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## Identification of basketball parameters for a simulation model

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### Abstract

We estimate values of the coefficients of air drag, friction, stiffness and damping for a dynamic model of basketball shots by analyzing ball position and orientation using video data for basketball-rim and basketball-backboard bounce tests. The dynamic model can calculate any configuration change of a basketball during shots. The simulation model is compared to actual bounce tests and the parameters are determined from these experiments. The dynamic model with the identified parameters calculates a ball trajectory similar to the actual measured results. Stiffness and damping coefficients are determined from basketball-backboard bounce tests.

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**Keywords:** Basketball; dynamic model; coefficient of friction; basketball stiffness and damping

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### 1. Introduction

A complete, detailed and accurate dynamic model with real parameters for basketball shots can be used in numerical simulations to replace actual dynamic experiments. In order to improve the model it will be necessary to compare it to actual physical experiments.

Some previous studies have measured basketball parameters. Okubo and Hubbard [1] did MTS tests to obtain the ball stiffness and damping coefficients. Okubo and Hubbard [2] also measured the coefficient of friction between the ball and rim, and between the ball and a polycarbonate backboard. Alaways et al. [3] introduced a new friction test apparatus to determine coefficients of friction for round balls including basketballs. Two dynamic models have been used to analyze basketball shots. Okubo and Hubbard [4][5] investigated and compared release conditions for capture in direct and bank shots using their dynamic model, which has six distinct sub-models. Silverberg et al. [6] analyzed direct and bank shots using a comparable dynamic model.

To our knowledge, no one has compared their dynamic model to actual basketball shots. We compare our dynamic model to actual data and improve it. Data on basketball shots are obtained in some simple bounce tests

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from the rim and backboard. By comparing ball trajectories and angular velocities, reasonable estimates are determined for the air drag coefficient, basketball stiffness and damping coefficients, and coefficients of friction between the ball and rim, and ball and backboard.

## 2. Methods

We assume that the basketball with radial compliance and damping maintains contact with the rim/backboard. The configuration has six degrees of freedom. Three determine ball position and the others describe ball orientation. A Coulomb friction force is assumed to act in the tangential plane either between the basketball and rim or between the ball and backboard.

An official NBA basketball was used in the experiment with a mass of 0.591 kg and a radius of 0.12 m. The official basketball rim had major inside diameter of 0.45 m and minor diameter of 0.019 m, and the official backboard was made from safety tempered glass. Ball inflation pressure was 8 psi. Gravity was estimated to be 9.7975 at a latitude of 35.68 degrees.

Data on basketball trajectories were obtained by filming free fall and bounce tests on the rim and backboard using a video camera. Ball position and angle were analyzed from the video data in order to identify ball parameters. Comparing trajectories between actual data and our dynamic simulation, we identify ball stiffness, damping, air drag coefficient, and coefficient of friction between the ball and rim and the ball and board.

## 3. Results

### 3.1 Air drag

A basketball was dropped from the height of the bottom of backboard as shown in Fig. 1. We calculated the trajectory of the ball using the DLT method. The camera was operated at 500 Hz. When the ball is assumed to be acted on by air resistance with constant air drag, the equation of motion is

$$-mg + \frac{1}{2} \rho A C_D v^2 = m\dot{v} \quad (1)$$

where  $m$  is basketball mass,  $g$  is gravity,  $\rho$  is air density,  $A$  is ball cross sectional area,  $C_D$  is the coefficient of drag and  $v$  is the vertical velocity. The vertical displacement  $h$  of the basketball from the floor can be expressed as

$$h = h_0 - \frac{a^2}{g} \ln \cosh\left(-\frac{g}{a} t\right) \quad (2)$$

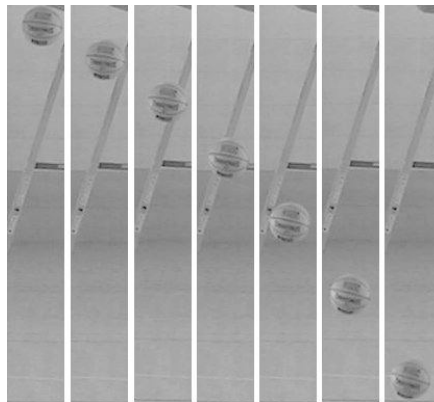


Fig.1: Film data of a basketball free fall.

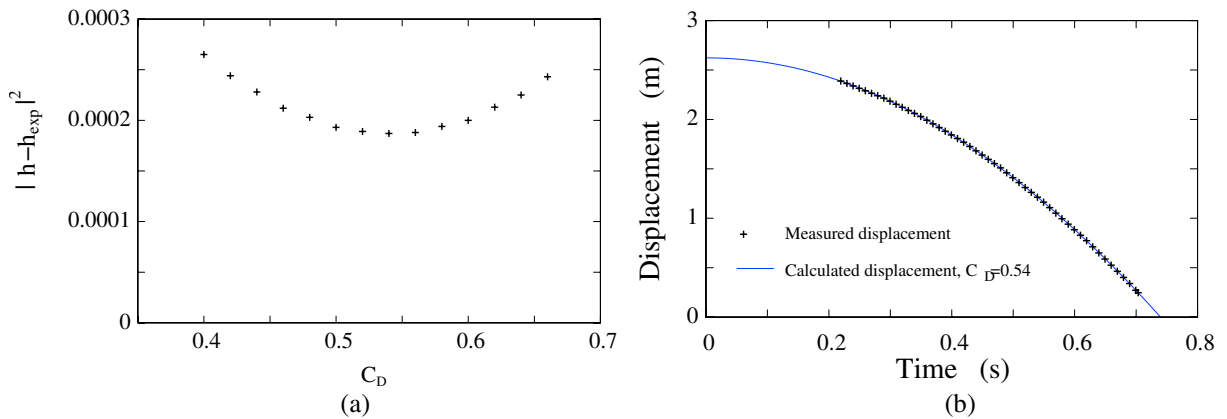


Fig.2: (a) Difference between the actual and calculated trajectories. (b) Trajectory of basketball in free fall with the coefficient of air drag of 0.54

where

$$a = \sqrt{\frac{2mg}{\rho A C_D}}$$

$h_0$  is the initial vertical position from the floor. Comparing the trajectory of Eq. (2) to the actual data of the vertical position between 0.24 and 2.383 m, measured from the bottom of basketball, as shown in Fig. 2(a), we estimate the coefficient of air drag to be 0.54. Figure 2(b) shows the basketball trajectory of free fall with a constant air drag coefficient of 0.54.

### 3.2 Stiffness and damping coefficients

Image data was also obtained by filming some bounce tests from the backboard for shots with back-spin, low-spin, and top-spin angular velocities. Two cameras operated at 1000 Hz. Figure 3 shows examples of the basketball-backboard interaction for a shot with back-spin.

Durations of ball-board contact were measured to be 0.011, 0.012, and 0.011 s for shots with back-spin, low, and top-spin, respectively. The stiffness  $k$  N/m can be calculated by

$$k = m \left( \frac{\pi}{t_c} \right)^2 \quad (3)$$

where  $t_c$  is the average of the duration of the contact. We adopt the average value of the contact duration for the value of  $t_c$  and estimate the stiffness to be  $45(10^3)$  N/m. This is close to the result measured quasi-statically from the result of force-deflection hysteresis curves [1]. Cross [7] measured the normal reaction force at a ball-contact point with his experimental arrangement for several balls, in which the duration of contact was approximately 15 ms. The durations for basketball-backboard and basketball-arrangement include the stiffness of the board and the arrangement, respectively. We expect the net stiffness of basketball to be larger than the values measured by Cross.

We also calculated the impact and rebound velocity of basketball in a ball-board bounce test. The equivalent radial damping coefficient can be determined from the impact and rebound velocities to the normal direction of the backboard. The combination of impact and rebound velocities ( $v_1, v_2$ ) are (3.02, -2.40), (2.30, -1.92), and (1.92, -1.73) m/s for the three types of shots with back-spin, low spin, and top-spin angular velocities, respectively. The damping ratio  $\zeta$  and coefficient can be calculated by

$$\zeta = \left( \ln \left| \frac{v_2}{v_1} \right| \right)^2 / \left[ \pi^2 + \left( \ln \left| \frac{v_2}{v_1} \right| \right)^2 \right] \quad (4)$$

$$c = 2m\zeta \sqrt{\frac{k}{m}} \quad (5)$$

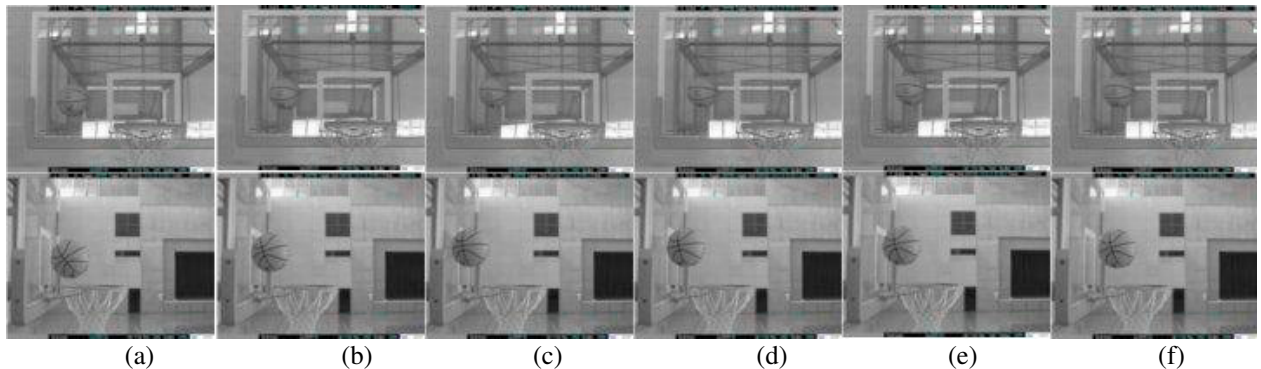


Fig. 3: Example of basketball-backboard bounce for a shot with back-spin. Ball positions at 80 ms intervals are shown. The ball moves in a normal plane to the backboard with spin axis approximately parallel to the backboard.

The damping coefficients are estimated to be 24, 19, and 11 Ns/m, respectively. These values of damping coefficient are slightly larger than those measured from the hysteresis curves of force-deflection [1] when the impact velocity is larger than about 3 m/s. The probable reason is that the actual backboard is not rigid and dissipates energy in addition to the ball.

### 3.3 Coefficient of friction

We previously measured the coefficient of friction between a leather basketball and rim to be 0.5 in a slip test [2]. The coefficient of friction between the basketball and a tempered glass backboard has been estimated to be approximately 0.6 with an inclined plane experiment.

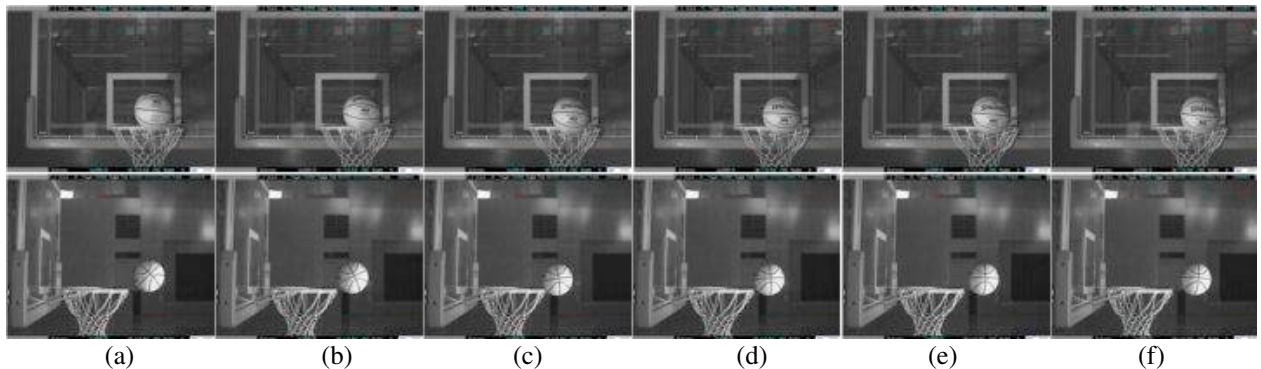


Fig. 4: Example of basketball-rim bounce for shot with back-spin. The ball positions at 80 ms intervals are shown. The ball moves in the YZ plane and the direction of spin is approximately about the X-axis.

Measured basketball trajectories are compared to calculated trajectories using our dynamic model with the estimated basketball parameters. Actual data are obtained from ball-board and ball-rim bounce tests of shots with back-spin, low, and top-spin angular velocities. The ball moves almost completely in a normal plane to the backboard. The direction of spin is approximately parallel to the backboard and floor. Figure 4 shows an example of the ball-rim interaction for a shot with back-spin angular velocity.

Position and orientation of the basketball are estimated from the video data. A Newtonian frame XYZ with the origin at the middle of hoop center has its XY plane horizontal and Y axis normal to the backboard. The position vector  $(x, y, z)$  denotes the basketball center. The dynamic equations with the estimated basketball parameters are used to calculate ball trajectories. The model allows slipping and non-slipping motion at a single ball-contact point and switches between slipping and non-slipping motions depending on the contact velocity and forces.

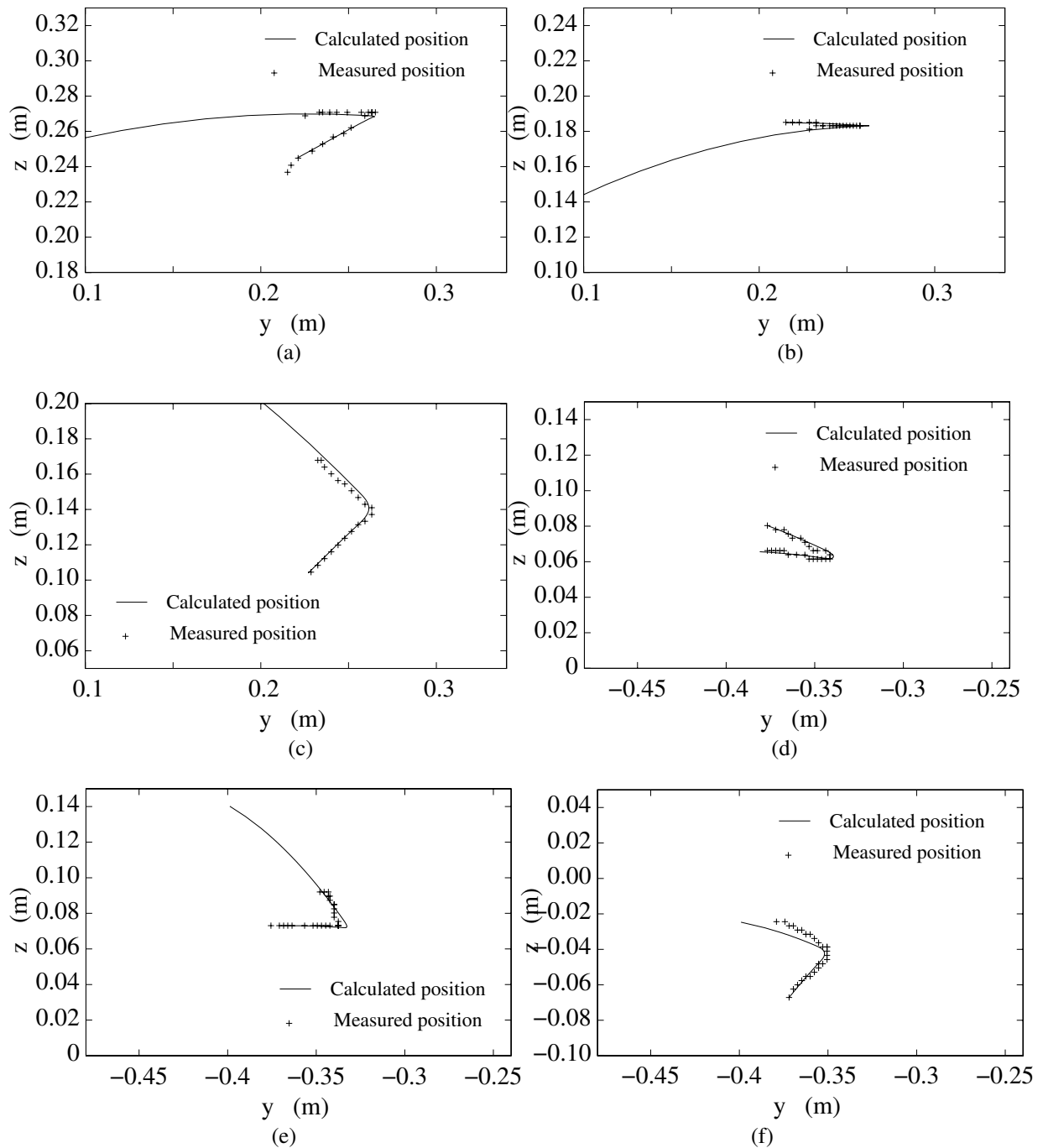


Fig. 5: Horizontal and vertical displacement of the basketball in bounce tests. Ball-board bounce for shots with (a) back-spin, (b) low, and (c) top-spin angular velocities. Ball-rim bounce for shots with (d) back-spin, (e) low, and (f) top-spin angular velocities. Ball positions at 2 ms intervals are plotted.

The actual and calculated displacements of ( $y, z$ ) are shown in Fig. 5(a)-(f). Ball positions at 2 ms intervals are plotted. The values of angular velocities of ball about the X-axis are shown in Table 1. Although the dynamic model with the measured parameters is not perfect, it calculates similar motions to measured ball trajectories. The probable

reason for differences between the calculation and actual motion is that the rim and backboard are completely rigid in the model. Also, measured parameters are average or typical values and therefore cannot cover any situation. The ball probably does not have the same coefficient of friction at all points on the ball surface. However, the dynamic model can be improved by optimizing the basketball parameters.

Table 1: Angular velocities (rad/s) before and after impact.

	ball-board			Ball-rim		
before impact	15.0	-1.75	-15.7	16.4	1.6	-13.6
after impact(measured)	-3.3	-0.2	-10.6	3.7	-4.6	-6.7
after impact(calculated)	-1.0	0.7	-14.6	5.9	-3.7	-8.0

#### 4. Conclusions

We have identified parameters for a simulation model of basketball shots by analyzing measured basketball position and orientation from video data in ball-rim and ball-bounce experiments. The coefficient of air drag has been estimated from a free-fall trajectory. The stiffness and damping coefficients are determined from ball-board bounce tests. The dynamic model with the measured basketball parameters calculates reasonable ball trajectories. The model may be able to help players improve their skills by using it to calculate optimal release conditions in any type of basketball shot and to expect optimal position for rebounds.

#### Acknowledgements

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