Lecture 1-3: Force, moments, and equillibriums

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1 Force

Important note: In this course, all the force is assumed to be applied to a *rigid body*, which is a body that does not deform under the force.

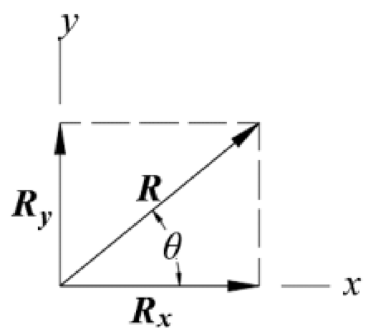


Figure 1: Force

Resolving a force via trigonometry:
$F_x = F\cos(\theta)$

$F_y = F\sin(\theta)$

Angle of the force, θ, can be calculated with:

$$\tan(\theta) = \frac{F_y}{F_x}$$

$$\theta = \arctan(\frac{F_y}{F_x})$$

Force can be represented with *cartesian vectors*, which uses the unit vectors $\hat{\imath}\ and\ \hat{\jmath}$.

1.1 Unit vector

A unit vector has a unit length of 1, and is usually denoted as $\hat{A}$.

It is defined by: $\hat{A} = \dfrac{A}{|A|}$

Where $|A|$ is the magnitude of the vector A, and defined as: $|A| = \sqrt{{A_x}^2 + {A_y}^2}$

Note that the unit vector, is also a vector in itself, as such is defined in x, y, and z directions. For example, $\hat{A}_x$ is the unit vector in the x direction, and is defined as $\hat{A}_x = \frac{A_x}{|A|}$.

1.2 Coordinate angles

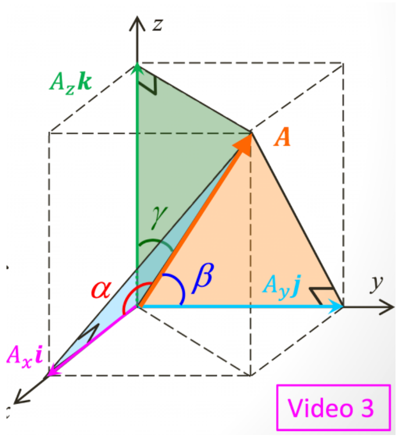


Figure 2: Coordinate direction angles

The direction of $\vec{A}$ is defined by the coordinate direction angles, α, β, and γ, which are measured between the tail of $\vec{A}$ and the x, y, z axes.

1.3 Position vectors

...What's the difference between unit vectors and position vectors?

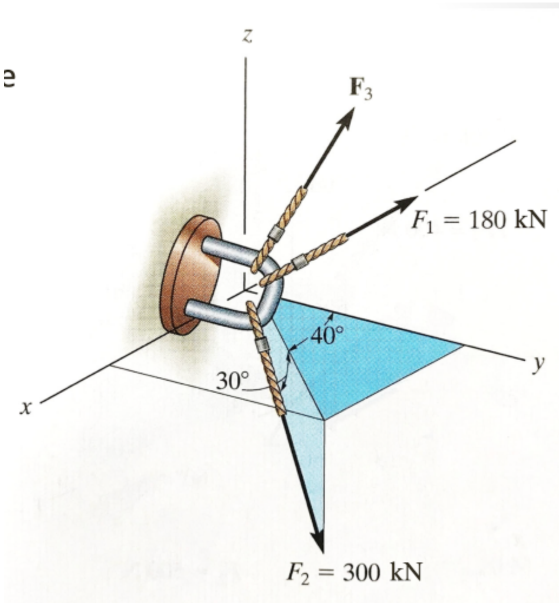
- A unit vector is vector used to specify only direction, direction does not have a magnitude therefore its length is always **one**.
- A position vector, meanwhile, is used to tell *how far and in which direction* a point is with respect to some arbitrarily chosen origin.

Position vectors are denoted by $\vec{F}$, and are defined as:

$$\begin{aligned}\vec{F} &= |F| \times \hat{F} \\ &= |F| \times \hat{r} \\ &= |F| \times \frac{\vec{r}}{|r|}\end{aligned}$$

1.3.1 Sample problem

Determine the magnitude and coordinate direction angles of $\mathbf{F_3}$ so that the resultant of the three forces acts along the positive y axis and has a magnitude of 600kN.



We know that the resultant vector should be $0\hat{\imath} + 600\hat{\jmath} + 0\hat{k}$.
First, notice that $\vec{F_1}$ acts along the x axis, and hence $\vec{F_1} = -180\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$.
Secondly, decompose $\vec{F_2}$ into its components:
$F_{2x} = F_1 \cos(30)\sin(40) = 300\cos(30)\sin(40) \approx 167$

$F_{2y} = F_1 \cos(30)\cos(40) = 300\cos(30)\cos(40) \approx 199$

$F_{2z} = F_1 \sin(30) = 300\cos(30) = 150$

$\vec{F_2} = 167\hat{\imath} + 199\hat{\jmath} + 150\hat{k}$

Through algebra, we can find that $\vec{F_3}$ is equal to $13\hat{\imath} + 401\hat{\jmath} + 150\hat{k}$.
Finding its coordinate direction angles can be done by:

$$\alpha = \arccos(\frac{13}{\sqrt{13^2 + 401^2 + 150^2}})$$
$$= \arccos(\frac{13}{428.33})$$
$$\approx 88°$$

The same can be done for β and γ.

2 Moment

Moment is defined as the tendency of a force to rotate an object about an axis or point.
It is defined as $M = F \times d$

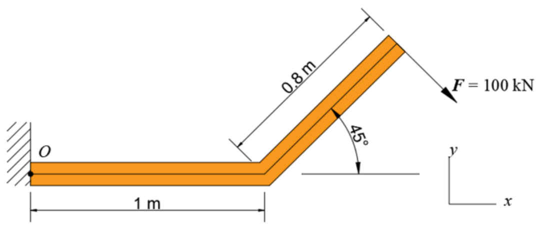
where d is the *perpendicular distance* from the line of action of the force to the axis of rotation.

In engineering, we assume that the moments are about the *same moment axis*, hence is equal to the algebraic sum of its magnitudes.

To find moment about a point, **decompose** the force into its components (x, y, and z axes), and find the moment of each component about the point. The moment will be the **sum of the moments of each component**.

2.0.1 Sample problem

A 100kN force acts on a structure as shown below. Find the moment of the force about point O.



$$
\begin{aligned}
h &= 0.8\sin(45) \\
d &= 1 + 0.8\cos(45) \\
F_x &= 100\cos(45) \\
F_y &= 100\sin(45) \\
\therefore M &= F_x \times h + F_y \times d
\end{aligned}
$$

2.1 Moment formulation by cross products

The moment of a force about point O, can be formulated as a **cross product** of two vectors:

We know that:

$|M_O| = |F|d$, and $d = |r|\sin(\theta)$

$\therefore |M_O| = |F||r|\sin(\theta) \iff \vec{M_O} = \vec{r} \times \vec{F}$

$\vec{r}$ is a position vector drawn from O to *any* point lying on the *line of action* of $\vec{F}$.

Note that we can choose any r we want, as the perpendicular distance towards point O (d) is constant and is always the same regardless of any point we choose.

2.2 Moment of a couple

A couple is defined as two forces of equal magnitude, opposite direction, and parallel lines of action.

Its magnitude is defined by $M = Fd$, where d is the *distance between the couple.*

Note that d does not have any relation to the axis of rotation, nor to the point of application. Instead, it is a *free vector*, and is the same about *any point*.

2.3 Equivalent systems

As long as two sets of loadings produce the same effect of translating and rotating a body, they are considered **equivalent**.

A system of several forces and moments can be reduced to an *equivalent* resultant force and couple.

In other words, the force might be applied in a different location and the couple might be applied in a different location, however the effect of the equivalent system we reduced it to will be the same as the original.

Force can be moved to a point provided a couple moment is added to the body.

The couple moment is a *free vector* and can be applied at any point, hence it can be directly added when calculating the resultant moment at any point.

$\mathbf{F_R} = \Sigma\mathbf{F}$

$\mathbf{M_R} = \Sigma\mathbf{M} + \Sigma\mathbf{M_o}$, where $\mathbf{M_o}$ is the couple moment, defined as $\mathbf{M_O} = \mathbf{r} \times \mathbf{F}$.

3 Equilibrium

This course covers only **static equilibrium** which is when a body is at rest.

A body is in equilibrium if the *sum of the forces* acting on it and the *sum of the moments* is **zero**.

$\Sigma\mathbf{F} = 0 \iff \Sigma\mathbf{F_{x,y,z}} = \mathbf{0}$

$\Sigma\mathbf{M} = 0 \iff \Sigma\mathbf{M_{x,y,z}} = \mathbf{0}$

3.1 Free body diagram

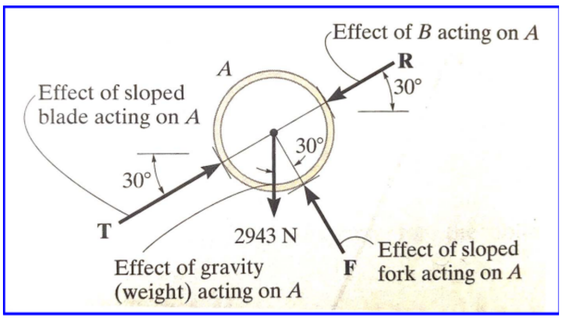


Figure 3: Free body diagram

A free body diagram is a diagram that shows all the forces and moments acting on a body.

The sketch consists of the outlined shape of the body *free* from its surroundings, with *all the forces and moments* acting on it.

3.1.1 Support reactions

A **support reaction** is a force that is exerted by a support on a body to keep it in equilibrium.

- A support prevents translation by exerting *force* in the opposite direction.
- A support prevents rotation by exerting *moment* in the opposite direction.
- A support reaction is a **reaction force** that is *equal* and *opposite* to the force applied by the body.