294 90905fa9563356d15f3760f3b8259985	58
3	7115262448dc747e5cdac7227da9bd9c 18aeb7aa794b3b66629448d575c7cebf	67
4	f867aee8b437a5210c24c1974cffeabc f81015f993c978a876ae017cb49e7eec	63
5	721eb200ba06206dcbd4bce704fa654e 5955c91b4e769f3cb4a94768e98d5267	81
6	0ad9d85689f9f77bc1c5f71185e5fb14 dc60a24d137662181e45b8d3726b2920	70
7	db18a8ffa16d30d5f88b08d777ba4eaa fe8343b8f88bef66cab7e977d005a03c	74
8	f91b4fbfe934c9bf8f2f85812b084989 da7dad581d1725c5b72fa0f9d9d1366a	67
9	cca104a13e678500ff59025f3bafaa34 0ccb4c66bbfd912f4b511d72996345e0	59
10	ff0b844a0853bf7c6934ab4364148fb9 fc8923ee501a7d207ab670686839996b	53

Equivalent Inverse Cipher

- AES decryption cipher is not identical to the encryption cipher
  - The sequence of transformations differs although the form of the key schedules is the same
  - Has the disadvantage that two separate software or firmware modules are needed for applications that require both encryption and decryption



Interchanging InvShiftRows and InvSubBytes

- InvShiftRows *affects the sequence* of bytes in State but *does not alter byte contents* and *does not depend on byte contents* to perform its transformation
- InvSubBytes *affects the contents* of bytes in State but *does not alter byte sequence* and *does not depend on byte sequence* to perform its transformation

Thus, these two operations commute and can be interchanged

Interchanging AddRoundKey and InvMixColumns

The transformations AddRoundKey and InvMixColumns do not alter the sequence of bytes in State

If we view the key as a sequence of words, then both AddRoundKey and InvMixColumns operate on State one column at a time

These two operations are linear with respect to the column input

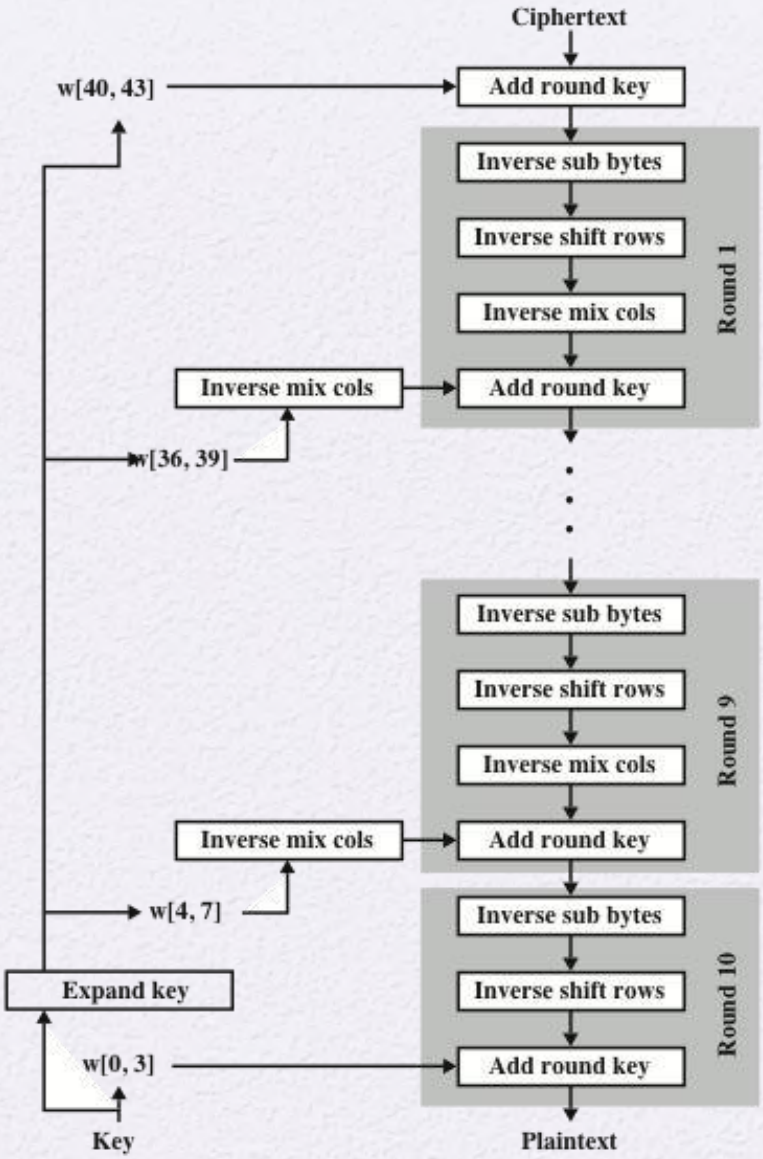


Figure 6.10 Equivalent Inverse Cipher

Implementation Aspects

- AES can be implemented very efficiently on an 8-bit processor
- AddRoundKey is a bytewise XOR operation
- ShiftRows is a simple byte-shifting operation
- SubBytes operates at the byte level and only requires a table of 256 bytes
- MixColumns requires matrix multiplication in the field GF(2^8), which means that all operations are carried out on bytes

Implementation Aspects

- Can efficiently implement on a 32-bit processor
  - Redefine steps to use 32-bit words
  - Can precompute 4 tables of 256-words
  - Then each column in each round can be computed using 4 table lookups + 4 XORs
  - At a cost of 4Kb to store tables
- Designers believe this very efficient implementation was a key factor in its selection as the AES cipher

Summary

- Finite field arithmetic
- AES structure
  - General structure
  - Detailed structure
- AES key expansion
  - Key expansion algorithm
  - Rationale

- AES transformation functions
  - Substitute bytes
  - ShiftRows
  - MixColumns
  - AddRoundKey
- AES implementation
  - Equivalent inverse cipher
  - Implementation aspects