

Subject _____ Date _____ No. _____

سلیمان

ج

علاء الدين

(1)

الحصاد والاحتمالات

Statistic and Propability

* Statistics : is the science of collecting data organizing or describing it can draw conclusion from it.

→ هو العلم الذي يقوم بجمع المعلومات وتنظيمها واستخراج النتائج

* types of statistic :-

1) descriptive statistic : الإحصاء الوصفي

→ Consist of method organizing and summarizing information using graph, table, calculating, chart, measures of location and variable.

→ يهتم بتنظيم وتناسب المعلومات باستخدام مجموعة من الطرق مثل

الرسم، الجداول التحварية، الـ اـ بـ اـ تـ، الـ اـ لـ عـ مـ، مقاييس التردد والتوزع.

• تردد مرئي ← وسط / متوسط / متوسط.

• مقاييس التوزع ← اطوى - الاختلاف المعياري، التباين.

2 Inferential statistic :- الاجراءات الاستنتاجية

consist of drawing conclusion about population based on information from a sample.

يتكون من الاجراءات التي ت時候م للتتنبأ بأهم ميزات المجتمع بالاعتماد على المعلومات التي حصلنا عليها من العينة.

★ describing data with graphs :-

وصف المعلومات بمحاسن:

- main concept :-

1-Variable :- is characteristics changes or varies over time and/or different individuals or object under consideration.

متغير متغير مع مرور الوقت ويتغير في متغير آخر.

Ex :- age , color , GPA , weight , ...

مقدمة * GPA : Grater Point Average .
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2- Experimental unit : وحدة المعاينة

(exp. unit) : is the individual or object in which variable is measured taken.

نأخذ القراءات للمجموع من

3- Population : - is the set of all measurement of interest need to be investigated.

جمع القراءات التي ترغب براميتها.

4- Sample :- is the subset from population

جزء من المجموع.

Ex sample → the student in the Irbid un.

population → all the student in the un.

variable → GPA

Exp. unit → A student

Graphs for categorial data: الرسم

- | | | |
|-----------------|--------------------------|---|
| 1) bar - chart | طريقة العمود | } |
| 2) pie - chart | طريقة المطالعات المثلثية | |
| 3) line - chart | طريقة الخط / الخطوط | |
| 4) dot - plot | طريقة النقاط | |
- طرق وصف البيانات باستخدام الرسوم

~~البيانات~~

Frequency $\Rightarrow (F)$ التكرار

أقصى

Relative Frequency $\Rightarrow (r.F)$ التكرار النسبي

أدنى

$$r.F = \frac{F}{\Sigma F}$$

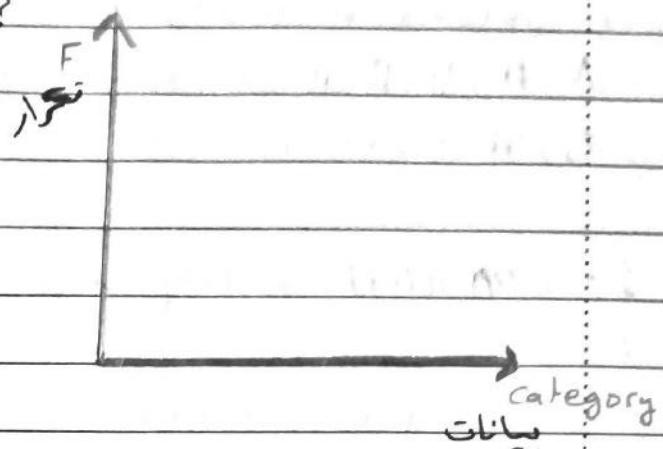
قيمة

$$\Sigma F = n$$

Percentage : النسبة المئوية

$$\text{Percentage} = r.F \times 100\%$$

* bar-chart كييفت ؟



* pie-chart كييفت ؟

1- draw circle.

2- the angle of sector ← زاوية القطاع المرئي

$$\theta_i = (r \cdot F) \cdot (360^\circ)$$

هذا القانون يباب زاوية

كل قطع من القطاع المرئي.

Ex Given the following Grades of 20 students in Jordan university :

A, A, B, B, B, C, C, C, A, B, C, A, B, B, C, C, A, C, B, A

1 - construct a Frequency table. كون جدول تكراري

2- draw bar - chart. ارسم طريقة الاعمدة

3- draw pie - chart. ارسم طريقة القطاع الدائري

① category	Tally	F	r.F تكراري	② المزدوجة
A		6	$\frac{6}{20} = 0.3$	$(0.3)(360) = 108^\circ$
B		7	$\frac{7}{20} = 0.35$	$(0.35)(360) = 126^\circ$
C		7	$\frac{7}{20} = 0.35$	$(0.35)(360) = 126^\circ$

$\Sigma F = n = 20$

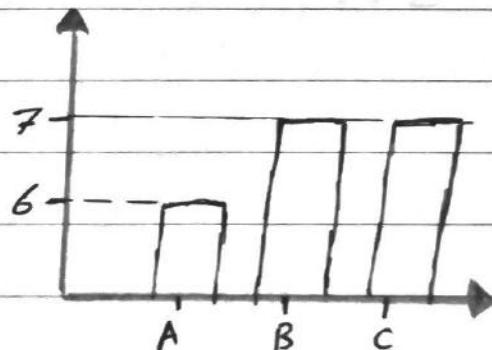
مجموع اعداد طلاب

$\Sigma \theta = 360^\circ$

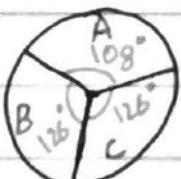
مجموع زوايا القطاعات الدائرية

يجب ان تبلغ 360°

② bar - chart



③ pie - chart



* يجب ان يكون التقسيم متساوياً
مترادفي

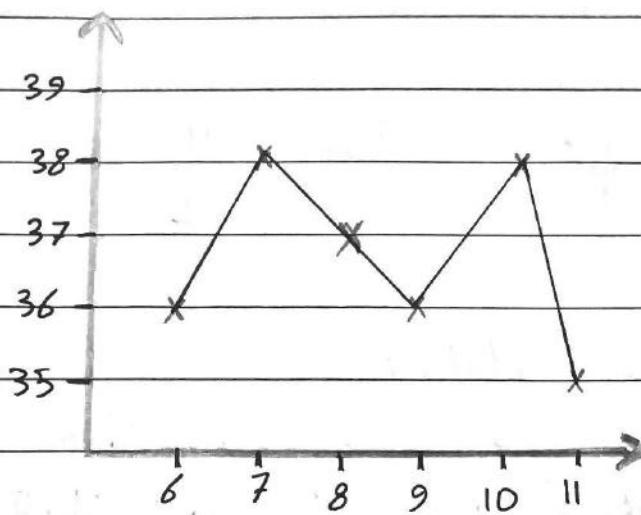
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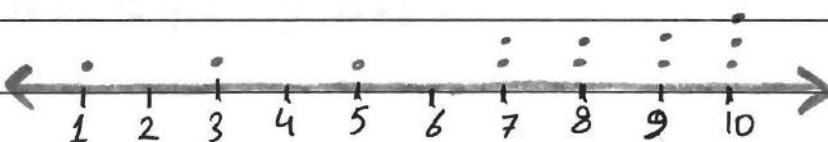
<u>Ex</u>	time	6	7	8	9	10	11
	temperature	36	38	37	36	38	35

draw this table by line - chart.



Ex 9, 8, 7, 7, 8, 9, 10, 10, 10, 1, 3, 5

draw dot - plot.



Stem and Leaf :

↑ تعرف بجمع الأرقام باعدها رقم لا يزيد عن المائة ← Stem
 ↓ تعرف للرقم الأعلى من المائة فقط ← Leaf

Ex

$$\begin{array}{c} 25.3 \\ \hline S \quad L \end{array}$$

$$\begin{array}{c} 27 \\ \hline S \quad L \end{array}$$

$$\begin{array}{c} 132.24 \\ \hline S \quad L \end{array}$$
Ex Given the following data::

3.2	2.1	2.9	5	6.9	4	4.2	2.2
4.3	4.2	3.4	5.7	3.6	4.4	4.8	4.6

Draw stem and leaf plot.

• لا يدخل المكرر لا يتكرر في الجدول.

• يجب أن يكون الترتيب تصاعديّ.

• يجب أن تكون البيانات مرتبة تصاعديّاً وغير مكررة.

• نحول جميع الأعداد الصحيحة إلى هنا التكال (المائة)
 $\frac{a \cdot b}{s} \quad I$
 بإضافة الأصفاد

لقد جمع الأعداد / الأرقام في المطالع بهذه الصيغة

$$5 \rightarrow 5.0$$

$$4 \rightarrow 4.0$$

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0, 2, 3, 4, 6, 8

5

0,7

6

9

- يجب أن يكون الترتيب تصاعدياً ↑

أمثلة التربية المعاصرة

Remark 3-

$$\frac{25 \cdot 3}{5} \rightarrow \text{leaf unit} = 0.1$$

$$\frac{27}{S \quad L} \rightarrow \text{leaf unit} = 0L \cdot 0 = L$$

$$\frac{132.24}{S \ L} \rightarrow \text{leaf unit} = 0.0L$$

* Leaf unit = O.L

Ex Given the following data:

12 11 2 25 21 24 23 39 36 38 29
34 35

Draw stem and leaf plot.

$$\frac{ab}{5} L$$

الشكل الغالب للرقم

$$* 2 \rightarrow 02$$

S

L

0

2

1

1, 2

2

1, 3, 4, 5, 9

3

4, 5, 6, 8, 9

$$\Rightarrow \text{Leaf unit} = 0L = L$$

Remark :- 0, 1 , 135, 2 , 12 , 24

لبيان حركة الارقام ونريد تحويلها إلى استعمل الغالب

0 0 0 , 1

1 3 5 , 2

0 1 2 , 0

0 2 4 , 0

كيفية تحويل
الارقام إلى استعمل
الغالب في المؤشر
بيانات اهمها

Shapes of distribution : أشكال التوزيعات

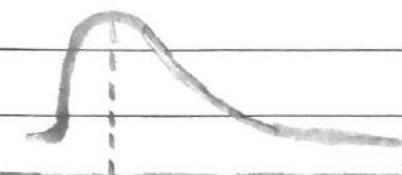
1] symmetric

ال對称



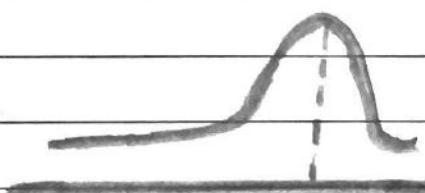
2] skewed to the right (positive skewed)

متل (ملتوه) إلى اليمين



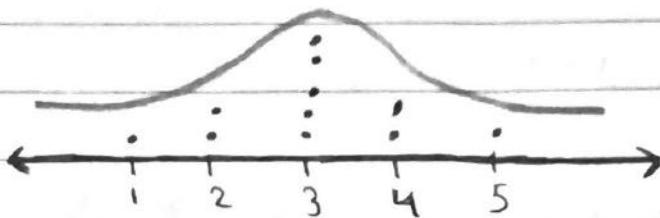
3] skewed to the left (negative skewed)

متل (ملتوه) إلى اليسار



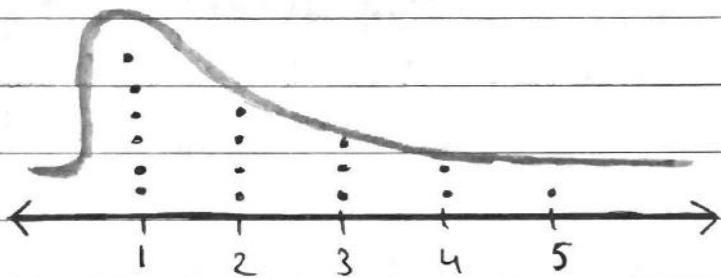
Ex Given the following dot plot :-

①



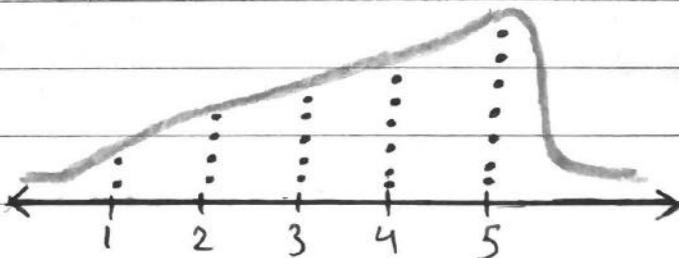
→ Symmetric

②



→ positive skewed.

③



→ negative skewed.

Frequency table with classes (Grouped Data) :

جدول تكراري للفئات

Ex Fill the frequency table :-

الفئات classes	الرقم المركب Frequency	نقطة المنتصف midpoint	ال FRE الfre. النسبية relative frequency	ال PER percent $(\text{RF}) \cdot 100\% = (\text{RF}) \cdot 100$	الحدود الفعلية actual limit	تكرار تراكمي cumulative frequency
4 - 8	4	$\frac{4+8}{2} = 6$	$\frac{4}{25} = 0.16$	16%	3.5 - 8.5	4
9 - 13	6	11	$\frac{6}{25} = 0.24$	24%	8.5 - 13.5	$4 + 6 = 10$
14 - 18	12	16	$\frac{12}{25} = 0.48$	48%	13.5 - 18.5	$4 + 6 + 12 = 22$
19 - 23	3	21	$\frac{3}{25} = 0.12$	12%	18.5 - 23.5	$4 + 6 + 12 + 3 = 25$
Total	25		1			

هذا الجدول فقط من معطيات

الحال ما تبقى هو اكمل

مجموع فئات

الجدول يجب أن تأتي

ـ

المقادير جميعها تكون

ΣF

* الحدود الفعلية \leftarrow نطرح من الحدودي 0.5
ونصف الحد الادنى 0.5

* نقطة المنتصف = الحد الادنى + الحد الادنى

2

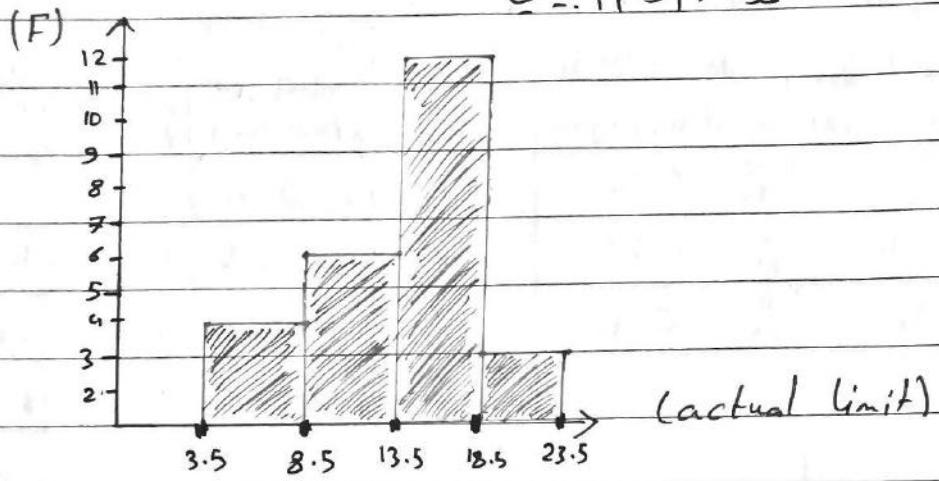
Sketch

الرسم

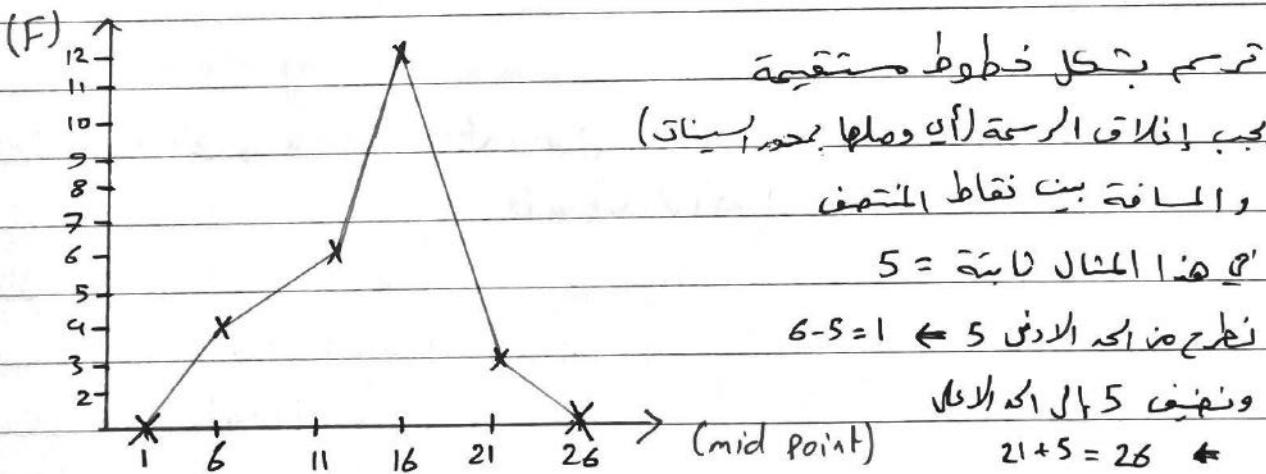
* تجاهل بيانات الأول السابق

1 Histograms

الدالة / الرسم البياني

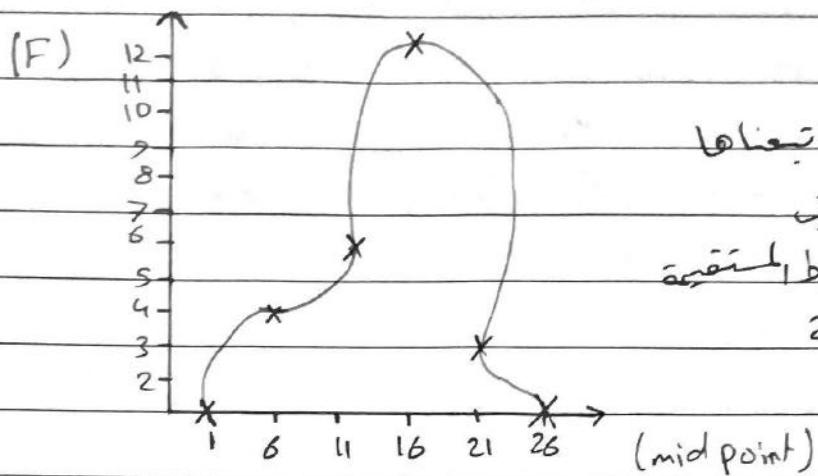

 $F \rightarrow$
 الترتيب
 ضروري
2 Polygons

المضلعات



3 Curve

المختبرات



نفس الخطوات التي اتبعتناها
في المخططات لكن الفرق
الواحد هو مكان الخطوط المستقيمة
نرسمها خطوط منحنية

$$\text{Class Length} = b - a + 1$$

(Δ) طول الفرة ^{الدائم} ^{اللدن}

$$\text{actual limit} = b - a = \text{class length}$$

الحدود الفعلية = الحد العلوي - الحد الدنيا

Ex Given the following grades for 40 student in statistic :

max								min			
94	82	77	70	62	89	57	35	92	80		
85	55	45	69	75	72	90	82	62	50		
44	67	53	82	90	91	57	47	82	78		
62	59	81	61	55	70	80	75	82			

Construct grouped frequency tables with 5 classes.

$$\textcircled{1} \text{ Number of classes} = 5$$

$$\textcircled{2} \text{ Range} = \text{max} - \text{min}$$

$$= 94 - 35 = 59$$

$$\textcircled{3} \text{ class length} = \frac{\text{Range}}{5} = \frac{59}{5} = 11.8 \approx 12$$

الخطوات \rightarrow

classes	F	
35 - 46	111	3
47 - 58	444 11	7
59 - 70	444 1111	9
71 - 82	444 444 1111	14
83 - 94	444 11	7

$$\Sigma F = 40$$

$$\text{min} + (12 - 1)$$

Measures of center

مقياسات النسبة المئوية

1) mean (average) (\bar{X}) \bar{x} bar

مُرْطِ

2) median

مُوَسِّط

3) mode

مُنْوَال

1 Single Data :-

بيانات مفردة

Ex 2, 5, 3, 7, 9, 11 find mean ?

$$\text{قانون} \quad \bar{X} = \frac{\sum X_i}{n} = \frac{2+5+3+7+9+11}{6} = \frac{37}{6} = 6.1$$

Ex 3, 2, 5, 7, 2, 7, 9, 2 find median ?

أولاً نقوم بترتيب الأرقام تصاعدياً
~~2, 7, 2, 3, 4, 5, 7, 7, 9~~

ثانياً نقوم بتنحى الأرقام من الطرف حتى يصل الرقم الذي في الوسط

المُرْطِ الذي في الوسط ← المُوَسِّط

median = 4

Ex 4, 5, 9, 2, 3, 7, 9, 1 Find median?

1, 2, 3, 4, 5, 7, 9, 9
اذا كان عدد البيانات زوجي

$$\rightarrow \text{median} = \frac{4+5}{2} = \frac{9}{2} = 4.5 \quad \begin{array}{l} \text{فيما يلي طبق جمع العددين} \\ \text{ونقسمها على 2} \end{array}$$

Ex 2, 5, 7, 2, 9, 1, 2, 7, 3, 5 Find mode?

1, 2, 2, 2, 3, 5, 5, 7, 7, 9
نكرار ← القمة المكررة

$$\rightarrow \text{mode} = 2$$

Ex 2, 4, 5, 4, 7, 5, 9 Find mode?

2, 4, 4, 5, 5, 7, 9
اذا كانت جميع القيم مكررة بنفس عدد التكرار

$$\text{mode 1} = 4$$

2, 2, 3, 3, 4, 4

$$\text{mode 2} = 5$$

لا يوجد هناك متوازن

Does not exist

Ex 2, 5, 9, 7, 14 Find mode?

d. n. e لا يوجد متوازن

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Ex Given the following data :

1, 2, 5, 2, 3, 4, 2, 9, 7, 6

find: 1) mean 2) median 3) mode.

Ans → 1, 2, 2, 2, 3, 4, 5, 6, 7, 9

$$\textcircled{1} \rightarrow \text{mean } \bar{x} = \frac{\sum x_i}{n} = \frac{41}{10} = 4.1$$

$$\textcircled{2} \rightarrow \text{median} = \frac{3+4}{2} = \frac{7}{2} = 3.5$$

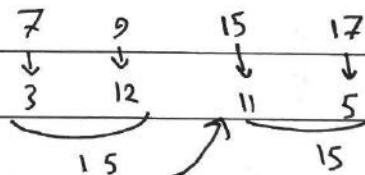
$$\textcircled{3} \rightarrow \text{mode} = 2$$

<u>Ex</u>	X	7	9	15	17	Total
F		3	12	11	5	31

find : 1) mean
2) median
3) mode.

$$\textcircled{1} \rightarrow \text{mean } \bar{x} = \frac{7 \times 3 + 9 \times 12 + 15 \times 11 + 17 \times 5}{31}$$

$$\textcircled{2} \rightarrow \text{median} = 15$$



$$\textcircled{3} \rightarrow \text{mode} = 9$$

2 Grouped Data (classes)

$$\textcircled{1} \quad \bar{x} = \frac{\sum (x_i f_i)}{\sum f_i} ; \quad x_i = \text{mid point} \quad (\text{أوسط}), \quad f_i = \text{frequency} \quad (\text{ال FREQUENCIES})$$

Ex find 1) mean 2) median 3) mode

of the following sample data:

classes	f	X	midpoint $f \times x$
4 - 6	3	5	15
7 - 9	4	8	32
10 - 12	2	11	22
13 - 15	1	14	14
Total	10		83

$$\textcircled{1} \quad \bar{X} = \frac{83}{10} = 8.3$$

$$\textcircled{3} \text{ mode } = \frac{7+9}{2} = \frac{16}{2} = 8$$

② median

دراة الوسيط (مكان تواجد)^٥

degree of median = $\frac{\sum f_i}{2}$

$$= \frac{10}{2} = 5$$

$$\frac{5-3}{7-3} = \frac{a-3.5}{6.5-3.5}$$

$$\Rightarrow a = \frac{6}{4} + 3.5 = 5$$

= median

Measure of Variability

معايير التشتت

- 1) Range (R) اطوال
- 2) Standard deviation (S.d) (S) الافتراضي المعياري
- 3) Variance (S^2) البيانات

I Single Data :-

$$1) R = \text{max} - \text{min}$$

$$2) S = \sqrt{\frac{\sum x_i^2 - n(\bar{x})^2}{n-1}}$$

$$3) S^2 = \frac{\sum x_i^2 - n(\bar{x})^2}{n-1}$$

نفسه لكن بدون جذر !!

الافتراضي
المعياري

Ex For the following sample find: 1) R 2) S 3) S^2

① 3, 2, 1, 4, 5

X	3	2	1	4	5	Total
X^2	9	4	1	16	25	55

$$1) R = \max - \min = 5 - 1 = 4$$

$$2) \bar{x} = \frac{\sum x_i}{n} = \frac{15}{5} = 3$$

$$\rightarrow s = \sqrt{\frac{\sum x_i^2 - n(\bar{x})^2}{n-1}} = \sqrt{\frac{55 - 5(3)^2}{5-1}} = \sqrt{\frac{10}{9}}$$

$$\textcircled{3} \quad s^2 = \left(\sqrt{\frac{10}{9}} \right)^2 = \frac{10}{9}$$

(2) 3, 3, 3, 3, 3, 3

X	3	3	3	3	3	3	Total
x^2	9	9	9	9	9	9	54

$$1) R = \max - \min = 3 - 3 = 0$$

$$2) \bar{x} = \frac{\sum x_i}{n} = \frac{18}{6} = 3$$

$$S = \sqrt{\frac{54 - 54}{5}} = 0$$

لذا كانت جميع الاعداد متساوية

مثل هذه المطالع تكون جميعها

الشّتّت =

$$\textcircled{3} \quad s^2 = 0$$

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2) Grouped Data (classes)

$$1) R = x_{\max} - x_{\min}$$

$$2) S = \sqrt{\frac{\sum x_i^2 f_i - \frac{(\sum x_i f_i)^2}{\sum f_i}}{\sum f_i - 1}}$$

$$3) S^2 = \frac{\sum x_i^2 f_i - \frac{(\sum x_i f_i)^2}{\sum f_i}}{\sum f_i - 1}$$

Remark :-

$$(\sum x_i)^2 \neq \sum x_i^2$$

Ex for the following sample find : 1) R 2) S 3) S^2

Classes	f	x_i	x_i^2	$x_i^2 f_i$	$x_i f_i$
0 - 4	5	2	4	20	10
5 - 9	3	7	49	147	21
10 - 14	1	12	144	144	12
15 - 19	2	17	289	578	34
Total	(11)	38		(889)	(77)

$$\textcircled{1} \quad R = x_{\max} - x_{\min} = 17 - 2 = 15$$

$$\textcircled{2} \quad S = \sqrt{\frac{889 - \frac{(77)^2}{11}}{11-1}} = \sqrt{35}$$

$$\textcircled{3} \quad S^2 = 35$$

Pearson's correlation coefficient: معاوی الارتباط بیرون
 ، لایل \rightarrow (P.C.C)

$$\rho = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \cdot \sum_{i=1}^n (y_i - \bar{y})^2}}$$

Ex The following data represent the grades
 5 student in the first exam (x) and the
 second exam (y)

Find the
 (P.C.C) ?

x	2	3	5	7	8
y	5	7	4	9	5

x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$\bar{x} = \frac{25}{5} = 5$
2	5	$2-5=-3$	-1	3	9	1	
3	7	-2	1	-2	4	1	
5	4	0	-2	0	0	4	
7	9	2	3	6	4	9	
8	5	3	-1	-3	9	1	$\bar{y} = \frac{30}{5} = 6$
Σ	25	30	0	0	26	16	

هذا
 ايجولان - يجب راجع
 ان يكون لها معنی

$$\rho = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \cdot \sum (y_i - \bar{y})^2}}$$

$$\rho = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \cdot \sum_{i=1}^n (y_i - \bar{y})^2}} = \frac{4}{\sqrt{26 \times 16}} = \frac{4}{\sqrt{416}} = \frac{4}{20} = 0.19$$

Regression

الخط

The linear regression equation is :-

$$\hat{y} = ax + b$$

$$\rightarrow a = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} \quad / \quad b = \bar{y} - a\bar{x}$$

Ex The following data represent the grades 5 student in the first exam (x) and the second exam (y):

x	2	3	5	7	8	find the Regression.
y	5	7	4	9	5	

x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$	$\bar{x} = \frac{25}{5} = 5$	$\bar{y} = \frac{30}{5} = 6$
2	5	$2-5 = -3$	-1	3	9		
3	7	-2	1	-2	4	$a = \frac{4}{26} = 0.15$	
5	4	0	-2	0	0		
7	9	2	3	6	4	$b = 5 - (0.15)(6)$	
8	5	3	-1	-3	9		= 4.08
Σ	25	30	0	0	26		

$$\rightarrow \hat{y} = 0.15x + 4.08$$

الخط بـ

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No.

Ex H.W The following data represent the grades
5 student in the first exam (x) and the
second exam (y)

x	1	5	6	8	10
y	6	7	8	9	10

x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})^2$	$(x-\bar{x})(y-\bar{y})$	$\bar{x} = \frac{30}{5} = 6$
1	6	-5	-2	25	10	
5	7	-1	-1	1	1	$\bar{y} = \frac{40}{5} = 8$
6	8	0	0	0	0	
8	9	2	1	4	2	
10	10	4	2	16	8	
Σ	30	40	0	46	21	

$$a = \frac{21}{46} = 0.45 \quad / \quad b = 6 - (0.45)(8) = 2.4$$

$$\rightarrow \hat{y} = 0.45x + 2.4$$

Ex Let $\sum_{i=1}^5 (x_i - \bar{x})^2 = 25$, $\bar{x} = 5$, $\bar{y} = 7$

$$\sum_{i=1}^5 (y_i - \bar{y})^2 = 16, \quad \sum_{i=1}^5 (x_i - \bar{x})(y_i - \bar{y}) = -15$$

Find : 1) P.C.C 2) Regression

$$\textcircled{1} r = \frac{\sum_{i=1}^5 (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^5 (x_i - \bar{x})^2 \cdot \sum_{i=1}^5 (y_i - \bar{y})^2}} = \frac{-15}{\sqrt{(25)(16)}} = \frac{-15}{5.4} = -0.75$$

$$\textcircled{2} a = \frac{-15}{25} = -0.325 / b = 5 - (-0.325)(7) \\ = 7.275$$

$$\rightarrow \hat{y} = -0.325x + 7.275$$

الاحتمالات
Subject Probability and probability distribution

* Random experiment التجربة العشوائية

- toss a coin قطعة نقد
- rolling (Throwing) a die حجر الرز
- Sample space (S) or (Ω) الفضاء العيني

Ex find (S):

① toss a coin $\rightarrow S = \{H, T\}$

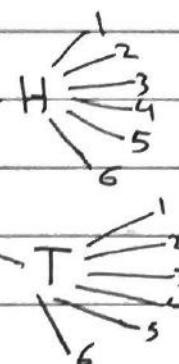
② rolling a die $\rightarrow S = \{1, 2, 3, 4, 5, 6\}$

③ toss a coin then roll a die

الامتحانات
الامتحانات
الامتحانات
الفعلة

طريقة التجربة

طريقة الجدول

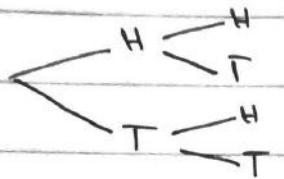


	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

$$S = \{H_1, H_2, H_3, H_4, H_5, H_6, T_1, T_2, T_3, T_4, T_5, T_6\}$$

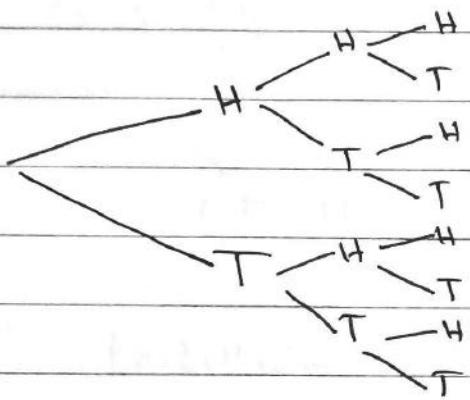
$$S = \{H_1, H_2, H_3, H_4, H_5, H_6, T_1, T_2, T_3, T_4, T_5, T_6\}$$

Ex toss two coins write S ?



$$S = \{HH, HT, TH, TT\}$$

Ex toss three coins write S ?



$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Def :- event (E) is a subset from S

الحدث

جزء (مجموعة جزئية)

Ex Roll a die, write the event :-

① A : odd number

② B : even number

③ C : number less than 4.

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$\textcircled{1} A = \{1, 3, 5\}$$

$$\textcircled{2} B = \{2, 4, 6\}$$

$$\textcircled{3} C = \{1, 2, 3\}$$

• حدث بسيط

* الحدث الذي ناتجه ١ ← حادث بسيط

* الحدث الذي ناتجه من بين الفحصاء العيني وليس كلهم ← حادث مركب

* الحدث الذي ناتجه شامل الفحصاء العيني ← حادث أكبر

* الحدث الذي ناتجه ليس متداخل الفحصاء العيني ← حادث مستقل

$$= S = \emptyset$$

* Type of event :-

1 Simple event حادث بسيط

Ex A = {1}, B = {10}, ...

2 Composite event حادث مركب

Ex A = {1, 2, 3, 4} ← مثلاً حجران زر

3 Sure event حادث اكيد

Ex A = {1, 2, 3, 4, 5, 6} ← مثلاً حجران زر

4 impossible event حادث مستحيل

Ex A = {} = \emptyset

Subject

Date

No.

Ex two die :-

A : sum of two numbers is 4

B : sum less than or equal 5

C : sum greater than 10.

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

$$A = \{(1,3), (2,2), (3,1)\}$$

$$B = \{(1,1), (1,2), (1,3), (2,1), (2,2), (1,4), (2,3), (3,1), (3,2), (4,1)\}$$

$$C = \{(5,6), (6,5), (6,6)\}$$

Def :- two events A, B are disjoint if $A \cap B = \emptyset = \{\}$

Ex Roll a die

$$A = \text{odd} = \{1, 3, 5\}$$

$$B = \text{even} = \{2, 4, 6\}$$

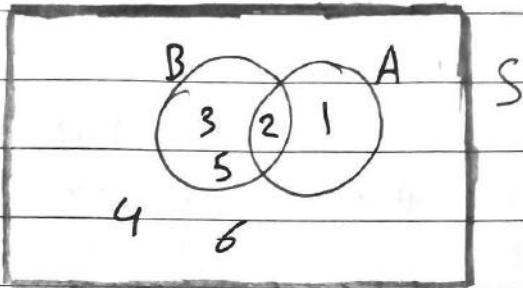
$$\rightarrow A \cap B = \emptyset \quad \therefore A, B \text{ disjoint.}$$

Venn diagram

أمثلة في

Ex Roll a die

$$A = \{1, 2\}, B = \{2, 3, 5\}$$



$$A \cup B = \{1, 2, 3, 5\}$$

$$A \cap B = \{2\}$$

Probability

$P(A)$: probability of event A

* $P(A) = \frac{E(A)}{E(S)}$ → عدد عناصر أكارت \rightarrow
عدد عناصر الفضاء العيني \rightarrow

* $0 \leq P(A) \leq 1$

* Σ of probability all simple events $S = 1$

Subject

Date

No.

Ex roll a die find probability that we get :

- 1) odd number
- 2) even number.
- 3) less than 3
- 4) greater than 1

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$\textcircled{1} \rightarrow A = \{1, 3, 5\} \rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$$

$$\textcircled{2} \rightarrow A = \{2, 4, 6\} \rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$$

$$\textcircled{3} \quad A = \{1, 2\} \rightarrow P(A) = \frac{2}{6} = \frac{1}{3}$$

$$\textcircled{4} \quad A = \{3, 4, 5, 6\} \rightarrow P(A) = \frac{5}{6}$$

Ex Let $S = \{E_1, E_2, E_3, E_4\}$, $P(E_1) = 0.1$, $P(E_2) = 0.3$, $P(E_3) = 0.2$, find $P(E_4)$.

$$P(E_1) + P(E_2) + P(E_3) + P(E_4) = 1$$

$$0.1 + 0.3 + 0.2 + P(E_4) = 1$$

$$0.6 + P(E_4) = 1 \rightarrow P(E_4) = 0.4$$

Ex $S = \{E_1, E_2, E_3, E_4, E_5\}$, $P(E_1) = P(E_2) = 0.15$
 $P(E_3) = 0.4$, $P(E_4) = 2P(E_5)$.

$$P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) = 1$$

$$0.15 + 0.15 + 0.4 + 2P(E_5) + P(E_5) = 1$$

$$\begin{array}{rcl} 0.15 + 3P(E_5) & = & 1 \\ -0.15 & & \\ \hline & -0.7 & \end{array}$$

$$3P(E_5) = 0.3 \rightarrow P(E_5) = 0.1$$

$$\rightarrow P(E_4) = 2 \times 0.1 = 0.2$$

Ex two coins find the ^{الى الامام}

1) probability observing exactly one head.

2) probability observing at least one head. ^{على الأقل}

3) probability observing at most one head. ^{على الأكثر}

$$S = \{\text{HH}, \text{HT}, \text{TH}, \text{TT}\}$$

$$\textcircled{1} A = \{\text{HT, TH}\} \rightarrow P(A) = \frac{2}{4} = \frac{1}{2}$$

$$\textcircled{2} A = \{\text{HH, HT, TH}\} \rightarrow P(A) = \frac{3}{4}$$

$$\textcircled{3} A = \{\text{HT, TH, TT}\} \rightarrow P(A) = \frac{3}{4}$$

Subject _____

Date _____

No. _____

Ex

1 blue

we select two balls one at time

2 red

جواب

1) without replacement بوناجاع

2) with replacement بجا

find the probability both balls are red.

 $p(r,r)$

① → without replacement

$$p(r,r) = \frac{2}{3} * \frac{1}{2} = \frac{1}{3}$$

احتمال اول
الاحتمال الثاني
الاحتمال الثالث
الاحتمال الرابع

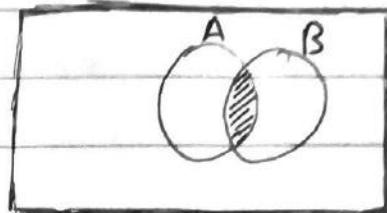
② → with replacement

$$p(r,r) = \frac{2}{3} * \frac{2}{3} = \frac{4}{9}$$

* Let A and B two event :-

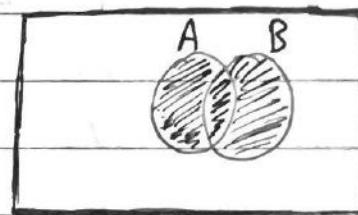
1] Intersection (\cap)

- event A and B occur
- or - both event occur



2] Union (\cup)

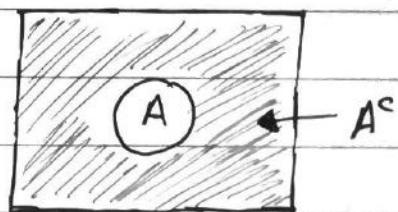
- event A or B occur
- or - either A or B or both
- or - at least one event occur



Complement

अन्तर्गत

$A^c, \bar{A}, \neg A, A^P, \dots$



Subject

Date

No.

Ex Roll a die, $A = \{1, 2, 3\}$, $B = \{3, 4, 6\}$, find :-

$$\textcircled{1} A \cup B = \{1, 2, 3, 4, 6\}$$

$$\textcircled{2} A \cap B = \{3\}$$

$$\textcircled{3} A^c = \{4, 5, 6\}$$

and find probability :-

$$\textcircled{1} A \text{ or } B \rightarrow P(A \cup B) = \frac{5}{6}$$

$$\textcircled{2} A \text{ and } B \rightarrow P(A \cap B) = \frac{1}{6}$$

$$\textcircled{3} \text{ not } B \rightarrow P(B^c) = \frac{3}{6} = \frac{1}{2}$$

$$B^c = \{1, 2, 5\}$$

$$\textcircled{4} A \text{ but } B \rightarrow P(A \cap B^c) = \frac{2}{6} = \frac{1}{3}$$

$$A \cap B^c = \{1, 2\}$$

$$\textcircled{5} \text{ neither } A \text{ and } B \rightarrow P(A^c \cap B^c) = \frac{1}{6}$$

$$(A^c \cap B^c) = \{5\}$$

Probability rules :-

$$1) P(S) = 1 \rightarrow \text{احداثي المضمار يعني} = 1$$

$$2) P(\emptyset) = 0$$

$$3) P(A^c) = 1 - P(A)$$

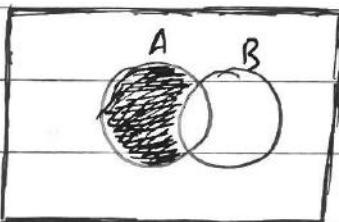
$$4) P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

ملاحظة \hookrightarrow if A and B disjoint then $P(A \cap B) = 0$

$$\Rightarrow P(A \cup B) = P(A) + P(B)$$

$$5) P(A \cap B^c) = P(\underline{A - B}) = P(A) - P(A \cap B)$$

Ex $P(B - A) = P(B) - P(B \cap A)$



$A - B$

* Demorgan laws : ① $(A \cup B)^c = A^c \cap B^c$

$$\textcircled{2} (A \cap B)^c = A^c \cup B^c$$

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- القواعد -

$$* P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$* A, B \text{ disjoint} \rightarrow P(A \cap B) = \emptyset$$

$$* P(\bar{A}) = 1 - P(A)$$

$$* P(\bar{A} \cup \bar{B}) = P(\bar{A} \cap \bar{B})$$

$$* P(\bar{A} \cap \bar{B}) = P(\bar{A} \cup \bar{B})$$

$$* P(A \cap \bar{B}) = P(A - B) = P(A) - P(A \cap B)$$

$$* P(B - A) = P(B) - P(B \cap A)$$

$$* P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Ex if $P(A) = 0.3$ $P(B) = 0.5$ $P(A \cap B) = 0.2$

$$\begin{aligned} \text{find : } 1) P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.3 + 0.5 - 0.2 \\ &= 0.6 \end{aligned}$$

$$\begin{aligned} 2) P(B - A) &= P(B) - P(A \cap B) \\ &= 0.5 - 0.2 = 0.3 \end{aligned}$$

$$3) P(\bar{A}) = 1 - P(A) = 1 - 0.3 = 0.7$$

$$\begin{aligned} 4) P(\bar{A} \cup B) &= P(\bar{A}) + P(B) - P(\bar{A} \cap B) \\ &= 1 - P(A) + P(B) - P(B - A) \\ &= 1 - P(A) + P(B) - (P(B) - P(A \cap B)) \\ &= 1 - P(A) + P(B) - P(B) + P(A \cap B) \\ &= 1 - 0.3 + 0.2 = 0.9 \end{aligned}$$

$$\begin{aligned} 5) P(\bar{A} \cup \bar{B}) &= 1 - P(A \cup B) = 1 - (P(A) + P(B) - P(A \cap B)) \\ &= 1 - (0.3 + 0.5 - 0.2) \\ &= 1 - 0.6 = 0.4 \end{aligned}$$

A b = 0.3
B b = 0.5

$$6) P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.5} = \frac{2}{5} = 0.4$$

$$7) P(\bar{A}/B) = \frac{P(\bar{A} \cap B)}{P(B)} = \frac{P(B - A)}{P(B)} = \frac{0.3}{0.5} = \frac{3}{5} = 0.6$$

Permutation

النباريل

أنتهاي → $P(n, k)$

$$P(n, k) = \frac{n!}{(n-k)!}, n > k$$

~~* Factorial~~ $n!$

$$\underline{\text{Ex}} \quad 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

$$1! = 1$$

$$0! = 1$$

$$\underline{\text{Ex}} \quad \text{Find: } ① P(5, 3) = \frac{5!}{(5-3)!} = \frac{120}{2} = 60$$

$$\underline{\text{حل}} = \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$$

$$\underline{\text{حل}} \quad P(5, 3) = \underline{5} \cdot \underline{4} \cdot \underline{3} = 120$$

$$\underline{\text{② }} P(\underline{10}, \underline{4}) = \underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7}$$

$$\underline{\text{③ }} P(\underline{7}, \underline{2}) = \underline{7} \cdot \underline{6}$$

النحوافق

$$\underline{1} \quad p(n, 0) = 1 \quad \underline{Ex} \quad p(3, 0) = 1 \\ p(5, 0) = 1$$

$$\underline{2} \quad p(n, 1) = n \quad \underline{Ex} \quad p(7, 1) = 7 \\ p(5, 1) = 5$$

$$\underline{3} \quad p(n, n) = 1 \quad \underline{Ex} \quad p(3, 3) = 1 \\ p(4, 4) = 1$$

Combination

(التوافقات)

أضطراب $\rightarrow \binom{n}{k}$

$$\binom{n}{k} = \frac{n!}{(n-k)! k!}$$

$$\binom{n}{k} = \frac{n!}{(n-k)! k!} = \frac{p(n, k)}{k!}$$

Subject

Date

No.

Ex Find: 1) $\binom{7}{3} = \frac{7!}{(7-3)!3!} = \frac{7!}{4!3!} = \frac{7 \times 6 \times 5 \times 4}{4!3!}$

$$= 35$$

2) $\binom{9}{3} = \frac{P(9,3)}{3!} = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = 84$

3) $\binom{10}{4} = \frac{P(10,4)}{4!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$

"ceşitlər"

$\cancel{\exists} (\hat{1}) = 1$

Ex $\binom{7}{7} = 1$

$\binom{8}{8} = 1$

$\cancel{\exists} (1) = 1$

Ex $\binom{7}{7} = 1$

$\cancel{\exists} (\hat{0}) = 1$

Ex $\binom{5}{0} = 1$

$\binom{7}{0} = 1$

Binomial distribution

الدالة ذات الحدين (جذري)

قانون

$$P(X=r) = \binom{n}{r} (p)^r (1-p)^{n-r}$$

Ex Let X has binomial distribution with $n=3$ and $p=\frac{1}{4}$

① write the probability function of X ?

$$n=3 \Rightarrow r=\{0,1,2,3\} \quad / \quad p=\frac{1}{4} \rightarrow 1-p=1-\frac{1}{4}=\frac{3}{4}$$

$$\Rightarrow P(X=r) = (r)^3 \cdot \left(\frac{1}{4}\right)^r \cdot \left(\frac{3}{4}\right)^{3-r}$$

$$\begin{aligned} \text{② Find } P(X=1) &= \binom{3}{1} \cdot \left(\frac{1}{4}\right)^1 \cdot \left(1-\frac{1}{4}\right)^{3-1} \\ &= 3 \cdot \frac{1}{4} \cdot \left(\frac{3}{4}\right)^2 \end{aligned}$$

الاعداد اكبر من ٣ فـ

$$P(X>2) = P(X=3) = \binom{3}{3} \cdot \left(\frac{1}{4}\right)^3 \cdot \left(\frac{3}{4}\right)^0 = 1 \cdot \left(\frac{1}{4}\right)^3$$

$$\begin{aligned} \text{الاعداد اكبر او يساوي } 1, 2, 3 &= \text{الاعداد اكبر من ٣} \quad P(X \geq 1) = P(X=1) + P(X=2) + P(X=3) \\ &= 1 - (P=0) \end{aligned}$$

$$\begin{aligned} &= 1 - \binom{3}{0} \cdot \left(\frac{1}{4}\right)^0 \cdot \left(\frac{3}{4}\right)^{3-0} = 1 - 1 \cdot 1 \cdot \left(\frac{3}{4}\right)^3 \\ &= 1 - \frac{27}{64} \end{aligned}$$

$$\begin{aligned} P(X<1) &= (P=0) = \binom{3}{0} \cdot \left(\frac{1}{4}\right)^0 \cdot \left(\frac{3}{4}\right)^{3-0} \\ &= 1 \cdot 1 \cdot \frac{27}{64} = \frac{27}{64} \end{aligned}$$

Subject

Date

No.

Remark :-

$$1) \text{ Variance } (S^2) = n * p * (1-p) \quad \text{الباين في حالات الحدود}$$

$$2) \text{ Standard deviation } (S.d) = \sqrt{S^2}$$

[من الحالات السابقة]

$$S^2 = 3 \cdot \frac{1}{4} \cdot \frac{3}{4} = \frac{9}{16}$$

$$S = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

Discrete Random Variableأختبار $\rightarrow (r.v)$

$E(P) = 1$

الجداول العشوائية المتغير

<u>Ex</u>	<u>X</u>	0	1	2
<u>P(X)</u>	$\frac{1}{7}$	$\frac{3}{7}$	$\frac{3}{7}$	

← التكمل العام
بالنسبة إلى باستهلاك ماء

Ex X | 0 | 1 | 2 \rightarrow find C.

<u>P(X)</u>	0.2	C	0.4
-------------	-----	---	-----

$$0.2 + 0.9 + C = 1$$

$$0.6 + C = 1$$

$$C = 0.4$$

Ex X | 1 | 2 | 3 \rightarrow find C.

<u>P(X)</u>	0.1	$3C+1$	0.5
-------------	-----	--------	-----

$$0.1 + (3C+1) + 0.5 = 1$$

$$0.6 + (3C+1) = 1$$

$$3C + X = 0.4 \Rightarrow 3C = -0.6 \Rightarrow C = -0.2$$

Ex

w	R
6	4

(2) balls at Random without replacement?

defined (r.v) to be the number of w ball.

find probability distribution of X.

$$n=2 \rightarrow n=\{0,1,2\} \quad / \quad P = \frac{6}{10} \leftarrow \begin{matrix} \text{ عدد } \\ \text{ عدد } \end{matrix}$$

$\Rightarrow \{\text{WW}, \text{WR}, \text{RW}, \text{RR}\}$

~~الإيجابية~~~~الإيجابية~~~~الإيجابية~~

$$\textcircled{1} \quad p(x=0) = p(\text{RR}) = \frac{4}{10} \cdot \frac{3}{9} = \frac{12}{90}$$

↓
ولا يهمنا

$$\textcircled{2} \quad p(x=1) = p(\text{WR}) + p(\text{RW}) = \frac{6}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{6}{9} = \frac{48}{90}$$

$$\textcircled{3} \quad p(x=2) = p(\text{WW}) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90}$$

لوكان مع الارجاع يمكن حل الفؤال باستخدام ذات الحقيقة.

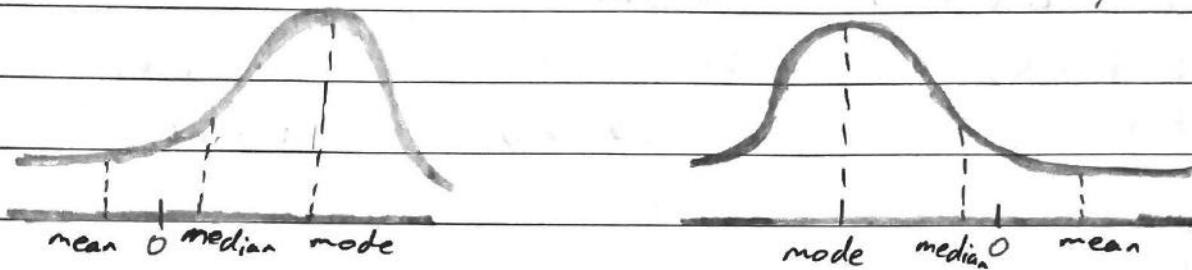
Normal Distribution

التوزيع الطبيعي

Def: a standard normal distribution denoted by

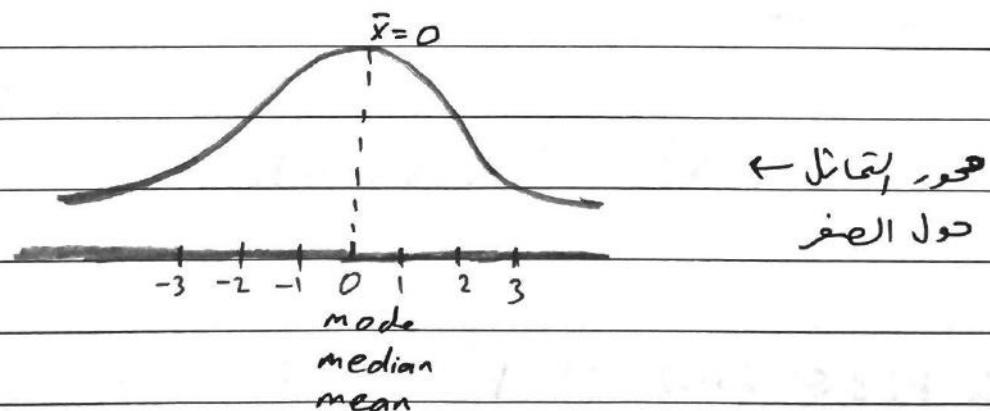
$$Z: N(0, 1)$$

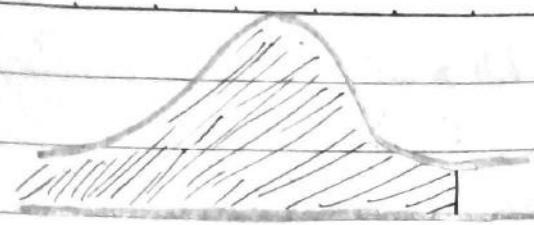
m, \bar{x} $s^2 = \sigma^2$



* أكمل قيمة باطنية المتوال
والترتيب في الرسمتين ثابت ووسط ووسط متوال .

كماءدا المحور المترافق هي الرسمة الوحيدة الذي يكون فيه
الوسط = الوسيط = المتوال = 0





حسب المساحة ←

بجاه اليسار

$$P(Z \leq z_0)$$

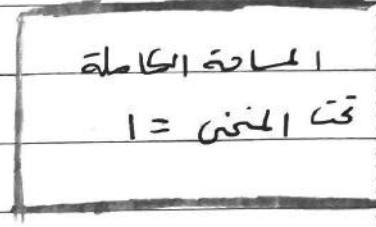
عدد

نكتة Ex $\therefore P(Z \leq 1.3) = 0.90320$

من اجل الحساب $\therefore P(Z \leq 0.14) = 0.55567$

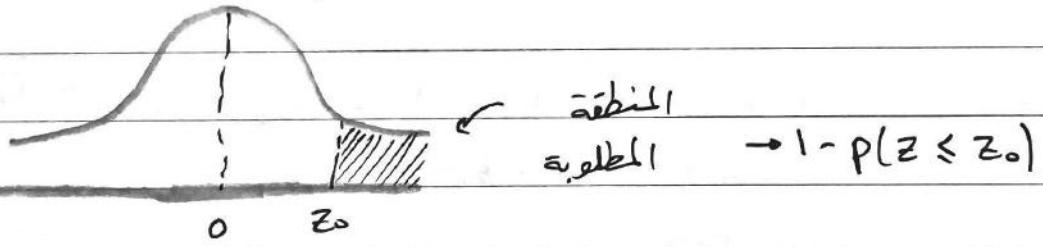
لما يزيد عن $\therefore P(Z \leq -0.23) = 0.40905$

هذا مطلب $\therefore P(Z \leq 2.31) = 0.98956$



Remark :-

$$P(Z \geq z_0) = 1 - P(Z \leq z_0)$$



المطلوبة

المطلوبة

$$\rightarrow 1 - P(Z \leq z_0)$$

Ex $\therefore P(Z \geq 1.3) = 1 - P(Z \leq 1.3)$

$$= 1 - 0.90320 = 0.0968$$

2 $P(Z \geq 2.15) = 1 - P(Z \leq 2.15)$

$$= 1 - 0.98422 = 0.01578$$

Remark :-

$$P(a \leq Z \leq b) = P(Z \leq b) - P(Z \leq a)$$

Ex $P(-0.1 \leq Z \leq 1.23)$

$$= P(Z \leq 1.23) - P(Z \leq -0.1)$$

$$= 0.89065 - 0.46017 = 0.43048$$

* لوَانَ مُعْنَا فِي الْبَرْوَلِ التَّوْجِيهِ :

1 $P(Z \leq -Z_0) = 1 - P(Z \leq Z_0) \Leftarrow$ قاعدة

Ex $P(Z \leq -1.34) = 1 - P(Z \leq 1.34)$

$$= 1 - 0.90988 = 0.09012$$

2 $P(Z \geq -Z_0) = P(Z \leq Z_0)$

Ex $P(Z \geq -1.35) = P(Z \leq 1.35)$

$$= 0.91149 .$$

* إذاً أعطانا X ولم يعطنا Z هنا نعمل $X \rightarrow Z$ حسب القانون التالي:

$$Z = \frac{X - \bar{x}}{S}$$

الخطوات
الاعمال المعاكير (S.D)

Ex if $\bar{x}=60$, $S=5$ find:

$$\textcircled{1} \quad P(X \leq 70) = P\left(\frac{X-\bar{x}}{S} \leq \frac{70-60}{5}\right) = P(Z \leq 2) \\ = 0.97725$$

~~$$\textcircled{2} \quad P(X \geq 50) = P\left(\frac{X-\bar{x}}{S} \geq \frac{50-60}{5}\right) = P(Z \geq -2) \\ = 1 - P(Z \leq 2) = 1 - 0.97725 \\ \rightarrow = P(Z \leq 2) \\ = 0.97725$$~~

$$\textcircled{3} \quad P(65 \leq X \leq 75) = P\left(\frac{65-60}{5} \leq \frac{X-\bar{x}}{S} \leq \frac{75-60}{5}\right) \\ = P(1 \leq Z \leq 3) \\ = P(Z \leq 3) - P(Z \leq 1) \\ = 0.99865 - 0.84134 = 0.15731$$

Subject

Date

No.

Ex Let $x: N(60, 16)$ find:

$$\mu = \bar{x} \quad \sigma^2 = s^2 \quad \rightarrow \bar{x} = 60$$

$$\textcircled{1} \quad P(x \geq 65)$$

$$s = \sqrt{16} = 4$$

$$\rightarrow P\left(\frac{x-\bar{x}}{s} \geq \frac{65-60}{4}\right) = P(Z \geq 1.25)$$

$$= 1 - P(Z \leq 1.25) = 1 - 0.89435 = 0.10565$$

$$\textcircled{2} \quad P(55 \leq x \leq 67)$$

$$P\left(\frac{55-60}{4} \leq \frac{x-\bar{x}}{s} \leq \frac{67-60}{4}\right) = P(-1.25 \leq Z \leq 1.75)$$

$$= P(Z \leq 1.75) - P(Z \leq -1.25)$$

$$= 0.95995 - 0.10565 = 0.8543$$

Ex let $x: N(2, 16)$ find A such that

$$P(x \leq A) = 0.9750.$$

$$P(x \leq A) = P\left(\frac{x-\bar{x}}{\sigma} \leq \frac{A-2}{4}\right) = P\left(Z \leq \frac{A-2}{4}\right) = 0.9750$$

Lipcszani

$$\rightarrow \frac{A-2}{4} = 1.96 \Rightarrow A-2 = 7.84$$

الرقم في الأعلى

$$\rightarrow A = 9.84 \quad \times$$

Ex Let $x: N(10, 1)$ find A if $P(Z \geq A) = 0.0228$

$$P(Z \geq A) = 1 - P(Z \leq A) = 0.0228$$

$$\rightarrow P(Z \leq A) = 1 - 0.0228 = 0.9772$$

$$\rightarrow A = 2.0 \quad \text{جواب}\downarrow$$

Ex Let $X: N(\mu, \sigma^2)$ and $P(X \geq 4) = 0.9772$

$$P(X \geq 5) = 0.9332 \quad \text{find } \mu, \sigma^2$$

$$P(X \geq 4) = 0.9772$$

$$P(X \geq 5) = 0.9332$$

$$P\left(\frac{X-\bar{x}}{s} \geq \frac{4-\bar{x}}{s}\right) = 0.9772$$

$$P\left(\frac{X-\bar{x}}{s} \geq \frac{5-\bar{x}}{s}\right) = 0.9332$$

$$P\left(Z \geq \frac{4-\bar{x}}{s}\right) = 0.9772$$

$$P\left(Z \geq \frac{5-\bar{x}}{s}\right) = 0.9332$$

$$1 - P\left(Z \leq \frac{4-\bar{x}}{s}\right) = 0.9772$$

$$1 - P\left(Z \leq \frac{5-\bar{x}}{s}\right) = 0.9332$$

$$\rightarrow P\left(Z \leq \frac{4-\bar{x}}{s}\right) = 0.02280$$

$$\rightarrow P\left(Z \leq \frac{5-\bar{x}}{s}\right) = 0.06680$$

جواب

$$\frac{4-\bar{x}}{s} = -2$$

$$\frac{5-\bar{x}}{s} = 1.5$$

$$\rightarrow 4 - \bar{x} = -2s \quad \dots \textcircled{1}$$

$$\rightarrow 5 - \bar{x} = 1.5s \quad \dots \textcircled{2}$$

$$\rightarrow 4 - \bar{x} = -2s$$

$$5 - \bar{x} = 1.5s \quad /-$$

$$+1 = +0.5s \quad \rightarrow \boxed{s = 2} \quad \rightarrow \sigma^2 = s^2 = (2)^2 = 4$$

$$4 - \bar{x} = -2s$$

$$4 + 4 = \bar{x} \quad \rightarrow \boxed{\bar{x} = 8} \quad \rightarrow \mu = \bar{x} = 8$$

Sampling distribution :-

Def :- the sampling distribution of statistic is the probability distribution for the possible values of the statistic that results when random samples of (n) repeated drawn from the population

جدول احتمالي للوحوش اي ↪

Ex Population $\{1, 2, 3, 4, 5\}$, $n=2$

(select all possible sample of size = 2)

1 Find the sampling distribution.

2 find $E. \bar{x} = (\Sigma \bar{x} \cdot P(\bar{x}))$

sol

	(1, 2)	(2, 3)	(3, 4)	(4, 5)
	(1, 3)	(2, 4)	(3, 5)	
	(1, 4)	(2, 5)		
	(1, 5)			

→ sampling

الكتل غير موحده

الكتل

العمليةsampling \bar{X}

(1, 2)	$\rightarrow \frac{1+2}{2} = 1.5$
(1, 3)	$\rightarrow \frac{1+3}{2} = 2$
(1, 4)	$\rightarrow 2.5$
(1, 5)	$\rightarrow 3$
(2, 3)	$\rightarrow 2.5$
(2, 4)	$\rightarrow 3$
(2, 5)	$\rightarrow 3.5$
(3, 4)	$\rightarrow 3.5$
(3, 5)	$\rightarrow 4$
(4, 5)	$\rightarrow 4.5$

<u>\bar{X}</u>	1.5	2	2.5	3	3.5	4	4.5
<u>$P(\bar{X})$</u>	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

الجدول الذي يمثل
العمرط الاحتمالي

$$[2] \rightarrow E \cdot \bar{X} = 1.5 * \frac{1}{10} + 2 * \frac{1}{10} + 2.5 * \frac{2}{10} + 3 * \frac{2}{10} + 3.5 * \frac{3}{10} + 4 * \frac{1}{10} + 4.5 * \frac{1}{10}$$

$$\rightarrow E \cdot \bar{X} = \underline{\underline{3}}$$

Remarks-

$$E \cdot \bar{X} = \mu = \bar{x}$$

بنتاج
السؤال الرابع
 $\Rightarrow 3$

$$\leftarrow \frac{1+2+3+4+5}{5} = \frac{15}{5} = \underline{\underline{3}}$$

Poisson distribution

توزيع بواسون

$$P(x=x) = \begin{cases} \frac{\mu^x \cdot e^{-\mu}}{x!}, & x=0, 1, 2, 3, \dots \\ 0, & \text{otherwise.} \end{cases}$$

الشرط
العادي

Ex if $\mu = 5$, by poisson distribution finds:-

$$\textcircled{1} P(x=0)$$

$$P(x=0) = \frac{5^0 \cdot e^{-5}}{0!} = \frac{1 \cdot e^{-5}}{1} = \frac{1}{e^5}$$

$$\textcircled{2} P(x < 2)$$

$$\begin{aligned} P(x < 2) &= P(x=0) + P(x=1) \\ &= \frac{1}{e^5} + \frac{5^1 \cdot e^{-5}}{1!} = \frac{1}{e^5} + \frac{5}{e^5} = \frac{6}{e^5} \end{aligned}$$

$$\textcircled{3} P(1 < x < 4)$$

$$\begin{aligned} P(1 < x < 4) &= P(x=2) + P(x=3) \\ &= \frac{5^2 \cdot e^{-5}}{2!} + \frac{5^3 \cdot e^{-5}}{3!} = \frac{25 \cdot e^{-5}}{2} + \frac{125 \cdot e^{-5}}{6} \\ &= \frac{75 + 125}{6e^5} = \frac{200}{6e^5} \end{aligned}$$

$$\textcircled{4} P(x=\frac{1}{2})$$

$$P(x=\frac{1}{2}) = 0$$

نهاية صيغة لذ

أعداد متحركة فقط $x=0, 1, 2, \dots$

ونتظم نفس الطريقة في حالة الاعداد الابدية

Ex if $P(A_1) = 0.7$, $P(A_2) = 0.5$, $P(A_1 \cap A_2) = 0.4$
finds.

$$\begin{aligned} \textcircled{1} P(A_1 \cup A_2) &= P(A_1) + P(A_2) - P(A_1 \cap A_2) \\ &= 0.7 + 0.5 - 0.4 = 0.8 \end{aligned}$$

$$\begin{aligned} \textcircled{2} P(A_1 - A_2) &= P(A_1) - P(A_1 \cap A_2) \\ &= 0.7 - 0.4 = 0.3 \end{aligned}$$

$$\begin{aligned} \textcircled{3} P(A_2 \cap \bar{A}_1) &= P(A_2 - A_1) = P(A_2) - P(A_1 \cap A_2) \\ &= 0.5 - 0.4 = 0.1 \end{aligned}$$

$$\textcircled{4} P(A_1 / A_2) = \frac{P(A_1 \cap A_2)}{P(A_2)} = \frac{0.4}{0.5} = \frac{4}{5}$$

$$\textcircled{5} P(A_1 / \bar{A}_2) = \frac{P(A_1 \cap \bar{A}_2)}{P(\bar{A}_2)} = \frac{P(A_1 - A_2)}{1 - P(A_2)} = \frac{0.3}{0.5} = \frac{3}{5}$$

$$\textcircled{6} P(A_1 \cup A_2) = 1 - P(A_1 \cap A_2) \\ = 1 - 0.8 = 0.2$$

Remark

$$\begin{aligned} A_1 \cup A_2 &= A_2 \cup A_1 \\ A_1 \cap A_2 &= A_2 \cap A_1 \end{aligned}$$

Random variable :

متغير عشوائي

Ex

①	X	0	1	2	find b ?
	P(X)	$\frac{1}{9}$	$\frac{2}{9}$	b	

$$\frac{1}{9} + \frac{2}{9} + b = 1 \rightarrow \left(\frac{3}{9}\right) + b = 1$$

$$\rightarrow b = \frac{6}{9}$$

$$\textcircled{2} \quad \{(0, 0.1), (1, 2b+1), (2, 0.5)\} \quad \text{find } b ?$$

نفس طريقة الفرج ① لكن مكان الجدول أزواج مترتبة.

$$0.1 + 2b+1 + 0.5 = 1$$

$$2b+1 + 0.6 = 1$$

$$2b+1 = 0.4 \quad \rightarrow \quad \frac{2b}{2} = \frac{-0.6}{2} \quad \rightarrow \quad b = -0.3$$

Binomial distribution :

Ex if $a = \frac{1}{3}$, $n = 3$ by binomial distribution
find:

$$\textcircled{1} \quad P(X=1) = \binom{3}{1} \cdot \left(\frac{1}{3}\right)^1 \cdot \left(1 - \frac{1}{3}\right)^{3-1} = 3 \cdot \frac{1}{3} \cdot \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\textcircled{2} \quad P(X > 2) = P(X=3) = \binom{3}{3} \cdot \left(\frac{1}{3}\right)^3 \cdot \left(1 - \frac{1}{3}\right)^{3-3} = 1 \cdot \frac{1}{27} \cdot 1 = \frac{1}{27}$$

$$\textcircled{3} \quad P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$\therefore = 1 - P(X=3) = 1 - \frac{1}{27} = \frac{26}{27}$$

Natural distribution :

جدول التوزيع الطبيعي

$$\textcircled{1} \quad P(Z < a)$$

$$\textcircled{2} \quad P(Z \geq a) = 1 - P(Z < a)$$

$$\textcircled{3} \quad P(a \leq Z \leq b) = P(Z \leq b) - P(Z \leq a)$$

$$\textcircled{4} \quad P(Z < -a) = 1 - P(Z \leq a)$$

$$\textcircled{5} \quad P(X \leq a) \rightarrow P\left(\frac{x-\bar{x}}{s} \leq \frac{a-\bar{x}}{s}\right) = P(Z \leq \frac{a-\bar{x}}{s})$$

Gama function : $(\Gamma(n))$

$$\Gamma(n) = (n-1)!$$

Ex find 8-

$$\textcircled{1} \quad \Gamma(5) = 4! = 4 * 3 * 2 * 1 = 24$$

$$\textcircled{2} \quad \Gamma(1) = 0! = 1$$

$$\textcircled{3} \quad \Gamma(7) = 6! = 720$$

$$\textcircled{4} \quad \frac{\Gamma(3)}{\Gamma(2)} = \frac{2!}{1!} = 2$$

$$\textcircled{5} \quad \frac{\Gamma(10)}{\Gamma(8)} = \frac{9!}{7!} = \frac{9 * 8 * 7!}{7!} = 72$$

Beta function $\beta(n, m)$

$$\beta(n, m) = \frac{\Gamma(n) \cdot \Gamma(m)}{\Gamma(n+m)}$$

Ex Find :

$$\textcircled{1} \quad \beta(5, 3) = \frac{\Gamma(5) \cdot \Gamma(3)}{\Gamma(5+3)} = \frac{4! \cdot 2!}{7!} = \frac{1}{105}$$

$$\textcircled{2} \quad \beta(3, 1) = \frac{\Gamma(3) \cdot \Gamma(1)}{\Gamma(3+1)} = \frac{2! \cdot 0!}{4!} = \frac{1}{12}$$