Subject Lecture $\boxed{1}$ Date 2/7 الثلاثاء No.

partial Diff Eqs

* Introduction:

متغير تابع — متغير مستقل $y = y(x)$

Let $y = f(x)$, $x \in D \subseteq \mathbb{R}$ be a function of one Variable. It's graph is curve in the xy plane and its derivatives are denoted by

y', y'', y''', $y^{(4)}$, ... or $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, $\frac{d^3y}{dy^3}$, ...

Ex: $y = 2x + 1$ معادلات جبرية تُحل بالتجريب

$y = \sin x$

$y = x^2$

$y = e^x$

Ordinary diff Equation (ODE):

an ordianry diff-Equ is an Equ that involves derivatives of an unknown function of one Variable.

Ex: 1) $y' + xy = 3$

2) $y'' + y' = \sin x$

3) $\frac{dy}{dx} + y = x$

4) $y^{(4)} + y''' + y'' + y = 0$

$y = F(x)$, $x \in D \subseteq R$, $y = y(x)$ كامل Domain

الجزء من ال Domain اسمه interval (فترة)

عندما يكون $u = u(x, y)$ الجزء من Domain اسمه region (منطقة).

(ODE) تحتوي على مشتقة واحدة عالأقل لو حذفنا هذه المشتقة ستتحول من معادلة تفاضلية إلى معادلة جبرية

$2x + 1 = 7$ معادلة جبرية لها حل واحد — $x^2 + 4 = 0$ جبرية تحل complex

$x + y = 5$ لها عدد لانهائي من الحلول وتحل بالتجريب

y''' triple prim

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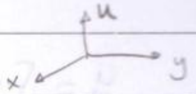
let $u = u(x,y)$, $(x,y) \in D \subseteq \mathbb{R} \times \mathbb{R}$ be a Function of two Variables. It's derivatives are partial derivatives and are denoted by



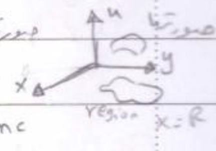
$$u_x = \frac{\partial u}{\partial x} \qquad u_y = \frac{\partial y}{\partial u}$$

$$u_{xx} = \frac{\partial^2 u}{\partial x^2} \qquad u_{yy} = \frac{\partial^2 u}{\partial y^2}$$

إذا زادت عدد متغيرين يُمثل Fun بالفراغ بـ سطح

صورتها غير مستوية

صورتها



$x = R$

تكون مستوية عندما Func بكون ثابت

$$u_{xy} = \frac{\partial^2 u}{\partial x \, \partial y} \qquad u_{yx} = \frac{\partial^2 u}{\partial y \cdot dx}$$

Its graph is a surface in the xyu-space

منطقة في الفراغ (الفراغ)

partial Diff Equ (PDE)

A partial diff Equation is an Equ that involves derivatives of an unknown function of serval Variables

مشتقة واحدة عالأقل

Ex

1) $u_x + 5u_y + u = x^2 + y$

2) $x\,u_{xx} + 3\,u_{xy} + u_{yy} = x^2 + u$

3) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = u$

4) $u_{xx} + u_{xy} = u$

Solve $u_x = 0$ تكامل

Sol $u(x,y) = F(y)$

تكامل الصفر اقتران بالنسبة لـ y

N O T E B O O K

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The general Form of a 1st order PDE:

$u = u(x,y)$

$F(x, y, u, u_x, u_y) = 0$

Ex:

1). $u_x + x u_y + u - xy + 5 = 0$

2). $u_x^2 + y^3 u_y - u^3 + 1 = 0$

3). $u_x + u_y = u + \sin x$

Linear 1st order PDE

$$a(x,y)\, u_x + b(x,y)\, u_y + c(x,y)\, u = d(x,y)$$

a, b, c : Function بدلالة x أو y

Ex 1). $x u_x + y^2 u_y + (\cos x) u = x^3$

$a = x$ $b = y^2$ $c = \cos x$ $d = x^3$ Linear

2). $\underset{a}{x^3}\, u_x + \underset{b}{\sqrt{y}}\, u_y + \underset{c=1}{u} = \underbrace{x + y + 4}_{d}$ Linear

مثلاً لو كانت u_y^2 ما بتكون Linear

مرتبطة بالزمن

Initial Value Problems: (IVPs) الشرط الابتدائي

$\boxed{1}$ In ODEs

The IVP is (ODE + IC)

$F(x) = y = y(x)$ التي نبني عليها المعادلة

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Ex: ① $y' + y = x - 1$; $y(0) = 1$ [معناها $y = 1$ at $x = 0$ $(0,1)$

1^{st} order [one condition]

② $y'' + x^2 y - 5 = 0$ $y(1) = 3$ ما بكفي شرط واحد لأنه

2^{nd} oredr [two condition] $y(2) = 5$ 2^n order

[2] In PDEs :

The IVP is (PDE + Ic)

Ex $u = u(x,y)$ التي تنبني عليها المعادلة

1). $u_x + 4u_y = u$; $u(1,0) = 3$

2). $u u_x + u_y^2 = 1$; $u(x,0) = x$ (x-axises $y=0$)

3). $u_x^2 + u_y^2 = u^2$; $u(0,y) = y^2$ (y-axis $x=0$)

not Linear

Ex: Solve the IVP:

$$\frac{dy}{dx} - 2x = 0 \quad ; \quad y(0) = 1$$

Sol: $\frac{dy}{dx} = 2x$ (seprable)

$dy = 2x\,dx \xRightarrow{\text{بالتكامل}} y = x^2 + c$ (g.s)

I.c. $y(0) = 1 \Rightarrow 1 = 0 + c \Rightarrow c = 1$

$y(x) = \boxed{y = x^2 + 1}$ particular solution at the point $(0,1)$

الحل الذي يقابل I.c

N O T E B O O K

$y = x^2 + 1$

The sol that passes through the point $(0,1)$

الرسم

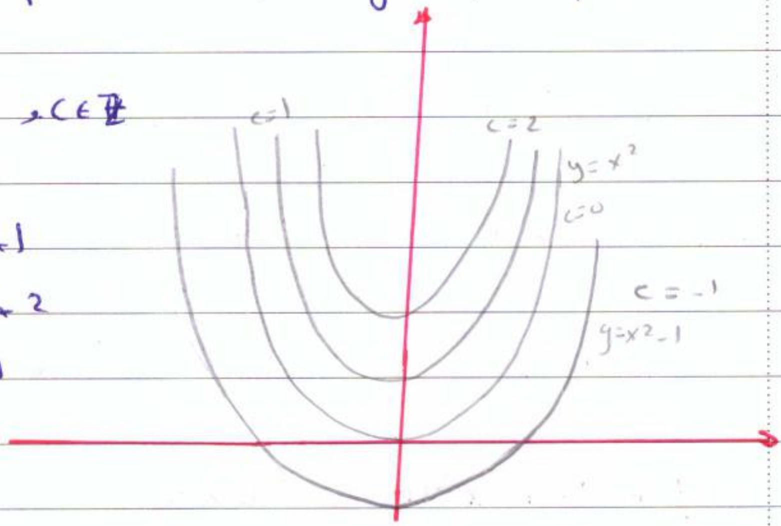
مثل الدوال $y = x^2 + c \quad , c \in \mathbb{R}$

$c = 0 \qquad y = x^2$

$c = 1 \qquad y = x^2 + 1$

$c = 2 \qquad y = x^2 + 2$

$c = -1 \qquad y = x^2 - 1$



1st order PDE "التصنيف"

classification of 1st order PDEs:

لينير $= xyu$ خطية متجانسة

$\boxed{1}$ $a(x,y)\,U_x + b(x,y)\,U_y = c(x,y)\,u$ "Linear homo"

مثل $u^2, \sqrt{u}, e^u$ نصف خطية

$\boxed{2}$ $a(x,y)\,U_x + b(x,y)\,U_y = c(x,y,u)$ "semi Linear"

شبه خطية

$\boxed{3}$ $a(x,y,u)\,U_x + b(x,y,u)\,U_y = c(x,y,u)$ "quasiLinear"

$\boxed{4}$ Eqs. involving powers of U_x or U_y other than 1 or multiplied terms as $U_x \cdot U_y$. "Non Linear" fully

قوى $(U)^2$ او مضروبة U_x بـ U_y

Ex: classify "سؤال امتحان فيرست"

1. $u_x + y u_y = x.u$ "Linear"

2. $2u_x + x u_y = \sqrt{xyu}$ "semi Linear"

3. $u.u_x + u_y = u$ "quasi Linear"

4. $u_x^3 + u_y - u^2 = 0$ "Non Linear"

5. $5u_x + u_y = x^2 u$ "Linear"

6. $u\, u_x u_y = 1$ "Non Linear"

7. $u^3 u_x + 7u_y = u^2$ "quasi Linear" لو شطبنا u^3 بتصير semi
لو شلنا u^2 حطينا x بتصير non

8. $u_x + u_y = e^u$ "semi Linear"

9. $u_x^2 + u_y^2 = u^2$ "Non Linear"

10. $(\sin x)\, u_x + u_y = 1$ "Linear"

The characteristic method

للمعادلات درجة أولى

Linear 1st order PDE

consider the IVP

PDE $a(x,y)\,u_x + b(x,y)\,u_y = c(x,y)\,u$

I.c $u(x,0) = f(x)$ x_axis

we will solve this problem using the char. method as follows:

- The coordinates x, y will be changed to new coordinates s, τ such that: (x,y) تتحول (s,τ)

  τ: will change along (على طول) the inital curve from $-\infty$ to ∞.

  s: will change along curves called the char. curves from 0 to ∞.

- In changing the coordinates our PDE becomes an ODE in s, τ [الجزئية تتحول إلى عادية $PDE \xrightarrow{\text{تتحول بدلالة } \tau, s} ODE$]

- we solve this ODE and go back to our solution $u(x,y)$

  التحويل العكسي

$u(x,y)$

$x = x(s,\tau) = x(s)$

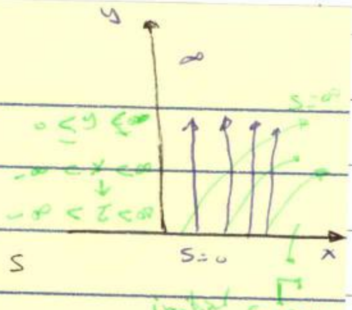
$y = y(s,\tau) = y(s)$

τ is treated as constant

(τ ثابت)

$0 < s < \infty$

$-\infty < \tau < \infty$



y يتغير على شكل أعمدة متوازية

s تتغير على شكل منحنيات

المنحنيات الناتجة اسمها char. curve

[النصف العلوي من المستوى (الربع ١ + ٢)]

τ: initial curve رسم أفقي

s: char curve رسم عمودية

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Func متغير

u تتبع متغير واحد $\{s\}$ u بالدالة $\{u(s)\}$

$$\frac{du}{ds} = U_x \frac{dx}{ds} + U_y \frac{dy}{ds}$$

let $\dfrac{dx}{ds} = a(x,y)$ $\qquad \dfrac{dy}{ds} = b(x,y)$

then the PDE becames

$$\frac{dx}{ds} U_x + \frac{dy}{ds} U_y = C(x,y)\, u$$

or $\dfrac{du}{ds} = c(x,y)u$ an ODE

Ex : Solve the following problem using the char. method

$$x U_x + y U_y = 0 \qquad u(x,1) = \cos x$$

Remark :

ODE x(s) لها y(s) لها ODE

$\boxed{1}$ $\dfrac{dx}{ds} = a(x,y)$ $\qquad \dfrac{dy}{ds} = b(x,y)$ are called the char. Eqs and their solution on is a family char. curves.

يمثل بـ

$\boxed{2}$. The initial curve Γ is parametrized by

$$u(x,0) = F(x)$$

$$\begin{cases} x(0) = \tau \\ y(0) = 0 \end{cases}$$

and. $u(0) = F(\tau)$.

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Ex: Solve the following problem the char. method
$$xU_x + yU_y = 0 \qquad u(x,1) = \cos x$$

Sol The initial curve Γ is parametrized by

$x(\overset{s}{0}) = \tau$

$y(0) = 1$

and $u(0) = \cos \tau$

The char. equ are

$$\frac{dx}{ds} = a \quad , \frac{dy}{ds} = b \quad , \frac{dx}{ds} = x \quad , \frac{dy}{ds} = y$$

$$\frac{dx}{x} = ds \Rightarrow \ln x = s + c_1 \Rightarrow e^{\ln x} = e^{s+c_2}$$

$$x(s) = e^{s} \cdot e^{c_2} \Rightarrow x(s) = c_1 e^{s}$$

I.c $x(0) = \tau$

$$x(0) = c_1 e^{0} \Rightarrow \tau = c_1 \quad \therefore \boxed{x = \tau e^{s}}$$

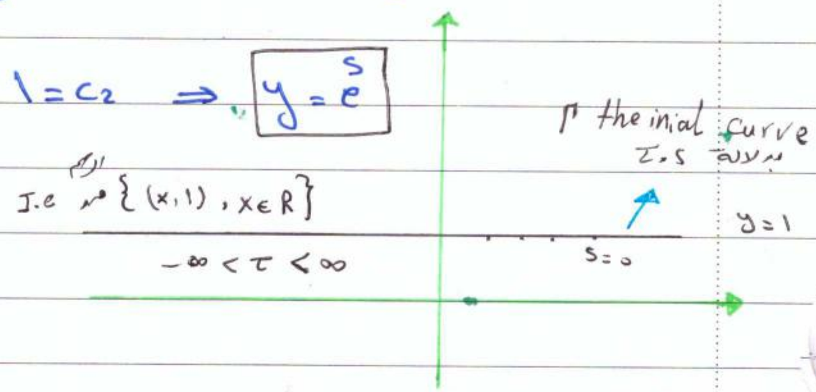
$$\frac{dy}{ds} = y \Rightarrow \frac{dy}{y} = ds \Rightarrow \ln y = s + c_2$$

$$e^{\ln y} = e^{s+c_2} \Rightarrow y(s) = e^{s} \cdot e^{c_2} \Rightarrow y(s) = c_2 \cdot e^{s}$$

I.c $y(0) = 1$

$$y(0) = c_2 e^{0} \Rightarrow 1 = c_2 \Rightarrow \boxed{y = e^{s}}$$

$$x \cdot U_x + y U_y = 0$$



$$\frac{du}{ds} = 0$$

$u(S) = C_3$

I.c $u(0) = \cos \tau$

$u(0) = C_3$

$\cos \tau = C_3$

$u(S) = \cos \tau$

sol بدلالة S.T

$$\boxed{u(S,\tau) = \cos \tau}$$

Going back to x,y:

$x = \tau \cdot e^{s}$

$y = e^{s} \Rightarrow \ln y = S$

$x = \tau \cdot y \Rightarrow \dfrac{x}{y} = \tau$

$$\boxed{u(x,y) = \cos \frac{x}{y}}$$ The solution of our problem

"الرسم"

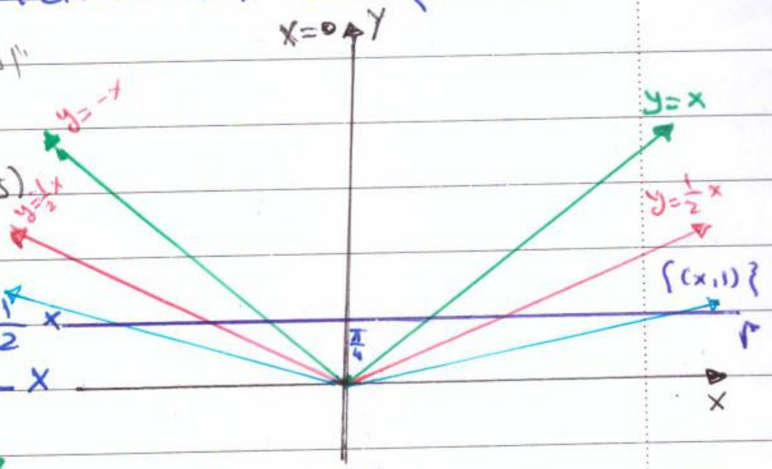
$x = \tau y$

$\tau = 0 \Rightarrow x = 0$ (y axsis)

$\tau = 1 \Rightarrow x = y \Rightarrow y = x$

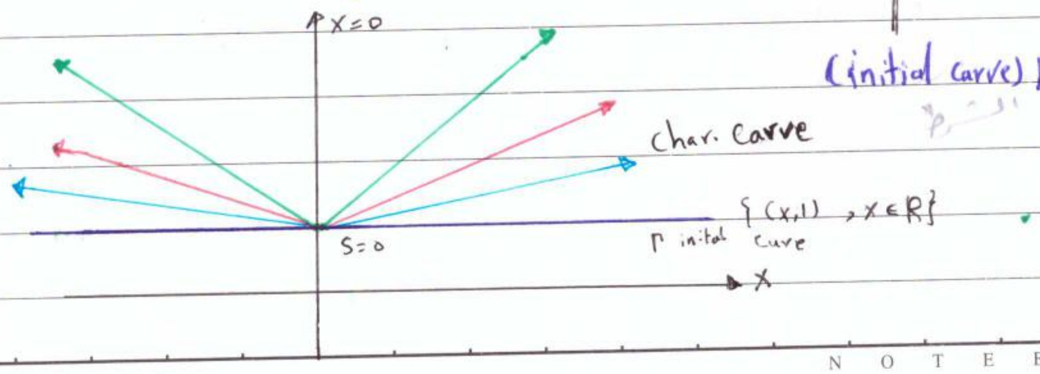
$\tau = 2 \Rightarrow x = 2y \Rightarrow y = \frac{1}{2}x$

$\tau = -1 \Rightarrow x = -y \Rightarrow y = -x$



المنحنيات لازم تبدأ من Γ (initial curve)

الرسم



Ex $u(x,2) = x^2$

$x(0) = \tau$

$y(0) = 2$

$u(0) = \tau^2$

$u(x,x^2) = x^3$

$x(0) = \tau$

$y(0) = \tau^2$

$u(0) = \tau^3$

$u(0,y) = y^2$

$x(0) = 0$

$y(0) = \tau$

$u(0) = \tau^2$

$y = mx + b$ (ميل m)

$y = x$ ميل 1

$y = \frac{1}{2}x$ ميل $\frac{1}{2}$

يعني الخط المستقيم كلها

الميل على x-axis (صفر)

الميل على y-axis (∞)

$y = x$ ينقسم الربع الأول بالنص

Ex: solve the following problem using the characteristic method then draw the solution PDE

$U_x + U_t + 2U = 0$

I.c $\quad u(x,0) = \sin x$

Sol: The Initial curve $\Gamma : \begin{cases} x(0) = \tau \\ t(0) = 0 \end{cases}$

and $u(0) = \sin \tau$

The char equ are

$\frac{dx}{ds} = 1$ مشتقة u_x , $\frac{dt}{ds} = 1$ مشتقة u_t

$dx = ds \Rightarrow x(s) = s + c_1 \xRightarrow{I.c} x(0) = 0 + c_1 \Rightarrow \tau = c_1$

$\Rightarrow \underline{x = s + \tau}$

$dt = ds \Rightarrow t(s) = s + c_2 \xRightarrow{I.c} t(0) = 0 + c_2 \Rightarrow 0 = c_2$

$\Rightarrow \underline{t = s}$

$u_x + u_t + 2u = 0 \Rightarrow \frac{du}{ds} + 2u = 0$

sparation

$\frac{du}{ds} = -2u \Rightarrow \frac{du}{u} = -2ds \Rightarrow \ln u = -2s + c_3$

$\Rightarrow e^{\ln u} = e^{-2s + c_3} \Rightarrow u(s) = c_3 e^{-2s}$

$\xRightarrow{I.c} u(0) = c_3 e^{0} \Rightarrow \sin \tau = c_3$

$u(s) = \sin \tau \, e^{-2s}$

$\Rightarrow \boxed{u(s, \tau) = \sin \tau \, e^{-2s}}$

$u(x,t) = ?$

$x = s + \tau \quad , t = s \Rightarrow x = t + \tau \Rightarrow$

$\underline{\tau = x - t}$

$\therefore$ The solution of the problem is

$u(x,t) = \sin(x-t)\, e^{-2t}$

$u(x,0) = \sin(x-0)\, e^{0} = \sin x$ I.c يعني الحل صحيح

الرسم

$u(x,t) = \sin(x-t)\, e^{-2t}$

$t=0 \Rightarrow u(x,0) = \sin x$

$t=1 \Rightarrow u(x,1) = \sin(x-1)\, e^{-2}$

$t=2 \Rightarrow u(x,2) = \sin(x-2)\, e^{-4}$ initial curve

إزاحة لليمين ← بيصغر

$\vdots$

$t=\infty \Rightarrow u(x,\infty) = \sin(x-\infty)\, e^{-\infty} = 0$

محدود ما بين ١ و -١ $\qquad \frac{1}{\infty} = 0$

char. curves

$x - t = \tau$

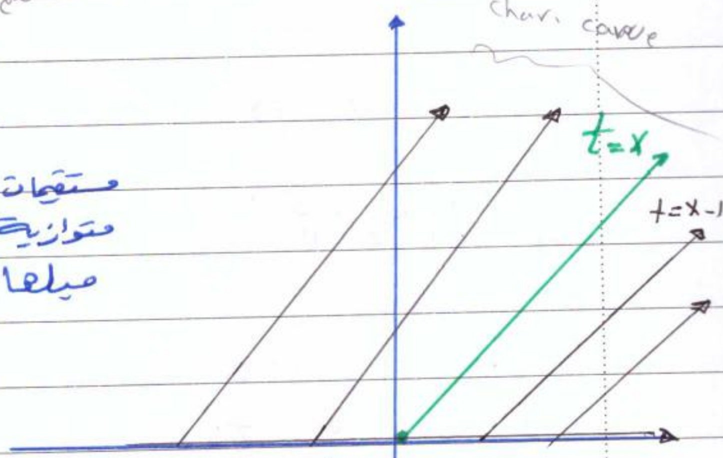
$\tau = 0 \;, x - t = 0 \Rightarrow x = t$

$\tau = 1 \;, x - t = 1 \Rightarrow t = x - 1$

$\tau = 2 \;, x - t = 2 \Rightarrow t = x - 2$

$\tau = -1 \;, x - t = -1 \Rightarrow t = x + 1$

مستقيمات متوازية ميلها 1



Γ: the initial curve (x-axis) $\{(x,0) \; x \in \mathbb{R}\}$.

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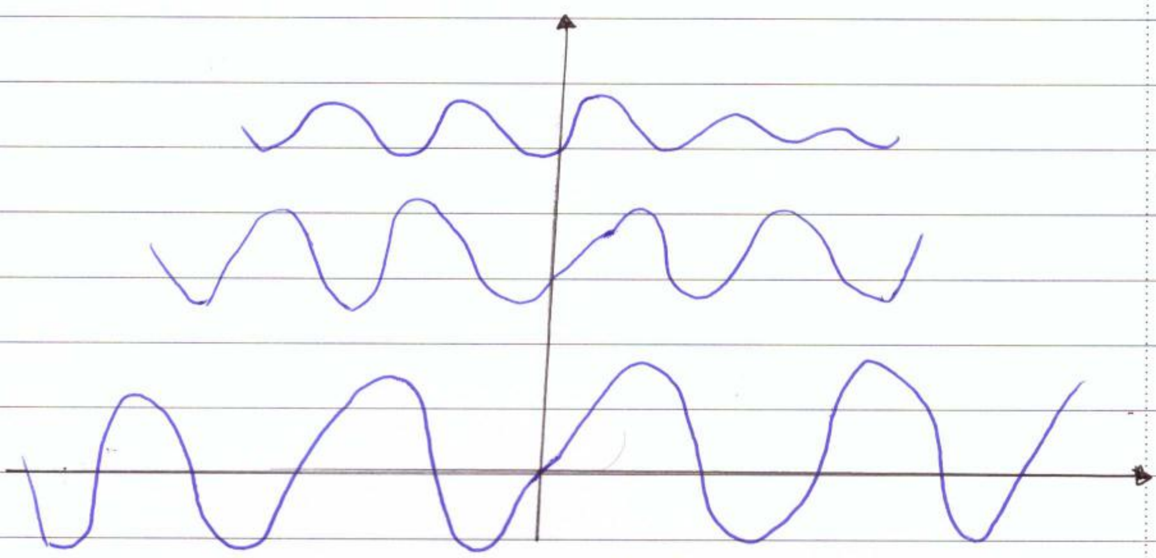

(initial wave)

The solution is wave that is shifted to the right and damping to zero.

تقترب من الصفر

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Semi Linear Eqs :

SemiLinear eqs can be solved by the char. method as Linear eqs.

Ex solve $U_x + 2U_y = u^2$, $u(x,0) = \ln x$

Sol

$$\Gamma \begin{cases} x(0) = \tau \\ y(0) = 0 \end{cases}$$

and $u(0) = \ln \tau$

char. equ

$$\frac{dx}{ds} = 1 \quad , \quad \frac{dy}{ds} = 2$$

$$dx = ds \Rightarrow x = s + c_1 \overset{I.c}{\Rightarrow} x(0) = 0 + c_1 \Rightarrow \tau = c_1$$

$$\Rightarrow \underline{x = s + \tau}$$

$$\frac{dy}{ds} = 2 \Rightarrow dy = 2ds \Rightarrow y = 2s + c_2 \overset{I.c}{\Rightarrow} y(0) = 0 + c_2$$

$$\Rightarrow c_2 = 0$$

$$\Rightarrow \underline{y = 2s}$$

$$u_x + 2u_y = u^2$$

$$\Rightarrow \frac{du}{ds} = u^2 \Rightarrow du = u^2 ds \Rightarrow u^{-2} du = ds$$

$$\frac{u^{-1}}{-1} = s + c_3 \Rightarrow \frac{-1}{u} = s + c_3 \Rightarrow \frac{1}{-u} = s + c_3 \Rightarrow \frac{u}{-1} = \frac{1}{s + c_3}$$

$$u(s) = \frac{-1}{s + c_3} \overset{I.c}{\Rightarrow} u(0) = \frac{-1}{0 + c_3}$$

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$$\ln \tau = \frac{-1}{c_3}$$

$$c_3 = \frac{-1}{\ln \tau}$$

$$u(s,\tau) = \frac{-1}{s - \frac{1}{\ln \tau}} = \frac{-1}{\frac{s \ln \tau - 1}{\ln \tau}}$$

$$u(s,\tau) = \frac{-\ln \tau}{s \ln \tau - 1}$$

$u(x,y) = ?$

$x = s + \tau \quad , y = 2s \Rightarrow \boxed{s = \frac{y}{2}}$

$x = \frac{y}{2} + \tau \Rightarrow \boxed{\tau = x - \frac{y}{2}}$

The solution is

$$u(x,y) = \frac{-\ln\left(x - \frac{y}{2}\right)}{\frac{y}{2}\ln\left(x - \frac{y}{2}\right) - 1} \qquad \#$$

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Quasi Linear Eqs.:

Consider the IVP:

$$a(x,y,u)\,u_x + b(x,y,u)\,u_y = c(x,y,u)$$

$$u(x,0) = F(x) \qquad x\text{-axis}$$

The char. eqs are

$$\frac{dx}{ds} = a, \quad \frac{dy}{ds} = b, \quad \frac{du}{ds} = c$$

their solution are

$$x = x(s,\tau), \quad y = y(s,\tau), \quad u = u(s,\tau)$$

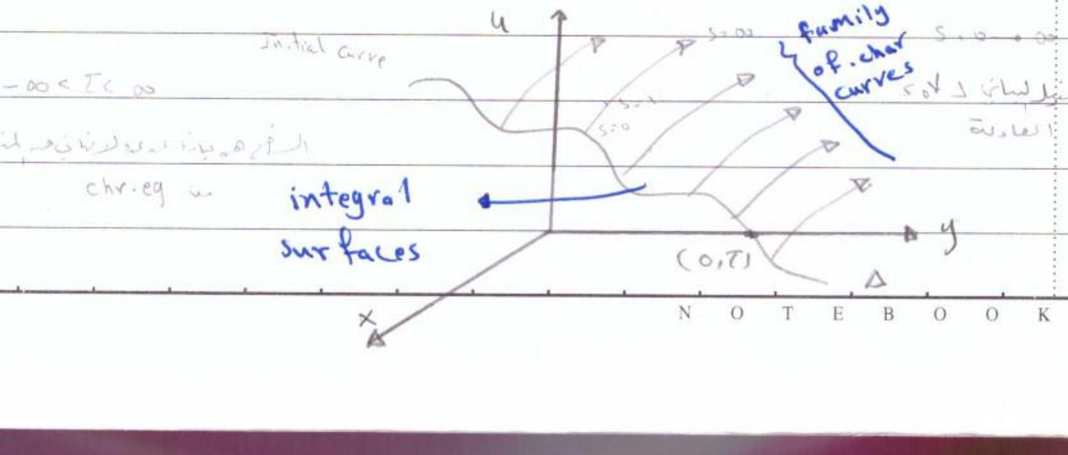
and the char. curves are in the xyu-Space

The initial curve is $\quad -\infty < \tau < \infty \quad s = 0$

$$\Gamma : \begin{cases} x(0) = x(0,\tau) = \phi(\tau) \quad \text{Func} \\ y(0) = y(0,\tau) = \psi(\tau) \\ u(0) = u(0,\tau) = h(\tau) \end{cases}$$

Which is a curve in the xyu-Space.

The char. curves form an integral surface Which is the graph of the Solution $u(x,y)$



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Ex: Solve

$$u\,u_x + u_y = 0 \qquad [\text{quasi Linear}]$$

$$u(x,0) = x^2$$

Sol

$$\Gamma: \begin{cases} x(0) = \tau \\ y(0) = 0 \\ u(0) = \tau^2 \end{cases}$$

char. eqs:

$$\frac{dx}{ds} = u \quad , \quad \frac{dy}{ds} = 1 \quad , \quad \frac{du}{ds} = 0$$

مبنلش منها لأنه فيها متغير

$\boxed{1}$ $\frac{dy}{ds} = 1 \Rightarrow dy = ds \Rightarrow y(s) = s + c_1$

$\overset{I.c}{\Longrightarrow} y(0) = 0 + c_1 \Rightarrow 0 = c_1 \Rightarrow \boxed{y = s}$

$\boxed{2}$ $\frac{du}{ds} = 0 \Rightarrow du = 0 \Rightarrow u(s) = c_2 \Rightarrow u(0) = c_2$

$\tau^2 = c_2$

$\boxed{u(s,\tau) = \tau^2}$

$\boxed{3}$ $\frac{dx}{ds} = u$

$\frac{dx}{ds} = \tau^2 \Rightarrow dx = \tau^2 ds \Rightarrow x(s) = \tau^2 s + c_3$

$x(0) = 0 + c_3 \Rightarrow \tau = c_3 \Rightarrow \boxed{x = \tau^2 s + \tau}$

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$u(x,y)$

$x = \tau^2 s + \tau \quad , y = s$

$x = \tau^2 y + \tau$

$y\tau^2 + \tau - x = 0$ using the quadratic formula

$$\tau = \frac{-b \mp \sqrt{b^2-4ac}}{2a} \quad ; a=y \quad b=1 \quad c=-x$$

$$\tau = \frac{-1 \mp \sqrt{1+4xy}}{2y}$$

The solution is:

$$u(x,y) = \left(\frac{-1 \mp \sqrt{1+4xy}}{2y}\right)^2$$

N O T E B O O K

Ex: solve [Quasi Linear]

$$u u_x + y u_y = x \qquad u(x,1) = 2x$$

Sol

The inital curve is $\Gamma \begin{cases} x(0) = \tau \\ y(0) = 1 \\ u(0) = 2\tau \end{cases}$

char. equ are

$$\frac{dx}{ds} = u \quad , \quad \frac{dy}{ds} = y \quad , \quad \frac{du}{ds} = x$$

$$\frac{dy}{ds} = y \Rightarrow dy = y\,ds \Rightarrow \frac{dy}{y} = ds \Rightarrow \ln y = s + c_1$$

$$y(s) = c_1 e^{s} \Rightarrow y(0) = c_1 e^{0} \Rightarrow 1 = c_1$$

$$\Rightarrow \boxed{y = e^{s}}$$

$$\frac{dx}{ds} = u \Rightarrow dx = u\,ds \Rightarrow \dots (1)$$

$$\frac{du}{ds} = x \Rightarrow du = x\,ds \dots (2)$$

Adding equ 1+2

$$dx + du = u\,ds + x\,ds = (x+u)\,ds$$

$$d(x+u) = (x+u)\,ds$$

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$$\frac{d(x+u)}{x+u} = ds$$

$\ln(x+u) = s + c_2$

$x + u = c_2 e^{s}$ ....(3) معادلة بمجهولين

ولدينا معادلة واحدة لذلك يلزمنا معادلة أخرى لذلك نطرح

subtracting 1 _ 2

$dx - du = u\,ds - x\,ds$

$d(x-u) = -(x-u)\,ds$

$$\frac{d(x-u)}{x-u} = \frac{-(x-u)}{x-u}\,ds = -ds$$

$\ln(x-u) = -s + c_3$

$x - u = c_3 e^{-s}$ ---- (4)

3،4 معادلتين بمجهولين x, u ⇐ نحل المعادلتين

solving equ (3) , (4)

$$\begin{array}{r} x + u = c_2 e^{s} \\ + \quad x - u = c_3 e^{-s} \\ \hline \end{array}$$

$$\begin{cases} x = \frac{c_2}{2} e^{s} + \frac{c_3}{2} e^{-s} \\ u = \frac{c_2}{2} e^{s} - \frac{c_3}{2} e^{-s} \end{cases}$$

$2x = c_2 e^{s} + c_3 e^{-s} \Leftrightarrow \left(2x(s) = c_2 e^{s} + c_3 e^{-s}\right)$

I.C: $x(0) = \tau \Rightarrow 2x(0) = c_2 e^{0} + c_3 e^{0} = c_2 + c_3$

$2\tau = c_2 + c_3$ ---- (5)

sub * in equ (3)

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$$x + u = c_2 e^{s}$$

$$\frac{c_2}{2} e^{s} + \frac{c_3}{2} e^{-s} + u = c_2 e^{s}$$

$$u(s) = \frac{c_2}{2} e^{s} - \frac{c_3}{2} e^{-s}$$

I.C $\quad u(0) = 2\tau$

$$u(0) = \frac{c_2}{2} e^{0} - \frac{c_3}{2} e^{0}$$

$$2\tau = \frac{c_2}{2} - \frac{c_3}{2}$$

$$4\tau = c_2 - c_3 \quad \ldots(6)$$

Solving equ 5, 6 for c_2, c_3

$$c_2 + c_3 = 2\tau \quad \ldots 5$$

$$c_2 - c_3 = 4\tau \quad \ldots 6$$

$$2c_2 = 6\tau$$

$$c_2 = 3\tau$$

Sub in equ (5) to find c_3

$$3\tau + c_2 = 2\tau$$

$$c_3 = -\tau$$

$$x - u = c_3 e^{-s}$$

$$x + u = c_2 e^{s}$$

$$\begin{cases} x(0) - u(0) = c_3 e^{-s} \\ x(0) + u(0) = c_2 e^{s} \end{cases}$$

$$\Rightarrow \tau - 2\tau = c_3 e^{0} \Rightarrow -\tau = c_3$$

$$\tau + 2\tau = c_2 e^{0} \Rightarrow 3\tau = c_2$$

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$\therefore \; \cancel{x} = \frac{3}{2}\tau e^{s} - \frac{\tau}{2} e^{-s}$

$u = \frac{3}{2}\tau e^{s} + \frac{\tau}{2} e^{-s}$

$u(s,\tau) = \frac{3}{2}\tau e^{s} + \frac{\tau}{2} e^{-s}$

$u(x,y)$:

$y = e^{s}, \quad x = \frac{3\tau}{2} e^{s} - \frac{\tau}{2} e^{-s}$

$x = \frac{3\tau}{2} y - \frac{\tau}{2} y^{-1}$

$x = \frac{3\tau}{2} y - \frac{\tau}{2}\frac{1}{y} = \frac{\tau}{2}\left[3y - \frac{1}{y}\right] = \frac{\tau}{2}\left[\frac{3y^2-1}{y}\right]$

$x = \frac{\tau}{2y}(3y^2-1) \Longrightarrow 2x\left(\frac{y}{3y^2-1}\right) = \tau$

$\tau = \frac{2xy}{3y^2-1}$

$\therefore$

$u(s,\tau) = \frac{\tau}{2}\left[3e^{s} + e^{-s}\right]$

$u(x,y) = \frac{1}{2}\cdot\frac{2xy}{3y^2-1}\left[3y + \frac{1}{y}\right] = \frac{xy}{3y^2-1}\left[\frac{3y^2+1}{y}\right]$

$$u(x,y) = \frac{x(3y^2+1)}{3y^2-1} \quad \#$$

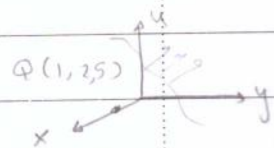
طريقة أخرى بلاقي c_2, c_3

$$x + u = c_2 e^{s}$$
$$x - u = c_3 e^{-s}$$

$$x(0) + u(0) = c_2 e^{s}$$
$$\tau + 2\tau = c_2 e^{s}$$
$$c_3 = 3\tau$$
$$\tau - 2\tau = c_3 e^{0} \Rightarrow c_3 = -\tau$$

General solution بدون I.C

we solve a quasilinear equ. to find a general solution using Lagrange method



$$a(x,y,u)\,u_x + b(x,y,u)\,u_y = c(x,y,u)$$

in the method we find two Function $\phi(x,y,u)$, $\psi(x,y,u)$ that are Linear along the characteristices from the nonparametrized characteristic equ

$\frac{dx}{ds} = a \Rightarrow \frac{dx}{a} = ds$

$\frac{dy}{ds} = b \Rightarrow \frac{dy}{b} = ds$

$$\frac{dx}{a} = \frac{dy}{b} = \frac{du}{c} \quad \text{and the solution is}$$

$$\phi = F(\psi)$$

بدون s nonparametrized equ

Ex: Find a general solution using Lagrange method

$u u_x + u_y = 0$

Sol

$$\frac{dx}{a} = \frac{dy}{b} = \frac{du}{c}$$

$$\frac{dx}{u} = \frac{dy}{1} = \frac{du}{0}$$

Taking the first and third terms

$$\frac{dx}{u} = \frac{du}{0} \Rightarrow \frac{u\,du}{u} = \frac{0 \cdot dx}{u}$$ قسمنا على u

$$u\,du = 0 \Rightarrow u = c_1$$

let $\phi(x, y, u) = u$

Taking the 1st and 2nd terms:

$$\frac{dx}{u} = \frac{dy}{1} \Rightarrow dx = c_1\,dy$$

$$c_1 y = x + c_2$$

$$uy = x + c_2 \Rightarrow uy - x = c_2$$

let $\psi(x, y, u) = uy - x$

G.S:

$\Phi = F(\psi)$

$u = F(\cancel{x}y - x)$ حل ضمني

نفسها (implicite) صريح ما بيكون

مثلاً $u = x^2 + y^3 + 1$

Note

general solution بيكون بدون condition

لكن حكماً تكون I.c وبتحدد particular solution لتوحيد F (مجهول)

Ex : Find a general solution using Lagrange method for

$u_x + u^2 u_y = 1$ quasi Linear

$$\frac{dx}{a} = \frac{dy}{b} = \frac{du}{c}$$

$$\frac{dx}{1} = \frac{dy}{u^2} = \frac{du}{1}$$

$$\frac{dx}{1} = \frac{du}{1} \Rightarrow dx = du \Rightarrow x = u + c_1$$

$$x - u = c_1$$

let $\phi(x, y, u) = x - u$

$$\frac{dy}{u^2} = \frac{du}{1} \Rightarrow u^2 du = dy$$

$$\frac{u^3}{3} = y + c_2 \Rightarrow \frac{u^3}{3} - y = c_2$$

let $\psi(x, y, u) = \frac{u^3}{3} - y$

The General solution

$\phi = F(\psi)$

حل ضمني

$$x - u = F\left(\frac{u^3}{3} - y\right)$$

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Ex : فكرة سؤال امتحان بالفيرست

Consider the PDE (quasi Linear)

$$u\,u_x + y\,u_y = x$$

(a) Find a general solution using Lagrange method

(b) Find a particular Solution given the I.C

$$u(x,1) = 1$$

Sol :

$$\frac{dx}{u} = \frac{dy}{y} = \frac{du}{x}$$

$$\frac{dx}{u} = \frac{du}{x} \Rightarrow x\,dx = u\,du$$

$$\frac{u^2}{2} = \frac{x^2}{2} + c_1$$

$$u^2 = x^2 + C_1 \Rightarrow C_1 = u^2 - x^2$$

let $\Phi(x,y,u) = u^2 - x^2$

$$\frac{dx}{u} = \frac{dy}{y}$$

$\boxed{\text{Lagrange + I.C = particualr}}$

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$$u^2 - x^2 = c_1$$

$$u^2 = c_1 + x^2$$

$$u = \sqrt{c_1 + x^2}$$

$$\frac{dx}{\sqrt{c_1 + x^2}} = \frac{dy}{y}$$

$$\int \frac{1}{\sqrt{c_1 + x^2}}\, dx = \frac{1}{y}\, dy$$

$$\int \frac{1}{\sqrt{c_1 + x^2}} \cdot \frac{x + \sqrt{c_1 + x^2}}{x + \sqrt{c_1 + x^2}}\, dx = \frac{dy}{y}$$

$$\int \frac{\frac{x}{\sqrt{c_1 + x^2}} + 1}{x + \sqrt{c_1 + x^2}}\, dx = \frac{dy}{y}$$

من البسط نطلع $\sqrt{c_1 + x^2}$

عامل مشترك

$$\frac{\cancel{\sqrt{c_1 + x^2}}\left[\frac{x}{\sqrt{c_1 + x^2}} + 1\right]}{\cancel{\sqrt{c_1 + x^2}} \cdot \left(x + \sqrt{c_1 + x^2}\right)}$$

$$\ln\left(x + \sqrt{c_1 + x^2}\right) = \ln y + \ln c_2$$

$$\ln(x + u) = \ln c_2 y$$

$$x + u = c_2 y \Rightarrow \frac{x + u}{y} = c_2$$

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let $\psi(x,y,u) = \dfrac{x+u}{y}$

G.S $\quad \phi = F(\psi)$

$$u^2 - x^2 = f\left(\frac{x+u}{y}\right)$$

I.C : $u(x,1) = 1$

$$1 - x^2 = F\left(\frac{x+1}{1}\right)$$ دائماً ما داخل F نفرضه Z

$$f(x+1) = 1 - x^2$$

نفرض $Z = x+1 \Rightarrow x = Z-1$

$$f(Z) = 1 - (Z-1)^2$$

$$f\left(\frac{x+u}{y}\right) = 1 - \left(\left(\frac{x+u}{y}\right) - 1\right)^2$$

$$f\left(\frac{x+u}{y}\right) = 1 - \left(\frac{x+u-y}{y}\right)^2$$

$\therefore u^2 - x^2 = 1 - \left(\dfrac{x+u-y}{y}\right)^2$ " particular sol "

حل ضمني

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Ex: (Linear)

Find a general solution for $u_x + u_y = u$

then find a particular solution using the

I.C $u(x,0) = 1$

Sol:

$$\frac{dx}{1} = \frac{dy}{1} = \frac{du}{u}$$

$$\frac{dx}{1} = \frac{dy}{1} \Rightarrow x = y + c_1$$

$$x - y = c_1$$

let $\phi(x,y,u) = x - y$

$$\frac{dx}{1} = \frac{du}{u} \Rightarrow x = \ln u + \ln c_2$$

$$x = \ln c_2 u \Rightarrow e^x = c_2 u \Rightarrow \frac{e^x}{u} = c_2$$

$$\psi(x,y,u) = \frac{e^x}{u}$$

G.S: $\phi = F(\psi)$

$$x - y = f\left(\frac{e^x}{u}\right)$$

NOTEBOOK

The partcular solution

I.C : $u(x,0) = 1$

$$x - 0 = f\left(\frac{e^x}{1}\right)$$

$$f(e^x) = x$$

$$Z = e^x \Rightarrow x = \ln Z$$

$f(z) = \ln Z$ Z هي فرض ما داخل F قبل التعويض ب I.C

$$\therefore f\left(\frac{e^x}{u}\right) = \ln \frac{e^x}{u} = \ln e^x - \ln u$$
$$= x - \ln u$$

$$\therefore x - y = x - \ln u .$$

$$\ln u = y \Rightarrow u = e^y$$

Ex: find a general solution for

$x U_x + U_y = u$ then find a partcuilar

Solution using $u(1,y) = y$

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Sol

$$\frac{dx}{x} = \frac{dy}{1} = \frac{du}{u}$$

$$\frac{dx}{x} = dy \Rightarrow \ln x = y + c_1 \Rightarrow c_1 = \ln x - y$$

$$\Rightarrow \phi(x, y, u) = \ln x - y$$

$$\frac{dx}{x} = \frac{du}{u} \Rightarrow \ln x = \ln u + \ln c_2$$

$$x = u c_2 \Rightarrow c_2 = \frac{x}{u}$$

$$\psi(x, y, u) = \frac{x}{u}$$

The General Sol $\phi = f(\psi)$

$$\ln x - y = f\left(\frac{x}{u}\right) \qquad u(1, y) = y$$

$$z = \frac{1}{y} \Rightarrow y = \frac{1}{z} \Rightarrow f(z) = 0 - \frac{1}{z}$$

$$f(z) = -\frac{1}{z}$$

G.S $\quad \ln x - y = \dfrac{-1}{\frac{x}{u}}$

$$\ln x - y = \frac{-u}{x}$$

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$$\ln x - y = \frac{-u}{x}$$

$$-u = x(\ln x - y)$$

$$u = -x(\ln x - y)$$

$$u = yx - x\ln x$$

char method [Linear - smei - quasi]

Lagrange [Linear - smei - quasi]

بدون I.C

$$\boxed{\text{Lagrange} + \text{I.C} = \text{particular}}$$

Non Linear : $F(x, y, u, p, q) = 0$ } نوجد

$u'(0) = p(0)\,x'(0) + q(0)\,y'(0)$ } 2. I.C p, q

general sol (Non Linear) $\rightarrow$ complete integral

$du = p\,dx + q\,dy$

بدلالة α, β

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NonLinear Eqs:

A nonLinear equ is the form $F(x, y, u, u_x, u_y) = 0$

or $F(x, y, u, p, q) = 0$ Where $p = u_x$, $q = u_y$
we shall solve this equ by the char. method
In this method we have 5 char. equs: تختلف عن char method السابقة

$$\frac{dx}{ds} = F_p \qquad \frac{dy}{ds} = F_q \qquad \frac{du}{ds} = pF_p + qF_q$$

المشتقة الجزئية لـ F بالنسبة لـ p

$$\frac{dp}{ds} = -F_x - pF_u \qquad \frac{dq}{ds} = -F_y - qF_u$$

But we have only 3 ICs:

$x(0) = \phi(\tau)$ [illegible] p,q [illegible]
$y(0) = \psi(\tau)$
$u(0) = h(\tau)$
we find ICs for p, q using the eqs

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$F(x(0), y(0), u(0), p(0), q(0)) = 0$ معادلتين بمجهولين

معلومة مجهول معلومة مجهول معلومة

$u'(0) = p(0)\,x'(0) + q(0)\,y'(0)$

Ex: Find ICs for p,q in the problem

$U_x U_y = x^2 \quad , u(x,0) = x$

Sol

$U_x U_y = x^2 \Rightarrow pq = x^2$

$F(x,y,u,p,q) = pq - x^2 = 0$

$(x(0), y(0), u(0)) = (\tau, 0, \tau)$

$F(x(0), y(0), u(0), p(0), q(0)) = 0$

$p(0) . q(0) - x^2(0) = 0 \Rightarrow p(0) q(0) = \tau^2 \quad \ldots (1)$

$u'(0) = p(0) . x'(0) + q(0) y'(0)$

$1 = p(0) . 1 + q(0) . 0 \Rightarrow \underline{p(0) = 1}$ (Ic for p)

sub in equ 1 $\Rightarrow 1 . q(0) = \tau^2 \Rightarrow \underline{q(0) = \tau^2}$ (Ic for q)

Ex: Find ICs for p, q

$u_x^2 + u_y = u \quad , u(x,1) = x^3$

Sol $p^2 + q = u$

$$F(x,y,u,p,q)=0 \Rightarrow p^2+q-u=0$$

$$(x(0),y(0),u(0)) = (\tau,1,\tau^3)$$

$$p^2(0)+q(0)=\tau^3 \quad \dots (1)$$

$$u'(0)=p(0)\,x'(0)+q(0)\,y'(0)$$

$$3\tau^2 = p(0).\,1 + q(0).\,0$$

$$\boxed{3\tau^2 = p(0)} \quad (\text{Ic for } p)$$

Sub in equ (1)

$$p^2(0)+q(0)=\tau^3$$

$$(3\tau^2)^2+q(0)=\tau^3 \Rightarrow 9\tau^4+q(0)=\tau^3$$

$$\boxed{q(0)=\tau^3-9\tau^4} \quad (\text{Ic for } q)$$

solve $\dfrac{dx}{ds}=2p$ $\quad \dfrac{dy}{ds}=1$ $\quad \dfrac{du}{ds}=2p^2+q$

$$\frac{dp}{ds}=p \qquad \frac{dq}{ds}=q$$

$$\frac{dq}{ds}=q \Rightarrow q=c_1 e^{s} \xRightarrow{I.c} \boxed{q=(\tau^3-9\tau^4)e^{s}}$$

Ex solve the following IVP using the char. method $u_x u_y = u$, $u(x,0) = x$

Sol: ① $pq = u \Rightarrow pq - u = 0 \Rightarrow F = pq - u$

② $(x(0), y(0), u(0)) = (\tau, 0, \tau)$

③ ICs for p.q

$p(0)\,q(0) - u(0) = 0 \Rightarrow p(0)\,q(0) - \tau = 0$

$p(0)\,q(0) = \tau$ ... (1)

$u'(0) = p(0)\,x'(0) + q(0)\,y'(0)$

$1 = p(0) \cdot 1 + q(0) \cdot 0$

$\boxed{p(0) = 1}$ (IC For p)

sub in equ 1

$1 \cdot q(0) = \tau \Rightarrow \boxed{q(0) = \tau}$ (IC. For q)

Char equ are

$\frac{dx}{ds} = F_p = q$

$$\frac{dy}{ds} = F_q = p$$

$$\frac{du}{ds} = pF_p + qF_q = pq + qp = 2pq$$

$$\frac{dp}{ds} = -F_x - pF_u = 0 - p(-1) = p$$

$$\frac{dq}{ds} = -F_y - qF_u = 0 - q(-1) = q$$

$$\frac{dx}{ds} = q \qquad \frac{dy}{ds} = p \qquad \frac{du}{ds} = 2pq$$

لدينا خمسة ٥ معادلات
معادلتين ينحلن

$$\frac{dp}{ds} = p \qquad \frac{dq}{ds} = q$$

We solve the char equ

$$\frac{dp}{ds} = p \Rightarrow \frac{dp}{p} = ds \Rightarrow \ln p = s + c$$

$$p(s) = c_1 e^{s} \xRightarrow{I.C} p(0) = 1$$

$$p(0) = c_1 e^{0} = 1 \Rightarrow c_1 = 1 \Rightarrow p(s) = e^{s}$$

$$\boxed{p = e^{s}}$$

$$\frac{dq}{ds} = q \Rightarrow \ln q = s + c_2$$

$$q(s) = c_2 e^{s} \overset{I.C}{\Longrightarrow} q(0) = \tau$$

$$q(0) = c_2 e^{0} = \tau \Rightarrow \tau = c$$

$$\boxed{q = \tau e^{s}}$$

$$\frac{dx}{ds} = q = \tau e^{s} \Rightarrow dx = \tau e^{s} ds$$

$$x = \tau e^{s} + c_3 \overset{I.C}{\Longrightarrow} x(0) = \tau$$

$$x(0) = \tau e^{0} + c_3 \Rightarrow \tau = \tau + c_3 \Rightarrow c_3 = 0$$

$$\boxed{x = \tau e^{s}}$$

$$\frac{dy}{ds} = p = e^{s} \Rightarrow dy = e^{s} ds \Rightarrow y = e^{s} + c_4$$

$$\overset{I.C}{\Longrightarrow} y(0) = 0 \Rightarrow 0 = e^{0} + c_4 \Rightarrow c_4 = -1$$

$$\boxed{y = e^{s} - 1}$$

$$\frac{du}{ds} = 2\underset{\sim}{\overset{2u}{pq}} = 2e^{s} \cdot \tau e^{s} = 2\tau e^{2s}$$

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$$du = 2\tau e^{2s} ds$$

$$u = 2\tau\left(\frac{1}{2}\right) e^{2s} + C_5$$

$$u = \tau e^{2s} + C_5 \Rightarrow u(0) = \tau$$

$$\tau = \tau e^{0} + C_5 \Rightarrow C_5 = 0 \Rightarrow u = \tau e^{2s}$$

بطريقة أخرى

$$\frac{du}{ds} = 2pq = 2u \quad (pq = u \text{ from the PDE})$$

$$\frac{du}{u} = 2ds \Rightarrow \ln u = 2s + C_5$$

$$u(s) = C_5 e^{2s}$$

$$IC: u(0) = \tau \Rightarrow u(0) = C_5 e^{0} \Rightarrow \tau = C_5$$

$$u(s) = \tau e^{2s}$$

$$\boxed{u(s, \tau) = \tau e^{2s}}$$

$u(x, y)$:

$$x = \tau e^{s}, \quad y = e^{s} - 1 \Rightarrow e^{s} = y + 1 \quad (\ln \text{ للطرفين})$$

$$\boxed{s = \ln(y+1)} \qquad x = \tau e^{s} \Rightarrow \tau = \frac{x}{y+1}$$

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$$u(s,\tau) = \tau e^{2s}$$

$$\therefore u(x,y) = \frac{x}{y+1} \cdot e^{2\ln(y+1)}$$

$$= \frac{x}{y+1} e^{\ln(y+1)^2}$$

$$= \frac{x(y+1)^2}{y+1} = x(y+1)$$

$\therefore u(x,y) = y\cdot x + x$ The solution of the problem

---

$$\frac{dp}{ds} = p \Rightarrow p = c_2 e^{s}$$

[illegible]

$$\xRightarrow{I.C} \boxed{p = 3\tau^2 e^{s}}$$

$$\frac{dy}{ds} = 1 \Rightarrow y = s + c_3 \xRightarrow{I.C} \boxed{y = s+1}$$

$$\frac{dx}{ds} = 2(3\tau^2 e^{s}) \Rightarrow x = 6\tau^2 e^{s} + c_4$$

$x(0) = \tau \Rightarrow \tau = 6\tau^2 + c_4$

$$\xRightarrow{I.C} x = 6\tau^2 e^{s} + \tau - 6\tau^2$$

$$\boxed{x = 6\tau^2(e^{s}-1) + \tau}$$

$$\frac{du}{ds} = 2(9\tau^4 e^{2s}) + (\tau^3 - 9\tau^4) e^{s}$$

$$u = 9\tau^4 e^{2s} + (\tau^3 - 9\tau^4) e^{s} + c_5$$

$$\xRightarrow{I.C} u(0) = \tau^3$$

$$\tau^3 = 9\tau^4 + \tau^3 - 9\tau^4 + c_5$$

$$\Rightarrow c_5 = 0$$

$$u(s,\tau) = 9\tau^4 e^{2s} + (\tau^3 - 9\tau^4) e^{s}$$

$u(x,y)$? $y = s+1 \Rightarrow \boxed{s = y-1}$

$$\tau^2(6e^{s} - 6) + \tau - x = 0$$

$$\tau = \frac{-1 \mp \sqrt{1 + 24x(e^{s}-1)}}{12(e^{s}-1)}$$

$$\tau = \frac{-1 \mp \sqrt{1 + 24x(e^{y-1}-1)}}{12(e^{y-1}-1)}$$

N O T E B O O K

Subject Lecture $\boxed{9}$ Date 16/7 No.

Ex: Solve the following cauchy problem using the char. method PDE:

$$u = x u_x + y u_y + \left(\frac{u_x^2 + u_y^2}{2}\right)$$

I.C: $u(x,0) = \frac{1-x^2}{2}$

Sol:

$$u - x u_x - y u_y - \left(\frac{u_x^2 + u_y^2}{2}\right) = 0$$

or

$$x u_x + y u_y + \left(\frac{u_x^2 + u_y^2}{2}\right) - u = 0$$

$$F = xp + yq + \left(\frac{p^2 + q^2}{2}\right) - u = 0$$

$$(x(0), y(0), u(0)) = \left(\tau, 0, \frac{1-\tau^2}{2}\right)$$

char. eqs:

$$\frac{dx}{ds} = F_p = x + p$$

$$\frac{dy}{ds} = F_q = y + q$$

$$\frac{du}{ds} = pF_p + qF_q = p(x+p) + q(y+q) = xp + p^2 + yq + q^2$$

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$$\frac{dp}{ds} = -F_x - pF_u = -p - p(-1) = 0$$

$$\frac{dq}{ds} = -F_y - qF_u = -q - q(-1) = 0$$

$\Rightarrow$ ICS for p, q:

$$x(0)\,p(0) + y(0)\,q(0) + \frac{p^2(0) + q^2(0)}{2} - u(0) = 0$$

$$\tau\, p(0) + 0 + \frac{p^2(0) + q^2(0)}{2} - \frac{1-\tau^2}{2} = 0$$

$$2\tau\, p(0) + p^2(0) + q^2(0) - 1 + \tau^2 = 0 \quad \ldots (1)$$

* $u'(0) = p(0)\,x'(0) + q(0)\,y'(0)$

$-\tau = p(0)\,(1) + q(0)\,(0)$

$\Rightarrow \boxed{p(0) = -\tau}$ sub in equ (1)

IC for p

$$2\tau(-\tau) + \tau^2 + q^2(0) - 1 + \tau^2 = 0$$

$$-2\tau^2 + 2\tau - 1 + q^2(0) = 0 \Rightarrow q^2(0) = 1$$

$\Rightarrow \boxed{q(0) = 1}$ IC for q

بنـ ds المعادلات

$$\frac{dx}{ds} = x + p \; ; \; \frac{dy}{ds} = y + q \; ; \; \frac{du}{ds} = p^2 + q^2 + xp + yq$$

$$\frac{dp}{ds} = \frac{dq}{ds} = 0$$

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4 Nodes

4) $\frac{dp}{ds} = 0 \Rightarrow dp = 0 \Rightarrow p(s) = c_1 \overset{I.c}{\Rightarrow} p(0) = -\tau$

$\Rightarrow -\tau = c_1 \quad \therefore \boxed{p = -\tau}$

5) $\frac{dq}{ds} = 0 \Rightarrow dq = 0 \Rightarrow q(s) = c_2 \overset{I.c}{\Rightarrow} q(0) = 1 \Rightarrow 1 = c_2$

$\therefore \boxed{q = 1}$

1) $\frac{dx}{ds} = x + p \Rightarrow \frac{dx}{ds} = \frac{x - \tau}{1} \Rightarrow dx = (x - \tau) ds$

$\frac{dx}{x - \tau} = ds \Rightarrow \ln(x - \tau) = s + c_3 \Rightarrow x - \tau = c_3 e^{s}$

$x(s) - \tau = c_3 e^{s} \overset{I.c}{\Rightarrow} x(0) - \tau = c_3 e^{0} \quad (x(0) = \tau)$

$\tau - \tau = c_3 \Rightarrow c_3 = 0 \Rightarrow x - \tau = 0$

$\therefore \boxed{x = \tau}$

2) $\frac{dy}{ds} = y + q \Rightarrow \frac{dy}{ds} = \frac{y+1}{1} \Rightarrow \frac{dy}{y+1} = ds \Rightarrow$

$\ln(y+1) = s + c_4 \Rightarrow y + 1 = c_4 e^{s} \Rightarrow y(s) + 1 = c_4 e^{s}$

$\overset{I.c}{\Rightarrow} (y(0) = 0) \Rightarrow 0 + 1 = c_4 e^{0} \Rightarrow c_4 = 1 \Rightarrow y + 1 = e^{s}$

$\therefore \boxed{y = e^{s} - 1}$

3) $\frac{du}{ds} = xp + p^2 + yq + q^2$

$= \tau(-\tau) + \tau^2 + y(1) + 1^2 = y = e^{s} - 1 + 1 = e^{s}$

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$$\frac{du}{ds} = e^{s} \Rightarrow du = e^{s} ds \Rightarrow u(s) = e^{s} + c_5$$

$$\overset{I.c}{\Rightarrow} u(0) = e^{0} + c_5 \Rightarrow \frac{1-\tau^2}{2} = 1 + c_5$$

$$\Rightarrow c_5 = \frac{1-\tau^2}{2} - \frac{2}{2} = \frac{-\tau^2-1}{2} = -\left(\frac{\tau^2+1}{2}\right)$$

$$\Rightarrow u = e^{s} - \left(\frac{\tau^2+1}{2}\right)$$

$$\Rightarrow u(s,\tau) = e^{s} + \left(\frac{-\tau^2-1}{2}\right)$$

$u(x,y)$?

$$x = \tau \quad , \quad y = e^{s} - 1 \Rightarrow y + 1 = e^{s}$$

$$u(x,y) = y + 1 + \left(\frac{-x^2-1}{2}\right)$$

$\Rightarrow$ The solution

$$u(x,y) = y - \frac{x^2+1}{2} + 1$$

Non Linear — 5 معادلات

1) $F(x,y,u,p,q) = 0$ Char-method

2) نوجد شروط p , q من

$$F(x(0), y(0), u(0), p(0), q(0)) = 0$$

$$u'(0) = x'(0)\,p(0) + y'(0)\,q(0)$$

3) solve

$$\frac{dx}{ds} = F_p \qquad \frac{dy}{ds} = F_q \qquad \frac{du}{ds} = pF_p + qF_q$$

$$\frac{dp}{ds} = -F_x - pF_u \qquad \frac{dq}{ds} = -F_y - qF_u$$

H.W: PDE: $U_x U_y = y$

I.c $u(x,1) = x^2$

N O T E B O O K

$$U_x U_y = y$$
$$u(x,1) = x^2$$

Sol $pq - y = 0$

$(x(0), y(0), u(0))$
$= (\tau, 1, \tau^2)$

$p(0) q(0) - y(0) = 0$

$p(0) q(0) - 1 = 0 \quad \ldots (1)$

$u'(0) = p(0) x'(0) + q(0) y'(0)$

$2\tau = p(0)\, 1 + 0$

$\boxed{p(0) = 2\tau}$ I.c of $p(0)$

$2\tau\, q(0) = 1 \Rightarrow$

$\boxed{q(0) = \frac{1}{2\tau}}$ I.c of q

The char equation

$\frac{dx}{ds} = q \quad \frac{dy}{ds} = p$

$\frac{du}{ds} = pq + qp = 2pq$

$\frac{dp}{ds} = -F_x - pF_u = 0$

$\frac{dq}{ds} = -F_y - qF_u = 1$

solve

$\frac{dp}{ds} = 0 \Rightarrow dp = 0 \cdot ds$

$p = c_1 \Rightarrow p(0) = 2\tau = c_1$

$\boxed{p = 2\tau}$

$\frac{dq}{ds} = 1 \Rightarrow dq = ds$

$q = s + c_2 \xRightarrow{I.c}$

$\frac{1}{2\tau} = 0 + c_2 \Rightarrow c_2 = \frac{1}{2\tau}$

$\boxed{q = s + \frac{1}{2\tau}}$

$\frac{dx}{ds} = q \Rightarrow dx = q\, ds$

$dx = \left(s + \frac{1}{2\tau}\right) ds$

$dx = s\, ds + \frac{ds}{2\tau}$

$x = \frac{s^2}{2} + \frac{s}{2\tau} + c_3$

$\xRightarrow{I.c} x(0) = \tau$

$\tau = c_3 \Rightarrow \boxed{x = \frac{s^2}{2} + \frac{s}{2\tau} + \tau}$

$\frac{dy}{ds} = p \Rightarrow dy = (2\tau)\, ds$

$y = 2\tau s + c_4 \xRightarrow{I.c} y(0) = 1$

$1 = c_4 \Rightarrow \boxed{y = 2\tau s + 1}$

$\frac{du}{ds} = 2pq = 2(2\tau)\left(s + \frac{1}{2\tau}\right)$

$= 4\tau s + \frac{4\tau}{2\tau}$

$du = (4\tau s + 2)\, ds$

$u = 4\tau\left(\frac{s^2}{2}\right) + 2s + c_5$

$u = 2\tau s^2 + 2s + c_5$

$\xRightarrow{I.c} u(0) = \tau^2$

$\tau^2 = c_5 \Rightarrow$

$\boxed{u(s,\tau) = 2\tau s^2 + 2s + \tau^2}$

$u(x,y)$?

5

إجعل من يراك يدعو لمن رباك

Subject

Class Name

PAGE	DESCRIPTION
	Partial second

[1] complete integral

[2] special type of 1st

[3] system of 1st order PDE (solve)

[4] classiy system (1st)

[5] classiy 2nd order

Rose

Subject Lecture $\boxed{10}$ Date 17 / 7 الأربعاء No.

بداية السيكند

Complete integrals (g.s) (non_Linear)

دليل تكامل

مثل $u_x u_y$, u_x^2, u_y^3 A complete integral is a general solution of the fully nonlinear equ. $F(x,y,u,p,q)=0$

Which is in the form $u(x,y,\alpha,\beta)$ α,β ثوابت مثل c_1, c_2

We use **Charpit's method** we find an integral

In this method We find an integral

$\underbrace{Q(x,}_{\text{متغير}}\underbrace{y,u,p,q)}_{\text{تفاضل}} = \underset{\text{ثابت}}{\alpha}$ From the equs

(Linear [مرة] lagrange, semi quasi [non] charpit's)

$$\frac{dx}{F_p} = \frac{dy}{F_q} = \frac{du}{pF_p + qF_q} = \frac{dp}{-F_x - pF_u} = \frac{dq}{-F_y - qF_u}$$

non parametrized method

then use $F(x,y,u,p,q)=0$ (the PDE)

to find $p = p(x,y,u,\alpha)$

$q = q(x,y,u,\alpha)$

and substitute in $\boxed{du = p\,dx + q\,dy}$ to find $u(x,y,\alpha,\beta)$

Ex Find a complete integral for the PDE

$$p^2 + qy - u = 0$$

Note

نعوض لا بدالة u_x, u_y

Sol.: The nonparametrized char eqs. are

$$\frac{dx}{F_p} = \frac{dy}{F_q} = \frac{du}{pF_p + qF_q} = \frac{dp}{-F_x - pF_u} = \frac{dq}{-F_y - qF_u}$$

F معطى بصورة مباشرة

$$F = p^2 + qy - u = 0$$

$$\frac{dx}{2p} = \frac{dy}{y} = \frac{du}{2p^2 + qy} = \frac{dp}{0 - p(-1)} = \frac{dq}{-q - q(-1)}$$

$$\frac{dx}{2p} = \frac{dy}{p} = \frac{du}{2p^2 + qy} = \frac{dp}{p} = \frac{dq}{0}$$

$$\frac{dp}{p} = \frac{dq}{0} \Longrightarrow p\,dq = 0 \xRightarrow{\text{نقسم } p} dq = 0 \Longrightarrow \underline{\underline{q = \alpha}}$$

Sub in $F = p^2 + qy - u = 0$ كي نجد p نعوض بدل $q = \alpha$

$p^2 + \alpha y - u = 0$

$p^2 = u - \alpha y \Rightarrow \underline{p = \sqrt{u - \alpha y}}$

$du = p\,dx + q\,dy$ نعوض p, q بالمعادلة

$du = \underbrace{\sqrt{u - \alpha y}}\,dx + \alpha\,dy$ نكامل الطرفين

ليست ثابت ما فينا نكامل لذلك نقسم عليها

$$\frac{du}{\sqrt{u - \alpha y}} = dx + \frac{\alpha}{\sqrt{u - \alpha y}}\,dy$$

$$\frac{du}{\sqrt{u-\alpha y}} - \frac{\alpha\, dy}{\sqrt{u-\alpha y}} = dx$$

$$\frac{du - \alpha\, dy}{\sqrt{u-\alpha y}} = dx \Rightarrow \frac{d(u-\alpha y)}{\sqrt{u-\alpha y}} = dx$$

مثل لما كان

$ع^{-\frac{1}{2}}\, d\, ع$

$(u-\alpha y)\ d(\alpha - u y)$

$$(u-\alpha y)^{-\frac{1}{2}}\, d(u-\alpha y) = dx$$ الجذر رفعناه

$$2(u-\alpha y)^{\frac{1}{2}} = x + \beta$$ نكامل ثم نقسم على 2

$$\sqrt{u-\alpha y} = \frac{x+\beta}{2} \Rightarrow u - \alpha y = \left(\frac{x+\beta}{2}\right)^2$$

$$u = \left(\frac{x+\beta}{2}\right)^2 + \alpha y$$

$\therefore\ u(x, y, \alpha, \beta) = \left(\frac{x+\beta}{2}\right)^2 + \alpha y$ a complete integral

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Ex: Find a complete integral for: $p^2 + q - u = 0$

sol: $F = p^2 + q - u = 0$

char. eqs.

$$\frac{dx}{F_p} = \frac{dy}{F_q} = \frac{du}{pF_p + qF_q} = \frac{dp}{-F_x - pF_u} = \frac{dq}{-F_y - qF_u}$$

$$\frac{dx}{2p} = \frac{dy}{1} = \frac{du}{2p^2 + q} = \frac{dp}{p} = \frac{dq}{q}$$

$$\frac{dp}{p} = \frac{dq}{q} \Rightarrow \ln p = \ln q + \ln \alpha \Rightarrow$$

$$\ln p = \ln \alpha q \Rightarrow \boxed{p = \alpha q}$$

sub in $p^2 + q - u = 0$

$$\alpha^2 q^2 + q - u = 0 \Rightarrow$$

we use the quadratic form. with $a = \alpha^2$ $b = 1$ $c = -u$

$$q = \frac{-1 \mp \sqrt{1 - 4(\alpha^2)(-u)}}{2\alpha^2} \Rightarrow q = \frac{-1 \mp \sqrt{1 + 4\alpha^2 u}}{2\alpha^2}$$

$$p = \alpha q = \alpha \cdot \left(\frac{-1 + \sqrt{1 + 4\alpha^2 u}}{2\alpha^2}\right) = \frac{-1 + \sqrt{1 + 4\alpha^2 u}}{2\alpha} = p$$

$$du = p\,dx + q\,dy$$ نعوض p, q في

$$du = \left(\frac{-1 + \sqrt{1 + 4\alpha^2 u}}{2\alpha}\right) dx + \left(\frac{-1 + \sqrt{1 + 4\alpha^2 u}}{2\alpha^2}\right) dy$$

$$2\alpha^2 \cdot du = \alpha\left(-1 + \sqrt{1 + 4\alpha^2 u}\right) dx + \left(-1 + \sqrt{1 + 4\alpha^2 u}\right) dy$$

$$\frac{2\alpha^2\, du}{-1 + \sqrt{1 + 4\alpha^2 u}} = \alpha\,dx + dy$$

$$\frac{1 + \sqrt{1 + 4\alpha^2 u}}{2u}\, du = \alpha\,dx + dy$$

$$\frac{2\alpha^2\, du}{\sqrt{1 + 4\alpha^2 u} - 1} = \alpha\,dx + dy$$

Special Tayps of 1st order PDE.

① Eqs involving p,q only (in this case $p=\alpha$)

$$F(p,q)=0$$

Ex Find a complete integral for the following eqs as special types.

ⓐ $p^2+q^2=1$

Sol:

عكس بكون ب U_x , V_y محولها

$$F=p^2+q^2-1=0$$

$$p=\alpha$$

$$\alpha^2+q^2-1=0 \implies q=\sqrt{1-\alpha^2}$$

لما يكون بالمعادلة p,q فقط نفرض $\boxed{p=\alpha}$ ونعوض ب $du=p\,dx+q\,dy$

$$du=p\,dx+q\,dy$$

$$du=\alpha\,dx+\sqrt{1-\alpha^2}\,dy \quad ; \ \alpha \text{ is constant}$$

$$u=\alpha x+\sqrt{1-\alpha^2}\,y+\beta \quad ; \text{ a complete integral}$$

ⓑ: $pq=3$

Sol $F=pq-3=0$, $p=\alpha$

$$\alpha q-3=0 \implies q=\frac{3}{\alpha}$$

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$$du = p\,dx + q\,dy$$

$$du = \alpha\,dx + \tfrac{3}{\alpha}\,dy$$

$u = \alpha x + \frac{3}{\alpha}y + \beta$ acomplete integral

② Eqs involving $p, q,$ and u

$F(p,q,u)=0$

in this case $p=\alpha q$

Ex: Find a complete integral

(a) $pq = u$

Sol $F = pq - u = 0$; $p = \alpha q$

$$\alpha q(q) - u = 0 \Rightarrow \alpha q^2 = u \Rightarrow q^2 = \frac{u}{\alpha}$$

$$q = \sqrt{\frac{u}{\alpha}} = \frac{\sqrt{u}}{\sqrt{\alpha}}$$

$$p = \alpha \cdot \frac{\sqrt{u}}{\sqrt{\alpha}} \Rightarrow p = \sqrt{\alpha}\sqrt{u}$$

$$du = p\,dx + q\,dy$$

$$= \sqrt{\alpha}\sqrt{u}\,dx + \frac{\sqrt{u}}{\sqrt{\alpha}}\,du$$

لما يكون بالمعادلة (p, q, u)
فقط نفرض $\boxed{p=\alpha q}$
بنطلع معادلتين p, q بدلالة u, α
نعوض بـ $du = p\,dx + q\,dy$

$$\frac{du}{\sqrt{u}} = \sqrt{\alpha}\, dx + \frac{1}{\sqrt{\alpha}}\, dy$$

$$2u^{\frac{1}{2}} = \sqrt{\alpha}\, x + \frac{1}{\sqrt{\alpha}} y + \beta$$

$$u^{\frac{1}{2}} = \frac{\sqrt{\alpha}\, x + \frac{1}{\sqrt{\alpha}} y + \beta}{2}$$

$$u = \left(\frac{\sqrt{\alpha}\, x + \frac{1}{\sqrt{\alpha}} y + \beta}{2}\right)^2$$ a complete integral

seprable Eqs (مافيه u)

منقدر نفصل شكل

$$f(x,p) = g(y,q)$$

$\downarrow$ نفرض $\downarrow$

α α

ونعوض بالمعادلة

$$du = p\,dx + q\,dy$$

complete integral

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③ $\boxed{\text{seprable Eqs.}}$ (مافيه u)

A separable Eqs can be written as

$$f(x,p) = g(y,q)$$

in this case $f(x,p) = g(y,q) = \alpha$

Ex: Find a complete integral

ⓐ $xq - yp = 0$

Sol $xq = yp$

$$\frac{x}{p} = \frac{y}{q} \qquad \left(\left(F(x,p) = \frac{x}{p} \quad g(y,q) = \frac{y}{q} \right)\right)$$

$$F(x,p) = \frac{x}{p} = \alpha \implies p = \frac{x}{\alpha}$$

$$g(y,q) = \frac{y}{q} = \alpha \implies q = \frac{y}{\alpha}$$

$$du = p\,dx + q\,dy$$

$$du = \frac{x}{\alpha}\,dx + \frac{y}{p}\,dy$$

$$u = \frac{x^2}{2\alpha} + \frac{y^2}{2\alpha} + \beta \quad \text{a complet integral}$$

(b). $p \sin y - q \cos x = 0$

$$p \sin y = q \cos x$$

$$\frac{p}{\cos x} = \frac{q}{\sin y} \implies \frac{\sin y}{q} = \frac{\cos x}{p}$$

$$F(x,p) = \frac{\cos x}{p} = \alpha \implies p = \frac{\cos x}{\alpha}$$

$$F(y,q) = \frac{\sin y}{q} = \alpha \implies q = \frac{\sin y}{\alpha}$$

$$du = p\,dx + q\,dy$$

$$du = \frac{\cos x}{\alpha}dx + \frac{\sin y}{\alpha}dy$$

$$u = \frac{\sin x}{\alpha} - \frac{\cos y}{\alpha} + \beta$$

$$u = \frac{\sin x - \cos y + \alpha\beta}{\alpha}$$ a complete integral

④. clairant Eq :

$$u = px + qy + F(p,q)$$

$p = \alpha \qquad q = \beta$

and the solution is

$u = \alpha x + \beta y + f(\alpha,\beta)$ a complete integral

Ex: Find a complete integeral

ⓐ $u - xp - yq - \sin pq = 0$

$$u = xp + yq + \underbrace{\sin pq}_{f(p,q)}$$

$p = \alpha \qquad q = \beta$

Clairant Equ

تكون شكل المعادلة

$u = px + qy + \underbrace{f(p,q)}$

اقتران بدلالة p, q

فإن g.s (complet integral)

يكون شكله

$u = \alpha x + \beta y + f(\alpha,\beta)$

$u = \alpha x + \beta y + \sin\alpha\beta$ a complete integral

(b) $u - xp + yq = \dfrac{1}{p^2+q^2}$

Sol

$p = \alpha \qquad q = \alpha$

$u = \alpha x + \beta y + \dfrac{1}{\alpha^2+\beta^2}$ a complete integral

Ex: find a complete integral

$p + q - u = 0$

Sol

$p = \alpha q$

$\alpha q + q - u = 0 \Rightarrow u = q(\alpha+1) \Rightarrow q = \dfrac{u}{\alpha+1}$

$du = p\,dx + q\,dy$

$du = \alpha q\,dx + \dfrac{u}{\alpha+1}\,dy$

$du = \dfrac{\alpha u}{\alpha+1}\,dx + \dfrac{u}{\alpha+1}\,dy$

$\dfrac{du}{u} = \dfrac{\alpha}{\alpha+1}\,dx + \dfrac{1}{\alpha+1}\,dy$

$\ln u = \dfrac{\alpha}{\alpha+1}x + \dfrac{1}{\alpha+1}y + \beta$

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$$\ln u = \frac{\alpha x + y + \beta(\alpha+1)}{\alpha+1}$$

$$u = e^{\frac{\alpha x + y + \beta(\alpha+1)}{\alpha+1}}$$

N O T E B O O K

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Systems of 1st order PDEs:

$U_x = \begin{bmatrix} u_x \\ v_x \end{bmatrix}$

consider the system $\vec{U}_x + A\vec{U}_y = 0$

where $\vec{U} = \begin{bmatrix} u \\ v \end{bmatrix}$, A a 2x2 matrix

we will solve this system by transforming it into a one with decoupled eqs.
then solve each eq.
separately using Lagrange method.

$\vec{U}_x + A\vec{U}_y = 0$ system طريقة أخرى لكتابة

$$\begin{bmatrix} u_x \\ v_x \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} u_y \\ v_y \end{bmatrix} = 0$$

$u_x + a_{11}u_y + a_{12}v_y = 0 \quad \dots (1)$

$v_x + a_{21}u_y + a_{22}v_y = 0 \quad \dots (2)$

The new system will be

$\vec{V}_x + D\vec{V}_y = 0$

Where $\vec{V} = \begin{bmatrix} w \\ z \end{bmatrix}$, $D = P^{-1}AP = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

$\boxed{\vec{U} = P\vec{V}}$ لـ نرجع

Where p is the matrix whose columns are the eigenvector of A.

Ex: $\overrightarrow{U}_x + \begin{bmatrix} 0 & 8 \\ 2 & 0 \end{bmatrix} \overrightarrow{U}_y = 0$

Solve the system.

Sol

$A = \begin{bmatrix} 0 & 8 \\ 2 & 0 \end{bmatrix}$

we find the eigenvalues of A

$|A - \lambda I| = 0$

$\begin{vmatrix} -\lambda & 8 \\ 2 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 16 = 0 \Rightarrow \lambda = \mp 4$

The eigenvalues $\lambda = 4$, $\lambda = -4$

((hyparpolic))

The eigenvalues are read and distinct

$\boxed{1}$ $\lambda_1 = 4$

$(A - \lambda_1 I)\overrightarrow{X} = 0$

$(A - 4I)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\begin{bmatrix} -4 & 8 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$ معادلتين متكافئتين [مع معادلة واحدة بمجهولين

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$2x_1 - 4x_2 = 0$

$x_1 - 2x_2 = 0 \Rightarrow x_1 = 2x_2$

نفرض Let $x_2 = s \Rightarrow x_1 = 2s$

any vector يعطي
Linearly independent يكون

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2s \\ s \end{bmatrix} = s\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

let $\vec{X}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ The eigenvector corresponding the eigen value $\lambda_1 = 4$

(2) $\lambda_2 = -4$

$(A - \lambda_2 I)\vec{X}_2 =$

$$\begin{bmatrix} 4 & 8 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = 0$$

$2\hat{x}_1 + 4\hat{x}_2 = 0 \Rightarrow \hat{x}_1 + 2\hat{x}_2 = 0 \Rightarrow \hat{x}_1 = -2\hat{x}_2$

let $\hat{x}_2 = t \Rightarrow \hat{x}_1 = -2t$

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} -2t \\ t \end{bmatrix} = t\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

NOTEBOOK

let $\vec{X_2} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ The eigenvector crrosponding to the eigenvalue $\lambda_2 = -4$

$$P = [\vec{X_1} \ \ \vec{X_2}] = \begin{bmatrix} 2 & -2 \\ 1 & 1 \end{bmatrix}$$

$$D = P^{-1} A P = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix} \qquad \begin{bmatrix} 3 & 1 \\ 4 & 0 \end{bmatrix}$$

The new system is :

$$\vec{V_x} + D \vec{V_y} = 0$$

$$\begin{bmatrix} w_x \\ z_x \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} w_y \\ z_y \end{bmatrix} = 0$$

langrang method

$W_x + 4W_y = 0 \quad \dots (3)$ معادلتين منفصلتين

lagrang method $Z_x - 4Z_y = 0 \quad \dots (4)$ كل معادلة بمجهول واحد

$$\vec{u_x} + \begin{bmatrix} 0 & 8 \\ 2 & 0 \end{bmatrix} \vec{u_y} = 0$$

$$\begin{bmatrix} u_x \\ v_x \end{bmatrix} + \begin{bmatrix} 0 & 8 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} u_y \\ v_y \end{bmatrix} = 0$$

بدنا U,V ما بتهمنا

$$U_x + 8V_y = 0 \quad \cdots (1)$$
$$V_x + 2U_y = 0 \quad \cdots (2)$$
The original system

we solve this system (3, 4) for w, z then use the transformation $\vec{U} = P\vec{V}$ to get u, v for the original system

we solve eq (3) by Lagrange method

$$w_x + 4w_y = 0$$

$$\frac{dx}{1} = \frac{dy}{4} = \frac{dw}{0}$$

$$dy = 4dx \Rightarrow y = 4x + c_1 \Rightarrow y - 4x = c_1$$

$$\Rightarrow \varphi_1(x, y, w) = y - 4x$$

$$\frac{dx}{1} = \frac{dw}{0} \Rightarrow dw = 0 \Rightarrow w = c_2$$

Fun بتكلع فحص

$$\psi_1(x, y, w) = w$$

G.S:

$$\psi_1 = F_1(\varphi_1)$$
$$w = F_1(y - 4x)$$

$Z_x - Zy = 0$

$$\frac{dx}{1} = \frac{dy}{-4} = \frac{dZ}{0}$$

$$\frac{dx}{1} = \frac{dy}{-4} \Rightarrow dy = -4\,dx \Rightarrow y = -4x + C_3$$

$$y + 4x = C_3 \Rightarrow \varphi_2(x, y, z) = y + 4x$$

$$\frac{dy}{-4} = \frac{dZ}{0} \Rightarrow -4dZ = 0 \Rightarrow dZ = 0$$

$$\Rightarrow Z = C_4$$

$$\Rightarrow \psi_2(x, y, z) = Z$$

G.S

$$\psi_2 = f_2(\varphi_2) \Rightarrow Z = f_2(y + 4x)$$

$$\boxed{\vec{U} = P\vec{V}}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ Z \end{bmatrix}$$

$$u = 2w - 2Z$$
$$= 2f_1(y - 4x) - 2f_2(y + 4x)$$

$$v = w + Z$$

$$v = f_1(y-4x) + f_2(y+4x)$$

u, v are solution of our system

بدلالة f_1, f_2 لازم نعطي 2 (IC)

1. eigen Values
2. eigen Vector
3. $P = [X_1 \ X_2]$, $D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$
4. $\vec{V}_x + D\,\vec{V}_y = 0$ بدلالة (Z, W)
6. نحل معادلتين Z, W system على طريقة (Lagrange) g.s
7. نحوّل $\vec{U} = P\vec{V}$

Ex: consider the system (سؤال مسكين)

$U_x + 2U_y + V_y = 0$ ... (1)

$V_x + 2U_y + V_y = 0$ ... (2)

(a) write this system in vector matrix Form

(b) Transform it into a one with decoupled eqs.

Sol:

$\boxed{a}$ $$\begin{bmatrix} U_x \\ V_x \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} U_y \\ V_y \end{bmatrix} = 0$$

$$\vec{U}_x + A\,\vec{U}_y = 0$$

$\boxed{b}$

$A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$, we finde the eigenvalues

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 1 \\ 2 & 1-\lambda \end{vmatrix} = 0 \Rightarrow$$

$(2-\lambda)(1-\lambda) - 2 = 0$

$2 - 2\lambda - \lambda + \lambda^2 - 2 = 0$

$\lambda^2 - 3\lambda = 0 \Rightarrow \lambda(\lambda - 3) = 0$

$\lambda = 0$, $\lambda = 3$ The eigen values

(hyparpolic)

1) For $\lambda_1 = 0$ we finde The eigen Vector crossponiding $\lambda_1 = 0$

$$(A - \lambda_1 I)\vec{X}_1 = 0$$

$$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$2x_1 + x_2 = 0 \Rightarrow x_2 = -2x_1$

let $x_1 = s \Rightarrow x_2 = -2s$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} s \\ -2s \end{bmatrix} = s\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\vec{X}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

2) For $\lambda_2 = 3$ we find the eigen Vector crosspoinding $\lambda = 3$

$$\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = 0$$

$-\hat{x}_1 + \hat{x}_2 = 0 \Rightarrow \hat{x}_1 = x_2$

let $\hat{x}_1 = t \Rightarrow \hat{x}_2 = t$

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} t \\ t \end{bmatrix} = t\begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \vec{X}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

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$$P = [\vec{x}_1 \ \vec{x}_2] = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

$$D = P^{-1}AP = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}$$

The new system is

$$\vec{V}_x + D\,\vec{V}_y = 0 \quad ; \quad \vec{V} = \begin{bmatrix} w \\ z \end{bmatrix}$$

$$\begin{bmatrix} w_x \\ z_x \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} w_y \\ z_y \end{bmatrix} = 0$$

$$\left.\begin{array}{l} w_x + 0 = 0 \\ z_x + 3z_y = 0 \end{array}\right\} \begin{array}{ll} w_x = 0 & \ldots(3) \\ z_x + 3z_y = 0 & \ldots(4) \end{array}$$

We solve equ (4) by lagrange method (general solution)

$$z_x + 3z_y = 0$$

$$\frac{dx}{1} = \frac{dy}{3} = \frac{dz}{0}$$

$$\frac{dx}{1} = \frac{dy}{3} \Rightarrow 3dx = dy \Rightarrow 3x = y + c_1$$

$$c_1 = 3x - y$$

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let $\Phi_2(x,y,z) = 3x-y$

$$\frac{dy}{3} = \frac{dz}{0} \Rightarrow 3\,dz = 0 \Rightarrow dz = 0$$

$$z = c_2 \Rightarrow \psi_2(x,y,z) = z$$

$$\Phi_2(x,y,z) = F_2(\psi_2) \Longrightarrow z = F_2(3x-y)$$

$w_x = 0$: we solve equ (3)

$$\int w_x(x,y) = \int 0\,dx \iff \int \frac{\partial w(x,y)}{\partial x} = \int 0\,dx$$

$w(x,y) = f_1(y)$ (Sol for equ 3)

we use the transformation $\vec{U} = P\vec{V}$ to get u, v

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} w \\ z \end{bmatrix}$$

$u = w + z \Rightarrow \boxed{u = f_1(y) + f_2(3x-y)}$

$v = -2w + z$

$\boxed{V = -2f_1(y) + f_2(3x-y)}$ general Sol

لو طالب particular sol بعطي 2. I.C

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<u>Ex</u>: consider the system:

$$3U_x + U_y + V_x + V_y = 0 \quad \text{...(1)}$$

$$V_x + V_y + U_y + U_x = 0 \quad \text{...(2)}$$

Write the system in Form $\vec{U}_x + A\vec{U}_y = 0$

<u>Sol</u>:

$$\underbrace{\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}}_{B} \begin{bmatrix} U_x \\ V_x \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} U_y \\ V_y \end{bmatrix} = 0$$

we find the inverse for martix B

we Find determent of $B = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$

$\Delta = 3 - 1 = 2$

$$\Rightarrow \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

$$B^{-1} \cdot B \; U_x + B^{-1} A \cdot U_y = \underbrace{B^{-1} \cdot 0}_{0}$$

We multiply the equ by $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$

$$B^{-1} \cdot B = I \Rightarrow \vec{U}_x + B^{-1} A \, \vec{U}_y = 0$$

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$$\begin{bmatrix} u_x \\ v_x \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u_y \\ v_y \end{bmatrix} = 0$$

$$\begin{bmatrix} u_x \\ v_x \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u_y \\ v_y \end{bmatrix} = 0$$

$$\vec{U}_x + A\vec{U}_y = 0$$

$$\begin{vmatrix} -\lambda & 0 \\ 1 & 1-\lambda \end{vmatrix} = 0$$

$\lambda = 0 \quad \lambda = 1$

For $\lambda_1 = 0$

$$\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = 0$$

$X_1 + X_2 = 0$

$X_2 = -X_1$

$X_1 = s \Rightarrow$

$$\vec{X}_1 = \begin{bmatrix} s \\ -s \end{bmatrix} = s\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

For $\lambda = 1$

$$\begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \hat{X}_1 \\ \hat{X}_2 \end{pmatrix} = 0$$

$-X_1 = 0 \quad X = 0 \Rightarrow X_1 = 0$

$X_2 = t \Rightarrow \vec{X}_2 = \begin{pmatrix} 0 \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\vec{P} = [X_1 \; X_2] = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$V_x + D\,V_y = 0$

$$\begin{bmatrix} w_x \\ z_x \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w_y \\ z_y \end{bmatrix} = 0$$

$w_x = 0 \quad \dots (3)$

$z_x + z_y = 0 \quad \dots (4)$

solve equ (3)

$\int w_x(x,y)\,dx = \int 0\,dx$

$$\boxed{w(x,y) = f_1(y)}$$

using Lagrange Method to solve $z_x + z_y = 0$

$$\frac{dx}{1} = \frac{dy}{1} = \frac{dz}{0}$$

$\alpha_2(x,y,z) = z$

$dx = dy \Rightarrow x = y + c_2$

$x - y = c_2 \Rightarrow$

$$\boxed{z = f_2(x-y)}$$

$\vec{u} = \vec{P}\vec{V}$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} w \\ z \end{bmatrix}$$

$u = w$

$v = -w + z$

$$\boxed{\begin{aligned} u &= f_1(y) \\ v &= -f_1(y) + f_2(x-y) \end{aligned}}$$

Classification of system

Consider the system:

$$\vec{U}_x + A\vec{U}_y = B\vec{u} + d$$

الطرف الأيمن لا يعتد بالتصنيف

To classify this system we find the eigen values and eigen vectors of A and we have

Linear independent

a If we have n eigenvalues, and n L.indep eigenvectors, then the system is of $\boxed{\text{hyperbolic}}$ Form and this happens in two cases

1. IF all the eigenvalues are real and distinct

2. IF A is real symmetric matrix

عندي فوق الفرضين اللي تحت

b IF we have n multiple eigenvalues and fewer than n L.indep eigenvectors, then it's of $\boxed{\text{parabolic}}$ type

Subject $\begin{bmatrix} \diagdown \end{bmatrix}$ upper $\begin{bmatrix} 0 & \\ & 0 \end{bmatrix}$ diegonal Date No.

eigen values (عناصر القطر الرئيسي)

$\boxed{C}$ If all the eigenvalues are complex, then it is of $\boxed{\text{elliptic}}$ type.

طالما قيم λ تكرر تكون vector

Ex: classify

$\boxed{1}$ $\vec{U}_x + \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \vec{U}_y = \vec{U} + \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ The eigenvalues are $\lambda = 1, \lambda = 3$ Because

A is upper triangolar matrix

The system is of hyp Type

(Two distinct real eigenvalues)

قيم له متختلفات مافي داعي نطالع eigen Vector

$\boxed{2}$ $\vec{U}_x + \vec{U}_y = \vec{U}$ ...

قيم له مكررة لازم نطلع eigen vector

$A = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

two repeated eigen values

The eigenvalues are $\{1, 1\}$

$(A - \lambda I)\vec{x} = 0$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Longrightarrow x_1, x_2$ are free

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let $x_1 = s \quad x_2 = t$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} s \\ t \end{bmatrix} = s \begin{bmatrix} 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

let $\vec{X_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \vec{X_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

two eigen vector and two eigen values

$\Rightarrow$ The system is of hyp. type

$\boxed{3}$ $A \vec{U}_x + B \vec{U}_y = C\vec{U} + \vec{d}$

where $A = \begin{bmatrix} 1 & 0 \\ -3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

(Lower)

Sol: we find A^{-1}:

$\det(A) = |A| = -1 - 0 = -1$

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} -1 & 0 \\ 3 & 1 \end{bmatrix} \Rightarrow -1 \begin{bmatrix} -1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & -1 \end{bmatrix}$$

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Multiply the equ by A^{-1}

$$\vec{U_x} + A^{-1}B\,\vec{U_y} = A^{-1}c\vec{u} + A^{-1}\vec{d}$$

$$\begin{bmatrix} 1 & 0 \\ -3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -5 & -2 \end{bmatrix} = \hat{A}$$

$$\therefore \vec{U_x} + \begin{bmatrix} 2 & 1 \\ -5 & -2 \end{bmatrix} \vec{U_y} = \hat{c}\,\vec{u} + \hat{\vec{d}}$$

we find the eigen values and eigenvector of $\hat{A}$

$$|\hat{A} - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 1 \\ -5 & -2-\lambda \end{vmatrix} = 0 \Rightarrow (2-\lambda)(-2-\lambda) + 5 = 0$$

$$-4 + \lambda^2 + 5 = 0 \Rightarrow \lambda^2 + 1 = 0 \Rightarrow \lambda = \mp i$$

$\Rightarrow \therefore$ It's elliptic.

(complex root)

Ex: classify the following system

$$\vec{U}_x + \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix} \vec{U}_y = 0$$

Solution:

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$$

The eigen values are $\lambda_{1,2} = 3$ (rpeated roots)

The eigen vectors

$$(A - \lambda I)\vec{X} = 0$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = 0$$

$X_2 = 0$, X_1 is free let $X_1 = S$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} S \\ 0 \end{bmatrix} = S \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\vec{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

عدد eigen vector أقل من عدد eigen values

$\therefore$ The system is parabolic

الجزء الرئيسي من المعادلة أو سستم

Remark: IF the principal part of the equ or System is Linear and nonLinearity is محصورة confined in lower order terms, then classification proceeds as befor.

Linear 2nd order eqs:

التصنيف يعتمد على الحدود العليا

$$\underbrace{a u_{xx} + b u_{xy} + c u_{yy}}_{\text{principal part}} + \underbrace{d u_x + e u_y + f u}_{\text{Lower order terms}} = g$$

مثلاً لو كان (u_y^2) ما بأثر عالتصنيف.

Ex:

classify $\boxed{1}$ $\nabla^2 u = e^u$

Sol:

$$\nabla^2 u = \underbrace{u_{xx} + u_{yy}}_{\text{pricipal part (Linear)}} = \underbrace{e^u}_{\text{non Linear}} \quad \text{(Laplace equation)} \quad \text{2nd order}$$

تُصنف بالمميز $b^2 - 4ac$

$a = 1 \quad b = 0 \quad c = 1$

$b^2 - 4ac = 0 - 4(1)(1) = -4 < 0 \Rightarrow$ it's elliptic

e^u is nonLinear but the principal part is Linear

Note

$D > 0$ hyparabolic زائدي

$D = 0$ parapolic مكافئ

$D < 0$ elliptic

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system

$\boxed{2}$ $u_x - \gamma u_y = u^2 + v$

$v_x + \gamma v_y = u \cdot v$

Sol: لما يكون system منفصل كل معادلة لحالها

$$\underbrace{u_x - \gamma u_y}_{\text{principal}} = \underbrace{u^2 + v}_{\text{Lower order terms}} \qquad \gamma \in \mathbb{R}$$

Linear   Non Linear

$$\underbrace{v_x - \gamma u_y}_{\text{principal part}} = \underbrace{u \cdot v}_{\text{Lower order terms}}$$

Linear   Non Linear

We write the system in vector matrix Form

$$\begin{bmatrix} u_x \\ v_x \end{bmatrix} + \begin{bmatrix} -\gamma & 0 \\ 0 & \gamma \end{bmatrix} \begin{bmatrix} u_y \\ v_y \end{bmatrix} = \begin{bmatrix} u^2 + v \\ u \cdot v \end{bmatrix}$$

$$\vec{U_x} + A \vec{U_y} = \vec{d}$$

The eigen values of A are $\gamma, -\gamma$

The eigen Values are real and distinct

$\therefore$ The system is of hyperbolic type.

فقط في حال تكرر eigen values منوجد eigen vector لنعرف عدمهم (hyp perbolic)

شكل التمييز

"Revision" مراجعة

Ex: Find a complete integral as speical type of equ.

1) $p^2+q^3=1$

Sol

$p^2+q^3-1=0$

Let $p=\alpha$

$\alpha^2+q^3-1=0 \Rightarrow q^3=1-\alpha^2 \Rightarrow q=\sqrt[3]{1-\alpha^2}$

$du = p\,dx + q\,dy$

$du = \alpha\,dx + \sqrt[3]{1-\alpha^2}\,dy$

$u = \alpha x + \sqrt[3]{1-\alpha^2}\,y + \beta$ is a complete integral

2) $p^2 y(1+x^2) = q x^2$

Sol : spreable equation

$p^2\left(\frac{1+x^2}{x^2}\right) = \frac{q}{y}$

$F(x,p) = \frac{p^2(1+x^2)}{x^2} = \alpha \Rightarrow \alpha x^2 = p^2(1+x^2)$

$p^2 = \frac{\alpha x^2}{1+x^2} \Rightarrow p = \sqrt{\frac{\alpha x^2}{1+x^2}} = \frac{\sqrt{\alpha}\,x}{\sqrt{1+x^2}}$

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$$F(y,q) = \frac{q}{y} = \alpha \Rightarrow q = \alpha y$$

$$du = p\,dx + q\,dy$$

$$= \frac{\sqrt{\alpha}\;x}{\sqrt{1+x^2}}\,dx + \alpha y\,dy$$

$$= \sqrt{\alpha}\,x\,(1+x^2)^{-\frac{1}{2}}\,dx + \alpha y\,dy$$

$$du = \underbrace{\frac{1}{2}\sqrt{\alpha}\;2x}_{g'(x)}\;\underbrace{(1+x)^{-\frac{1}{2}}}_{g^n(x)}\,dx + \alpha y\,dy$$

$$u = \frac{1}{2}\sqrt{\alpha}\,(1+x^2)^{\frac{1}{2}}\,x + \frac{\alpha y^2}{2} + \beta$$

$$u(x,y,\alpha,\beta) = \sqrt{\alpha}\,\sqrt{1+x^2} + \frac{\alpha y^2}{2} + \beta$$

<u>Note</u>: $\int g'(x)\,(g(x))^n\,dx = \frac{(g(x))^{n+1}}{n+1} + c$

$\int e^{\sin x}\,dx$

[2] Consider the system

$$U_x + 3U_y + 5V_y = 0 \quad \dots (1)$$

$$V_x + V_y = 0 \quad \dots (2)$$

[1] Write this system in vector matrix Form.

[2] Trans Form it into a one with decoupled equ:

Solution

$$\begin{bmatrix} U_x \\ V_x \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} U_y \\ V_y \end{bmatrix} = 0 \quad ; \quad A = \begin{bmatrix} 3 & 5 \\ 0 & 1 \end{bmatrix}$$

$$\vec{U}_x + A\vec{U}_y = 0$$

$$|A - \lambda I| = 0 \quad , \quad \begin{vmatrix} 3-\lambda & 5 \\ 0 & 1-\lambda \end{vmatrix} = 0 \quad , \quad (3-\lambda)(1-\lambda) = 0$$

Then eigen values are $\lambda = 1$ $\lambda = 3$

(real distinct $\Rightarrow$ hy par polic)

Trans form يعني فقط التحويل $\vec{V}_x + D\vec{V}_y = 0$ بدون حل

For $\lambda = 1$ The eigen vector

$$(A - \lambda I)\vec{X} = 0$$

$$\begin{bmatrix} 2 & 5 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = 0 \Rightarrow 2X_1 + 5X_2 = 0$$

$$2X_1 = -5X_2 \Rightarrow X_1 = \frac{-5}{2}X_2 \quad , \text{Let } X_2 \text{ free} , X_2 = S$$

$$\vec{X}_1 = \begin{bmatrix} -\frac{5}{2}S \\ S \end{bmatrix} = S\begin{bmatrix} -\frac{5}{2} \\ 1 \end{bmatrix} \Rightarrow \vec{X}_1 = \begin{bmatrix} -\frac{5}{2} \\ 1 \end{bmatrix}$$

for $\lambda = 3 \Rightarrow$ The eigen Vetor

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$$\begin{bmatrix} 0 & 5 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = 0$$

$\Rightarrow x_2 = 0$ and x_1 ... free let $x_1 = t$

$$\vec{x}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} t \Rightarrow \vec{x}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\vec{P} = \begin{bmatrix} -\frac{5}{2} & 1 \\ 1 & 0 \end{bmatrix}$$

(Trans form)
هنا الحل

$\vec{V}_x + D\,\vec{V}_y = 0$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

هنا المطلوب

$$\begin{bmatrix} w_x \\ Z_x \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} w_y \\ Z_y \end{bmatrix} = 0$$

$w_x + w_y = 0$ ... (3) decoupled equations

$Z_x + 3Z_y = 0$ ... (4)

$dx = dy \Rightarrow x = y + c_2$

$w = f(x-y)$

$3dx = dy \Rightarrow 3x - y = c_4$

$w_x + w_y = 0 \Rightarrow \frac{dx}{1} = \frac{dy}{1} = \frac{dw}{0}$

$\boxed{w = f_1(x-y)}$

$Z_x + 3Z_y = 0 \Rightarrow \frac{dx}{1} = \frac{dy}{3} = \frac{dZ}{0}$

$\boxed{Z = f_2(3x-y)}$

$$\vec{U} = P\,\vec{V} \Rightarrow \begin{bmatrix} U \\ V \end{bmatrix} = \begin{bmatrix} -\frac{5}{2} & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ z \end{bmatrix}$$

$U = -\frac{5}{2} f_1(x-y) + f_2(3x-y)$

$V = f_1(x-y)$

N O T E B O O K

مادة إلفاينل

2nd ـــ system 1st order
عبادليت

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اختزال (تبسيط)

Reducation of 2nd order PDE to a system of 1st order eqs:

2nd order PDEs can be reduced using Factorization techniques to system of 1st order PDEs

التحليل إلى عوامل

طريقة

Ex: Reduce the equ

$$u_{tt} - \alpha^2 u_{xx} = 0$$

$\frac{d}{dx}$ عملية الاشتقاق بالنسبة لـ x

مؤثر oprenter

رمز $D = \frac{\partial}{\partial x}$ عملية الاشتقاق الجزئي بالنسبة لـ x (oprentor)

D_x [مشتقة جزئية بالنسبة لـ x]

$D_y = \frac{\partial}{\partial y}$

$D_x^2 = \frac{\partial^2}{\partial x^2}$

$\frac{\partial^2}{\partial x \partial y} = D_x D_y$

Sol: Wave equation

$u_{tt} = D_t^2$ اشتقاق مرتين بالنسبة لـ t

$u_{xx} = D_x^2$ اشتقاق مرتين بالنسبة لـ x

$$(D_t^2 - \alpha^2 D_x^2) u = 0$$ فرق بين مربعين

$$(D_t - \alpha D_x)\underbrace{(D_t + \alpha D_x) u}_{\text{نفرضو}} = 0$$

$$(D_t + \alpha D_x) u = v \quad \dots (1)$$

$$(D_t - \alpha D_x) v = 0 \quad \dots (2)$$

N O T E B O O K

هلأ ننتقل من لغة opreators إلى لغة اشتقاقات

$$U_t + \alpha U_x = V \quad \ldots (3)$$

$$V_t - \alpha V_x = 0 \quad \ldots (4)$$

<u>Ex</u> : Reduce

Dam Wave equation
المتلاشية

$$U_{tt} - \alpha^2 U_{xx} - \lambda^2 u = 0$$

<u>Sol</u> :

$$U_{tt} - \alpha^2 U_{xx} = \lambda^2 u$$

$$(D_t^2 - \alpha^2 D_x^2)\, u = \lambda^2 u$$

$$(D_t - \alpha D_x)(D_t + \alpha D_x)\, u = \lambda^2 u$$

$$(D_t + \alpha D_x)\, u = V \quad \ldots (1)$$

$$(D_t - \alpha D_x)\, V = \lambda^2 u \quad \ldots (2)$$

$$\left.\begin{aligned} u_t + \alpha u_x &= V \quad \ldots (3) \\ V_t - \alpha V_x &= \lambda^2 u \quad \ldots (4). \end{aligned}\right\} \text{The system}$$

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Ex: Reduce

$$U_{xx} - 7x\, U_{xy} + 10\, x^2\, U_{yy} - U_x - U = 0$$

Sol

$$U_{xx} - 7x\, U_{xy} + 10x^2\, U_{yy} = U_x + U$$

$$\left(D_x^2 - 7x\, D_x D_y + 10x^2 D_y^2\right) u = u_x + u$$

$$(D_x - 2x D_y)(D_x - 5x D_y)\, u = u_x + u$$

$$(D_x - 5x D_y)\, u = \cancel{u_x + u}\ V \quad \dots (1)$$

$$(D_x - 2x D_y)\, V = U_x + U \quad \dots (2)$$

$$U_x - 5x\, U_y = \cancel{u_x + u}\ V \quad \dots (3)$$

$$V_x - 2x\, V_y = U_x + U \quad \dots (4)$$

} System

N O T E B O O K

المحاضرة الأخيرة

Subject Lecture 18 Date الثلاثاء ٦/٨ No.

"نموذج سؤال فاينل"

Ex: Reduce the following PDE into a system of 1st order equation

$$x^2 U_{xx} + 9xy^2 U_{xy} + 14y^4 U_{yy} - U_x + U - x = 0$$

Solution:

$$x^2 U_{xx} + 9xy^2 U_{xy} + 14y^4 U_{yy} = U_x - U + x$$

$$(x^2 D_x^2 + 9xy^2 D_x D_y + 14y^4 D_y^2) u = U_x - U + x$$

$$(xD_x + 2y^2 D_y)(xD_x + 7y^2 D_y)\ u = U_x - U + x$$

$$(xD_x + 7y^2 D_y)\ u = V$$

$$(xD_x + 2y^2 D_y)\ V = U_x - U + x$$