PROJECT PROPOSAL: SYMMETRIC INDEFINITE SOLVERS

CHEN GREIF & PAUL LIU

This project will focus on developing fast and robust solvers for symmetric indefinite linear systems. Such systems often appear in the form of saddle-point systems associated with constrained optimization problems [1]. The matrices typically have a special structure, which can be exploited to decrease memory costs and time complexity during computation [1, 3].

We will focus on the following aspects of the problem:

(1) *Structurally symmetric ordering strategies.* Our focus here will be on utilizing graph algorithms to re-order the unknowns in a way that facilitates a more efficient use of Gaussian elimination solvers. Such techniques are based on analyzing the nonzero patterns of a matrix and its respective decompositions. These techniques generally try to minimize the bandwidth of the matrix, reduce the fill-in, or dissect the graph of the matrix into manageable sub-graphs [2].

(2) *Specialized block factorizations.* Here the main difficulty lies in the fact that standard elimination procedures may be numerically unstable. Therefore, techniques based on block $\mathrm{LDL}^{\mathrm{T}}$ decompositions must be used [3].

(3) *Connection to preconditioning.* Ultimately, we will aim to use the code that we generate for computing incomplete factorizations as preconditioners for iterative solvers. Here the challenge is in performing the elimination procedure in a way that allows for dropping elements while preserving symmetry and keeping the computational cost at bay.

Any of the above items has been considered in the literature, but sophisticated solvers that are based on all the above are not very common. Practitioners often prefer to eliminate the indefiniteness (whenever possible), since indefinite solvers involve many subtleties and are nontrivial to implement. Maintaining the indefiniteness may potentially provide flexibility and robustness and keep the structure of the underlying (often continuous) intact.

References

1. Michele Benzi, Gene H. Golub, and Jörg Liesen, *Numerical solution of saddle point problems*, Acta Numer. **14** (2005), 1–137.
2. Timothy A. Davis, *Direct methods for sparse linear systems*, Fundamentals of Algorithms, vol. 2, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2006.
3. Nicholas J. Higham, *Accuracy and stability of numerical algorithms*, second ed., Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2002.