Derivetive : المشتقات

$\boxed{1}$ $f(x) = c \longrightarrow f'(x) = 0$

Ex $f(x) = 5 \longrightarrow f'(x) = 0$

$f(x) = \sqrt{3} \longrightarrow f'(x) = 0$

$\boxed{2}$ $f(x) = x^n \longrightarrow f'(x) = n\,x^{(n-1)}$

Ex $f(x) = x^5 \longrightarrow f'(x) = 5x^4$

$f(x) = x^2 \longrightarrow f'(x) = 2x$

$f(x) = x \longrightarrow f'(x) = 1$

$\boxed{3}$ $f(x) = a \cdot x^n \longrightarrow f'(x) = a \cdot n\, x^{n-1}$

Ex $f(x) = 5x^3 \longrightarrow f'(x) = 15x^2$

$f(x) = 2x^{-3} \longrightarrow f'(x) = -6x^{-4}$

$\boxed{4}$ $f(x) = x^{\frac{n}{m}} \longrightarrow f'(x) = \frac{n}{m}\, x^{\frac{n-m}{m}}$

Ex $f(x) = x^{\frac{3}{5}} \longrightarrow f'(x) = \frac{3}{5}\, x^{\frac{-2}{5}}$

$f(x) = 2\,x^{\frac{1}{4}} \longrightarrow f'(x) = \frac{1}{2}\, x^{\frac{-3}{4}}$

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$\boxed{5}$ $f(x) = g(x) \pm h(x) \longrightarrow f'(x) = g'(x) \pm h'(x)$

Ex $f(x) = x^5 + 4x^2 - 3 \longrightarrow f'(x) = 5x^4 + 8x$

$f(x) = x^{\frac{3}{2}} + 4x^5 - 4 \longrightarrow f'(x) = \frac{3}{2}x^{\frac{1}{2}} + 20x^4$

$\boxed{6}$ $f(x) = g(x) \cdot h(x) \longrightarrow f'(x) = g(x) \cdot h'(x) + h(x) \cdot g'(x)$

Ex $f(x) = (4x^2+5)(2x-1) \longrightarrow f'(x) = 2(4x^2+5) + (8x)(2x-1)$

$\boxed{7}$ $f(x) = \frac{g(x)}{h(x)} \longrightarrow f'(x) = \frac{h(x) \cdot g'(x) - g(x) \cdot h'(x)}{(h(x))^2}$

Ex $f(x) = \frac{5x^2-3}{2x^3+4} \longrightarrow f'(x) = \frac{(2x^3+4)(10x) - (5x^2-3)(6x^2)}{(2x^3+4)^2}$

Remark :-

① $f(x) = \frac{a}{g(x)} \longrightarrow f'(x) = \frac{-a \cdot g'(x)}{(g(x))^2}$

Ex $f(x) = \frac{5}{(x^3+3x+1)} \longrightarrow f'(x) = \frac{-5(3x^2+3)}{(x^3+3x+1)^2}$

② $f(x) = \frac{g(x)}{a} \longrightarrow f'(x) = \frac{g'(x)}{a}$

Ex $f(x) = \frac{x^2+5x^3-3}{7} \longrightarrow f'(x) = \frac{2x+15x^2}{7}$

$\boxed{8}$ $f(x) = (g(x))^n \longrightarrow f'(x) = n\,(g(x))^{n-1} \cdot g'(x)$

Ex $f(x) = (5x^2 + 7x - 3)^7 \longrightarrow f'(x) = 7(5x^2 + 7x - 3)^6 (10x + 7)$

$\boxed{9}$ $f(x) = \sqrt{g(x)} \longrightarrow f'(x) = \dfrac{g'(x)}{2\sqrt{g(x)}}$

Ex $f(x) = \sqrt{x^2 + 5x - 4} \longrightarrow f'(x) = \dfrac{2x + 5}{2\sqrt{x^2 + 5x - 4}}$

Remark

$f(x) = \sqrt[3]{(x^2 + 5x - 3)^7}$

تحويل من شكل جذر الى شكل الاس

$f(x) = (x^2 + 5x - 3)^{\frac{7}{3}}$

ثم نحل على القاعدة 8

$f'(x) = \frac{7}{3}(x^2 + 5x - 3)^{\frac{4}{3}} (2x + 5)$

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$\boxed{10}$

الاقترانات المثلثية

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$

Ex find $f'(x)$:-

1) $f(x) = \sin(5x^2+1) \longrightarrow f'(x) = (10x)\cos(5x^2+1)$

2) $f(x) = \tan 7x \longrightarrow f'(x) = 7\sec^2(7x)$

3) $f(x) = \sin^5(3x) \longrightarrow f'(x) = 3 * 5 \sin^4(3x)\cos(3x)$

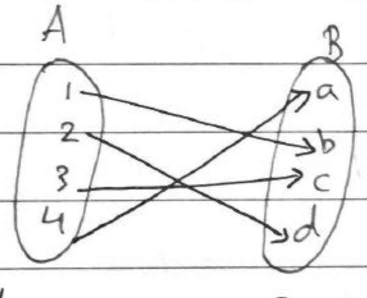
4) $f(x) = \cos^7(9x^2) \longrightarrow f'(x) = 7\cos^6(9x^2) \cdot 18x \cdot -\sin(9x^2)$

Def: A function with domain A is called a one to one function if no two elements of A have the same image, that is:

$$f(x_1) = f(x_2) \text{ wherever } x_1 = x_2$$

$$f(x_1) \neq f(x_2) \text{ wherever } x_1 \neq x_2$$

Ex



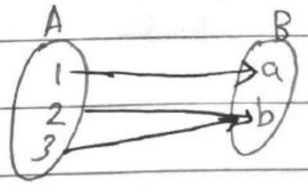
domain Image

$f(1) = b$
$f(2) = d$
$f(3) = c$
$f(4) = a$

→ one-one function.

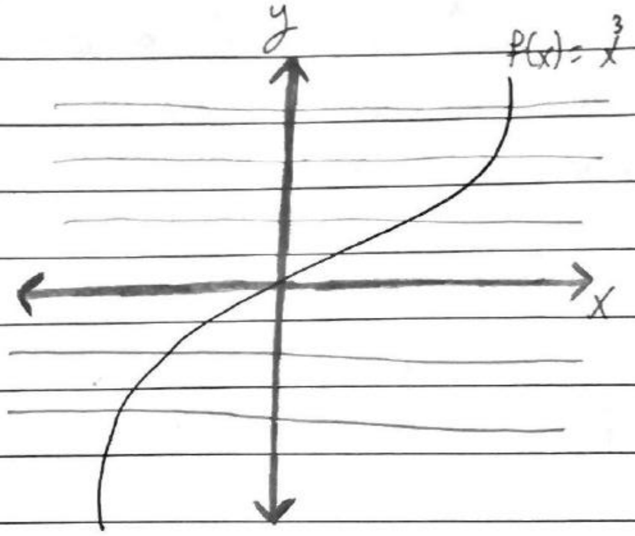
السبب ← كل عنصر في المجال (domain) له صورة واحدة فقط في المدى (Image) (range)

Ex



→ not one-one function

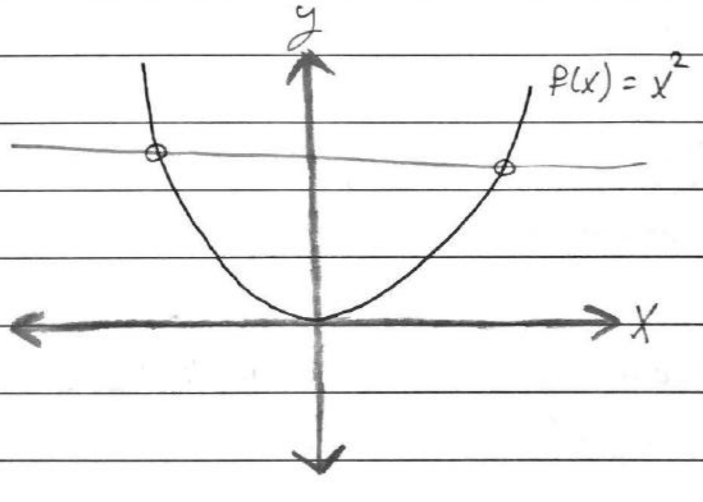
* Horizantal line test :- اختبار الخط الافقي



* الخطوط الحمراء
اختبار الخط الافقي

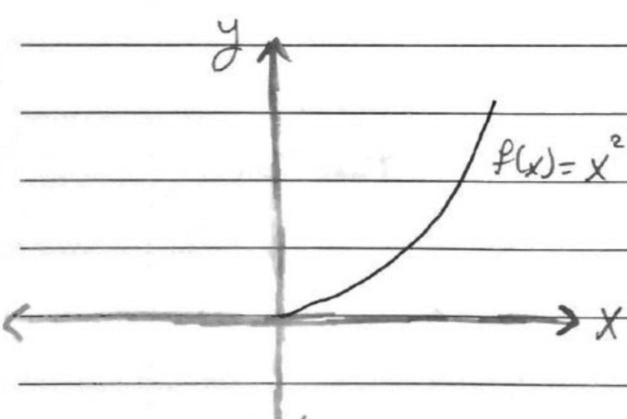
→ one-one function

يقطع عند نقطة واحدة ←



→ not (one-one) function

* في بعض الاحيان عند قسم الشكل
من النصف نستطيع ان نصبح اقتران (1-1)
مثل المثال السابق



→ one-one function

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Ex determind if the function (1-1) or not:-

① $f(x) = 2x + 3$

$f(x_1) = f(x_2)$

$2x_1 + \cancel{3} = 2x_2 + \cancel{3} \rightarrow \frac{2x_1}{2} = \frac{2x_2}{2} \rightarrow x_1 = x_2$
$-\cancel{3} \qquad -\cancel{3}$

$\therefore$ (1-1) function.

② $f(x) = 3x^3 + 5$

$f(x_1) = f(x_2)$

$3x_1^3 + \cancel{5} = 3x_2^3 + \cancel{5} \rightarrow \frac{\cancel{3}x_1^3}{\cancel{3}} = \frac{\cancel{3}x_2^3}{\cancel{3}} \rightarrow \sqrt[3]{x_1^3} = \sqrt[3]{x_2^3}$
$-\cancel{5} \qquad -\cancel{5}$

$\rightarrow x_1 = x_2$

$\therefore$ (1-1) function

N O T E B O O K

③ $f(x) = 3x^2 + 1$

$f(x_1) = f(x_2)$

$$3x_1^2 + 1 = 3x_2^2 + 1 \quad \rightarrow \quad \frac{3x_1^2}{3} = \frac{3x_2^2}{3} \quad \rightarrow \quad \sqrt{x_1^2} = \sqrt{x_2^2}$$
$$\quad -1 \qquad\qquad -1$$

$$\rightarrow \quad |x_1| = |x_2| \quad \Rightarrow \quad x_1 = x_2$$
أو $x_1 = -x_2$ → جوابين

$\therefore$ not (1-1) function.

④ show that the function is (1-1) ?

$f(x) = \frac{1-2x}{x} \quad , x \notin \{0\}$

$f(x_1) = f(x_2)$

$$\frac{1-2x_1}{x_1} = \frac{1-2x_2}{x_2} \quad \rightarrow \quad x_2(1-2x_1) = x_1(1-2x_2)$$

$$\rightarrow \quad x_2 - \cancel{2x_1x_2} = x_1 - \cancel{2x_2x_1} \quad \rightarrow \quad x_2 = x_1$$
$$\quad + \cancel{2x_1x_2} \qquad\qquad + \cancel{2x_1x_2}$$

$\therefore$ One-one function.

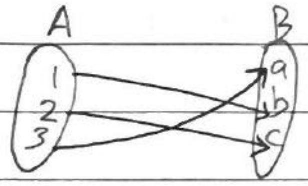
Inverse : المعكوس

Def :- Let f be a one to one function with domain A and range B then the inverse function $f^{-1}(x)$ has domain B and range A is defind by :-

$$f^{-1}(x) = x \iff f(x) = y$$

* يطبق inverse في حال كان الاقتران فقط (1-1)

Ex



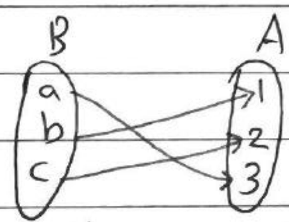
← $f(x)$

← (1-1) function

$f(1) = b$
$f(2) = c$
$f(3) = a$

domain $f(x) = \{1,2,3\}$
range $f(x) = \{a,b,c\}$

∴ →



← $f^{-1}(x)$

$f^{-1}(a) = 3$
$f^{-1}(b) = 1$
$f^{-1}(c) = 2$

domain $f^{-1}(x) = \{a,b,c\}$
range $f^{-1}(x) = \{1,2,3\}$

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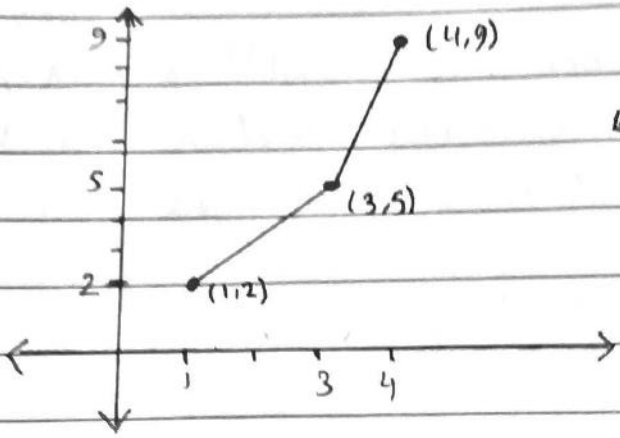
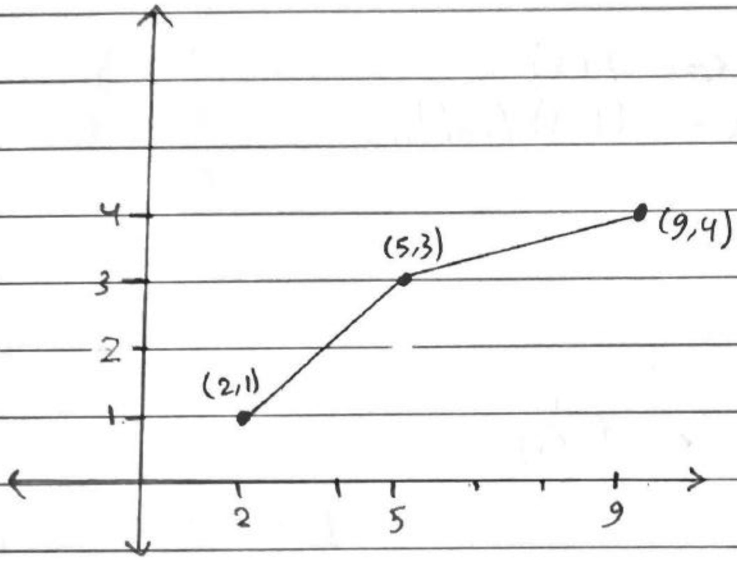
Ex if the following $f(x)$, find $f^{-1}(x)$



$f(x)$

$\leftarrow$ (1-1) function

$f(x)$	$\longrightarrow$	$f^{-1}(x)$
(1,2)	$\longrightarrow$	(2,1)
(3,5)	$\longrightarrow$	(5,3)
(4,9)	$\longrightarrow$	(9,4)



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Ex find the $f^{-1}(x)$ in the following :-

$\boxed{1}$ $f(x) = 3x+5$

$$x = 3y+5 \Rightarrow 3y = x-5 \Rightarrow y = \frac{x-5}{3}$$
$$-5 \qquad -5$$

$$\Rightarrow f^{-1}(x) = \frac{x-5}{3}$$

$\boxed{2}$ $f(x) = 3x^3+4$

$$x = 3y^3+4 \Rightarrow \frac{3y^3}{3} = \frac{x-4}{3} \Rightarrow \sqrt[3]{y^3} = \sqrt[3]{\frac{x-4}{3}} \Rightarrow y = \sqrt[3]{\frac{x-4}{3}}$$
$$-4 \qquad -4$$

$$\Rightarrow f^{-1}(x) = \sqrt[3]{\frac{x-4}{3}}$$

$\boxed{3}$ $f(x) = (3x-2)^5+2$

$$x = (3y-2)^5+2 \Rightarrow \sqrt[5]{(3y-2)^5} = \sqrt[5]{x-2} \Rightarrow 3y-2 = \sqrt[5]{x-2}$$
$$-2 \qquad -2 \qquad\qquad +2 \qquad +2$$

$$\Rightarrow \frac{3y}{3} = \frac{\sqrt[5]{x-2}+2}{3} \Rightarrow y = \frac{\sqrt[5]{x-2}+2}{3}$$

$$\Rightarrow f^{-1}(x) = \frac{\sqrt[5]{x-2}+2}{3}$$

تكملة ⟶

$\boxed{4}$ $f(x) = \sqrt{3-x}$:

$x = \sqrt{3-y} \Rightarrow$ تربيع الطرفين $\rightarrow x^2 = 3-y \Rightarrow y = 3-x^2$

$\Rightarrow f^{-1}(x) = 3-x^2$

$\boxed{5}$ $f(x) = \dfrac{3x-5}{4-2x}$

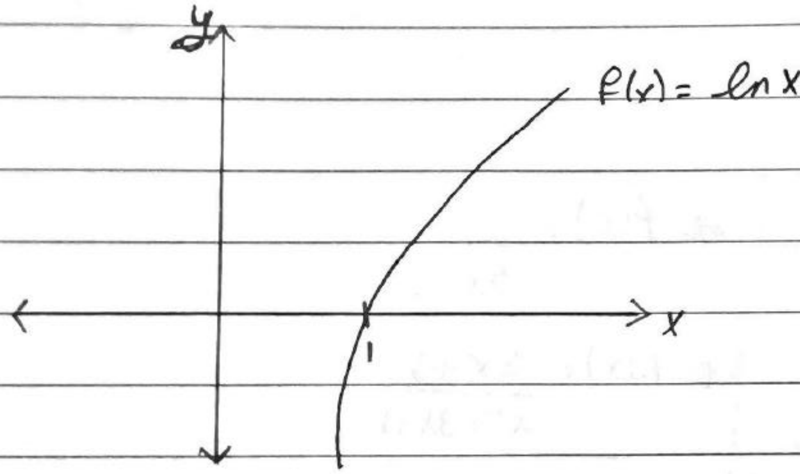
$x = \dfrac{3y-5}{4-2y} \Rightarrow x(4-2y) = 3y-5 \Rightarrow 4x - 2xy = 3y-5$

$\Rightarrow 4x+5 = 3y+2xy \Rightarrow 4x+5 = y(3+2x)$

$\Rightarrow y = \dfrac{4x+5}{3+2x} \Rightarrow f^{-1}(x) = \dfrac{4x+5}{3+2x}$

Natural logaritm اللوغاريتم الطبيعي

* $\log_e f(x) = \ln$



تقترب من الصفر ←
لكن لا تساويه أو تقطعه

$\text{Dom} = (0, \infty)$, $\text{Range} = (-\infty, \infty)$

* properties :-

1) $\ln(1) = 0$

2) $\ln(e) = 1$

3) $\ln(a \cdot b) = \ln(a) + \ln(b)$

4) $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

5) $\ln x^n = n \ln x$

6) $\ln\left(\frac{1}{x}\right) = \ln(x^{-1}) = -1 \ln(x)$

Dervetive of logaritm :- مشتقة اللوغاريتم

If $f(x) = \ln(g(x))$ then $\boxed{f'(x) = \dfrac{g'(x)}{g(x)}}$

Ex find the following :-

[1] $f(x) = \ln(5x-3) \Rightarrow f'(x) = \dfrac{5}{5x-3}$

[2] $f(x) = \ln(x^2+3x-1) \Rightarrow f'(x) = \dfrac{2x+3}{x^2+3x+1}$

[3] $f(x) = \ln\left((x^2+3)^5 \cdot (x-4)^6\right)$

$= \ln(x^2+3)^5 + \ln(x-4)^6 \leftarrow$ باستخدام الخاصية ٣

$= 5\ln(x^2+3) + 6\ln(x-4) \leftarrow$ باستخدام الخاصية ٥

$\Rightarrow f'(x) = 5 * \dfrac{2x}{x^2+3} + 6 * \dfrac{1}{x-4}$

$f'(x) = \dfrac{10x}{x^2+3} + \dfrac{6}{x-4}$

[4] $f(x) = \ln\sqrt[3]{\dfrac{x-3}{x^2+5}}$

$= \ln\left(\dfrac{x-3}{x^2+5}\right)^{\frac{1}{3}} = \dfrac{1}{3}\ln\left(\dfrac{x-3}{x^2+5}\right) = \dfrac{1}{3}\left(\ln(x-3) - \ln(x^2+5)\right)$

$\Rightarrow f'(x) = \dfrac{1}{3}\left(\dfrac{1}{x-3}\right) - \dfrac{1}{3}\left(\dfrac{2x}{x^2+5}\right)$

$\boxed{5}$ $y^3 = \ln\left(\frac{x-3}{x^2+4}\right)$ اشتقاق ضمني

$y^3 = \ln(x-3) - \ln(x^2+4)$

$3y^2 y' = \frac{1}{x-3} - \frac{2x}{x^2+4} \Rightarrow y' = \frac{1}{3y^2}\left(\frac{1}{x-3} - \frac{2x}{x^2+4}\right)$

$\boxed{6}$ $f(x) = x^3 \cdot \ln x$

Remark :-

$\ln(x^2) \neq (\ln x)^2$

$f'(x) = x^3 \cdot \frac{1}{x} + 3x^2 \cdot \ln x$

$f'(x) = x^2 + 3x^2 \cdot \ln x$

$\boxed{7}$ $f(x) = \frac{x^2+5}{\ln(x)}$

$f'(x) = \frac{\ln(x) * 2x - (x^2+5) \cdot \frac{1}{x}}{(\ln(x))^2}$

$\boxed{8}$ $y = \frac{(x+3)^2 (x^2+5)^6}{(x-3)^7 (x^2+2)^9}$ $\leftarrow$ أخذ ln للطرفين

$\ln y = \ln\left(\frac{(x+3)^2 (x^2+5)^6}{(x-3)^7 (x^2+2)^9}\right)$

$\ln(y) = \ln\left((x+3)^2 (x^2+5)^6\right) - \ln\left((x-3)^7 (x^2+2)^9\right)$

$\ln(y) = \ln(x+3)^2 + \ln(x^2+5)^6 - \ln(x-3)^7 - \ln(x^2+2)^9$

$\ln(y) = 2\ln(x+3) + 6\ln(x^2+5) - 7\ln(x-3) - 9\ln(x^2+2)$

نشتق

$\frac{y'}{y} = \frac{2}{x+3} + \frac{12x}{x^2+5} - \frac{7}{x-3} - \frac{18x}{x^2+2}$

$y' = y\left(\frac{2}{x+3} + \frac{12x}{x^2+5} - \frac{7}{x-3} - \frac{18x}{x^2+2}\right)$

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Integral التكاملات

1. $\boxed{\int a\,dx = ax + C}$ → ثابت تكامل

Ex $\int 5\,dx = 5x + c$

$\int \frac{1}{2}\,dx = \frac{1}{2}x + c$

$\int \sqrt{2}\,dx = \sqrt{2} + c$

2. $\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + c}$

Ex $\int x^5\,dx = \frac{x^6}{6} + c$

$\int x^{-3}\,dx = \frac{x^{-2}}{-2} + c$

$\int x^7\,dx = \frac{x^8}{8} + c$

3. $\boxed{\int a x^n\,dx = \frac{a x^{n+1}}{n+1} + c}$

Ex $\int 2x^3\,dx = \frac{2x^4}{4} + c = \frac{x^4}{2} + c$

$\int 3x^2\,dx = x^3 + c$

4. $\boxed{\int x^{\frac{n}{m}}\,dx = \frac{m\, x^{\frac{n+m}{m}}}{n+m} + c}$

Ex $\int x^{\frac{3}{2}}\,dx = \frac{2\,x^{\frac{5}{2}}}{5} + c$

$\int x^{\frac{1}{2}}\,dx = \frac{2}{3} * x^{\frac{3}{2}} + c$

$\int 2x^{\frac{-1}{3}}\,dx = \cancel{2} * \frac{3}{\cancel{2}}\, x^{\frac{2}{3}} + c = 3x^{\frac{2}{3}} + c$

$$\boxed{5} \quad \boxed{\int \left(f(x) \pm g(x) \pm L(x)\right) dx = \int f(x)\,dx \pm \int g(x)\,dx \pm \int L(x)\,dx}$$

Ex $\int (2x^4 + 3x^2 + 5x - 4)\, dx = \frac{2x^5}{5} + x^3 + \frac{5x^2}{2} - 4x + C$

★ التكامل يوزع على عملية الجمع والطرح فقط
لايوزع على عملية الضرب والقسمة

Ex find the following:-

① $\int (2x+3)(5x-4)\, dx = \int (10x^2 - 8x + 15x - 12)\, dx$

$= \int (10x^2 + 7x - 12)\, dx$

$= \frac{10x^3}{3} + \frac{7x^2}{2} - 12x + c$

② $\int (2x+3)^2\, dx = \int (4x^2 + 12x + 9)\, dx$

$= \frac{4x^3}{3} + 6x^2 + 9x + c$

③ $\int \frac{5}{x^2}\, dx = \int 5x^{-2}\, dx = \frac{5x^{-1}}{-1} + c = \frac{-5}{x} + c$

④ $\int \sqrt[3]{x}\, dx = \int (x)^{\frac{1}{3}}\, dx = \frac{3}{4} * x^{\frac{4}{3}} + c$

⑤ $\int (5\sqrt{x} + \frac{4}{x^5} + 9x + 3)\, dx = \int (5x^{\frac{1}{2}} + 4x^{-5} + 9x + 3)\, dx$

$= 5 * \frac{2}{3} x^{\frac{3}{2}} + \frac{4x^{-4}}{-4} + \frac{9x^2}{2} + 3x + C$

$= \frac{10}{3} x^{\frac{3}{2}} - x^{-4} + \frac{9x^2}{2} + 3x + C$

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الاقترانات الدائرية	تكاملها
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
قا $\sec^2 x$	$\tan x$
قتا $\csc^2 x$	$-\cot x$
$\sec x \tan x$	$\sec x$
$\csc x \cot x$	$-\csc x$

Ex ① $\int (5\cos x + 4\sin x - 3\sec^2 x + 4x - 3)\,dx$

$= 5\sin x + 4(-\cos x) - 3\tan x + 2x^2 - 3x + c$

② $\int (4\sec^2 x + 3\sec x\tan x + 7)\,dx$

$= 4\tan x + 3\sec x + 7x + c$

Remark :-

$$\int \sin(ax+b)\,dx = \frac{-\cos(ax+b)}{a} + c$$

مقسوم على معامل x

Ex :- $\int \sin(4x+3)\,dx = -\frac{\cos(4x+3)}{4} + c$

$\int \cos(4x)\,dx = \frac{\sin(4x)}{4} + c$

$\int \sec^2(5x-3)\,dx = \frac{\tan(5x-3)}{5} + c$

Remark

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

Ex ① $\int \sin^2 x \, dx$

$= \int (\frac{1}{2} - \frac{1}{2}\cos 2x) dx = \frac{1}{2}x - \frac{1}{2}\frac{\sin 2x}{2} + c$

② $\int \cos^2 x \, dx$

$= \int (\frac{1}{2} + \frac{1}{2}\cos 2x) dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + c$

③ $\int \sin^4 x \, dx$.

$= \int \sin^2 x \sin^2 x \, dx = \int (\frac{1}{2} - \frac{1}{2}\cos 2x)(\frac{1}{2} - \frac{1}{2}\cos 2x)$

$= \int \frac{1}{4}(1 - 2\cos 2x + 2(1 + \cos 4x)) dx$

$= \frac{1}{4}(x - \sin 2x + 2(x + \frac{\sin 4x}{4}) + c$

H.W $\int \cos^4 x \, dx$

$= \int (\frac{1}{2}(1+\cos 2x))^2 dx = \int \frac{1}{4}(1+\cos 2x)^2 dx = \frac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x) dx$

$= \frac{1}{4}\int (1 + 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x) dx$

$= \frac{1}{4}(x + \sin 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x) + c$

Remark

① $\int \frac{1}{x}\,dx = \ln|x| + c$

② $\int \frac{f'(x)}{f(x)} = \ln|f(x)| + c$

Ex Find the following :-

① $\int \frac{2x}{x^2+3}\,dx = \ln|x^2+3| + c$

② $\int \frac{8x}{4x^2+5}\,dx = \ln|4x^2+5| + c$

③ $\int \frac{-2\sin x}{2\cos x+4}\,dx = \ln|2\cos x+4| + c$

④ $\int \frac{x}{2x^2+5}\,dx = \frac{1}{4}\ln|2x^2+5| + c$

⑤ $\int \frac{\cos x}{3\sin x+7}\,dx = \frac{1}{3}\ln|3\sin x+7| + c$

⑥ $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\ln|\cos x| + c$

Subject Date No.

تحل بضرب المرافق (7) $\int \sec x \, dx = \int \sec x \cdot \frac{(\sec x + \tan x)}{(\sec x + \tan x)} \cdot dx$

$= \int \frac{\sec^2 x + \tan x \sec x}{\sec x + \tan x} \, dx = \ln|\sec x + \tan x| + c$

مشتقة المقام = البسط

H.W

* $\int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx = \ln|\sin x| + c$

* $\int \csc x \, dx = \int \csc x \cdot \frac{(\csc x + \cot x)}{(\csc x + \cot x)} \, dx$

$= \int \frac{\csc^2 x + \cot x \csc x}{\csc x + \cot x} \, dx = -\ln|\csc x + \cot x| + c$

Subject Date No.

Exponantial function :- الاقتران الاسي

$e^{f(x)}$ → عدد طبيعي

$a^{f(x)}$ → الاساس رقم
والاس اقتران

$(f(x))^{h(x)}$ → اقتران اس
اقتران

* Properties :- خصائص الاسس

1) $a^n \cdot a^m = a^{n+m}$ ← بشرط ان يكون الاساس متساوي

2) $\frac{a^n}{a^m} = a^{n-m}$

3) $(a^n)^m = a^{n \cdot m}$

4) $a^0 = 1$ → عدد اس صفر $= 1$

5) $a^{-n} = \frac{1}{a^n}$

6) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

$\boxed{1}$ $e^{f(x)}$ → Natural Exponential Function :-

1 Derivetive الاشتقاق

$$\boxed{f(x) = e^{h(x)} \longrightarrow f'(x) = h'(x) \cdot e^{h(x)}}$$

Ex find the followings:-

① $f(x) = e^{x^2+3} \rightarrow f'(x) = 2x \cdot e^{x^2+3}$

② $f(x) = e^{x^3-4x+5} \rightarrow f'(x) = (3x^2-4) \cdot e^{x^3-4x+5}$

③ $f(x) = (e^{2x+3})^2 = e^{4x+6} \rightarrow f'(x) = 4 \cdot e^{4x+6}$

④ $f(x) = \sqrt{e^{4x+8}} = (e^{4x+8})^{\frac{1}{2}} = e^{2x+4} \rightarrow f'(x) = 2 \cdot e^{2x+4}$

⑤ $f(x) = \sqrt[3]{e^{3x+12}} \cdot \sqrt{e^{4x+6}} = (e^{3x+12})^{\frac{1}{3}} \cdot (e^{4x+6})^{\frac{1}{2}} = e^{x+4} \cdot e^{2x+3}$

$= e^{3x+7}$

$\rightarrow f'(x) = 3 \cdot e^{3x+7}$

⑥ $f(x) = e^{5x} \cdot \ln|2x| \rightarrow f'(x) = e^{5x} \cdot \frac{2}{2x} + \ln(2x) \cdot 5e^{5x}$

⑦ $f(x) = \frac{e^{x^2} + 3x}{x^3+1} \rightarrow f'(x) = \frac{(x^3+1) \cdot (2x\, e^{x^2} + 3) - (e^{x^2} + 3x) \cdot 3x^2}{(x^3+1)^2}$

2 Integral التكامل

(1) $$\boxed{\int e^{ax+b} \cdot dx = \frac{e^{ax+b}}{a} + c}$$

Ex
① $\int e^{5x+4} \cdot dx = \frac{e^{5x+4}}{5} + c$

② $\int e^{7x-2} \cdot dx = \frac{e^{7x-2}}{7} + c$

③ $\int e^{x} \cdot dx = e^{x} + c$

④ $\int e^{5-4x} \cdot dx = \frac{e^{5-4x}}{-4} + c$

⑤ $\int \sqrt{e^{8x-12}} \cdot dx = \int (e^{8x-12})^{\frac{1}{2}} dx = \int e^{4x-6} \cdot dx$
$= \frac{e^{4x-6}}{4} + c$

(2) $$\boxed{\int f'(x)\, e^{f(x)} \cdot dx = e^{f(x)} + c}$$

Ex
① $\int 2x\, e^{x^2+5} \cdot dx = e^{x^2+5} + c$

② $\int x^2 \cdot e^{x^3+4} \cdot dx = \frac{1}{3} e^{x^3+4} + c$

Subject Date No.

$\boxed{2}$ $a^{f(x)}$

1 Derivetive الاشتقاق

$$\boxed{f(x) = a^{h(x)} \quad \text{then} \quad f'(x) = h'(x) \cdot a^{h(x)} \cdot \ln|a|}$$

Ex ① $f(x) = 5^{x^2} \rightarrow f'(x) = 2x \cdot 5^{x^2} \cdot \ln|5|$

② $f(x) = 7^{x^3+5x-1} \rightarrow f'(x) = (3x^2+5) \cdot 7^{x^3+5x-1} \cdot \ln|7|$

③ $f(x) = 5^{(2x+7)} \cdot 5^{(3x-2)} = 5^{5x+5} \rightarrow f'(x) = 5 \cdot 5^{5x+5} \cdot \ln|5|$

④ $f(x) = 5^{2x+2}$

$\rightarrow f'(x) = \cancel{5}\ 2 \cdot 5^{2x+2} \cdot \ln|5|$

Subject Date No.

$\boxed{3}$ $(f(x))^{g(x)}$

Derivative : الاشتقاق . 3

Ex if $f(x) = x^x$ find $f'(x)$

$$y = x^x \rightarrow \ln y = \ln x^x$$

اشتقاق الضمني $\rightarrow$ $\ln y = x \ln x$

$$\frac{y'}{y} = x \cdot \frac{1}{x} + \ln x$$

$$y' = y(1 + \ln x)$$

$$y' = x^x(1 + \ln x)$$

Ex if $f(x) = (x^2+1)^{x-1}$ find $f'(x)$

$$y = (x^2+1)^{x-1}$$

$$\ln y = \ln (x^2+1)^{x-1}$$

$$\ln y = (x-1) \ln (x^2+1)$$

$$\frac{y'}{y} = (x-1) \cdot \frac{2x}{(x^2+1)} + \ln (x^2+1)$$

$$\rightarrow y' = y\left(\frac{2x(x-1)}{x^2+1} + \ln(x^2+1)\right)$$

$$y' = (x^2+1)^{x-1}\left(\frac{2x(x-1)}{(x^2+1)} + \ln(x^2+1)\right)$$

Remark

$$\log_b a = \frac{\ln a}{\ln b}$$

$$\boxed{\log_5 7}$$

$5 \rightarrow$ الأساس

Ex $f(x) = \log_7 (x^2+5)$ find $f'(x)$

$$f(x) = \frac{\ln (x^2+5)}{\ln(7)}$$

$\leftarrow$ المقام ثابت

$$\rightarrow f'(x) = \frac{\frac{2x}{x^2+5}}{\ln(7)}$$

Subject   Date   No.

Definite Integral :-   تكامل محدود

$$\int_a^b f'(x)\,dx = f(x)\Big]_a^b = f(b) - f(a)$$

Ex find :-

$\boxed{1}$ $\int_1^3 2x\,dx = x^2\Big]_1^3 = 9 - 1 = 8$

$\boxed{2}$ $\int_0^1 (2x + 3x^2 + 6x^3)\,dx = (x^2 + x^3 + \frac{3}{2}x^4)\Big]_0^1$

$= 1 + 1 + \frac{3}{2} = 2 + \frac{3}{2} = \frac{7}{2}$

$\boxed{3}$ $\int_1^{16} \sqrt{x}\,dx = \int_1^{16} (x)^{\frac{1}{2}}\,dx = \frac{2}{3}x^{\frac{3}{2}}\Big]_1^{16}$

$= \frac{2}{3}(\sqrt{x})^3\Big]_1^{16} = \frac{2}{3}\cdot(4)^3 - \frac{2}{3}$

$= \frac{2}{3}\cdot 64 - \frac{2}{3}$

* Properties of Definite Integral : خصائص التكامل المحدود

1) $\boxed{\int_a^a f(x)\,dx = 0}$

Ex [1] $\int_1^1 (5x+3)\,dx = 0$

[2] $\int_7^7 (\sqrt{x} + 4x - 1)\,dx = 0$

[3] $\int_{5a+3}^{9} f(x)\,dx = 0$ find a. $\rightarrow$ $5a+3 = 9$

$-3 \quad -3$

$\frac{5a}{5} = \frac{6}{5} \rightarrow \boxed{a = \frac{6}{5}}$

[4] $\int_{2a+4}^{7a-2} f(x)\,dx = 0$ find a.

$\rightarrow 7a\cancel{-2} = 2a+4$ $\rightarrow 7a = \cancel{2a}+6$

$\cancel{+2} \quad +2$ $\quad -2a \quad \cancel{-2a}$

$\rightarrow 5a = 6 \rightarrow \boxed{a = \frac{6}{5}}$

2) $\boxed{\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx}$

Ex [1] if $\int_1^3 f(x)\,dx = 7$ find $\int_3^1 f(x)\,dx = -7$

[2] if $\int_2^3 5f(x)\,dx = 15$ find $\int_3^2 f(x)\,dx$

$= \frac{5}{5}\int_2^3 f(x)\,dx = \frac{15}{5} \rightarrow \int_2^3 f(x)\,dx = 3$

$\rightarrow \int_3^2 f(x)\,dx = -3$

$\boxed{3}$ if $\int_2^4 (3f(x)-2)\,dx = 7$ find:

① $\int_4^2 f(x)\,dx$

② $\int_2^4 5f(x)\,dx$.

$\Rightarrow \int_2^4 (3f(x)-2)\,dx = 7 \Rightarrow \int_2^4 3f(x)\,dx - \int_2^4 2\,dx = 7$

$\Rightarrow 3\int_2^4 f(x)\,dx - 2(4-2) = 7$

$\Rightarrow 3\int_2^4 f(x)\,dx - \cancel{4} = 7 \Rightarrow \frac{\cancel{3}\int_2^4 f(x)\,dx}{\cancel{3}} = \frac{11}{3}$
$\quad\quad\quad\quad\quad\quad\quad\quad +\cancel{4} \quad\quad +4$

$\rightarrow \int_2^4 f(x)\,dx = \frac{11}{3}$

① $\rightarrow \int_4^2 f(x)\,dx = \frac{-11}{3}$

② $\rightarrow \int_2^4 5f(x)\,dx = 5 \cdot \frac{11}{3} = \frac{55}{3}$

$$3)\ \boxed{\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx}$$

$$\boxed{\int_a^{b} + \int_{b}^{c} = \int_a^c}$$

Ex [1] if $\int_1^3 f(x)\,dx = 5$

$\int_3^7 f(x)\,dx = 9$

find $\int_1^7 f(x)\,dx$

$$\to \int_1^7 f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^7 f(x)\,dx$$

$$= 5 + 9 = \underline{\underline{14}}$$

[2] if $\int_1^3 2f(x)\,dx = 6$

$\int_1^9 (3f(x)+1)\,dx = 10$

find $\int_3^9 f(x)\,dx$

$$\to \int_1^9 (3f(x)+1)\,dx = 10$$

$$\int_1^9 3f(x)\,dx + \int_1^9 1\,dx = 10$$

$$3\int_1^9 f(x)\,dx + 1(9-1) = 10$$

$$-\cancel{8} \qquad -8$$

$$\frac{\cancel{3}\int_1^9 f(x)\,dx}{\cancel{3}} = \frac{2}{3}$$

$$\int_1^9 f(x)\,dx = \frac{2}{3}$$

$$\to \int_1^3 2f(x)\,dx = 6$$

$$\frac{\cancel{2}\int_1^3 f(x)\,dx}{\cancel{2}} = \frac{6}{2}$$

$$\int_1^3 f(x)\,dx = 3$$

$$\to \int_3^1 f(x)\,dx = -3$$

$$\Rightarrow \int_3^9 f(x)\,dx = \int_3^1 f(x)\,dx + \int_1^9 f(x)\,dx = \frac{2}{3} - 3 = \underline{\frac{-7}{3}}$$

$\boxed{3}$ find $\int_{-5}^{3} |2x+4|\, dx$

اعادة تعريف القيمة المطلقة

$2x+4=0$

$2x=-4$

$x=-2$

$\xleftarrow{\quad -(2x+4) \qquad (2x+4) \quad}$

$- \; - \;\; \underset{-2}{|} \;\; + \; +$

$$\Rightarrow = \int_{-5}^{-2} (-2x-4)\,dx + \int_{-2}^{3} (2x+4)\, dx$$

$$= \int_{-5}^{-2} (-2x)\,dx - \int_{-5}^{-2} 4\, dx + \int_{-2}^{3} 2x\,dx + \int_{-2}^{3} 4\, dx$$

$$= -x^2\Big]_{-5}^{-2} - 4(-2+5) + x^2\Big]_{-2}^{3} + 4(3+2)$$

$$= -4 + 25 - 12 + 9 - 4 + 20$$

$$= 2 + 25 + 5 = 32$$

Subject Date No.

Integration by substitution

تكامل باستخدام التعويض
(تكامل بالتعويض)

① $\int (\quad)(\underbrace{\text{non linear}}_{y})^{n}\, dx$

نسميها

Ex find ① $\int (2x+1)(\underbrace{x^2+x+5}_{\text{non-linear}})^9\, dx$

$y = x^2 + x + 5$

$\frac{dy}{dx} = 2x+1 \rightarrow dx = \frac{dy}{(2x+1)}$

$\Rightarrow \int \cancel{(2x+1)} \cdot y^9 \cdot \frac{dy}{\cancel{(2x+1)}} = \int y^9 \cdot dy$

$= \frac{y^{10}}{10} + C = \frac{(x^2+x+5)^{10}}{10} + C$

② $\int x^2 (x^3+2)^7\, dx$

$y = x^3 + 2$

$\frac{dy}{dx} = 3x^2 \longrightarrow dx = \frac{dy}{3x^2}$

$\rightarrow \int \cancel{x^2} (y)^7 \cdot \frac{dy}{3\cancel{x^2}}$

تكملة

$\rightarrow = \frac{1}{3} \int y^7\, dy$

$= \frac{1}{3} \cdot \frac{y^8}{8} + C$

$= \frac{1}{24} \cdot (x^3+2)^8 + C$

③ $\int \frac{(x^3+5)}{\sqrt[3]{x^4+20x}}\,dx = \int (x^3+5)(x^4+20x)^{-\frac{1}{3}}\,dx$

$y = x^4+20x$

$\frac{dy}{dx} = 4x^3+20 \rightarrow dx = \frac{dy}{4(x^3+5)}$

$\Rightarrow \int \cancel{(x^3+5)} \cdot y^{-\frac{1}{3}} \frac{dy}{4\cancel{(x^3+5)}} = \frac{1}{4}\int y^{-\frac{1}{3}}\,dy.$

$= \frac{1}{4} \cdot \frac{3}{2} \cdot y^{\frac{2}{3}} + c = \frac{3}{8} \cdot (x^4+20x)^{\frac{2}{3}} + c$

④ $\int x^5 \sqrt{x^3+4}\,dx = \int x^5 (x^3+4)^{\frac{1}{2}}\,dx$

$y = x^3+4 \rightarrow x^3 = y-4$

$\frac{dy}{dx} = 3x^2 \rightarrow dx = \frac{dy}{3x^2}$

$\Rightarrow \int x^3 \cdot \cancel{x^2} \cdot y^{\frac{1}{2}} \cdot \frac{dy}{3\cancel{x^2}} = \frac{1}{3}\int (y-4) \cdot y^{\frac{1}{2}} \cdot dy.$

$= \frac{1}{3}\int (y^{\frac{3}{2}} - 4y^{\frac{1}{2}})\,dy. = \frac{1}{3}\left(\frac{2}{5}y^{\frac{5}{2}} - \frac{2}{3}.4\,y^{\frac{3}{2}}\right) + c$

$= \frac{1}{3}\left(\frac{2}{5}(x^3+4)^{\frac{5}{2}} - \frac{8}{3}(x^3+4)^{\frac{3}{2}}\right) + c$

(5) $\int \frac{x^3}{\sqrt{1+x^2}}\, dx = \int x^3 (1+x^2)^{-\frac{1}{2}}\, dx$

$y = 1 + x^2 \rightarrow x^2 = y - 1$

$\frac{dy}{dx} = 2x$

$dx = \frac{dy}{2x} \Rightarrow \int x^2 \cdot \cancel{x} \cdot y^{-\frac{1}{2}} \cdot \frac{dy}{2\cancel{x}}$

$= \frac{1}{2}\int (y-1) \cdot y^{-\frac{1}{2}} \cdot dy.$

$= \frac{1}{2}\int (y^{\frac{1}{2}} - y^{-\frac{1}{2}})\, dy.$

$= \frac{1}{2}\left(\frac{2}{3} y^{\frac{3}{2}} + 2 y^{\frac{1}{2}}\right) + C$

$= \frac{1}{2}\left(\frac{2}{3}(1+x^2)^{\frac{3}{2}} + 2(1+x^2)^{\frac{1}{2}}\right) + C$

$= \frac{(1+x^2)^{\frac{3}{2}}}{3} + (1+x^2)^{\frac{1}{2}} + C$

(2) Sin, cos, tan, ...

إذا كانت الزاوية غير خطية (non-linear)

نفرض الزاوية.

Ex find ① $\int x \sin(x^2+3)\, dx$

$y = x^2 + 3$

$\frac{dy}{dx} = 2x \rightarrow dx = \frac{dy}{2x}$

$\rightarrow \int \cancel{x} \sin y \cdot \frac{dy}{2\cancel{x}} = \frac{1}{2}\int \sin y \, dy.$

$= \frac{1}{2} \cdot -\cos y + c$

$= \frac{-\cos(x^2+3)}{2} + c$

② $\int \frac{x \sec^2\sqrt{x^2+1}}{\sqrt{x^2+1}}\, dx$

$y = \sqrt{x^2+1}$

$\frac{dy}{dx} = \frac{\cancel{2}x}{\cancel{2}y} \rightarrow dx = \frac{y\,dy}{x}$

$\rightarrow \int \frac{\cancel{x} \sec^2 y}{\cancel{y}} \cdot \frac{\cancel{y}}{\cancel{x}}\, dy = \int \sec^2 y \, dy$

$= \tan y + c$

$= \tan\sqrt{x^2+1} + c$

③ $\int \frac{1}{x^2} \sec^2 \frac{1}{x} \, dx$

$y = \frac{1}{x}$

$\frac{dy}{dx} = \frac{-1}{x^2} \quad \rightarrow dx = -x^2 dy.$

$\rightarrow \int \frac{1}{\cancel{x^2}} \cdot \sec^2 y \cdot -\cancel{x^2} dy$

$= \int -\sec^2 y \, dy \quad = -\tan y + c$

$= -\tan(\frac{1}{x}) + c$

---

③ $\int (\sin x)^m \cos x \, dx$

فردي فردي ← الأكبر

فردي زوجي ← زوجي

زوجي وزوجي ← متطابقات

Ex ① $\int \sin x \cos^4 x \, dx$

$y = \cos x$

$\frac{dy}{dx} = -\sin x \quad \rightarrow dx = \frac{-dy}{\sin x}$

$\rightarrow \int \cancel{\sin x} \cdot y^4 \cdot \frac{-dy}{\cancel{\sin x}} \quad = -1 \int y^4 \, dy.$

$= \frac{-y^5}{5} + c \quad = \frac{-(\cos^5 x)}{5} + c$

Subject Date No.

② $\int \tan x \sec^2 x \, dx$

الطريقة 1	الطريقة 2
$y = \tan x$	$y = \sec x$
$\frac{dy}{dx} = \sec^2 x \rightarrow dx = \frac{dy}{\sec^2 x}$	$\frac{dy}{dx} = \sec x \tan x \rightarrow dx = \frac{dy}{\sec x \tan x}$
$\rightarrow \int y \cdot \cancel{\sec^2 x} \cdot \frac{dy}{\cancel{\sec^2 x}}$	$\rightarrow \int \cancel{\tan x \sec x} \cdot y \cdot \frac{dy}{\cancel{\sec x \tan x}}$
$= \frac{y^2}{2} + c = \frac{\tan^2 x}{2} + c$	$= \int y \, dy = \frac{y^2}{2} + c$
	$= \frac{\sec^2 x}{2} + c$

التكامل يعطي أكثر من إجابة في بعض الاحيان مثل هذا سؤال.

③ $\int \cos^3 x \sin^4 x \, dx$ ← نفرض الزوجي

$y = \sin x$

$\frac{dy}{dx} = \cos x \rightarrow dx = \frac{dy}{\cos x}$

$\rightarrow \int \cancel{\cos x} \cdot \cos^2 x \cdot y^4 \cdot \frac{dy}{\cancel{\cos x}}$ $\boxed{\sin^2 x + \cos^2 x = 1}$

$\int (1 - \underbrace{\sin^2 x}_{y^2}) \cdot y^4 \, dy = \int (y^4 - y^6) \, dy = \frac{y^5}{5} - \frac{y^7}{7} + c$

$= \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + c$

NOTEBOOK

Ex ① $\int \frac{(2x+3)^9}{x^{11}}\,dx$

$= \int \frac{(2x+3)^9}{x^9 \cdot x^2}\,dx = \int \left(\frac{2x+3}{x}\right)^9 \cdot \frac{1}{x^2} \cdot dx$

$= \int (2+\frac{3}{x}) \cdot \frac{1}{x^2}\,dx$

$y = 2 + \frac{3}{x}$

$\frac{dy}{dx} = \frac{-3}{x^2} \rightarrow dx = \frac{x^2\,dy}{-3}$

$\rightarrow \int (y)^9 \cdot \frac{1}{\cancel{x^2}} \cdot \frac{\cancel{x^2}\,dy}{-3} = \frac{-1}{3} \int y^9\,dy$

$= \frac{-1}{3} \cdot \frac{y^{10}}{10} + C = \frac{-(2+\frac{3}{x})^{10}}{30} + C$

② $\int \frac{x}{1+x\tan x}\,dx$

$= \int \frac{x}{1+x\frac{\sin x}{\cos x}}\,dx = \int \frac{x}{\frac{\cos x + x\sin x}{\cos x}}\,dx$

$= \int \frac{x \cdot \cos x}{\cos x + x\sin x}\,dx$

$u = \cos x + x\sin x$

$\frac{du}{dx} = \cancel{-\sin x} + x\cos x + \cancel{\sin x}$

$dx = \frac{du}{x\cos x}$

$\rightarrow \int \frac{\cancel{x\cos x}}{u} \cdot \frac{du}{\cancel{x\cos x}}$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \ln|\cos x + x\sin x| + C$

$$③ \int \frac{\ln\sqrt{x+5}}{x+5}\, dx$$ (الاس الخطي بالاجزاء)

$$\int \frac{\ln(x+5)^{\frac{1}{2}}}{x+5}\, dx = \int \frac{\frac{1}{2}\ln(x+5)}{x+5}\, dx$$

$u = \ln(x+5)$ ← نعوض ln كاملة

$$\frac{du}{dx} = \frac{1}{x+5} \rightarrow dx = du\,(x+5)$$

$$\rightarrow \int \frac{\frac{1}{2}u}{\cancel{x+5}} \cdot \cancel{(x+5)}\, du$$

$$= \frac{1}{2}u\, du = \frac{u^2}{4} + c = \frac{(\ln(x+5))^2}{4} + c$$

$$④ \int (x^2+4)\, e^{x^3+12x}\, dx$$ ← اس غير خطي بالتعويض

$y = x^3 + 12x$

$$\frac{dy}{dx} = 3x^2 + 12 \rightarrow dx = \frac{dy}{3(x^2+4)}$$

$$\rightarrow \int \cancel{(x^2+4)}\, e^y \cdot \frac{dy}{3\cancel{(x^2+4)}}$$

$$\frac{1}{3}\int e^y \cdot dy = \frac{1}{3}e^y + c = \frac{1}{3}e^{x^3+12x} + c$$

Subject Date No.

Integration by part تكامل بالاجزاء

Remark :-

$\boxed{*}$ $\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a \cdot (n+1)} + c$

Ex ① $\int (2x+1)^5 \, dx = \frac{(2x+1)^6}{12} + c$

② $\int (3-2x)^7 \, dx = \frac{(3-2x)^8}{-16} + c$

③ $\int (\sqrt[3]{4x+2}) \, dx = \int (4x+2)^{\frac{1}{3}} \, dx$

$= \frac{3(4x+2)^{\frac{4}{3}}}{16}$

$\boxed{\text{II}}$ $\int (f(x)) (ax+b)^n \, dx$

Ex →

Ex ① $\int \underbrace{2x}_{u} \, \underbrace{(5x+7)^8 \, dx}_{dv}$

الطريقة (1)

$u = 2x \qquad + \qquad \int dv = \int (5x+7)^8 \, dx$

$du = 2 \, dx \qquad - \qquad v = \dfrac{(5x+7)^9}{45}$

$$= \frac{2x \cdot (5x+7)^9}{45} - \int \frac{2(5x+7)^9}{45} \, dx$$

$$= \frac{2 \cdot (5x+7)^9}{45} - \frac{2}{45} \cdot \frac{(5x+7)^{10}}{50} + C$$

الطريقة (2)

$\int \underset{\sim}{2x} \, \underset{\sim}{(5x+7)^8} \, dx$

تنجح هذه الطريقة في حالة كانت المشتقة
قادر على ايصالها الى الصفر

نشتق		نكامل
$2x$	$+$	$(5x+7)^8$
2	$-$	$\dfrac{(5x+7)^9}{45}$
0		$\dfrac{(5x+7)^{10}}{45 \cdot 50}$

$$= \frac{2x(5x+7)^9}{45} - \frac{2(5x+7)^{10}}{45 \cdot 50} + C$$

② $\int 3x^2 (x+5)^4 \, dx$

حل باستخدام الطريقة ج

$\int 3x^2 (x+5)^4 \, dx$

تفاضل		تكامل
		$(x+5)^4$
$3x^2$	$+$	$\frac{(x+5)^5}{5}$
$6x$	$-$	$\frac{(x+5)^6}{5 \cdot 6}$
6	$+$	$\frac{(x+5)^7}{5 \cdot 6 \cdot 7}$
0		

$$= \frac{3x^2 (x+5)^5}{5} - \frac{6x(x+5)^6}{5 \cdot 6} + \frac{6(x+5)^7}{5 \cdot 6 \cdot 7} + c$$

Remark :-

$$\int \sin(\underbrace{ax+b}_{\text{خطي}}) \, dx = \frac{-\cos(ax+b)}{a} + c$$

$$\int \cos(\underbrace{ax+b}_{\text{خطي}}) \, dx = \frac{\sin(ax+b)}{a} + c$$

Ex ① $\int \sin(5x+3) \, dx = \frac{-\cos(5x+3)}{5} + c$

② $\int \cos(2x) \, dx = \frac{\sin 2x}{2} + c$

③ $\int \sec^2(4x) \, dx = \frac{\tan(4x)}{4} + c$

$\boxed{2}$ $\int f(x) \cdot \sin(ax+b)\, dx$

$\int f(x) \cdot \cos(ax+b)\, dx$

$\vdots$

Ex find

① $\int 4x \sin 3x \, dx$

مشتق		تكامل
$4x$	$+$	$\sin 3x$
4	$-$	$\frac{-\cos 3x}{3}$
0		$\frac{-\sin 3x}{9}$

$$= \frac{-4x\cos 3x}{3} + \frac{4\sin 3x}{9} + C$$

② $\int (5x+3) \cos 3x \, dx$

مشتق		تكامل
$5x+3$	$+$	$\cos 3x$
5	$-$	$\frac{\sin 3x}{3}$
0		$\frac{-\cos 3x}{9}$

$$= \frac{(5x+3)\sin 3x}{3} + \frac{5\cos 3x}{9} + C$$

Subject　　　　Date　　　　No.

Remark:

$$\int e^{ax+b}\,dx = \frac{e^{ax+b}}{a} + c$$

Ex $\int e^{5x+3}\,dx = \frac{e^{5x+3}}{5} + c$

$\int e^{-2x}\,dx = \frac{e^{-2x}}{-2} + c$

$\boxed{3}$ $\int \underbrace{f(x)}_{u}\ \underbrace{e^{ax+b}\,dx}_{dv}$

Ex ① $\int 2x\ e^{5x+3}\,dx$

مشتق		تكامل
$2x$	$+$	e^{5x+3}
2	$-$	$\frac{e^{5x+3}}{5}$
0		$\frac{e^{5x+3}}{25}$

$$= \frac{2x\,e^{5x+3}}{5} - \frac{2\,e^{5x+3}}{25} + c$$

Subject Date No.

② $\int \underbrace{\ln x}_{u} \; \underbrace{dx}_{v}$ ← هالا تنفع الطريقة ؟

$u = \ln x$ $\quad + \quad$ $\int dv = \int 1 \, dx$

$du = \frac{1}{x} \, dx$ $\quad - \quad$ $V = x$

$= x \ln x - \int x \cdot \frac{1}{x} \, dx \qquad = x \ln x - x + c$

③ $\int \underbrace{x}_{v} \; \underbrace{\ln x}_{u} \; dx$

$u = \ln x$ $\qquad \int dv = \int x \, dx$

$du = \frac{1}{x} \, dx$ $\qquad V = \frac{x^2}{2}$

$= \frac{x^2}{2} \ln x - \int \frac{x^{\cancel{2}}}{2} \cdot \frac{1}{\cancel{x}} \, dx$

$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + c$

Subject Date No.

④ $\int \frac{\ln x}{x^2} dx$ H.W

$= \int \underbrace{x^{-2}}_{v} \underbrace{\ln x}_{u} dx$

$u = \ln x$ $\int dv = \int x^{-2} dx$

$du = \frac{1}{x} dx$ $v = -x^{-1}$

$= -\frac{\ln x}{x} + \int x^{-1} \cdot \frac{1}{x} \cdot dx$

$= \frac{-\ln x}{x} + \int x^{-2} dx$ $= \frac{-\ln x}{x} + \frac{x^{-1}}{-1} + c$

$= \frac{-\ln x}{x} - \frac{1}{x} + c$

Subject Date No.

* Remark :

$$\int \frac{\ln x}{x^n}\, dx \rightarrow$$ تكل بالاجزاء

but $$\int \frac{\ln x}{x}\, dx \rightarrow$$ تكل بالتعويض

Ex $$\int \frac{\ln x}{x}\, dx$$

$$y = \ln x \rightarrow \frac{dy}{dx} = \frac{1}{x} \Rightarrow x\,dy = dx$$

$$\Rightarrow \int \frac{y}{\cancel{x}} \cdot \cancel{x}\,dy = \int y\, dy = \frac{y^2}{2} + c$$

$$= \frac{(\ln x)^2}{2} + c$$

Subject Date No.

هناك بعض الامثلة تحل بالتعويض أولاً ثم بالاجزاء

Ex ① $\int \sin \underbrace{\sqrt{x}}_{\text{غير خطي}} \, dx$

$y = \sqrt{x} \rightarrow dx = 2y\,dy$

$= \int \sin y \cdot 2y\,dy \quad = \int \underbrace{2y}_{u} \, \underbrace{\sin y \, dy}_{dv}$

نشتق		نكامل
$2y$	$+$	$\sin y$
2	$-$	$-\cos y$
0		$-\sin y$

$= -2y\cos y + 2\sin y + C$

$= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$

Subject Date No.

② $\int e^{\sqrt{x+2}}\,dx$ ← الاس غير خطي

$y = \sqrt{x+2} \rightarrow dx = 2y\,dy$

$= \int e^{y} \cdot 2y\,dy \quad = \int \underbrace{2y}_{u}\, \underbrace{e^{y} dy}_{dv}$

مشتق	تكامل
$2y$	e^{y}
2	e^{y}
0	e^{y}

$= 2y e^{y} - 2e^{y} + C$

$= 2\sqrt{x+2}\, e^{\sqrt{x+2}} - 2 e^{\sqrt{x+2}} + C$

Subject Date No.

③ $\int x^5 \cos x^3 \, dx$

$y = x^3 \rightarrow dx = \dfrac{dy}{3x^2}$

$\rightarrow \int x^3 \cdot \cancel{x^2} \cdot \cos y \cdot \dfrac{dy}{3\cancel{x^2}} = \frac{1}{3}\int \underbrace{y}_{u} \underbrace{\cos y \, dy}_{dv}$

مشتق	تكامل
$\frac{1}{3}y$ $+$	$\cos y$
$\frac{1}{3}$ $-$	$\sin y$
0	$-\cos y$

$= \frac{1}{3} y \sin y + \frac{1}{3} \cos y + C$

$= \frac{1}{3} x^3 \cdot \sin x^3 + \frac{1}{3} \cos x^3 + C$

Subject Date No.

(4) $\int e^{x^3+\ln x^2}\,dx$

$$\boxed{e^{\ln x} = x} \qquad \boxed{\ln(e)^x = x}$$

$= \int e^{x^3} \cdot e^{\ln x^2}\,dx$

$= \int x^2 e^{x^3}\,dx$

$y = x^3 \rightarrow \frac{dy}{dx} = 3x^2 \longrightarrow dx = \frac{dy}{3x^2}$

$\rightarrow \int \cancel{x^2} \cdot e^y \cdot \frac{dy}{3\cancel{x^2}} \quad = \frac{1}{3}\int e^y\,dy = \frac{1}{3} e^y + c$

$= \frac{1}{3} e^{x^3} + c$

Subject Date No.

Ex $\int \underset{u}{e^x} \underbrace{\cos x \, dx}_{dv}$ تكامل دوار

$u = e^x \qquad \int dv = \int \cos x \, dx$

$du = e^x \, dx \qquad V = \sin x$

$= e^x \sin x - \int \underset{u}{e^x} \underbrace{\sin x \, dx}_{dv}$

$u = e^x \qquad \int dv = \int \sin x \, dx$

$du = e^x \, dx \qquad V = -\cos x$

$= e^x \sin x - \left(-e^x \cos x + \underbrace{\int e^x \cos x \, dx}_{I}\right)$

نفس السؤال

$I = e^x \sin x + e^x \cos x - \underbrace{\int e^x \cos x \, dx}_{I}$

$\frac{1}{\cancel{2}} \times \cancel{2}I = (e^x \sin x + e^x \cos x) \times \frac{1}{2}$

$I = \frac{1}{2}(e^x \sin x + e^x \cos x)$

Subject Date No.

* $\int \underbrace{e^x}_{u} \underbrace{\sin x}_{dv}\, dx$ **H.W**

$u = e^x \qquad \int dv = \int \sin x\, dx$

$du = e^x \qquad v = -\cos x$

$I = -e^x \cos x + \int \underbrace{e^x}_{u} \underbrace{\cos x\, dx}_{dv}$

$u = e^x \qquad \int dv = \int \cos x\, dx$

$du = e^x \qquad v = \sin x$

$I = -e^x \cos x + e^x \sin x - \underbrace{\int e^x \sin x\, dx}_{\text{السؤال نفسه}}$

$\frac{2I}{2} = \frac{(-e^x \cos x + e^x \sin x)}{2}$

$\Rightarrow I = \frac{1}{2}(-e^x \cos x + e^x \sin x)$

Subject Date No.

Ex $\int \cos(\underline{\ln x})\, dx$

غير خطي

$y = \ln x \qquad dx = x\, dy$

$e^y = e^{\ln x} \rightarrow e^y = x$

$\rightarrow \int \cos y \cdot x \cdot dy = \int e^y \cdot \cos y \cdot dy$

تحل نفس المثالين السابقين (تكامل دوار)

$* \int \sin(\underline{\ln x})\, dx$ H.W

$y = \ln x \rightarrow dx = x\, dy$

$\rightarrow e^y = e^{\ln x} \Rightarrow e^y = x$

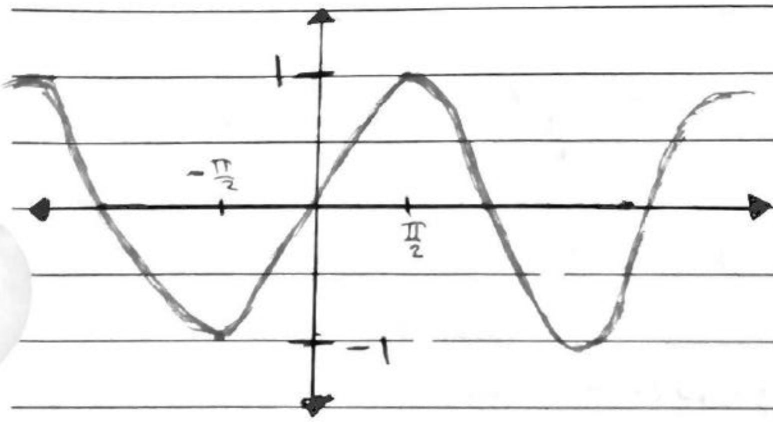
$\rightarrow \int \sin y \cdot x \cdot dy = \int \sin y \cdot e^y \cdot dy$

تكملة الحل (تكامل بالاجزاء) (تكامل دوار)

Subject   Date   No.

Inverse of Trigonometric function :-

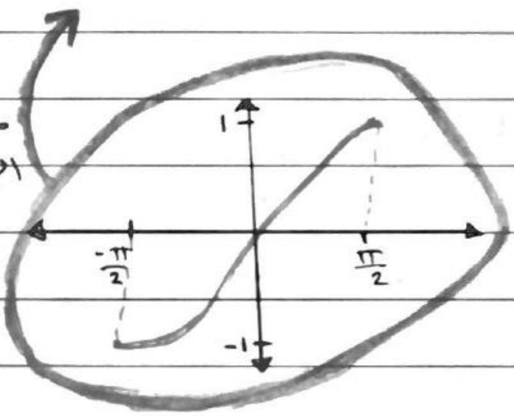
1 Sin X :-



(not (1-1) function)

<u>Dom</u>		<u>Range</u>
$\mathbb{R}$	$\longrightarrow$	$[-1,1]$

جزء من الرسمة



$\sin x \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1,1]$

(one-one function)

$\sin^{-1} x : [-1,1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Dom → Range ال بحول ← (-1)
Range → Dom الو

Subject Date No.

* $\sin^{-1} x = \text{arc}\sin x$

* $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$

مشتقة
وعكسها التكامل

* $(\sin^{-1}(g(x)))' = \dfrac{g'(x)}{\sqrt{1-(g(x))^2}}$

$\int \dfrac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

* $\int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + C$

Ex

① $(\sin^{-1}(x^3))' = \dfrac{3x^2}{\sqrt{1-(x^3)^2}}$

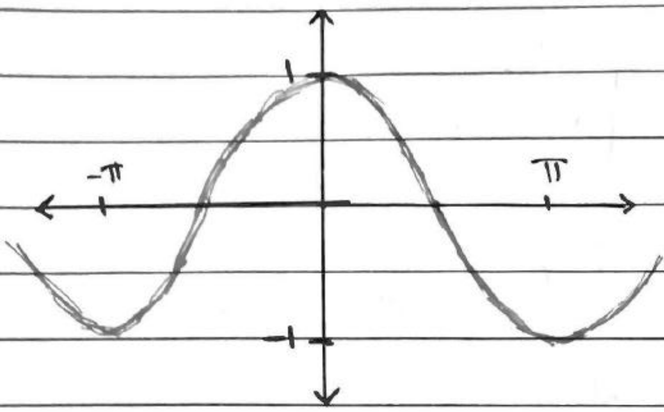
② $(\sin^{-1}(3x^4))' = \dfrac{12x^3}{\sqrt{1-(3x^4)^2}}$

③ $\int \dfrac{1}{\sqrt{4-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{2}\right) + C$

④ $\int \dfrac{1}{\sqrt{3-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{\sqrt{3}}\right) + C$

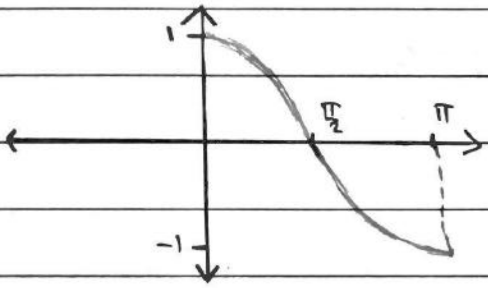
Subject Date No.

$\boxed{2}$ $\cos x$:-



$\mathbb{R} \to [-1,1]$

not one-one function



$\cos x : [0,\pi] \to [-1,1]$

one-one function

$\cos^{-1} x : [-1,1] \to [0,\pi]$

Subject Date No.

$* \cos^{-1} x = \arc \cos x$

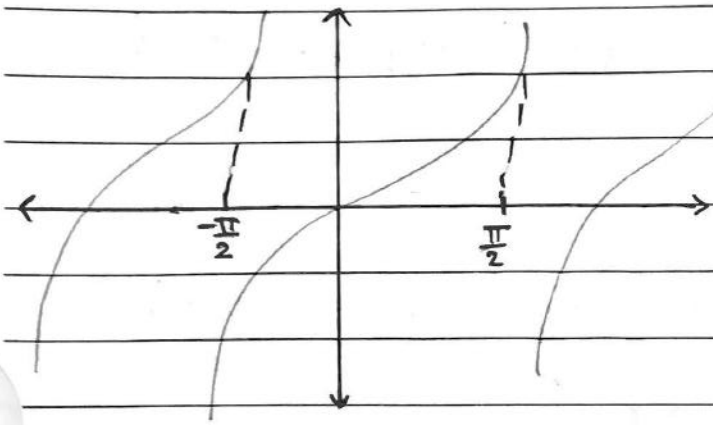
$* (\cos^{-1} x)' = \dfrac{-1}{\sqrt{1-x^2}} = -(\sin^{-1} x)'$

$* (\cos^{-1}(g(x)))' = \dfrac{-g'(x)}{\sqrt{1-(g(x))^2}}$

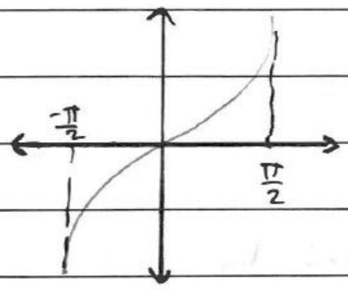
Ex $(\cos^{-1}(2x^3))' = \dfrac{-6x^2}{\sqrt{1-(2x^3)^2}}$

← تكامله مثل تكامل $\sin^{-1}$ ويكون الناتج $-\sin^{-1}$

$\boxed{3}$ $\tan x$:-



not one-one function



one-one function

$\tan x : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$

$\tan^{-1} x : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$* \ (\tan^{-1} x)' = \dfrac{1}{1+x^2}$

$* \ (\tan^{-1} g(x))' = \dfrac{g'(x)}{1+(g(x))^2}$

تكامل

$* \displaystyle\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + c$

$* \displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

Ex $\displaystyle\int \frac{1}{7+x^2}\,dx = \frac{1}{\sqrt{7}} \cdot \tan^{-1}\left(\frac{x}{\sqrt{7}}\right) + c$

Subject Date No.

$\boxed{4}$ Sec x :-

Dom $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi] \rightarrow$ Range $(-\infty, -1] \cup [1, \infty)$

(1-1) function

$\sec^{-1} x : (-\infty, -1] \cup [1, \infty) \rightarrow [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

* $(\sec^{-1} x)' = \frac{1}{|x| \sqrt{x^2 - 1}}$

* $(\sec^{-1}(g(x)))' = \frac{g'(x)}{|g(x)| \sqrt{(g(x))^2 - 1}}$

* $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1}\left(\frac{|x|}{a}\right) + c$

Ex ① $\frac{d}{dx} (\sec^{-1}(\ln x)) = \frac{\frac{1}{x}}{|\ln x| \sqrt{(\ln x)^2 - 1}}$

② $\int \frac{dx}{x\sqrt{x^2 - 1}} = \sec^{-1}(|x|) + c$

③ $\int \frac{dx}{x\sqrt{x^2 - 4}} = \frac{1}{2} \sec^{-1}\left(\frac{|x|}{2}\right) + c$

Subject Date No.

Remark :-

① $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}} = -\frac{d}{dx}(\sin^{-1}x)$

② $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2} = -\frac{d}{dx}(\tan^{-1}x)$

③ $\frac{d}{dx}(\csc^{-1}x) = \frac{-1}{|x|\sqrt{x^2-1}} = -\frac{d}{dx}(\sec^{-1}x)$

Subject Date No.

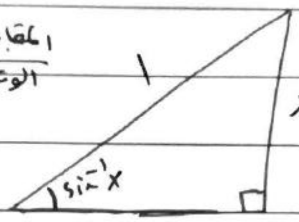
كما لو انها زاوية

Remark :- $\underbrace{\sin(\sin^{-1}x)}$ $= x$

$\underbrace{\cos(\cos^{-1}x)} = x$

$\sin(\underbrace{\sin^{-1}x}_{\text{الزاوية}}) = x$

$\sin\theta = \dfrac{\text{المقابل}}{\text{الوتر}}$



هذا الضلع ← a

باستخدام قانون فيثاغورس

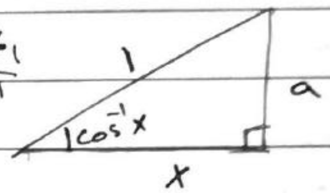
$(1)^2 = x^2 + a^2$

$a^2 = 1 - x^2$

$a = \sqrt{1-x^2}$

$\cos(\underbrace{\cos^{-1}x}_{\text{الزاوية}}) = x$

$\cos\theta = \dfrac{\text{المجاور}}{\text{الوتر}}$



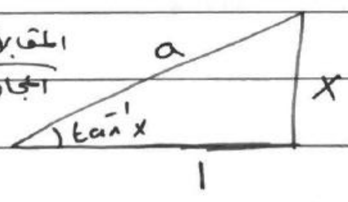
$(1)^2 = x^2 + a^2$

$a^2 = 1 - x^2$

$a = \sqrt{1-x^2}$

$\tan(\tan^{-1}x) = x$

$\tan\theta = \dfrac{\text{المقابل}}{\text{المجاور}}$



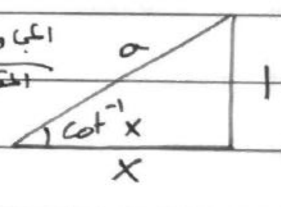
$a^2 = 1^2 + x^2$

$a^2 = 1 + x^2$

$\rightarrow a = \sqrt{1+x^2}$

$\cot(\cot^{-1}x) = x$

$\cot\theta = \dfrac{\text{المجاور}}{\text{المقابل}}$



$a^2 = x^2 + 1^2$

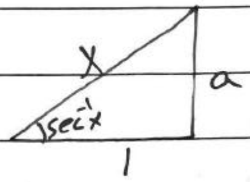
$a^2 = x^2 + 1$

$\rightarrow a = \sqrt{1+x^2}$

Subject Date No.

$\sec(\sec^{-1} x) = x$

$\sec\theta = \frac{\text{الوتر}}{\text{المجاور}}$



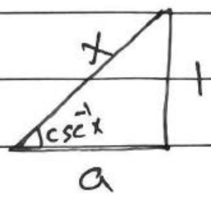
$x^2 = a^2 + 1^2$

$a^2 = x^2 - 1$

$\rightarrow a = \sqrt{x^2-1}$

$\csc(\csc^{-1} x) = x$

$\csc\theta = \frac{\text{الوتر}}{\text{المجاور}}$



$x^2 = 1^2 + a^2$

$a^2 = x^2 - 1$

$\rightarrow a = \sqrt{x^2-1}$

Subject Date No.

Hyperbolic functions :- الاقترانات الزائدية

[1] $\sinh x = \frac{e^{x} - e^{-x}}{2}$ ← اقتران زائدي لـ sin

Dom = $\mathbb{R}$ / Range = $\mathbb{R}$

[2] $\cosh x = \frac{e^{x} + e^{-x}}{2}$

Dom = $\mathbb{R}$ / Range = $[0, \infty)$

* $(\sinh x)' = \cosh x$

* $(\cosh x)' = \sinh x$

* $\int \sinh x \, dx = \cosh x + c$

* $\int \cosh x \, dx = \sinh x + c$

$\boxed{3}$ $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$Dom = \mathbb{R}$

$\boxed{4}$ $\coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

$Dom = \mathbb{R} - \{0\}$

$\boxed{5}$ $\operatorname{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}}$

$Dom = \mathbb{R}$

$\boxed{6}$ $\operatorname{csch} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}}$

$Dom = \mathbb{R} - \{0\}$

Remark :-

* $(\tanh x)' = \operatorname{sech}^2 x$

* $(\coth x)' = -\operatorname{csch}^2 x$

* $(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x$

* $(\operatorname{csch} x)' = -\operatorname{csch} x \coth x$

فردي * $\sinh(-x) = -\sinh x \rightarrow$ odd

زوجي * $\cosh(-x) = \cosh x \rightarrow$ even

Subject Date No.

متطابقات

① $\cosh^2 x - \sinh^2 x = 1$

② $1 - \tanh^2 x = \operatorname{sech}^2 x$

③ $\coth^2 x - 1 = \operatorname{csch}^2 x$

④ $\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y.$

⑤ $\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y.$

Ex ① $\int \coth 5x \, dx$

$$= \int \frac{\cosh 5x}{\sinh 5x} \, dx \qquad u = \sinh 5x$$

$$\frac{du}{dx} = 5 \cosh 5x$$

$$\rightarrow \int \frac{\cancel{\cosh 5x}}{u} \cdot \frac{du}{5\cancel{\cosh 5x}} = \frac{1}{5} \ln|u| + c$$

$$= \frac{1}{5} \ln|\sinh 5x| + c$$

② $\int \frac{\cosh x}{2 + 3\sinh x} \, dx$ **H.W**

$u = 2 + 3\sinh x$

$$\frac{du}{dx} = 3\cosh x \Rightarrow \int \frac{\cancel{\cosh x}}{u} \cdot \frac{du}{3\cancel{\cosh x}} = \int \frac{1}{3u} \, du$$

$$= \frac{1}{3} \ln|u| + c = \frac{1}{3} \ln|2 + 3\sinh x| + c$$

③ $\int \sinh^3 x \, dx$

$$= \int \sinh^2 x \, \sinh x \, dx \quad \rightarrow \boxed{\cosh^2 x - \sinh^2 x = 1}$$

$$= \int (\cosh^2 x - 1) \sinh x \, dx$$

$u = \cosh x$

$$\frac{du}{dx} = \sinh x \qquad \rightarrow \int (u^2 - 1) \cancel{\sinh x} \frac{du}{\cancel{\sinh x}} = \int (u^2 - 1) \, du$$

$$= \frac{u^3}{3} - u + c = \frac{\cosh^3 x}{3} - \cosh x + c$$

④ $\int e^{x} \cdot \sinh x \, dx$

$= \int e^{x} \cdot \left(\frac{e^{x} - e^{-x}}{2} \right) dx = \frac{1}{2} \int e^{x} (e^{x} - e^{-x}) \, dx$

$= \frac{1}{2} \int (e^{2x} - e^{0}) \, dx = \frac{1}{2} \int (e^{2x} - 1) \, dx$

$= \frac{1}{2} \left(\frac{e^{2x}}{2} - x \right) + c$

⑤ $\int \operatorname{csch}(\ln x) \, dx$

$\boxed{\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^{x} - e^{-x}}}$

$\rightarrow = \int \frac{2}{e^{\ln x} - e^{-\ln x}} \, dx = \int \frac{2}{x - \frac{1}{x}} \, dx$

$= \int \frac{2}{\frac{x^2 - 1}{x}} \, dx = \int \frac{2x}{x^2 - 1} \, dx$

$= \ln |x^2 - 1| + c$

Subject Date No.

Remark :

* $\sinh^{-1} x = \ln\left(x + \sqrt{1+x^2}\right)$

* $\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$, $x \geq 1$

* $\tanh^{-1} x = \frac{1}{2} \ln\left|\frac{1+x}{1-x}\right|$, $|x| < 1$

Ex $f(x) = \sinh^{-1} x \rightarrow f'(x) = \frac{1}{\sqrt{x^2+1}}$

$f(x) = \cosh^{-1} x \rightarrow f'(x) = \frac{1}{\sqrt{x^2-1}}$

$f(x) = \tanh^{-1} x \rightarrow f'(x) = \frac{1}{1-x^2}$

Ex find $\int \underbrace{\sinh^{-1} x}_{u} \, \underbrace{dx}_{dv}$ ← بالأجزاء

$u = \sinh^{-1} x$ $\qquad \int dv = \int 1 \, dx$

$du = \frac{1}{\sqrt{x^2+1}}$ $\qquad v = x$

$= x \sinh^{-1} x - \int \frac{x}{\sqrt{x^2+1}} dx \qquad = x \sinh^{-1} x - \int x (x^2+1)^{-\frac{1}{2}} dx$

$y = x^2+1$ $\qquad \rightarrow = x \sinh^{-1} x - \int \cancel{x} (y)^{-\frac{1}{2}} \cdot \frac{dy}{2\cancel{x}}$

$\frac{dy}{dx} = 2x$

$= x \sinh^{-1} x - \frac{1}{\cancel{2}} \cdot y^{\frac{1}{2}} \cdot \cancel{2} + c$

$= x \sinh^{-1} x - y^{\frac{1}{2}} + c$

$= x \sinh^{-1} x - (x^2+1)^{\frac{1}{2}} + c$

H.W ① $\int \cosh^{-1} x \, dx$

② $\int \tanh^{-1} x \, dx$

① $\int \underbrace{\cosh^{-1} x}_{u} \; \underbrace{dx}_{dv}$

$u = \cosh^{-1} x$ $\qquad \int dv = \int 1 \, dx$

$du = \frac{1}{\sqrt{x^2-1}}$ $\qquad v = x$

$= x\cosh^{-1} x - \int \frac{x}{\sqrt{x^2-1}} \, dx = x\cosh^{-1} x - \int x \underbrace{(x^2-1)}_{\text{فرض}}{}^{-\frac{1}{2}} \, dx$

$y = x^2 - 1$

$\frac{dy}{dx} = 2x \quad \rightarrow \quad = x\cosh^{-1} x - \int \cancel{x} \cdot y^{-\frac{1}{2}} \cdot \frac{dy}{2\cancel{x}}$

$= x\cosh^{-1} x - \frac{1}{2} \int y^{-\frac{1}{2}} \, dy \quad = x\cosh^{-1} x - \frac{1}{\cancel{2}} y^{\frac{1}{2}} \cdot \cancel{2} + c$

$= x\cosh^{-1} x - (x^2-1)^{\frac{1}{2}} + c$.

② $\int \underbrace{\tanh^{-1} x}_{u} \; \underbrace{dx}_{dv}$

$u = \tanh^{-1} x$ $\qquad \int dv = \int 1 \, dx$

$\frac{du}{dx} = \frac{1}{1-x^2}$ $\qquad v = x$

$= x\tanh^{-1} x - \int \frac{x}{1-x^2} \, dx \quad = x\tanh^{-1} x + \frac{1}{2} \ln|1-x^2| + c$

Trignometric function

substitution الفرض

① $\sqrt{a^2 - x^2} \longrightarrow x = a\sin\theta$

② $\sqrt{x^2 - a^2} \longrightarrow x = a\sec\theta$

③ $\sqrt{x^2 + a^2} \longrightarrow x = a\tan\theta$

Ex ① $\int \sqrt{1-x^2}\, dx$

$x = 1 \cdot \sin\theta$
$dx = \cos\theta \, d\theta$ $\rightarrow = \int \sqrt{1-\sin^2\theta} \cdot \cos\theta \, d\theta$

$\boxed{\cos^2\theta = 1 - \sin^2\theta} \rightarrow = \int \sqrt{\cos^2\theta} \cdot \cos\theta \, d\theta$

$= \int \cos\theta \cdot \cos\theta \, d\theta = \int \frac{1}{2}(1 + \cos 2\theta)\, d\theta$

$= \frac{1}{2}\left(\theta + \frac{\sin 2\theta}{2}\right) + c \qquad \Rightarrow x = \sin\theta$

$\rightarrow \sin^{-1} x = \cancel{\sin^{-1}}(\cancel{\sin}\,\theta) = \theta$

ملاحظة : بالتكامل غير المحدود التربيع يذهب مع الجذر، وتبقى القيمة الموجبة فقط. أما في حال التكامل المحدود تصبح القيمة متشعبة.

② $\int \sqrt{x^2 - 9}\; dx$

$x = 3 \sec\theta$

$dx = 3 \sec\theta \tan\theta \, d\theta$

$\rightarrow = \int \sqrt{(3\sec\theta)^2 - 9} \cdot 3 \sec\theta \tan\theta \, d\theta$

$= \int \sqrt{9\sec^2\theta - 9} \cdot 3 \sec\theta \tan\theta \, d\theta$

$= \int \sqrt{9(\sec^2\theta - 1)} \cdot 3 \sec\theta \tan\theta \, d\theta$

$= \int \sqrt{9 \tan^2\theta} \cdot 3 \sec\theta \tan\theta \, d\theta$

$= \int 3\tan\theta \cdot 3\sec\theta \cdot \tan\theta \cdot d\theta$

$= \int 9 \sec\theta \tan^2\theta \, d\theta$

$= 9 \int \sec\theta \, (\sec^2\theta - 1) \, d\theta$

$= 9 \int (\sec^3\theta - \sec\theta) \, d\theta$

$= 9\Big(\underbrace{\int \sec^3\theta \, d\theta}_{\text{أجزاء}} - \underbrace{\int \sec\theta \, d\theta}_{\text{مرافق}} \Big)$

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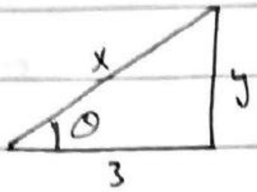
حل آخر لنفس السؤال السابق $\int \sqrt{x^2-9}\, dx$ على طريقة المثلثات

$x = 3\sec\theta$

$\frac{x}{3} = \sec\theta$



$x^2 = 3^2 + y^2$

$x^2 - 9 = y^2$

$y = \sqrt{x^2-9}$

$\rightarrow \tan\theta = \frac{\sqrt{x^2-9}}{3}$

$\rightarrow 3\tan\theta = \sqrt{x^2-9}$

$= \int 3\tan\theta\, dx$

$\hookrightarrow x = 3\sec\theta \rightarrow dx = 3\sec\theta \tan\theta\, d\theta$

$= \int 9\tan^2\theta \sec\theta\, d\theta$

Integral by partial fraction الكسور الجزئية

$\boxed{1}$ $\frac{P(x)}{q(x)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}$ ← إذا كانت درجة البسط أقل من درجة المقام والمقام يحلل بالصورة أولية

Ex ① $\int \frac{1}{x^2+4x-5}\,dx$

$= \int \frac{1}{(x-1)(x+5)}\,dx = \frac{A}{x+5} + \frac{B}{x-1}$

$\frac{1}{(x+5)(x-1)} = \frac{A(x-1) + B(x+5)}{(x+5)(x-1)}$

$\rightarrow 1 = A(x-1) + B(x+5)$

$x=1 \qquad x=-5$

$x=1$	$x=-5$
$1 = 0 + 6B$	$1 = -6A + 0$
$B = \frac{1}{6}$	$A = -\frac{1}{6}$

$\rightarrow = \frac{-1}{6}\ln|x+5| + \frac{1}{6}\ln|x-1| + C$

② $\int \frac{2x-3}{x^2-2x-3}\,dx$ H.W

$= \int \frac{2x-3}{(x-3)(x+1)}\,dx = \frac{A}{(x+1)} + \frac{B}{(x-3)}$

$\frac{2x-3}{(x-3)(x+1)} = \frac{A(x-3)+B(x+1)}{(x+1)(x-3}$

$\rightarrow 2x-3 = A(x-3) + B(x+1)$

$x=3 \qquad x=-1$

$\underline{\underline{x=3}}$ $\qquad$ $\underline{\underline{x=-1}}$

$3 = 0 + 4B \qquad -5 = -4A + 0$

$B = \frac{3}{4} \qquad A = \frac{5}{4}$

$\Rightarrow = \frac{5}{4} \ln|x+1| + \frac{3}{4} \ln|x-3| + C$

③ $\int \frac{5x-3}{x^2-2x-3}\,dx$

$= \int \frac{5x-3}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{x-3}$

$\frac{5x-3}{(x+1)(x-3)} = \frac{A(x-3)+B(x+1)}{(x+1)(x-3)}$

$\rightarrow 5x-3 = A(x-3) + B(x+1)$

$x=3 \qquad x=-1$

$\underline{\underline{x=3}}$ $\qquad$ $\underline{\underline{x=-1}}$

$12 = 0 + 4B \qquad -8 = -4A + 0$

$B = 3 \qquad A = 2$

$\rightarrow = 2\ln|x+1| + 3\ln|x-3| + C$

(4) $\int \frac{\cos x}{\sin^2 x + 4\sin x - 5} dx \rightarrow y = \sin x$

$\frac{dy}{dx} = \cos x$

$\rightarrow \int \frac{\cancel{\cos x}}{y^2 + 4y - 5} \cdot \frac{dy}{\cancel{\cos x}} = \int \frac{1}{(y+5)(y-1)} dy = \frac{A}{y+5} + \frac{B}{y-1}$

$\frac{1}{(y+5)(y-1)} = \frac{A(y-1) + B(y+5)}{(y+5)(y-1)}$

$\rightarrow 1 = A(y-1) + B(y+5)$

$y=1$ $y=-5$

$\underline{y=1}$ $\underline{y=-5}$

$1 = 0 + 6B$ $1 = -6A + 0$

$B = \frac{1}{6}$ $A = \frac{-1}{6}$

$\rightarrow = \frac{-1}{6} \ln|y+5| + \frac{1}{6} \ln|y-1| + c$

$= \frac{1}{6} \ln|\sin x + 5| + \frac{1}{6} \ln|\sin x - 1| + c$

$\boxed{2}$ إذا كان درجة البسط أكبر أو يساوي درجة المقام
هنا قسمة ثم كسور جزئية

Ex ① $\int \frac{x^2-5x+7}{x^2-5x+6}\, dx$

$$\begin{array}{r|l} & 1 \\ x^2-5x+6 & \cancel{x^2} - \cancel{5x} + 7 \\ & \mp\cancel{x^2} \pm \cancel{5x} \mp 6 \\ \hline & 1 \end{array}$$

$= \int \left(\text{الناتج} + \frac{\text{الباقي}}{\text{المقسوم عليه}} \right)$ ← قانونها

$= \int \left(1 + \frac{1}{x^2-5x+6}\right) dx = \int 1\, dx + \int \frac{1}{x^2-5x+6}\, dx$

تكامل مباشر ← ... كسور جزئية ←

$\boxed{3}$ إذا كان المقام مكرر

$$\frac{p(x)}{q(x)} = \frac{A}{x+a} + \frac{B}{(x+a)^2}$$

Ex $\displaystyle\int \frac{2x+5}{x^3+2x^2}\,dx = \int \frac{2x+5}{x^2(x+2)}\,dx = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+2)}$

$$\frac{2x+5}{x^2(x+2)} = \frac{Ax(x+2) + B(x+2) + Cx^2}{x^2(x+2)}$$

$$\rightarrow 2x+5 = Ax(x+2) + B(x+2) + Cx^2$$

$\underline{x=0}$

$5 = 0 + 2B + 0$

$B = \frac{5}{2}$

$\underline{x=-2}$

$1 = 0 + 0 + 4C$

$C = \frac{1}{4}$

هنا نعوض قيم B و C

نعوض أي رقم غير اصفار المقام لإيجاد A $\rightarrow$ $\underline{\underline{x=1}}$

$7 = 3A + \frac{15}{2} + \frac{1}{4}$

$3A = 7 - \frac{31}{4}$

$A = \frac{-1}{4}$

$$\rightarrow \int \left(\frac{\frac{-1}{4}}{x} + \frac{\frac{5}{2}}{x^2} + \frac{\frac{1}{4}}{x+2} \right) dx$$

نرفعها للبسط

$$\rightarrow = \frac{-1}{4} \ln|x| + \frac{1}{4} \ln|x+2| - \frac{5}{2} x^{-1} + C$$

$$= \frac{-1}{4} \ln|x| + \frac{1}{4} \ln|x+2| - \frac{5}{2x} + C$$

Ex $\displaystyle\int \frac{x^2}{(x-1)^2(x+1)}\,dx$

$$\rightarrow \frac{x^2}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$$

$$= \frac{A(x-1)(x+1) + B(x+1) + C(x-1)^2}{(x-1)^2(x+1)}$$

$$\rightarrow x^2 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

$x=1$	$x=-1$	$x=0$
$1 = 0+2B+0$	$1 = 0+0+4C$	$0 = -A + \frac{1}{2} + \frac{1}{4}$
$\rightarrow B = \frac{1}{2}$	$\rightarrow C = \frac{1}{4}$	$\rightarrow A = \frac{3}{4}$

$$\rightarrow = \int \frac{\frac{3}{4}}{(x-1)}\,dx + \int \frac{\frac{1}{2}}{(x-1)^2}\,dx + \int \frac{\frac{1}{4}}{(x+1)}\,dx$$

نرفعها للبسط

$$= \frac{3}{4}\ln|x-1| + \frac{-1}{2(x-1)} + \frac{1}{4}\ln|x+1| + C$$

$\boxed{4}$ إذا كان المقام لا يحلل

$$\frac{p(x)}{q(x)} = \frac{A}{x+a} + \frac{B}{x^2+Bx+y}$$ لا يحلل

↳ $\Delta =$ سالب

المميز $b^2 - 4ac < 0$

$\boxed{x^2+a^2 \Rightarrow \text{لا يحلل}}$

Ex $\displaystyle\int \frac{1}{(x+2)(x^2+4)}\,dx$

$$\frac{1}{(x+2)(x^2+4)} = \frac{A}{(x+2)} + \frac{Bx+c}{(x^2+4)}$$ ← هنا المقام تربيعي لم البسط أقل درجة من المقام

$$= \frac{A(x^2+4) + (Bx+c)(x+2)}{(x+2)(x^2+4)}$$

$\rightarrow 1 = A(x^2+4) + (Bx+c)(x+2)$

هنا نأخذ أي عدد

$x=-2$	$x=0$	$x=1$
$1 = 8A + 0$	$1 = \frac{1}{8}\cdot 4 + 0 + 2c$	$1 = \frac{5}{8} + 3(B+\frac{1}{4})$
$\rightarrow A = \frac{1}{8}$	$\rightarrow c = \frac{1}{4}$	$\rightarrow B = \frac{-1}{8}$

$$\rightarrow \int \frac{\frac{1}{8}}{x+2}\,dx + \int \frac{\frac{-1}{8}x + \frac{1}{4}}{x^2+4}\,dx$$

$\tan^{-1} x = \int \frac{1}{x^2+1}$

$\tan^{-1}(\frac{x}{a}) = \int \frac{1}{x^2+a^2}$

$$= \frac{1}{8}\ln|x+2| + \int \frac{\frac{-1}{8}}{x^2+4}\,dx + \int \frac{\frac{1}{4}}{x^2+4}\,dx$$

بالتعويض

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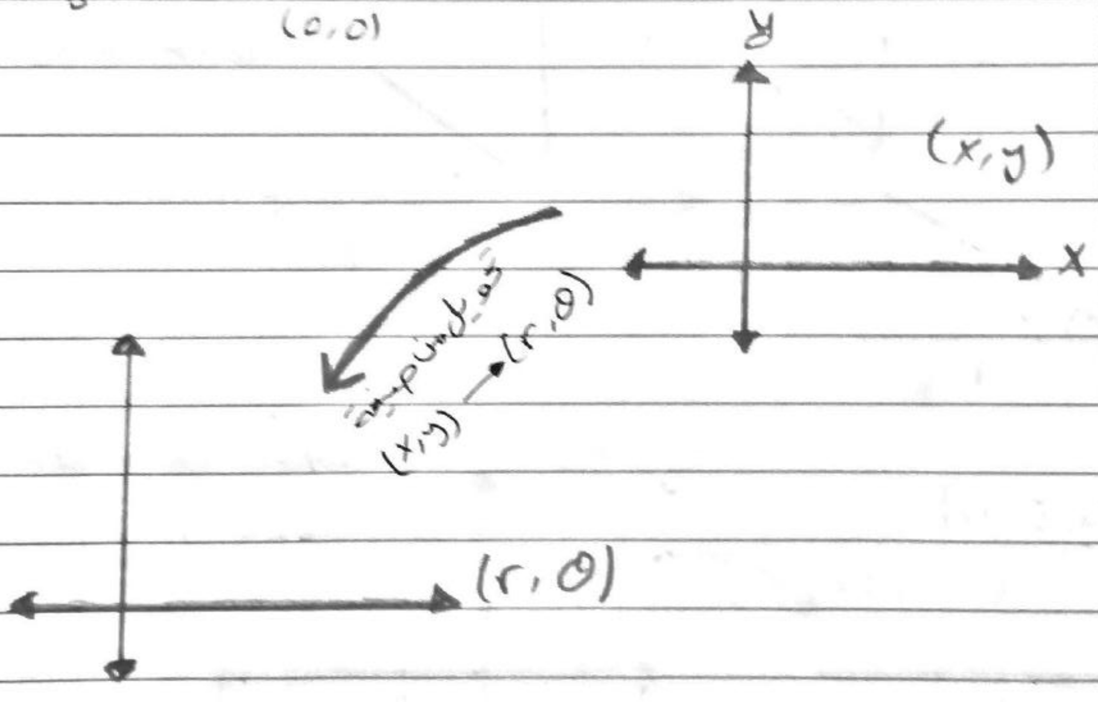
Polar coordinates الاحداثيات القطبية

Def: The polar coordinate system is two dimensional coordinate system in which each point P on a plane is determined by a distance r from a fixed point O that is called the pole (origin) and angle θ from a fixed direction.

* العلاقة بين r (المسافة) و θ (الزاوية)

تبدأ بالرسم من نقطة الاصل

origin → نقطة الاصل
(0,0)

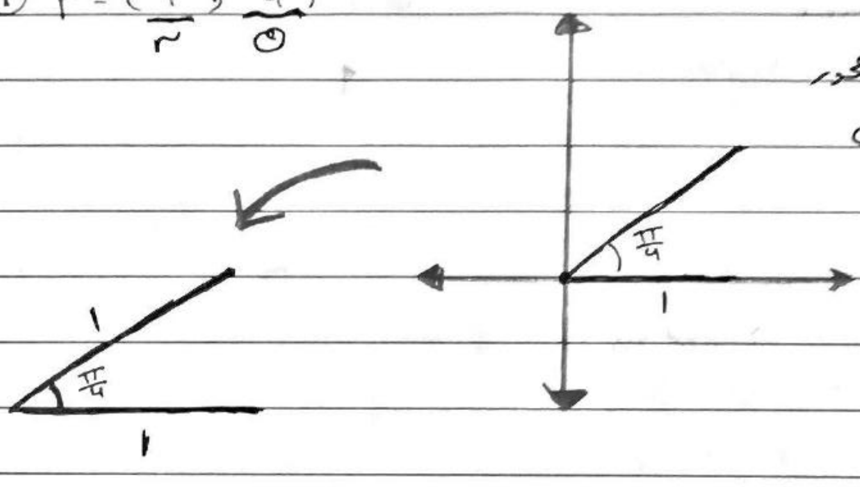


Remark :-

The point P is represent by the order point $P(r, \theta)$ and r, θ called polar coordinate.

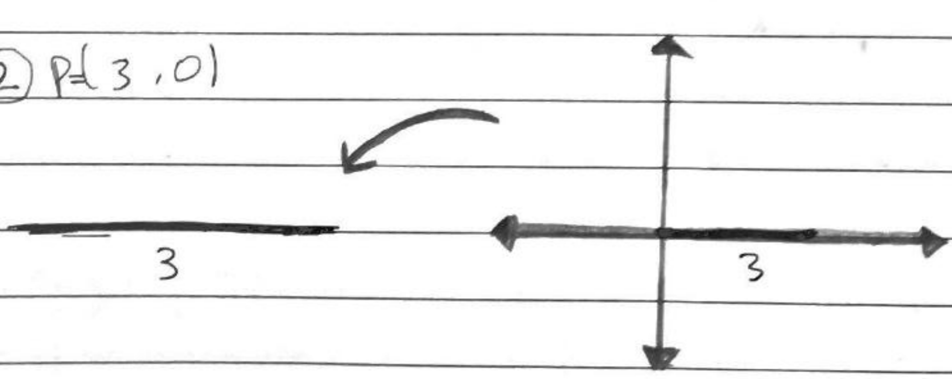
Ex plote the point whose polar coordinate are given.

① $P = (\underset{r}{1}, \underset{\theta}{\frac{\pi}{4}})$



يبدأ الرسم من محور
السينات الموجب

② $P = (3, 0)$



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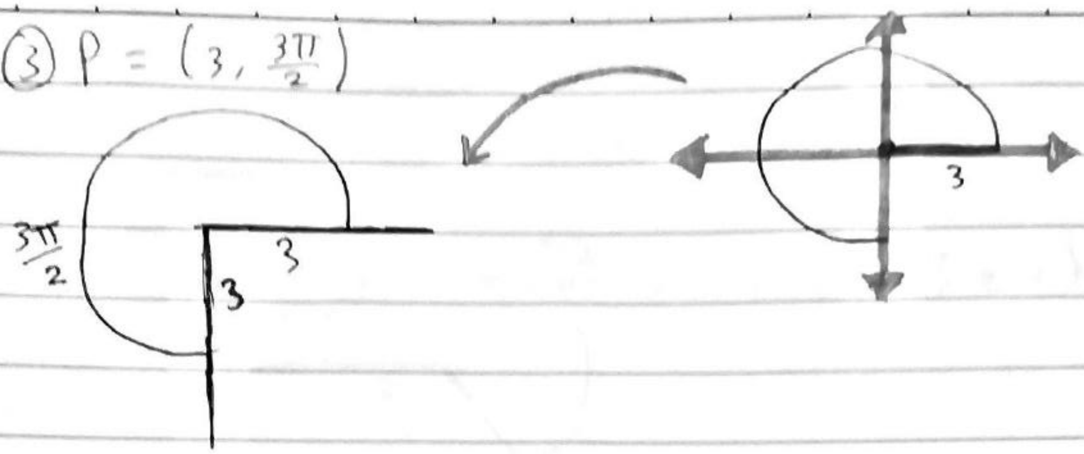
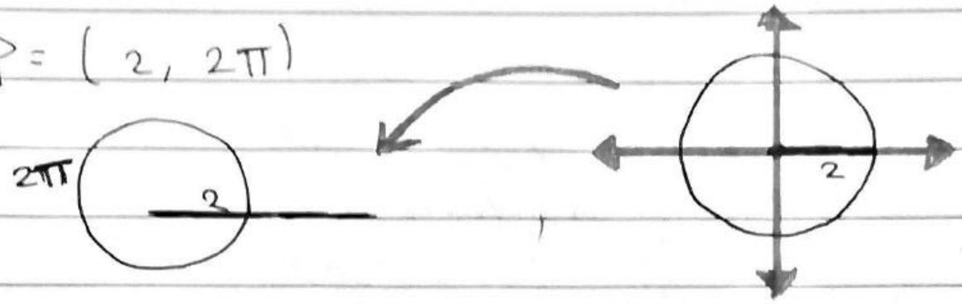
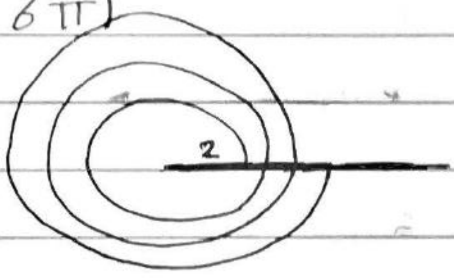
③ $P = \left(3, \frac{3\pi}{2}\right)$



④ $P = (2, 2\pi)$



⑤ $P = (2, 6\pi)$



Remarks:-

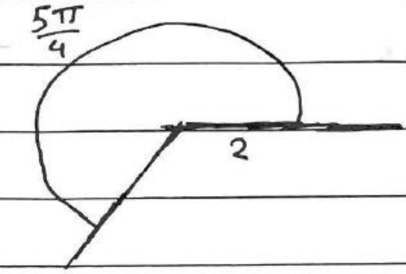
$$P = (r, \theta) = (r, \theta + 2\pi n) \;;\; n: 1, 2, 3, \ldots$$

عدد اللفات

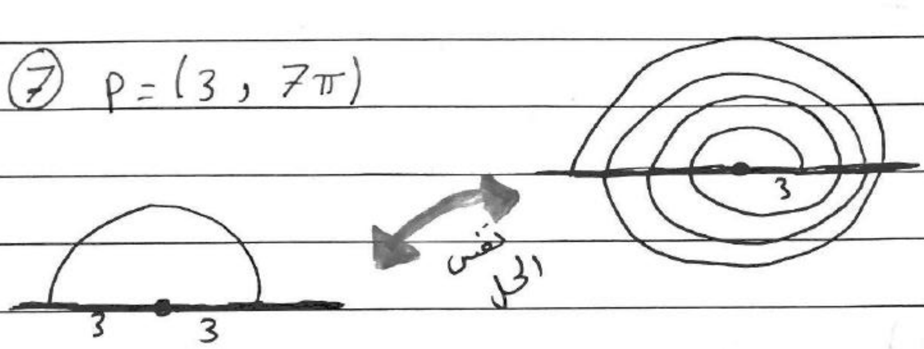
$(-r, \theta) = (r, \theta + \pi)$ ← تستخدم هذه القاعدة في حال كان يوجد سالب في r

⑥ $p = (-2, \frac{\pi}{4})$

$\Rightarrow p = (2, \frac{\pi}{4} + \pi)$

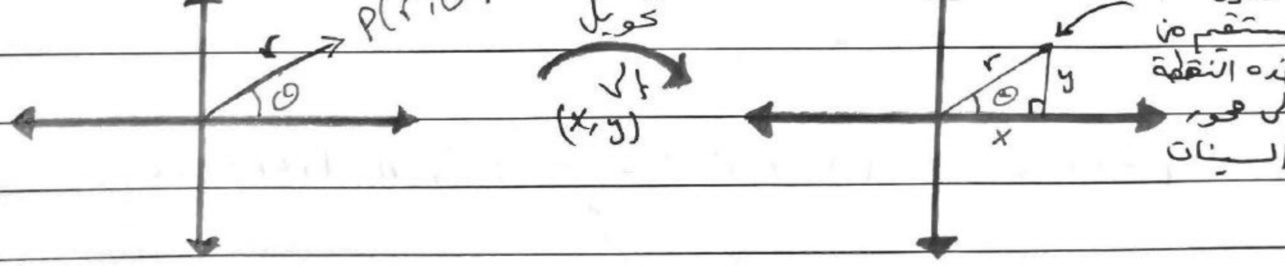


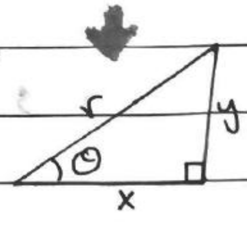
⑦ $p = (3, 7\pi)$



* The relation between polar coordinate and cartesian.

العلاقة بين الاحداثيات القطبية و المستوى الديكارتي.





$\sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta$

$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$

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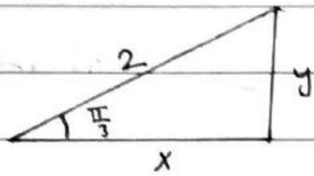
$\therefore \boxed{(x,y) = (r\cos\theta \,,\, r\sin\theta)}$

$\star \boxed{r^2 = x^2 + y^2}$ ← قانون فيثاغورس

Ex convert the point from polar coordinate to Cartesian. $(x,y) \leftarrow (r,\theta)$ حول من *

① $P = (2, \frac{\pi}{3})$



$x = r\cos\theta$

$= 2 \cdot \cos\frac{\pi}{3} = 2 \cdot \frac{1}{2} = 1$

$y = r\sin\theta$

$= 2\sin\frac{\pi}{3} = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$

$\therefore (r,\theta) \rightarrow (x,y)$

$(2, \frac{\pi}{3}) \rightarrow (1, \sqrt{3})$

② $P = (3, \frac{\pi}{2})$

$x = r \cdot \cos\theta = 3 \cdot 0 = 0$

$y = r \cdot \sin\theta = 3 \cdot 1 = 3$

$\therefore (r, \theta) \longrightarrow (x, y)$

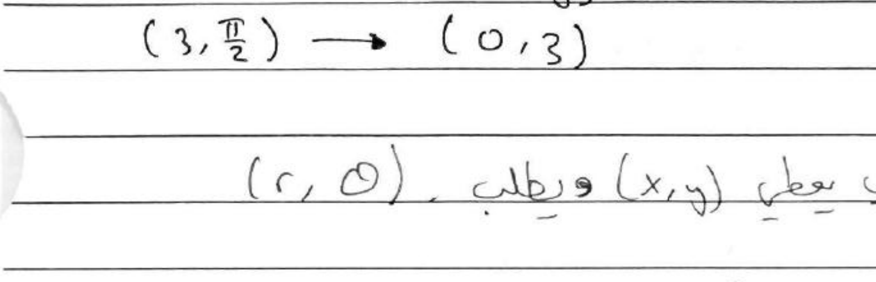
$(3, \frac{\pi}{2}) \longrightarrow (0, 3)$

* لو أعطى العكس أي يعطي (x,y) ويطلب (r, θ) .

<u>Ex</u> convert the points from cartesian to polar coordinate. * تحويل من $(x,y) \leftarrow (r,\theta)$

① $(1, 1)$ — x, y

$\rightarrow r^2 = x^2 + y^2 \rightarrow r^2 = 1 + 1 \rightarrow r = \sqrt{2}$



$\rightarrow \sin\theta = \frac{1}{\sqrt{2}}$

$\rightarrow \theta = \frac{\pi}{4}$

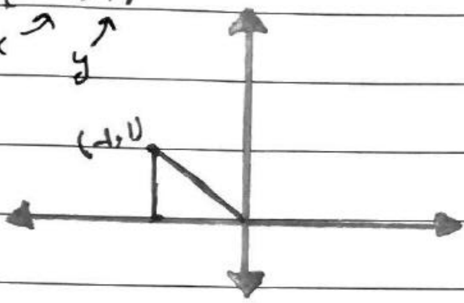
$\therefore (x, y) \longrightarrow (r, \theta)$

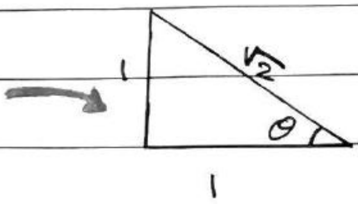
$(1, 1) \longrightarrow (\sqrt{2}, \frac{\pi}{4} + \underbrace{2k\pi})$

عدد اللفات (نكتبها من أجل تعميم الزاوية لأنه غير معروف عدد اللفات)

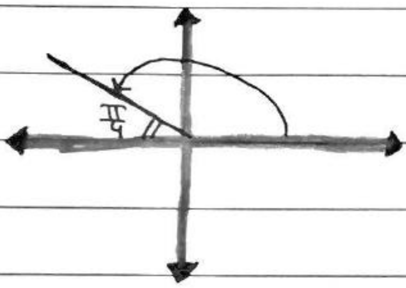
② $(-1, 1)$
$x \nearrow$ $y \nearrow$

$r^2 = x^2 + y^2 \rightarrow r^2 = 2 \rightarrow r = \sqrt{2}$





$\sin\theta = \frac{1}{\sqrt{2}} \rightarrow \theta = \frac{\pi}{4}$ ← هذه الزاوية هي القمة



$\rightarrow \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$(x, y) \rightarrow (r, \theta)$
$(-1, 1) \rightarrow (\sqrt{2}, \frac{3\pi}{2} + 2k\pi)$

---

*Express the equation in polar coordinate :-

← تحويل معادلات من صيغة (x, y) إلى صيغة (r, θ)

① $x = 1$ ← هذه معادلة وليست نقطة. حولها لمعادلة من صيغة $(r,\theta) \leftarrow (x,y)$

$r\cos\theta = 1$

② $x^2 = 4y$

$(r\cos\theta)^2 = 4(r \cdot \sin\theta)$

$\frac{r^2\cos^2\theta}{r} = \frac{4 \cdot r \cdot \sin\theta}{r}$

$r\cos^2\theta = 4\sin\theta$