The spaces of (**multitype**) **Λ-coalescents** and **continuous state branching processes** are **homeomorphic**. We can construct an **explicit** homeomorphism.

Multitype Λ-coalescents and continuous state branching processes

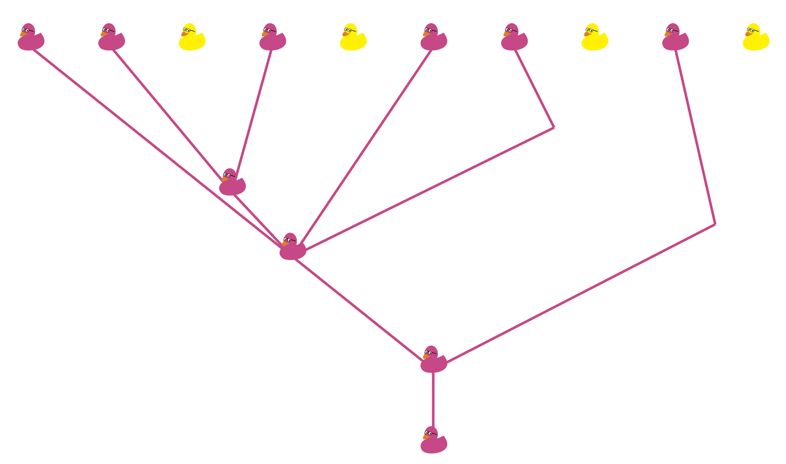
Imanol Nuñez Morales
Centro de Investigación en Matemáticas, A.C.

Joint work with Adrián González Casanova, Noemi Kurt & José-Luis Pérez

Seminar on Stochastic Processes 2025

Λ-coalescents (L)

Exchangeable, consistent, coalescent processes **without multiple** simultaneous coalescent events



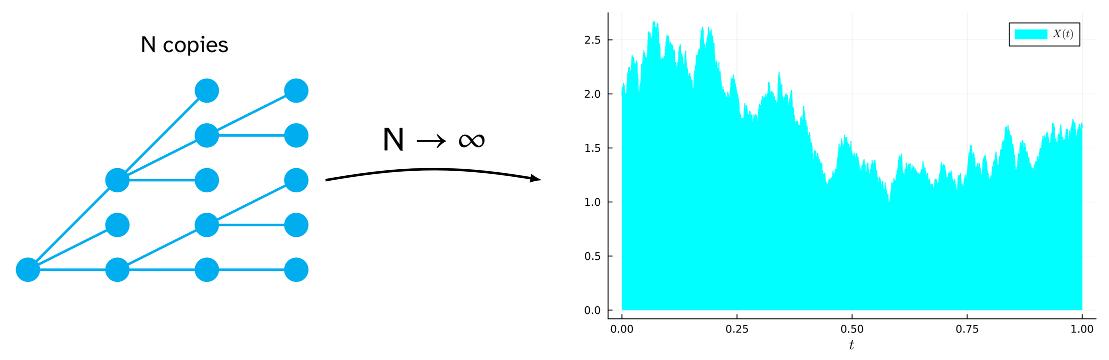
Characterized by (ρ, Λ)

$$\lambda_{b,k} = \rho 1_{\{k=2\}} + \int_{(0,1]} u^k (1-u)^{b-k} \frac{\Lambda(du)}{u^2}$$

$$\lambda_{b,k} = \lambda_{b+1,k+1} + \lambda_{b+1,k}$$

Continuous state branching processes (Ψ)

Rescaling limits of Galton–Watson processes



Characterized by (σ, μ)

$$\mathcal{L}f(x) = x\sigma f''(x) + x \int_{(0,\infty)} (f(x+w) - f(x) - (1 \wedge w) f'(x)) \mu(dw)$$

Key results

ALPHA-STABLE BRANCHING AND BETA-COALESCENTS

M. BIRKNER, J. BLATH, M. CAPALDO,
A. ETHERIDGE, M. MÖHLE, J. SCHWEINSBERG, A. WAKOLBINGER

STOCHASTIC FLOWS ASSOCIATED TO COALESCENT PROCESSES III: LIMIT THEOREMS

JEAN BERTOIN AND JEAN-FRANÇOIS LE GALL

A SMALL-TIME COUPLING BETWEEN Λ-COALESCENTS AND BRANCHING PROCESSES

BY JULIEN BERESTYCKI[1], NATHANAËL BERESTYCKI[2] AND VLADA LIMIC[3]

THE RELATIVE FREQUENCY BETWEEN TWO CONTINUOUS-STATE BRANCHING PROCESSES WITH IMMIGRATION AND THEIR GENEALOGY

MARÍA EMILIA CABALLERO, ADRIÁN GONZÁLEZ CASANOVA, JOSÉ LUIS PÉREZ

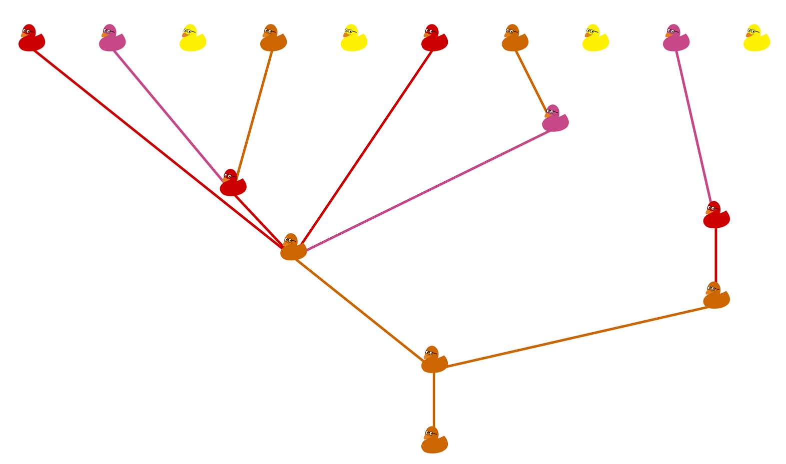
C-GC-P Theorem

The spaces **L** and **Ψ** are **homeomorphic**. Given $z > 0$, an **explicit** homeomorphism mapping a CSBP to a Λ-coalescent is constructed by

$$\rho = 2\frac{\sigma}{z} \quad \text{and} \quad \Lambda(du) = zu^2(\mu \circ T_z^{-1})(du),$$

where $T_z : w \mapsto w/(w+z)$

What is a multitype Λ-coalescent?



Multitype Λ-coalescent ($\mathbf{L_d}$)

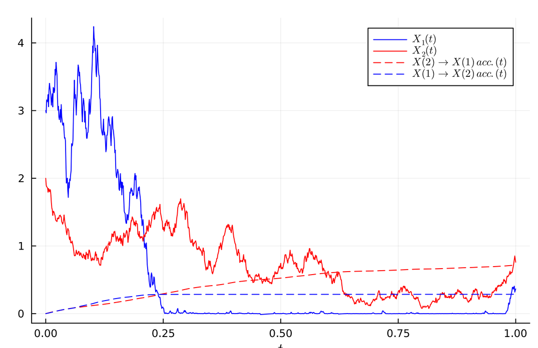
No simultaneous mutation events!

Characterized by $(\rho, \mathcal{Q})$

$$\lambda^{(i)}_{\vec{b},\vec{k}} = \rho_{ii} 1_{\{\vec{k}=2e_i\}} + \sum_{j=1, j\neq i}^{d} \rho_{ij} 1_{\{\vec{k}=e_j\}} + \int_{[0,1]^d \setminus \{0\}} \prod_{j=1}^{d} u_j^{k_j} (1-u_j)^{b_j - k_j} \mathcal{Q}_j(d\vec{u})$$

Multitype CSBP ($\mathbf{\Psi_d}$)

Characterized by $(\mathbf{B}, \vec{\sigma}, \vec{\mu})$



$$\mathcal{L}f(\vec{x}) = \langle \mathbf{B}\vec{x}, \nabla f(\vec{x}) \rangle + \sum_{i=1}^{d} x_i \sigma_i \partial_i^2 f(\vec{x}) + \sum_{i=1}^{d} x_i \int_{[0,\infty)^d \setminus \{0\}} (f(\vec{x}+\vec{w}) - f(\vec{x}) - (1 \wedge w_i) \partial_i f(\vec{x})) \mu_i(d\vec{w})$$

Main result

The spaces $\mathbf{L}_d$ and $\mathbf{\Psi}_d$ are **homeomorphic**. Given $\vec{z} \in (0,\infty)^d$, an **explicit** homeomorphism mapping a MCSBP to a multitype Λ-coalescent is constructed by

$$\rho_{ii} = 2\frac{\sigma_i}{z_i}, \quad \rho_{ij} = b_{ij}\frac{z_i}{z_j}$$

and

$$\mathcal{Q}_i(d\vec{u}) = z_i(\mu_i \circ T_{\vec{z}}^{-1})(d\vec{u}),$$

where $T_{\vec{z}} : \vec{w} \mapsto (w_1/(w_1+z_1), \ldots, w_d/(w_d+z_d))$

Scan QR code to get the full paper

Ciencia y Tecnología
Secretaría de Ciencia, Humanidades, Tecnología e Innovación
CIMAT