Understanding the role of seedbanks in populations under selection

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Motivation

Cyclic selection in nature. Side-blotched lizards exhibit rock–paper–scissors frequency-dependent competition among male morphs (Sinervo and Lively, 1996). These dynamics motivate multitype models with competitive feedback.

Ecological memory via seedbanks. *Daphnia* produce resting eggs that can hatch after many seasons, effectively sampling ancestors far in the past and slowing evolutionary turnover.

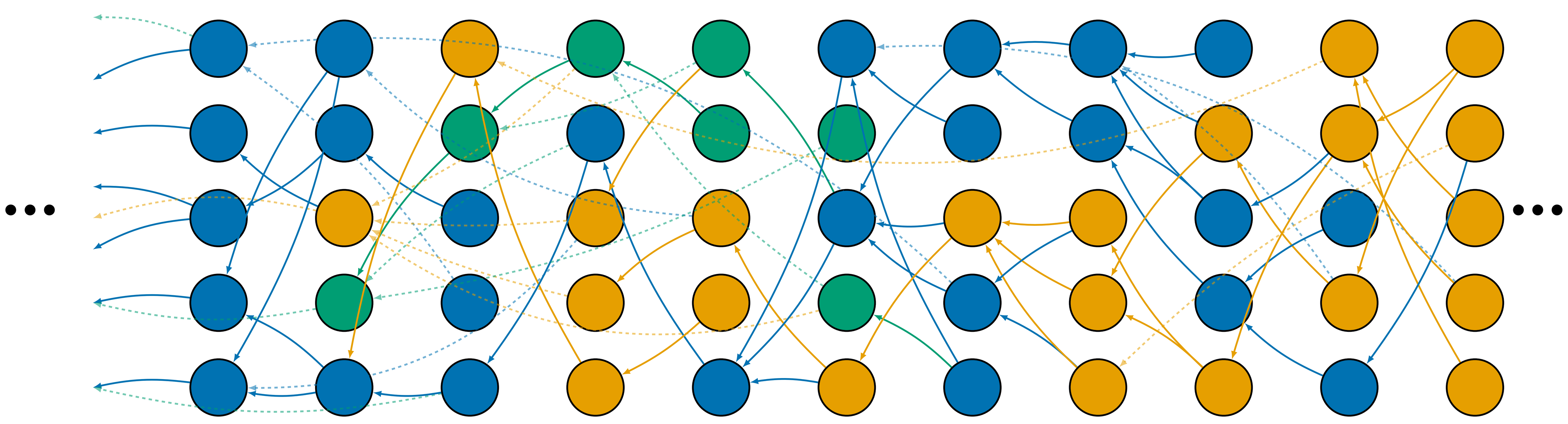
Mathematical motivation. Classical multitype Wright–Fisher models assume reproduction only from the previous generation. Recent work incorporates seedbanks or frequency-dependent selection.

Our contribution. We study a multitype model with both frequency-dependent selection and seedbank: individuals choose parents from the current generation and a deep ancestral pool. This framework captures coexistence, fixation, and oscillatory behavior under ecological memory.

Model Description

- N individuals at each generation $g \in \mathbb{Z}$ with types in $[K]$.
- Each individual v at generation g has K_v (usually 1 or 2) potential parents, each chosen:
  - Uniformly from generation $g-1$ with probability r_N.
  - Uniformly from generation $g-1-k$ with probability $(1-r_N)q_N(1-q_N)^{k-1}$.
- The type of individual v is selected according to a coloring rule (dependent on the types of the potential parents).

Graphical Construction



Frequency Process

We study the frequency process $(X^N(g) : g \in \mathbb{Z})$ where $X_i^N(g) := \frac{1}{N}\#\{v \in V_g : \mathrm{type}(v) = i\}$. We use the auxiliary process $(Y^N(g) : g \in \mathbb{Z})$ given by $Y_i^N(g) = \sum_{k=1}^{\infty} q_N(1-q_N)^{n-k} X_i(g-k)$.

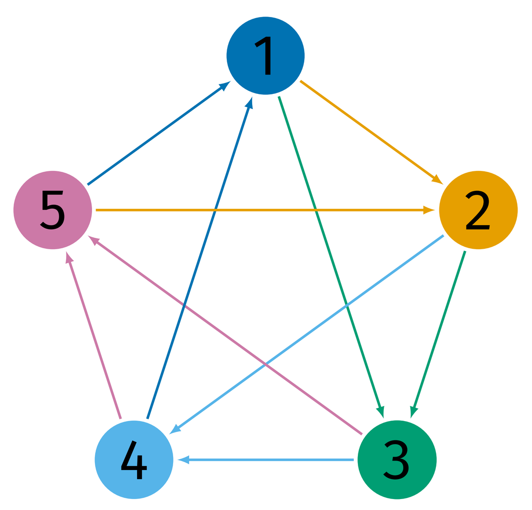
Under certain conditions, for large N, we can approximate (X^N, Y^N) by $(\mathcal{X}, \mathcal{Y})$, the solution of

$$d\mathcal{X}_i(t) = \kappa\big(\mu_i(\mathcal{X}(t)) + c_1(\mathcal{Y}_i(t) - \mathcal{X}_i(t))\big)dt + \sigma\sqrt{\mathcal{X}_i(t)}dB_i(t) - \sigma\mathcal{X}_i(t)\sum_{j=1}^{K}\sqrt{\mathcal{X}_j(t)}dB_j(t)$$

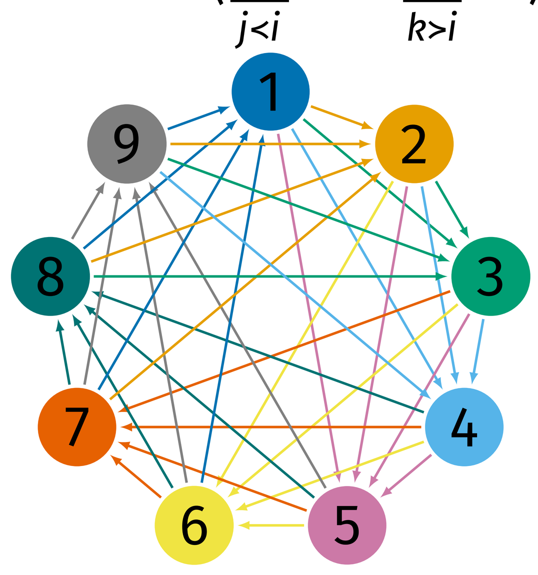
$$d\mathcal{Y}_i(t) = c_2(\mathcal{X}_i(t) - \mathcal{Y}_i(t))dt$$

Rock–Paper–Scissors Coloring Rules

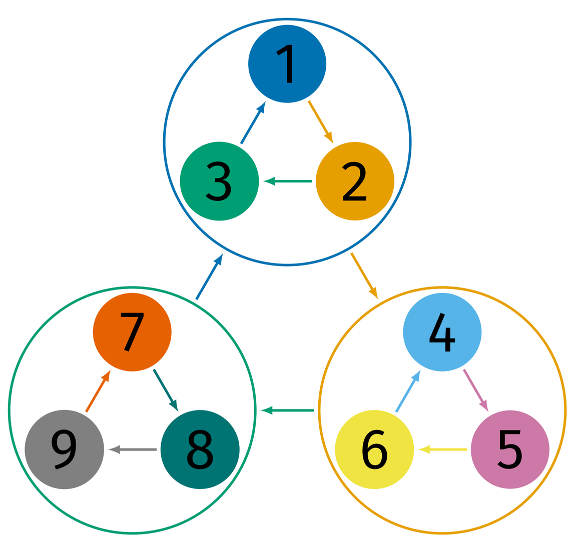
$$\mu_i(x) = x_i\Big(\sum_{j<i} x_j - \sum_{k>i} x_k\Big)$$



5-RPS



9-RPS



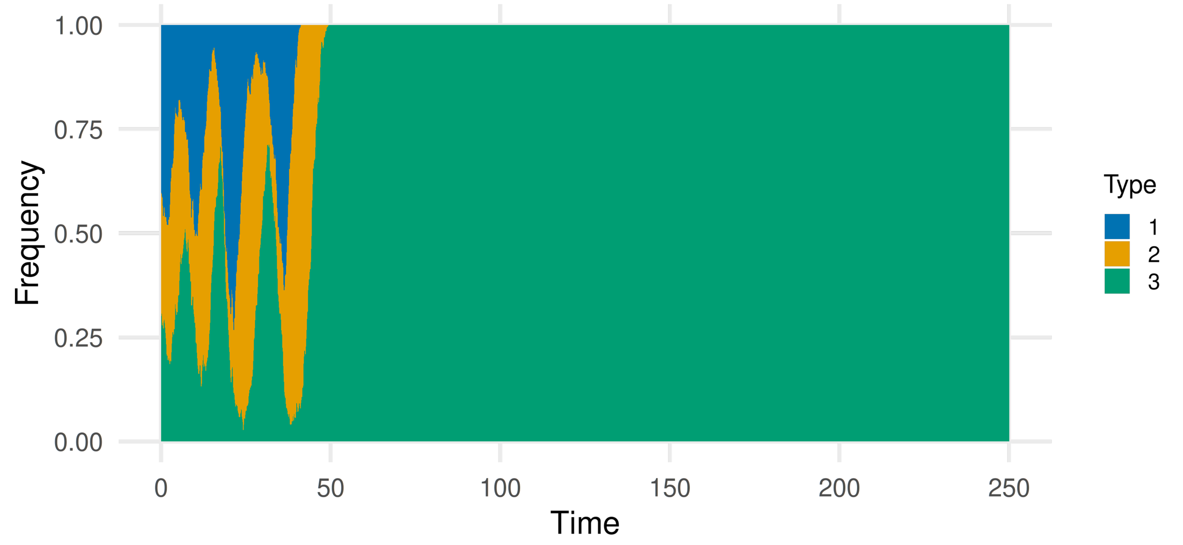
Hierarchical RPS

Lizard & Daphnia

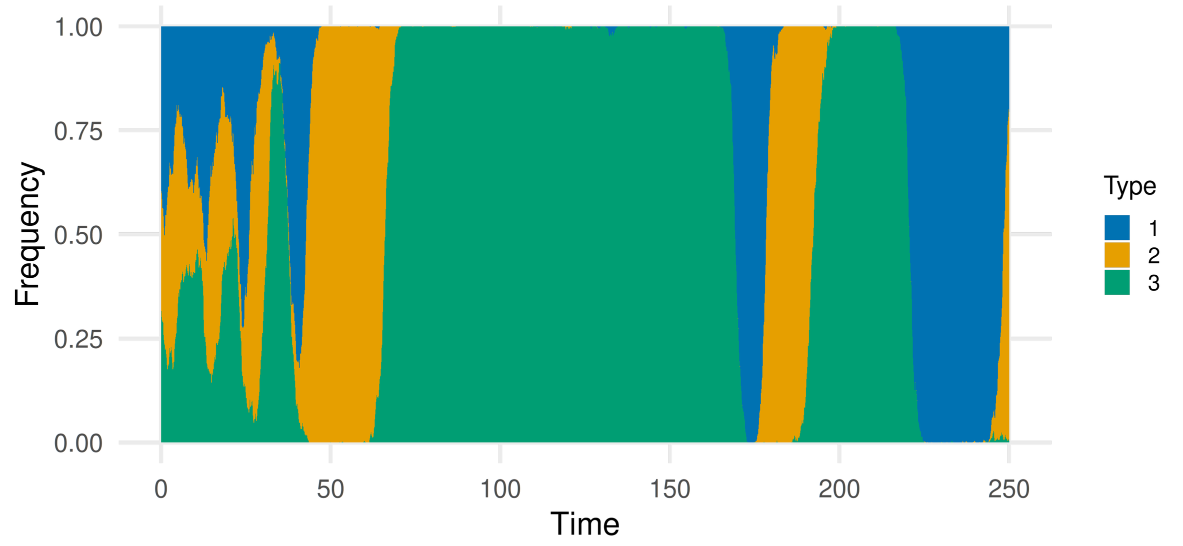
Photo: Judy Gallagher / CC BY 2.0

Photo: Hajime Watanabe / CC BY 2.5

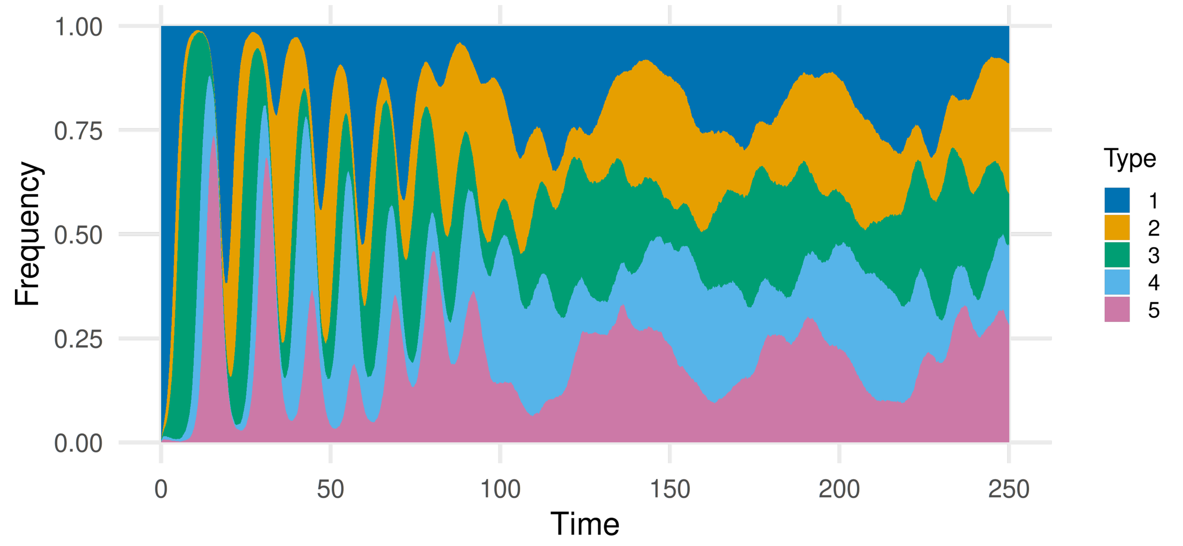
RPS with no seedbanks



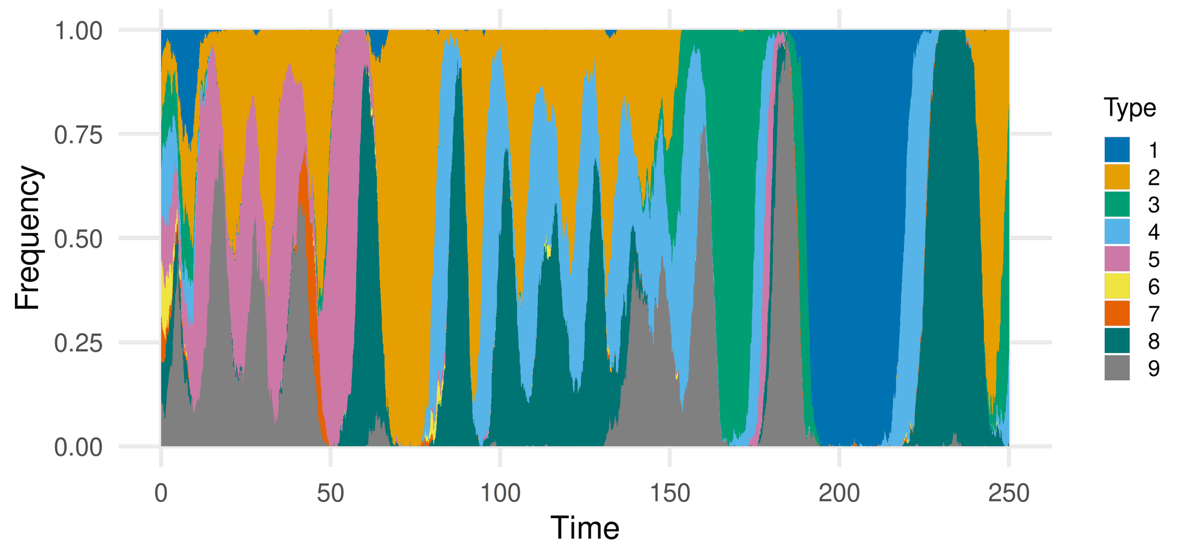
RPS with seedbanks



5-RPS



Hierarchical RPS



References

González Casanova, A. and Smadi, C. (2020). On Λ-Fleming–Viot processes with general frequency-dependent selection. *Journal of Applied Probability* **57**(4), 1162–1197.

Sinervo, B., Lively, C. M. (1996). Rock–paper–scissors and the evolution of alternative strategies. *Nature* 380, 240–243.

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